



## Kidney: Weibull regression with random effects

McGilchrist and Aisbett (1991) analyse time to first and second recurrence of infection in kidney patients on dialysis using a Cox model with a multiplicative frailty parameter for each individual. The risk variables considered are age, sex and underlying disease (coded other, GN, AN and PKD). A portion of the data are shown below.

Patient Number	Recurrence time t	Event (2 = cens)	Age at time t	Sex (1 = female)	Disease (0 = other; 1 = GN; 2 = AN; 3 = PKD)
1	8,16	1,1	28,28	0	0
2	23,13	1,2	48,48	1	1
3	22,28	1,1	32,32	0	0
4	447,318	1,1	31,32	1	0
.....					
35	119,8	1,1	22,22	1	1
36	54,16	2,2	42,42	1	1
37	6,78	2,1	52,52	1	3
38	63,8	1,2	60,60	0	3

We have analysed the same data assuming a parametric Weibull distribution for the survivor function, and including an additive random effect  $b_i$  for each patient in the exponent of the hazard model as follows

$$t_{ij} \sim \text{Weibull}(r, \mu_{ij}) \quad i = 1, \dots, 38; \quad j = 1, 2$$

$$\log \mu_{ij} = \alpha + \beta_{\text{age}} \text{AGE}_{ij} + \beta_{\text{sex}} \text{SEX}_i + \beta_{\text{disease1}} \text{DISEASE}_{i1} + \beta_{\text{disease2}} \text{DISEASE}_{i2} + \beta_{\text{disease3}} \text{DISEASE}_{i3} + b_i$$

$$b_i \sim \text{Normal}(0, \tau)$$

where  $\text{AGE}_{ij}$  is a continuous covariate,  $\text{SEX}_i$  is a 2-level factor and  $\text{DISEASE}_{ik}$  ( $k = 1, 2, 3$ ) are dummy variables representing the 4-level factor for underlying disease. Note that the the survival distribution is a truncated Weibull for censored observations as discussed in the mice example. The regression coefficients and the precision of the random effects  $\tau$  are given independent "non-informative" priors, namely

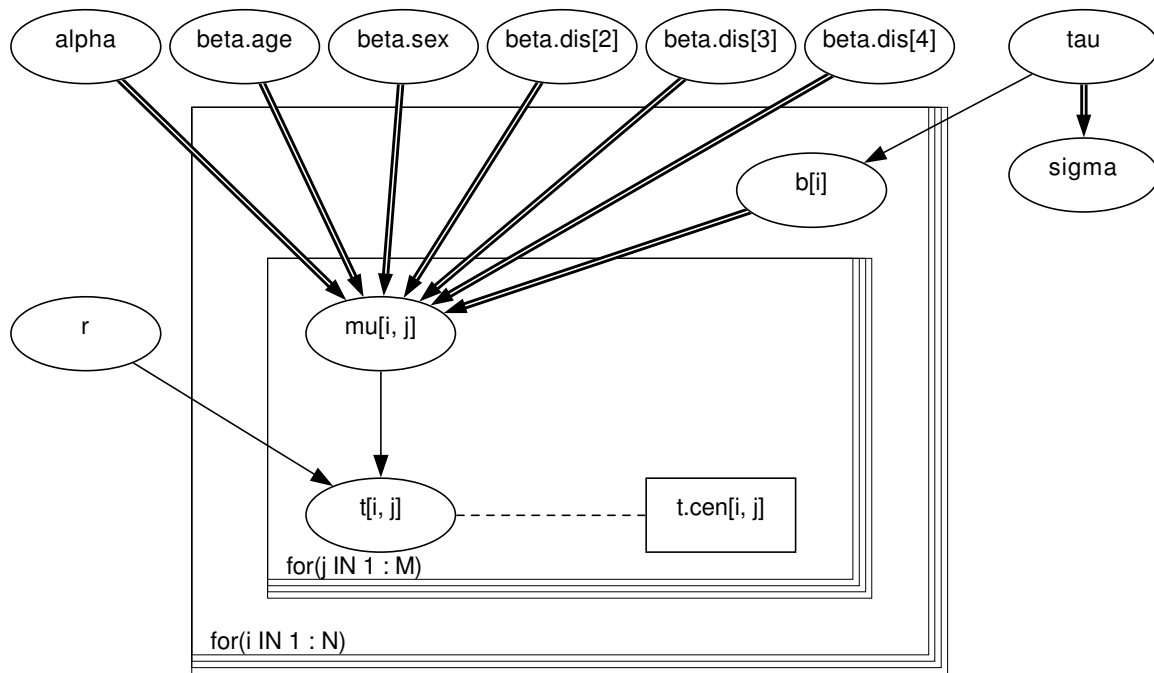
$$b_k \sim \text{Normal}(0, 0.0001)$$

$$\tau \sim \text{Gamma}(0.0001, 0.0001)$$

The shape parameter of the survival distribution  $r$  is given a  $\text{Gamma}(1, 0.0001)$  prior which is slowly decreasing on the positive real line.

The graphical model and *BUGS* language are given below.

*Graphical model for kidney example:*



*BUGS language for kidney example*

```

model
{
  for (i in 1 : N) {
    for (j in 1 : M) {
      # Survival times bounded below by censoring times:
      t[i,j] ~ dweib(r, mu[i,j]) I(t.cen[i, j], );
      log(mu[i,j]) <- alpha + beta.age * age[i, j]
        + beta.sex * sex[i]
        + beta.dis[disease[i]] + b[i];
    }
    # Random effects:
    b[i] ~ dnorm(0.0, tau)
  }
  # Priors:
  alpha ~ dnorm(0.0, 0.0001);
}

```

```

beta.age ~ dnorm(0.0, 0.0001);
beta.sex ~ dnorm(0.0, 0.0001);
# beta.dis[1] <- 0; # corner-point constraint
for(k in 2 : 4) {
  beta.dis[k] ~ dnorm(0.0, 0.0001);
}
tau ~ dgamma(1.0E-3, 1.0E-3);
r ~ dgamma(1.0, 1.0E-3);
sigma <- 1 / sqrt(tau); # s.d. of random effects
}

```

[Data](#) ( click to open )

[Inits](#) ( click to open )

## Results

A 1000 update burn in followed by a further 10000 updates gave the parameter estimates

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
alpha	-4.529	0.9036	0.06244	-6.348	-4.473	-2.932	1001	10000
beta.dis[2]	0.1265	0.5679	0.01859	-0.9922	0.1201	1.3	1001	10000
beta.dis[3]	0.5995	0.5781	0.02205	-0.5284	0.5863	1.815	1001	10000
beta.dis[4]	-1.198	0.8483	0.03147	-2.805	-1.206	0.5525	1001	10000
beta.sex	-1.945	0.5019	0.028	-3.054	-1.906	-1.042	1001	10000
r	1.205	0.1711	0.01523	0.9005	1.2	1.541	1001	10000
sigma	0.6367	0.3802	0.03159	0.04092	0.6494	1.366	1001	10000