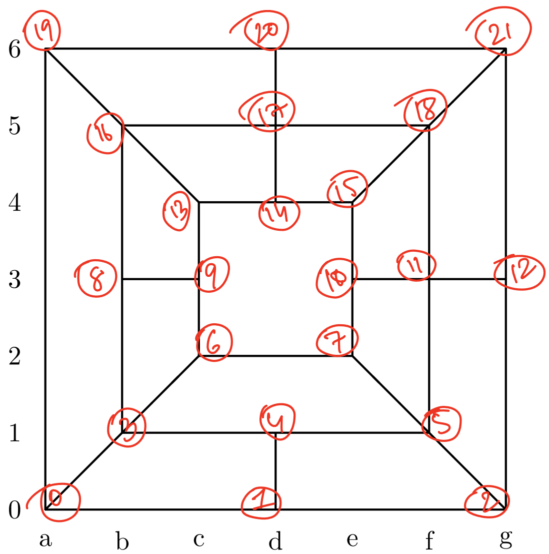
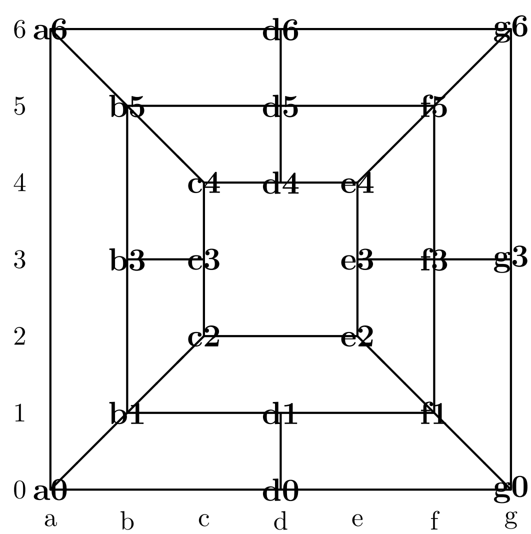
Morris Game Variant

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
a0	d0	g0	b1	d1	f1	c2	e2	b3	c3	e3	f3	g3	c4	d4	e4	b5	d5	f5	a6	d6	g6





Nine Men's Morris

Define mill : horizontal vertical diagonal

Nine Men's Morris is a board game between two players: White and Black. There are many online implementations available online. See, e.g., first link, or second link.

The **Morris Game Variant** is a variant of Nine Men's Morris game. Each player has 9 pieces, and the game board is as shown above. Pieces can be placed on intersections of lines. (There are a total of 22 locations for pieces.) The goal is to remove opponent's pieces by getting three pieces on a single line (a mill). The winner is the first player to reduce the opponent to only 2 pieces, or block the opponent from any further moves. The game has three distinct phases: opening, midgame, and endgame.

Opening: Players take turns placing 9 pieces - one at a time - on any vacant board intersection spot.

Midgame: Players take turns moving one piece along a board line to any adjacent vacant spot.

Endgame: A player down to only three pieces may move a piece to any open spot, not just an adjacent one (hopping).

Mills: At any stage if a player gets three of their pieces on the same straight board line (a mill), then one of the opponent's isolated pieces is removed from the board. An isolated piece is a piece that is not part of a mill.

Even Opening stage ?

who decides which one removed ?

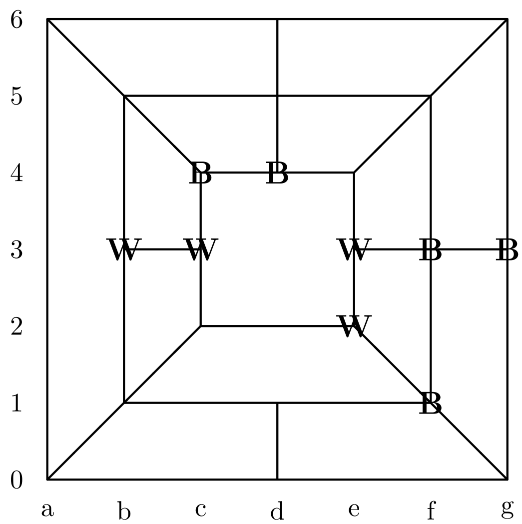
A computer program that plays Variant

The basic components of a computer program that plays Variant are a procedure that generates moves, a function for assigning static estimation value for a given position, and a MiniMax or AlphaBeta procedure.

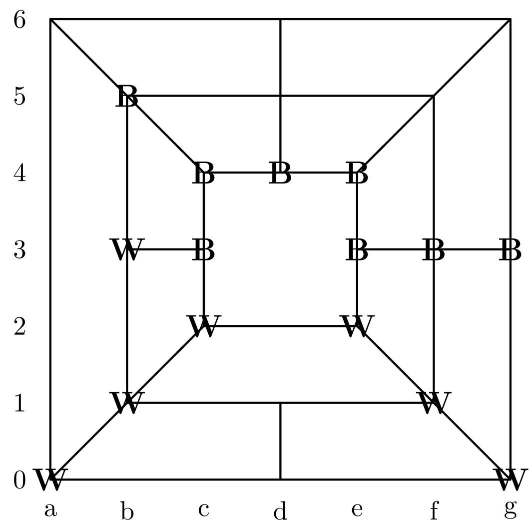
Representing board positions

One way of representing a board position is by an array of length 22 , containing the pieces as the letters W, B, x. (The letter x stands for a "non-piece".) The array specifies the pieces starting from bottom-left and continuing left-right bottom up. Here are a two examples:

xxxxxBxWWWWWBBBBxxxxxxx



WxWWxWWWWWBBBBBBBBxxxxx



Move generator

A move generator gets as input a board position and returns as output a list of board positions that can be reached from the input position. In the next section we describe a pseudo-code that can be used as a move generator for White. A move generator for Black can be obtained by the following procedure:
Input: a board position b.
Output: a list L of all positions reachable by a black move.
?! **1.** compute the board **tempb** by swapping the colors in b. Replace each W by a B, and each B by a W.
2. Generate L containing all positions reachable from **tempb** by a white move.
3. Swap colors in all board positions in L, replacing W with B and B with W.

A move generator for White

A pseudo-code is given for the following move generators: **GenerateAdd**, generates moves created by adding a white piece (to be used in the opening). **GenerateMove**, generates moves created by moving a white piece to an adjacent location (to be used in the midgame). **GenerateHopping**, generates moves created by white pieces hopping (to be used in the endgame). These routines get as an input a board and generate as output a list L containing the generated positions. They require a method of generating moves created by removing a black piece from the board. We name it **GenerateRemove**.

Generate Add : Opening : add white
Generate Move : Midgame : move white piece to an adjacent location
Generate Hopping : Endgame : white hopping when 3
Generate Remove : remove a

Board → L : generated positions

MiniMax {

minMaxOpening(){

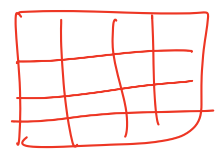
GenerateMovesOpening

Input: a board position
Output: a list L of board positions
Return the list produced by **GenerateAdd** applied to the board.

GenerateMovesMidgameEndgame

Input: a board position
Output: a list L of board positions
if the board has 3 white pieces Return the list produced by **GenerateHopping** applied to the board. Otherwise return the list produced by **GenerateMove** applied to the board.

GenerateAdd



Input: a board position
Output: a list L of board positions
L = empty list
for each location in board:
    if board[location] == empty {
    b = copy of board; b[location] = W
    if closeMill(location, b) generateRemove(b, L)
    else add b to L
    }
return L

GenerateHopping

Input: a board position
Output: a list L of board positions
L = empty list
for each location α in board
if board[α] == W {
    for each location β in board
    if board[β] == empty {
        b = copy of board; b[α] = empty; b[β] = W
        if closeMill(β, b) generateRemove(b, L)
        else add b to L
    }
}
return L

GenerateMove

Input: a board position
Output: a list L of board positions

```
L = empty list
for each location in board
if board[location]==W {
        n = list of neighbors of location
        for each j in n
        if board[j] == empty {
                b = copy of board; b[location] = empty; b[j]=W
                if closeMill(j, b) GenerateRemove(b, L)
                else add b to L
        }
}
return L
```

GenerateRemove

Input: a board position and a list L
Output: positions are added to L by removing black pieces

```
for each location in board:
        if board[location]==B {
        if not closeMill(location, board) {
                b = copy of board; b[location] = empty
                add b to L
        } }
If no positions were added (all black pieces are in mills) add b to L.
```

player to be removed

neighbors and closeMill

The proposed coding of the methods neighbors and closeMill is by "brute force". The idea is as follows.

neighbors

Input: a location j in the array representing the board
Output: a list of locations in the array corresponding to j's neighbors

```
switch(j) {
        case j==0 (a0) : return [1,3,16]. (These are d0,b1,a6.)
        case j==1 (d0) : return [0,4,2]. (These are a0,d1,g0.)
        etc.
}
```

closeMill
Input: a location j in the array representing the board and the board b
Output: true if the move to j closes a mill

```
C = b[j]; C must be either W or B. Cannot be x.
switch(j) {
        case j==0 (a0) : return true if
                (b[1]==C and b[2]==C)
                    or (b[3]==C and b[6]==C)
              else return false
        case j==1 (d0) : return true if
                (b[0]==C and b[2]==C)
              else return false
        etc.
}
```

Static estimation

revert static estimation for black

The following static estimation functions are proposed. Given a board position b compute:
numWhitePieces = the number of white pieces in b.
numBlackPieces = the number of black pieces in b.
L = the MidgameEndgame positions generated from b by a black move.
numBlackMoves = the number of board positions in L.

is this for maximising player or case player

A static estimation for MidgameEndgame:
if (**numBlackPieces** $\leq$ 2) return(10000)
else if (**numWhitePieces** $\leq$ 2) return(-10000)
else if (**numBlackMoves**==0) return(10000)
else return (1000(**numWhitePieces** − **numBlackPieces**) - **numBlackMoves**)

A static estimation for Opening:
return (**numWhitePieces** − **numBlackPieces**)

MiniMax Opening = Move generator + static estimation
MiniMax Game

AB Opening
AB Game

MiniMax Opening Black
MiniMax Game Black

MiniMax Opening Improved
MiniMax Game Improved