Model predictive control simulation for a two-link robotic arm

ECE – ME - 853 – Optimal control Project

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Introduction

- The mechanical structure of articulated rigid body robots is not complex. But their non-linear dynamics makes the task of control more difficult.
- One of the simplest possible robots in this classification is the 2-link planar manipulator.
- The structure of the robot is mechanically simple. But possesses nonlinear dynamics.
- Many approaches are available in the literature to control the 2-link manipulator including fuzzy, robust and adaptive control schemes.
- Model predictive control provides better performance for most non-linear systems because it re-optimizes the control strategy at every time step.

Introduction

- The project demonstrates a model predictive control of a two-link manipulator robot.
- The technique consists of linearizing a nonlinear dynamic model of the robot by using a feedback linearization control.
- Once, the linear model has been obtained, a predictive control approach is developed by minimizing a quadratic criterion.
- The objective is to control the robot from an initial configuration to the final configuration using a predictive control approach with the states being the angle and angular velocity of each link.

System model

- θi, Li and Mi {i = 1,2} are respectively the joint angle, length and the mass of each link.
- Nonlinear dynamic (geometric) model of the robot is given as:

$$x = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2)$$
$$y = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2)$$

- The robot motion equations are found using formalism of Lagrange: L = E −U. Where E is the total Kinetic and U is the total potential energy of the robot.

$$E = \frac{1}{2}(M_1 + M_2)L_1^2\dot{\theta}_1^2 + \frac{1}{2}M_2 L_2^2 \dot{\theta}_1^2 + M_2 L_2^2 \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2}M_2 L_2^2 \dot{\theta}_2^2 + M_2 L_1 L_2 \left(\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1^2\right)\cos(\theta_2)$$

$$U = M_1 g L_1 \cos(\theta_1) + M_2 g \left(L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2)\right)$$

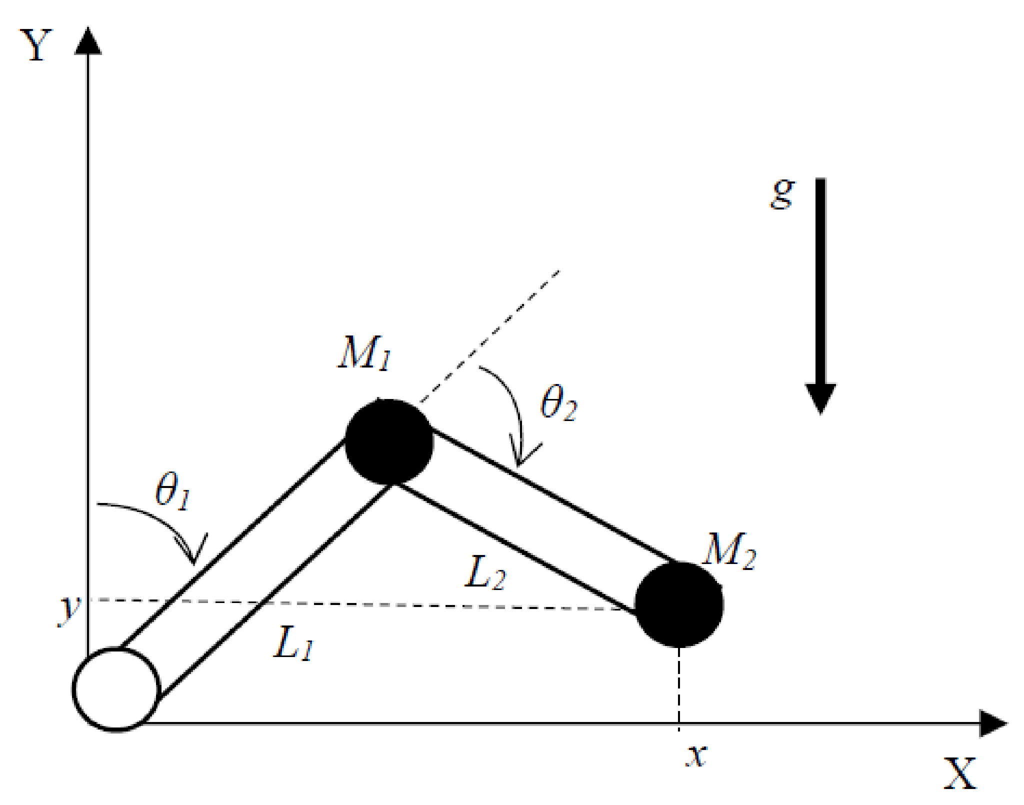


Fig. 1. Two link robot arm

System model

- The equations of motion and are defined as:

$$\tau = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i}$$

- Where τ is the vector of control inputs and θ and $\dot{\theta}$ are joint angle and velocity.
- Developing the equation gives the non-linear dynamical model of the robot:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta}) + G(\theta) = \tau$$
$$Y = \theta$$

- With M being the inertia matrix, C the vector of Coriolis and centrifugal forces and G the vector of gravity torques.

$$G(\theta) = \begin{bmatrix} -(M_1+M_2)gL_1\sin(\theta_1) - M_2gL_2\sin(\theta_1+\theta_2) \\ -M_2gL_2\sin(\theta_1+\theta_2) \end{bmatrix}$$

$$C(\theta,\dot{\theta}) = \begin{bmatrix} -M_2L_1L_2\left(2\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_1^2\right)\sin(\theta_2) \\ -M_2L_1L_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_2) \end{bmatrix}$$

$$M(\theta) = \begin{bmatrix} D_1 & D_2 \\ D_3 & D_4 \end{bmatrix}$$

$$D_1 = (M_1+M_2)L_1^2 + M_2L_2^2 + 2M_2L_1L_2\cos(\theta_2)$$
$$D_2 = M_2L_2^2 + M_2L_1L_2\cos(\theta_2)$$
$$D_3 = D_2$$
$$D_4 = M_2L_2^2$$

Feedback Linearization Control

- Feedback linearization provides exact linearization. As opposed to Jacobian linearization in which HOT are ignored in a series expansion.
- Control input which linearizes the system is found by deriving "synthetic" states.
- System need to be in the form:

$$\dot{x} = Ax + B\gamma(x)[u - \alpha(x)]$$

- If not, differentiate the output Y until the control input τ appears. This will ensure the above condition.
- In the case of the 2-link robot:

$$\ddot{Y} = \ddot{\theta} = M(\theta)^{-1}\left(-C\left(\theta,\dot{\theta}\right) - G(\theta) + \tau\right) = v$$

- v is a synthetic control (states) vector.
- The feedback linearized control thus becomes:

$$\tau = M(\theta)v + C\left(\theta,\dot{\theta}\right) + G(\theta)$$

Model Predictive Control

- The synthetic states obtained using feedback linearization can be expressed as: $$\ddot{\theta}_1 = v_1$$ $$\ddot{\theta}_2 = v_2$$
- Which is a decoupled linear system. Considering only joint 1. (Similar equations are also derived for joint 2)
- The system can be re-written in state space form as:
- $$\dot{x}_1(t) = x_2(t)$$ $$\dot{x}_2(t) = v_1(t)$$ $$Y(t) = x_1(t)$$ With $[x_1 \quad x_2]^T = [\theta_1 \quad \dot{\theta}_1]^T$
- Assuming V1 (t) = v1 (constant) in the time interval [t, t + h] ,where h is the horizon time of prediction. we get the prediction model as follows:

$$\dot{\theta}_1(t+h) = v_1 h + \dot{\theta}_1(t)$$

$$\theta_1(t+h) = \frac{1}{2} v_1 h^2 + \dot{\theta}_1(t) h + \theta_1(t)$$

Model Predictive Control

- Given the reference angle of the first link θ1d (constant), the proposed one-horizon time quadratic cost function for stabilizing the system is defined by:

$$J = e_1^2(t+h) + \rho \dot{e}_1^2(t+h)$$

- Where, $e_1(t+h) = \theta_{1d} - \theta_1(t+h)$ and $\dot{e}_1(t+h) = 0 - \dot{\theta}_1(t+h)$
- ρ (weight factor) and *h* are chosen later.
- Substituting the above prediction model in equation of *J* and minimizing w.r.t to V1 yields:

$$v_1(t) = k_3\theta_{1d} - k_1\theta_1(t) - k_2\dot{\theta}_1(t)$$

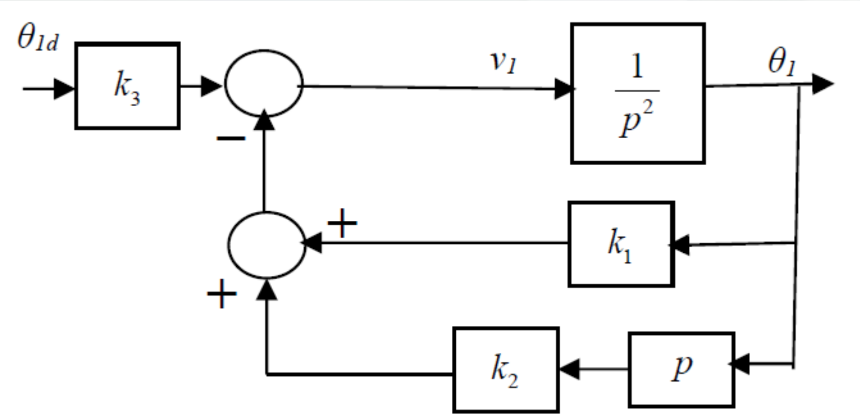
- The equation is a function of $\theta 1d$, $\theta 1$ and $\dot{\theta} 1$. As the **system** is linearized, The **constraints** to be applied are linear and the **cost function** is quadratic, this is a convex optimization problem and thus has a global optimum, which is found by the given equation.

Model Predictive Control

- Multiple solutions result for K1,K2 and K3. But the control gains chosen are given by:

$$k_1 = k_3 = \frac{2}{h^2 + 4\rho} \text{ and } k_2 = \frac{2h^2 + 4\rho}{h^3 + 4\rho h}$$

- The block diagram for the resulting closed loop system can be given as:



$$v_1(t) = k_3\theta_{1d} - k_1\theta_1(t) - k_2\dot{\theta}_1(t)$$

$$\frac{\theta_1(p)}{\theta_{1d}(p)} = \frac{k_3}{p^2 + k_2 p + k_1}$$

- The constraints of the system are defined in terms of ζ (damping factor) and ω0 (natural frequency) of a second order system by comparing it to the general form of a second order system. $\frac{w_0^2}{p^2 + 2\zeta w_0 p + w_0^2}$

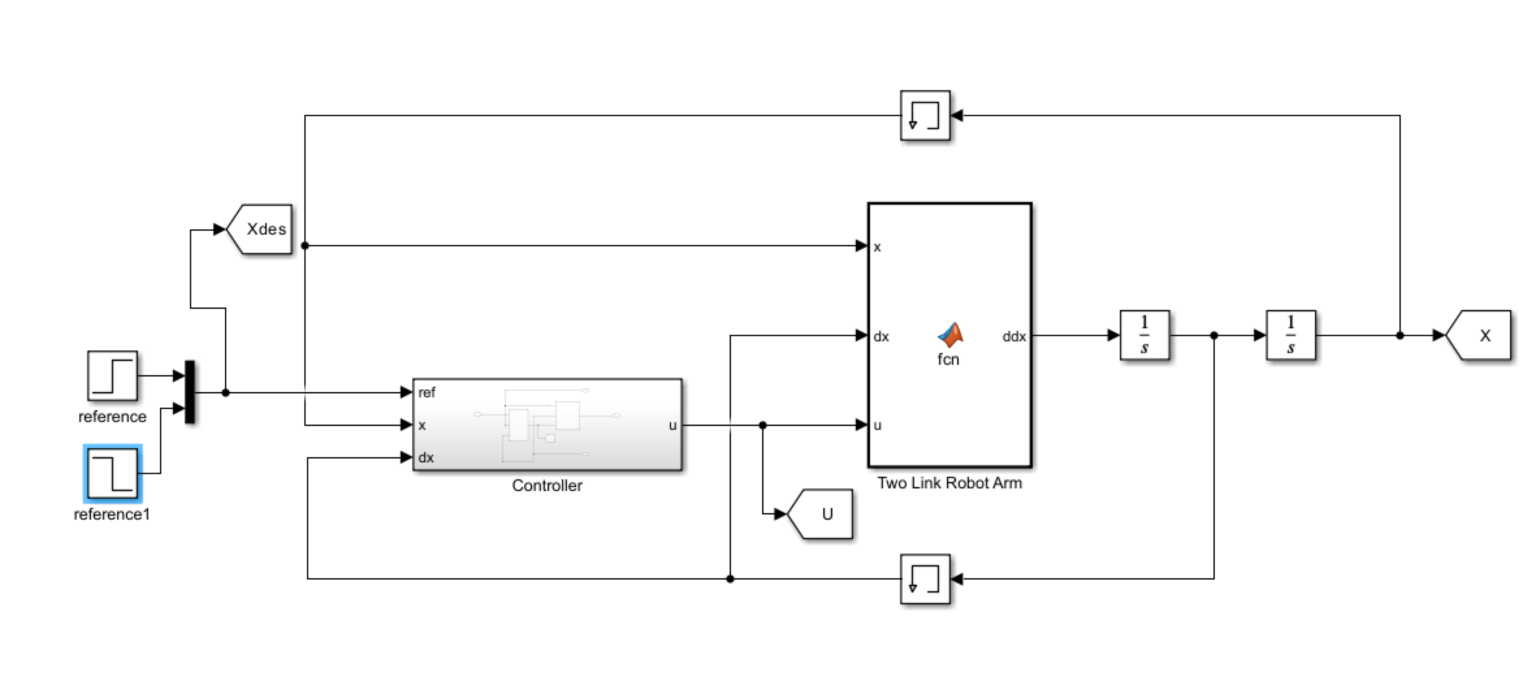
Model Predictive Control

- Giving:

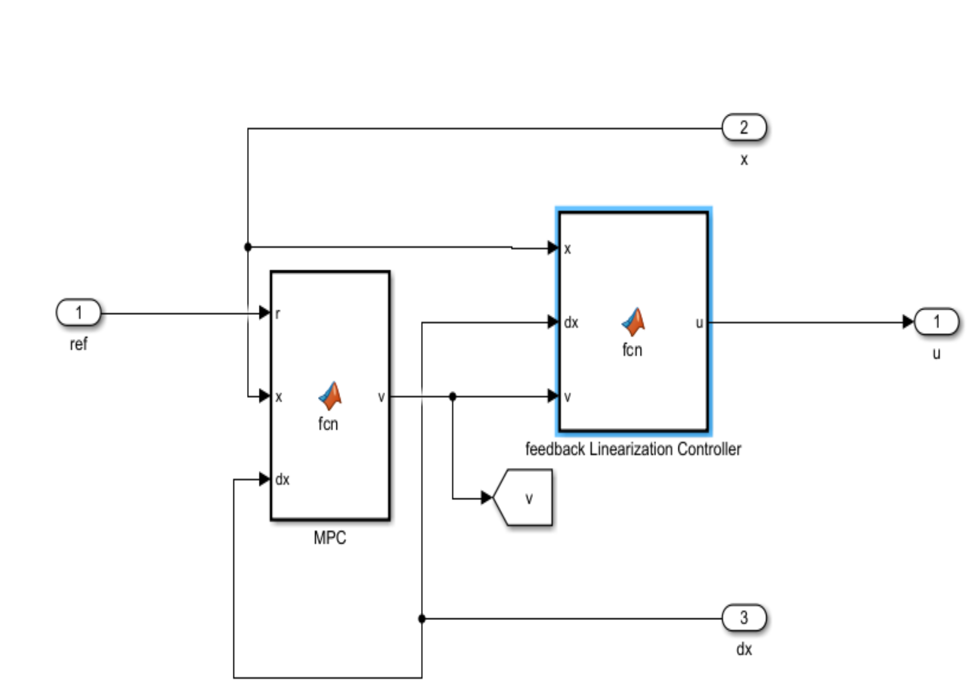
$$\rho = \frac{2-(w_0 h)^2}{4w_0^2}$$ and $$w_0^2 h^2 - 4\zeta w_0 h + 2 = 0$$

- For the simulation ζ (damping factor) is chosen as $\frac{\sqrt{2}}{2}$ and w0 = 4. From which ρ and h are found to be $1.14\ X\ 10^{-9}$ and 0.35 respectively.
- Yielding, $\mathrm{k1} = 15.99$; $\mathrm{k2} = 5.65$; and $\mathrm{k3} = \mathrm{k1}$

Simulink Model



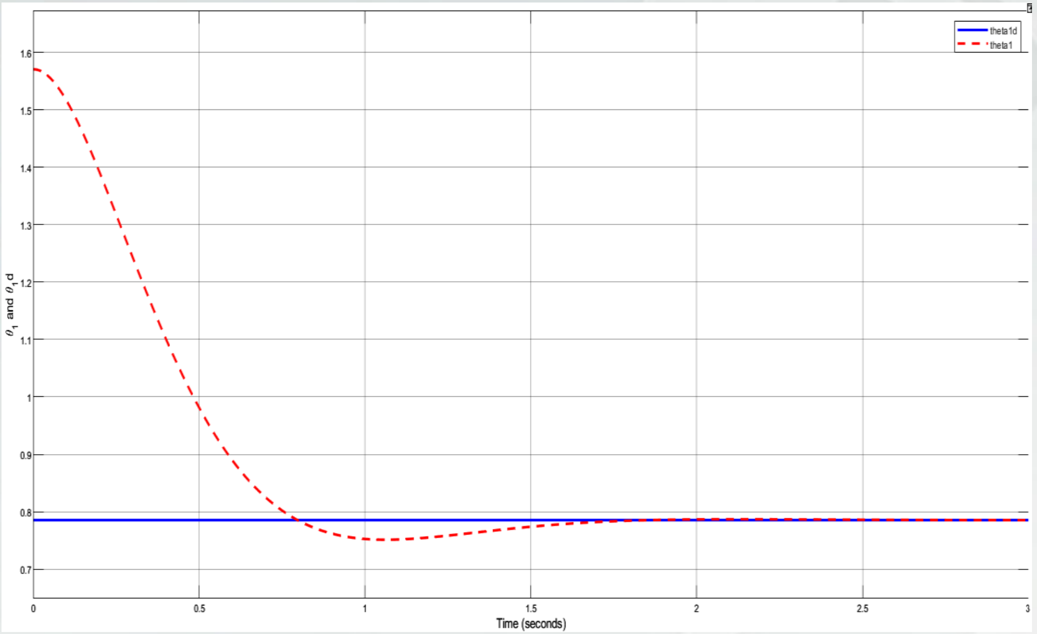
System Model



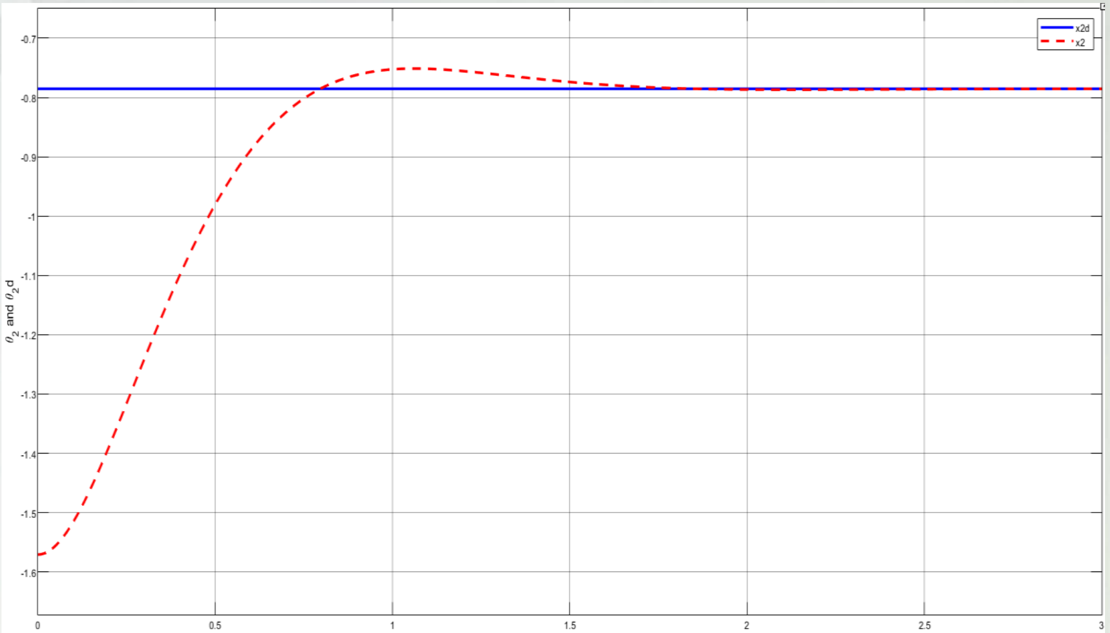
Controller model

Simulation Results

- Given: The mass of links $M_{i = 1,2}$ = 1 Kg and Link length $L_{i = 1,2}$ = 1 m.
- The initial and the desired orientations of the first and the second links of the robot arm are $\theta_{1(0)} = 0$, $\theta_{2(0)} = 0$, $\theta_{1d} = \pi/4$ and $\theta_{2d} = -\pi/4$.

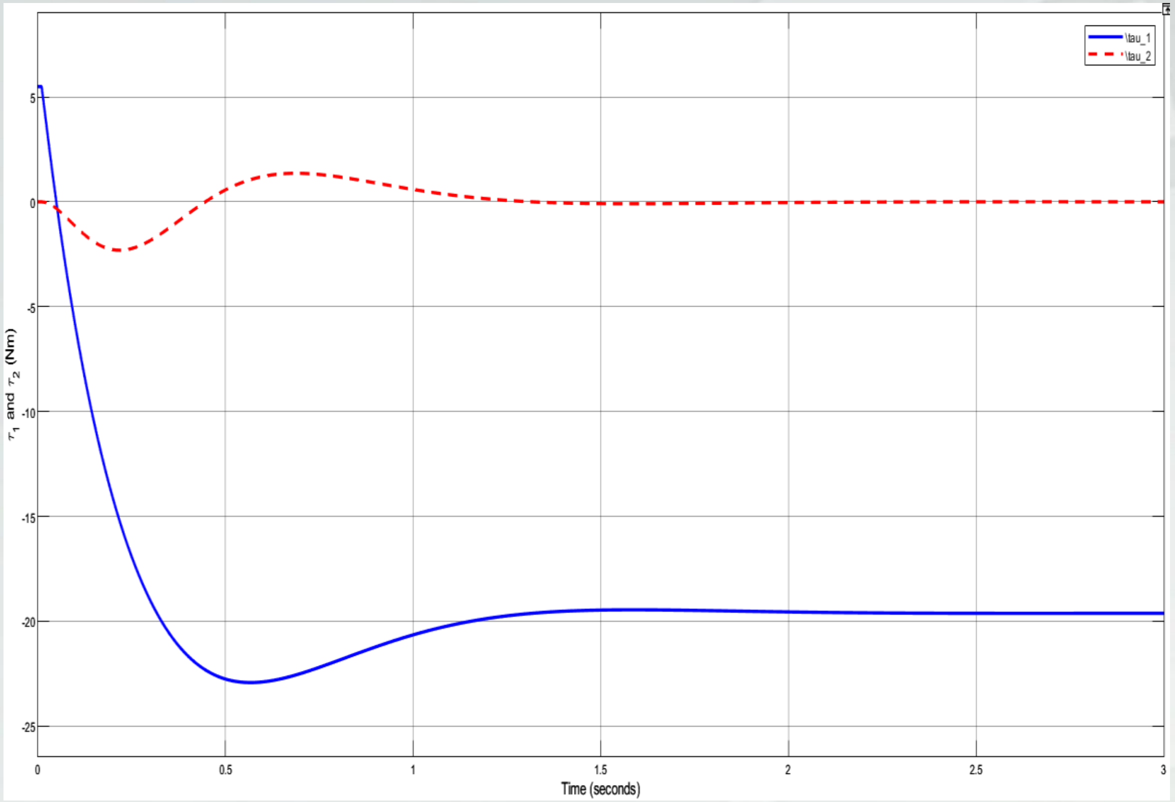


$\Theta1$ and $\Theta1_d$

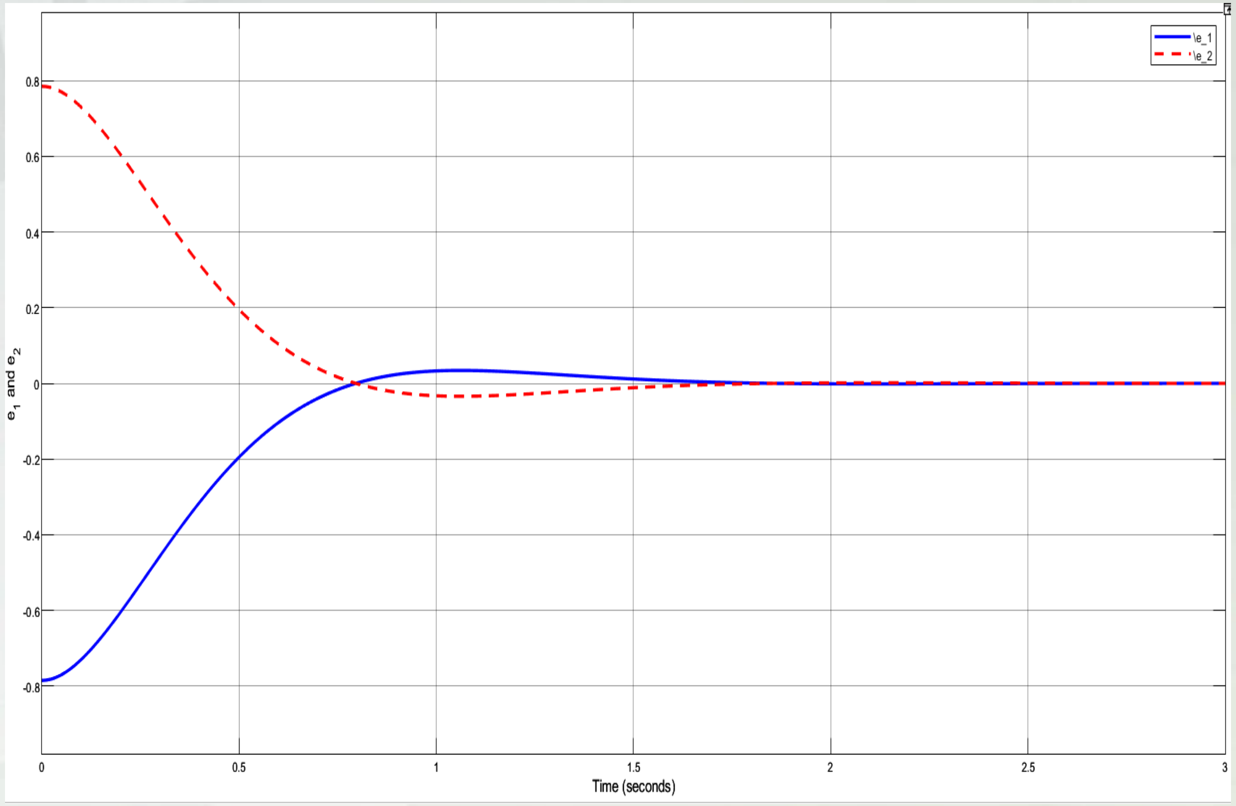


$\Theta2$ and $\Theta2_d$

Simulation Results



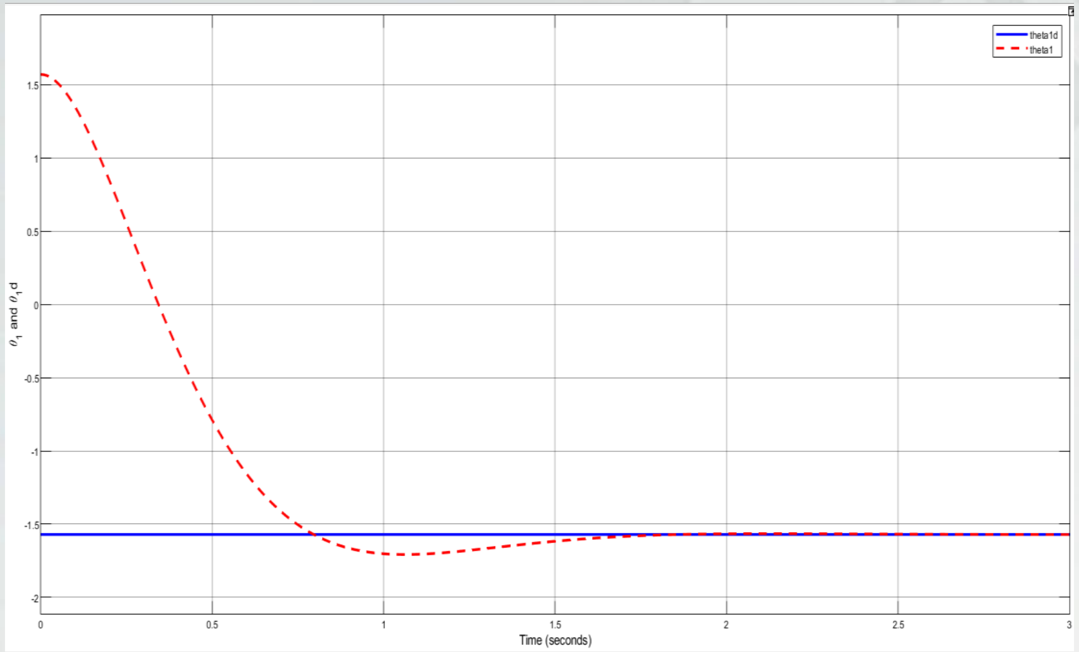
τ and τ $_d$



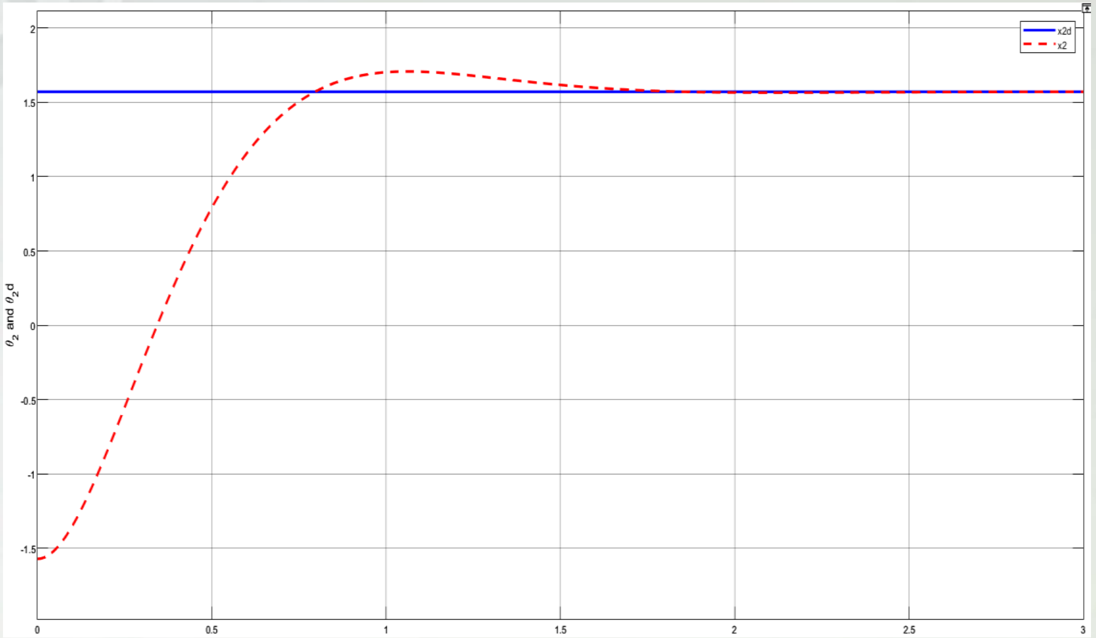
e_1 and e_2

Simulation Results

- Given: The mass of links $M_{i=1,2}$ = 1 Kg and Link length $L_{i=1,2}$ = 1 m.
- The initial and the desired orientations of the first and the second links of the robot arm are $\theta_{1(0)} = \pi/2$, $\theta_{2(0)} = -\pi/2$, $\theta_{1d} = -\pi/2$ and $\theta_{2d} = \pi/2$.

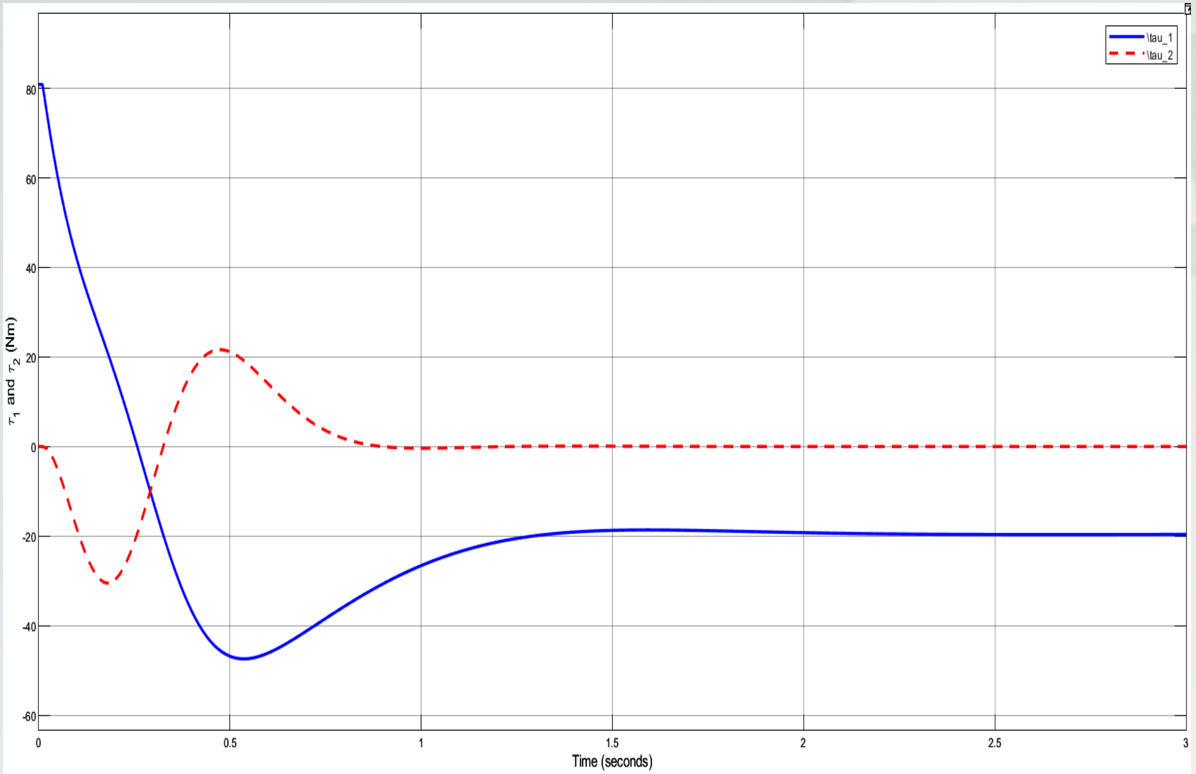


$\Theta 1$ and $\Theta 1_d$

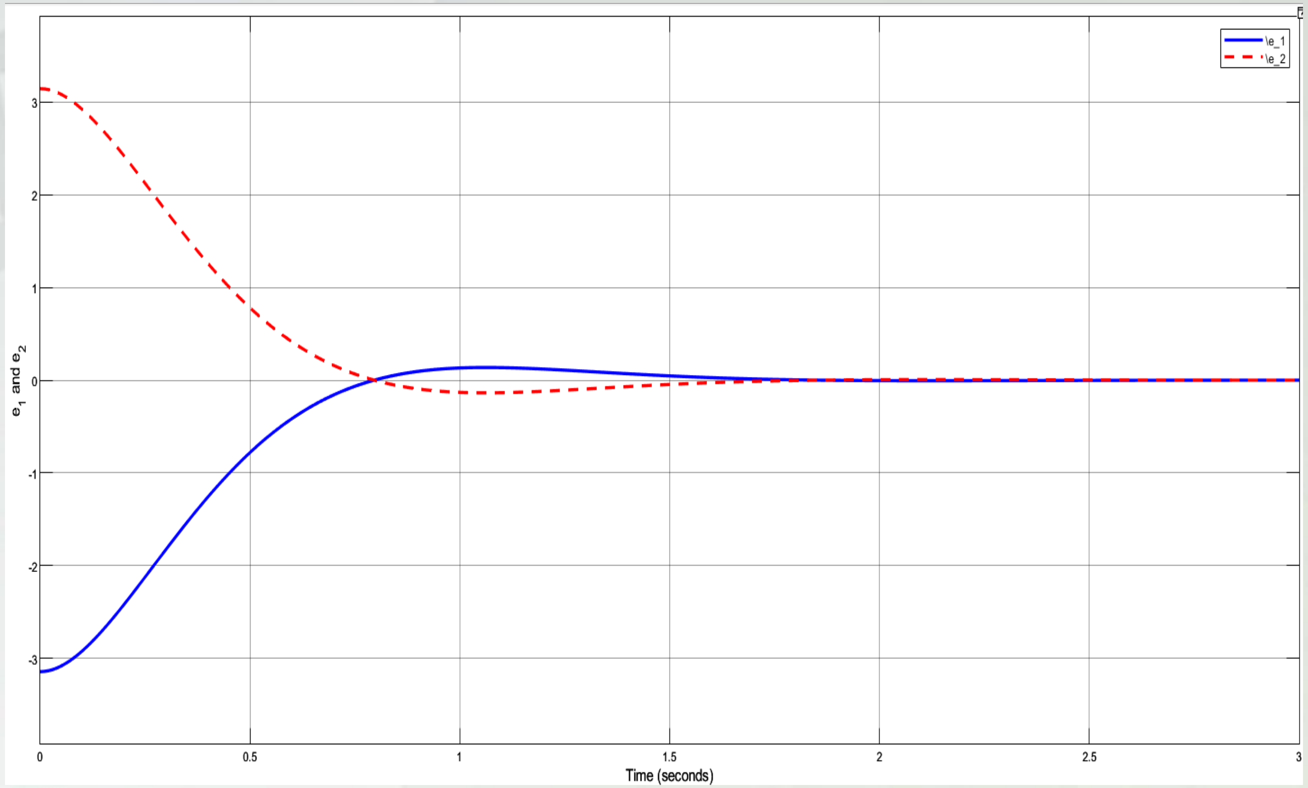


$\Theta 2$ and $\Theta 2_d$

Simulation Results



τ and τ d



e_1 and e_2

References

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