PROJECT REPORT

Interest rate modeling

SABER model

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1 Intoduction

The pricing with Black and Scholes of European forward option call requires different parameters. Each of these parameters affects the price of the call. One such setting is the implied volatility. Intuitively, implied volatility, which represents the future assumption on the price change of the asset, should depend on the strike price, yet in the Black and Scholes formula, volatility is considered constant. Various models have been developed to correct for these discrepancies. In this report, we will study one of these models which aims to reproduce the variation of the implied market volatility, It also corrects the resulting price dynamics, especially that of local volatility models.

The model that will be discussed is the SABR model which falls within the family of stochastic volatility models.

2 Volatility models for forward option call pricing

2.1 Constant implied volatility

Implied volatility is usually taken as a constant for a given asset in the Black-Scholes formula for the forward call price, and can be expressed as follows:

$$V_{call} = D(t_{set})[f\mathbf{N}(d1) - K\mathbf{N}(d1)]$$

With V_{call} the price of the call

$d_{1,2} = \frac{log(f/K) \pm \frac{1}{2}\sigma_B^2 t_{ex}}{\sigma_B \sqrt{t_{ex}}}$

t_{ex} is the expiry time and t_{set} settlement date.

$D(t)$ the discount factor.

f the value of the forward at time zero and K is the strike value.

In reality, this is hardly true, since volatility hinges on the strike, and different strikes tend to exhibit dissimilar volatilities. Under some circumstances, this may not be an issue, however, for some exotic options, it subsequently impacts the price of the option, especially when the exotic option depends on different strikes. Looking at the market, it became evident that the volatility depends on strike and expiry of the option, necessitating, in the case of constant implied volatility, the use of separate models for different strikes and expiries, which becomes unmanageable when dealing with risk. This is why it is preferable to use implicit volatility models that function for dissimilar strikes.

2.2 Local volatility model

The local volatility model modifies the way in which the evolution of the asset price under the probability of risk neutrality is expressed and introduces a volatility dependent on the forward asset price. The forward price of the option is a martingale under the appropriate conditions and can be written as :

$$dF_t = \sigma_B F_t dW_t$$

The local volatility model assumes that the σ depends on time and forward price. The volatility is then calibrated with the actual market values of the forward price for different strikes and maturities. The equation is denoted as stated by Dupire:

$$dF_t = \sigma_{loc}(t, F_t)F_t dW_t$$

As price values are only available for specific expiries. Volatility is generally kept constant between two maturities and calibrated in each interval to match different strikes.

This approach mimics real market values and the shape of the smile. This procedure is also arbitrage-free.

However, the problem arises when calculating the implied volatility that appears in the Black-Scholes formula. Implied volatility is expressed as a function of local volatility. The dynamics of the resulting functions do not correspond to the market. When the underlying price rises, the smile shape moves to the left, which is equivalent to a fall in price, and vice versa, the correlation between the spot and the volatility is negative. Far from the actual market dynamics, which asserts the contrary. Stochastic models adjust this dynamic.

Hedging (covering risk and price fluctuation) can become problematic as the risk incorporates a correction term that depends on the change in implied volatility $\frac{\partial \sigma_B(K,f)}{\partial f}$ that does not reflect the true change observed in the market. The usual Black and Scholes formula is much more effective for risk management even if it doesn't much real market fluctuation on the implied volatility. Proper model selection is key.

2.3 Stochastic models

Stochastic models such as SABER and Heston model volatility as a stochastic process, as is the case in reality and what is observed in real markets. In particular, SABER is widely adopted to fix vanilla rates (caps, swaptions, ...).

The volatility designated by α is a stochastic process and the dynamics of the forward price and volatility are as follows :

$$\begin{cases} dF_t = \alpha_t F_t^{\beta} dW_t^1 & F(0) = f \\ d\alpha_t = \nu_t \alpha_t dW_t^2 & \alpha(0) = \alpha \end{cases}$$

and $\mathbf{E}(dW_t^1 dW_t^2) = \rho t$

By identification in the Black-Scholes formula and by approximation, the implied volatility formula is expressed as follows :

$$\sigma_B(K, f) = \frac{\alpha}{f^{1-\beta}}[1 - \frac{1}{2}(1 - \beta - \rho\lambda)log(K/f) + \frac{1}{12}[(1 - \beta^2) + (2 - 3\rho^2)]\lambda^2]log^2(K/f) + ...$$

with $\lambda = \frac{\nu}{\alpha} f^{1-\beta}$ provided the strike and the underlying are close.

$\beta \in [0, 1], \rho \in [-1, 1]$, $\nu > 0$ and $\alpha > 0$

3 SABER calibration and practical example

3.1 Swaptions and SABER model calibration

A swaption is an over-the-counter (OTC) contract where the owner has the option to enter into a preset swap option. There are two types of swaptions, the buyer swaption (where he pays the fixed leg of the swap and receives the floating leg) and the receiver swaption (which is the opposite). The actual implied volatility of swaptions is approximated with the SABER volatility formula.

The implied volatility formula is approximated to the market's by minimizing the root mean square error between the implied volatility function and the market values of the implied volatility summed at all strikes, as to obtain the values of the minimization parameters for a fixed tenor and maturity. The cost function can be written as such :

$$\underset{\beta,\alpha,\rho,\nu}{min} \sum_i (\sigma_i - \sigma_B(\beta, \alpha, \rho, \nu, K_i, f))^2$$

Calibration is tested for four pairs of tenors expiries. The result obtained can be seen below:

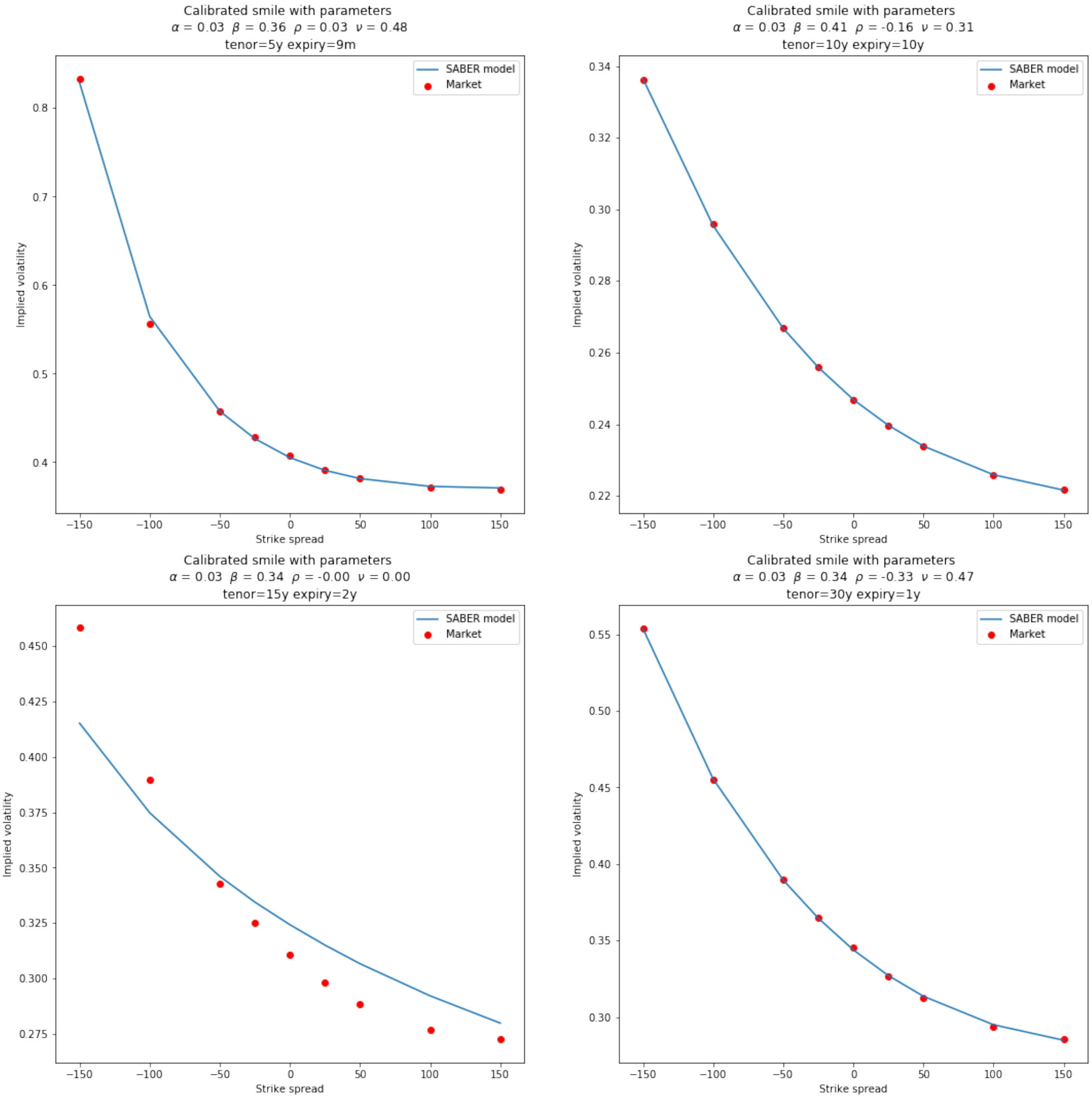

Figure 1: Testing the effectiveness for the calibration for different tenors and expiries

Hagan argues that β, if fixed, has little to no effect on the adjustment process. In order to test this idea, we experiment by tuning the value of β. I have made changes to the code for fixing individual parameters conveniently, given the name of the parameter and its values, the model of the saber will be readjusted to the market data considering the fixed value of the parameter.

After testing the readjustment under various scenarios, it appears that beta tends not to have a strong influence on the fitting process (see below for an example of readjustment for different values of β), however, it can be noted that in this case of β=0, the volatility curve of the saber does not exactly match that of the market and that β=0.5 is the preferable candidate in this case.

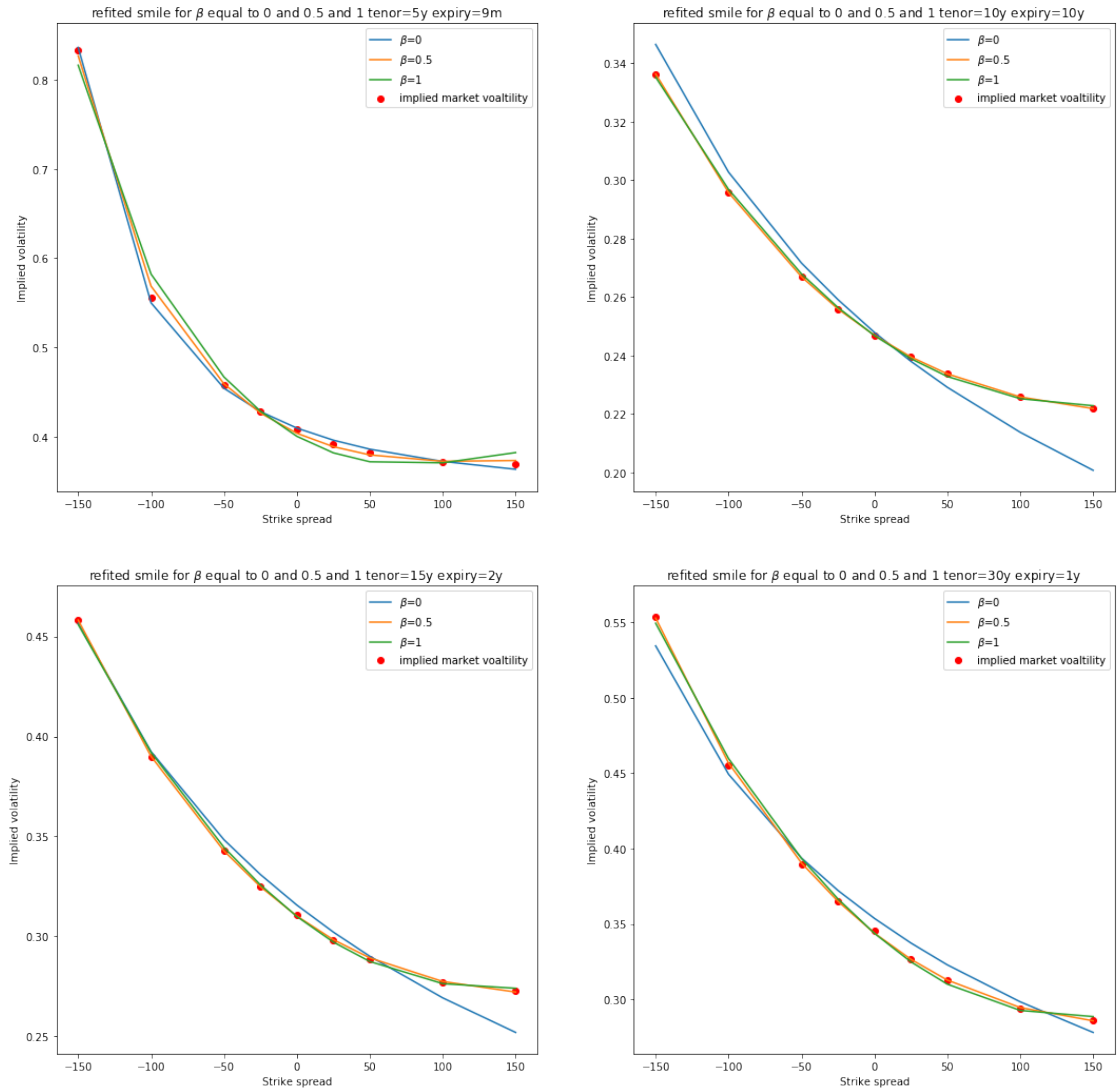

Figure 2: Calibration of SABER volatility model using a fixed β value

The β can be chosen in a purely aesthetic approach so as to not adversely influence the Greeks. Indeed, depending on the type of option, a different value for β is required. β is chosen in most cases either 0, 0.5 or 1. 0 indicates a normal model, since F disappears in the expression of the dynamics of F, 0.5 is the CIR model and 1 is the log normal model.

The model is calibrated for a tenor and a maturity, each with an underlying price. For a given tenor and maturity. The parameters obtained are then entered into the SABER volatility formula and one parameter is modified at a time to evaluate its effect on the shape of the smile (the graph of implied volatility versus strikes).

Implied volatility relates to different metrics: β, α, ν and ρ. They all influence the volatility graph in different ways. The sections below review the impact of changes on the shape of the smile (Implied volatility).

3.2 Impact of the β parameter

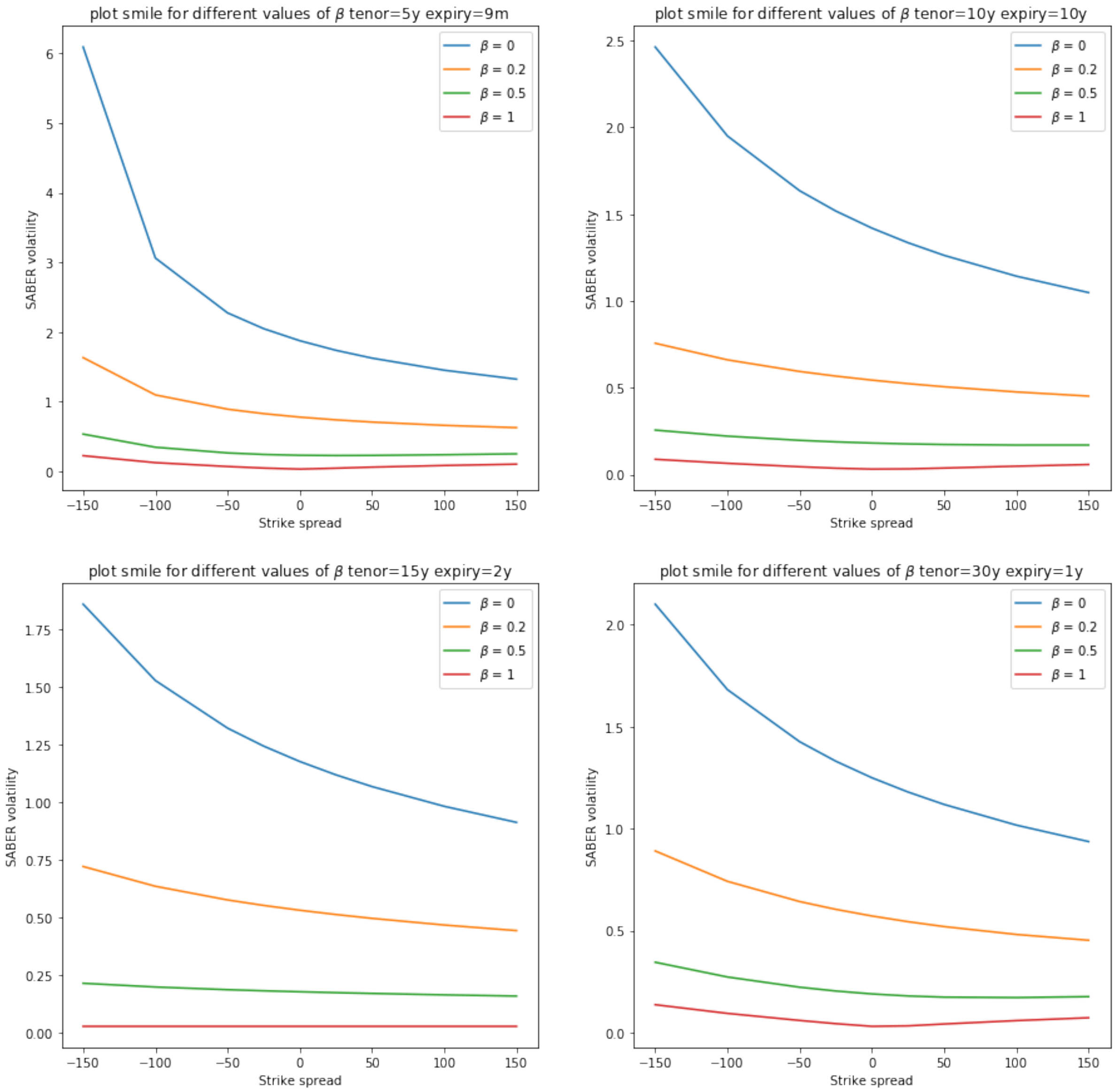

Figure 3: Visualisation of the affect of β on smile shape

The parameter β controls the flatness of the curve. A higher value of β causes the curve to be flatter, while a lower value of β causes the curve to have a greater convexity. In fact, β affects the slope since the slope becomes less noticeable as the value of β increases.

3.3 Impact of the ν parameter

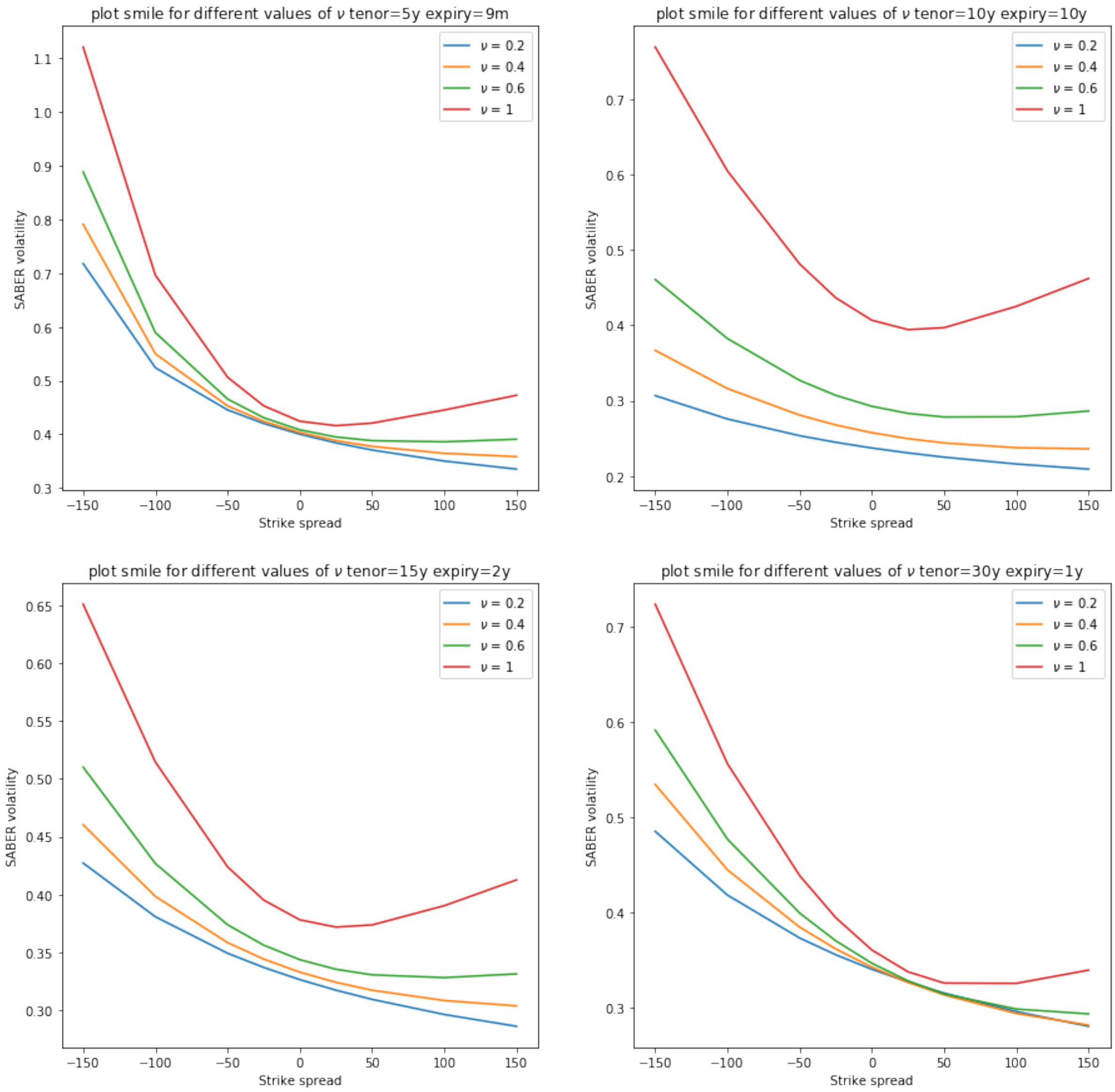

Figure 4: Visualisation of the affect on ν on smile shape

The smile is much more visible when the ν parameter increases. In other words, the value of ν has an upward effect on the volatility of options in money and out of money. It also has an effect on flattening in the vicinity of the implied volatility at money. This means that the curve is flatter around the at the money point when the value of ν decreases.

3.4 Impact of the α parameter

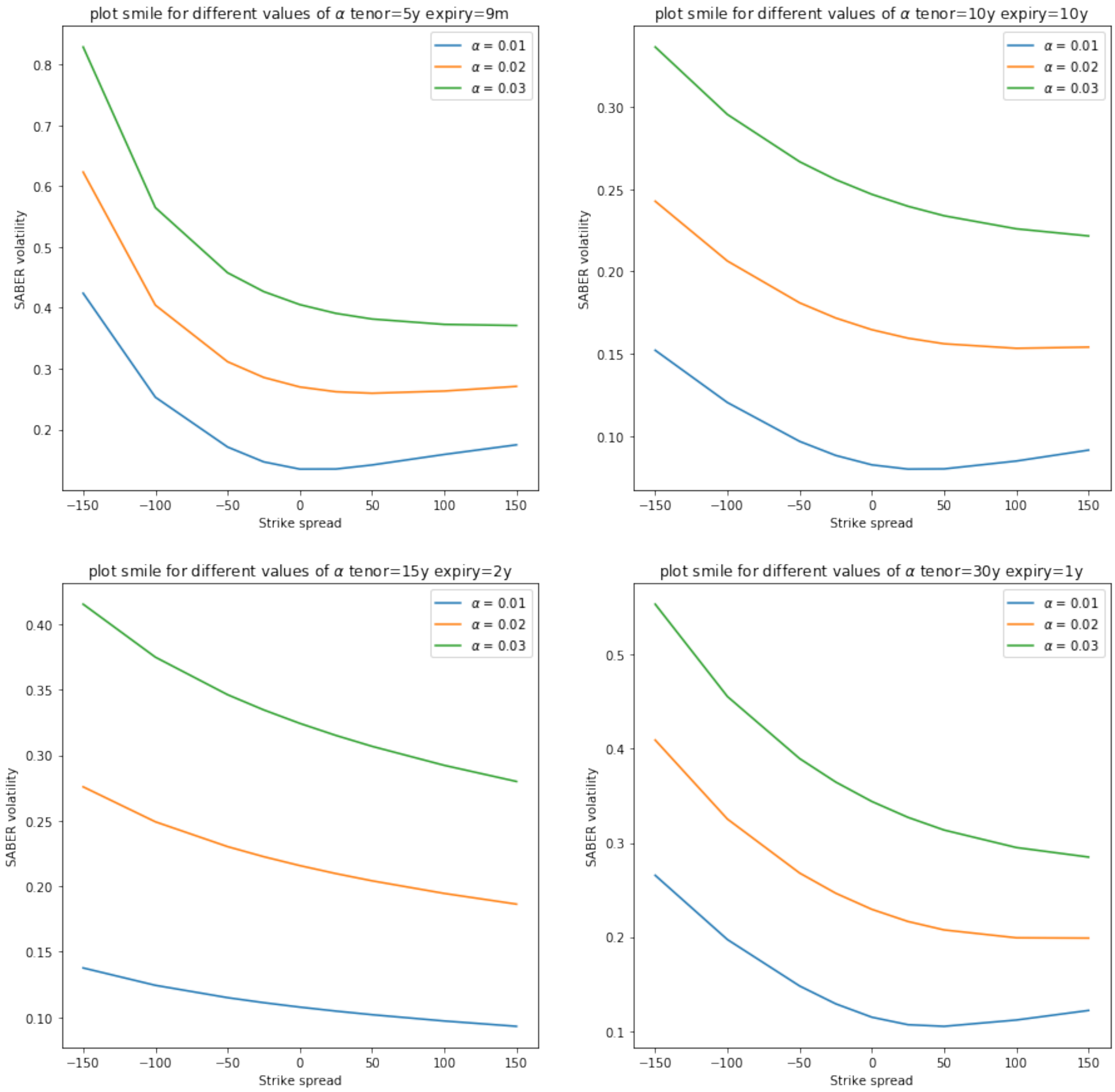

Figure 5: Visualisation of the affect of α on smile shape

The values of α are taken close to zero to avoid having a curve that does not account for the fact that the implied volatility is limited to one. When we take larger values of α, the curve takes very high values.

The shape of the implied volatility curve is not much affected when the value of the α is altered. Except that the curve changes position and moves either up or down depending on the value of α, when the α increases, it causes an upward shift clearly visible in the graphs. The graphs appear to be parallel for different values of α. The height or, in other words, the positions of the curve are controlled by α, which is consistent with the paper by Hagan et al.

3.5 Impact of the ρ parameter

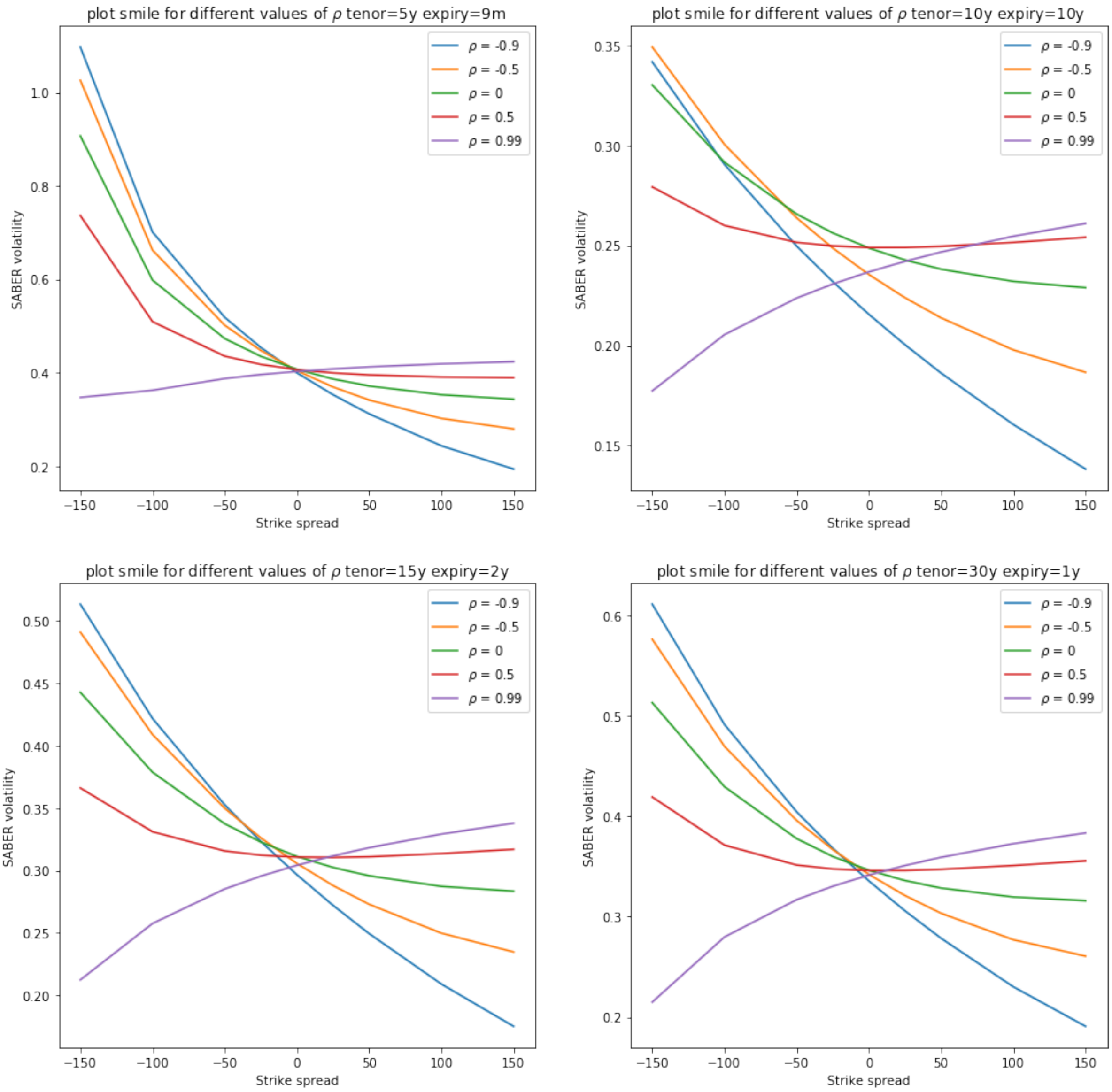

Figure 6: Visualisation of the affect of ρ on smile shape

ρ seems to turn the curve in a different direction depending on shifts in the value of ρ. That is, either clockwise as the value of ρ decreases, or vice versa. This in turn affects the curvature of the curve and produces an effect similar to that of β on the slope. This can be corroborated by the graphs above. It also appears to have a center of rotation close to the value at money.

4 Conclusion

The SABER stochastic model of implied volatility offers greater flexibility and relies on parameters that effectively control different characteristics on the curve, thus enabling a more accurate reflection of the market. It is used extensively to value Vanilla options and is therefore an excellent introduction to the world of interest rate modeling.