I. INTRODUCTION TO CONSUMER OPTIMIZATION

Why Consumer Optimization Is Important

- It has implications for how we view the desirability of market outcomes.

- It can help us to understand the many choices that consumers make.

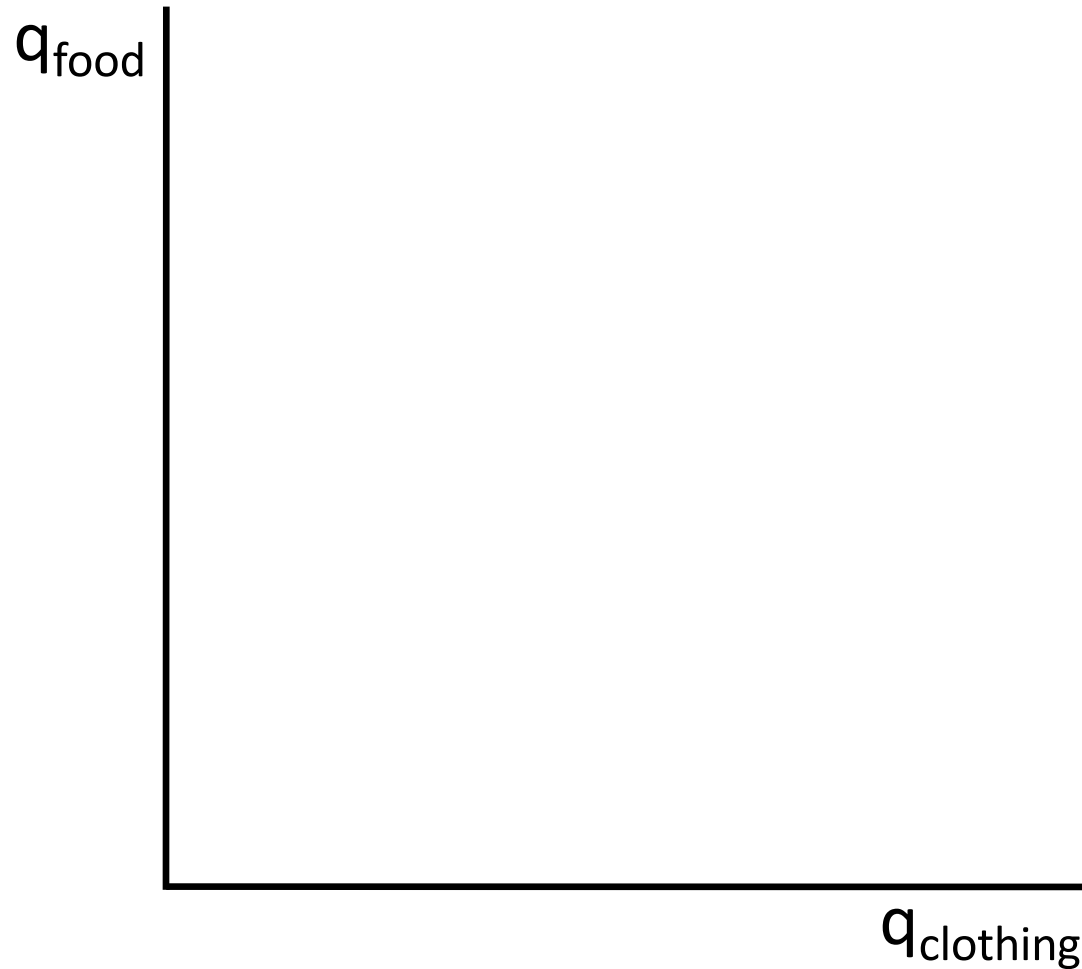# II. THE BUDGET CONSTRAINT

A Household's Budget Constraint

- **In words:** The total amount the household spends cannot exceed its income.

- **In symbols:**

$$P_a \cdot q_a + P_b \cdot q_b + P_c \cdot q_c + \ldots + P_z \cdot q_z = Income,$$

where the P's are the market prices of the various goods, and the q's are the quantities that the individual household buys.
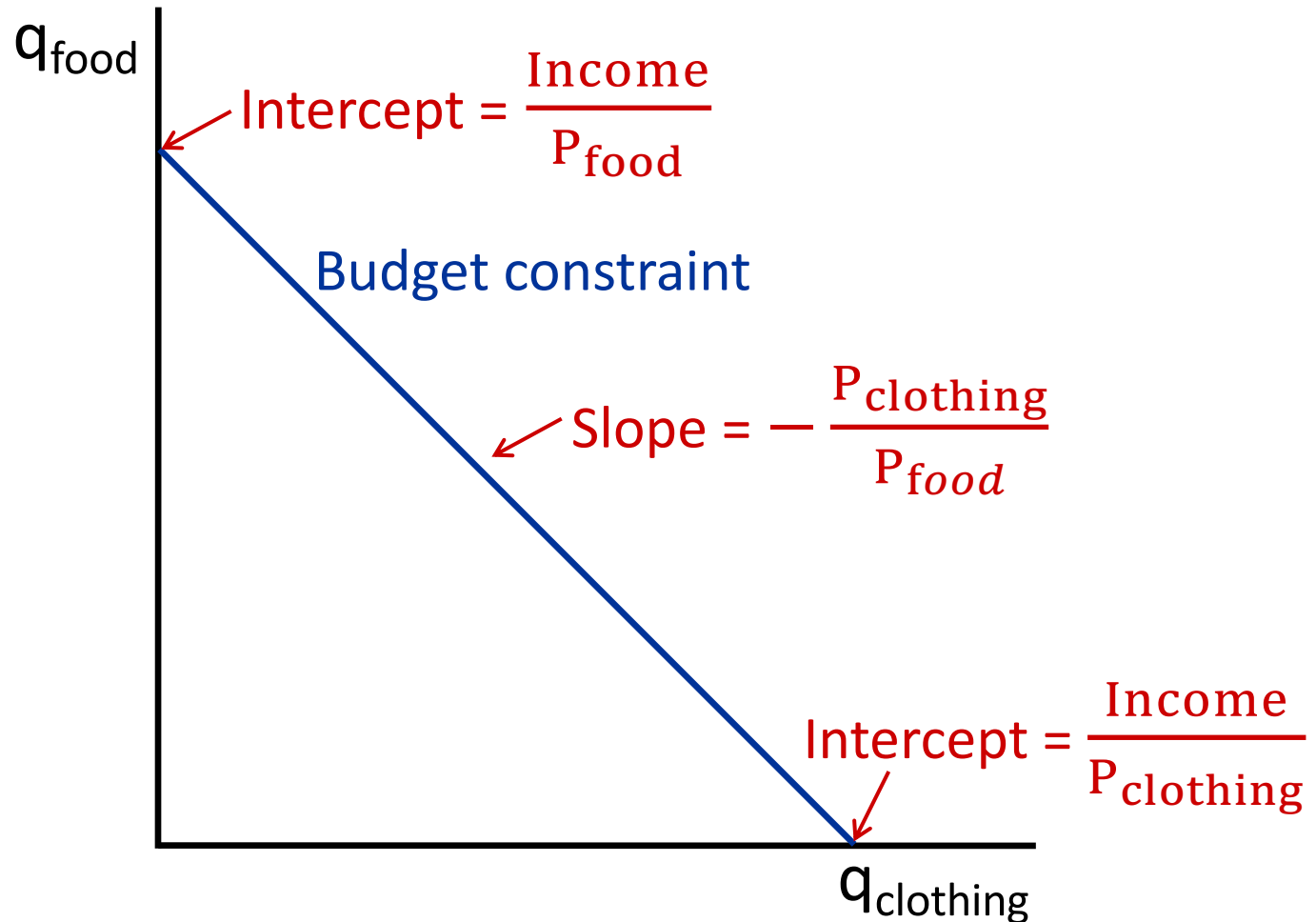
Budget Constraint for the Case of Two Goods
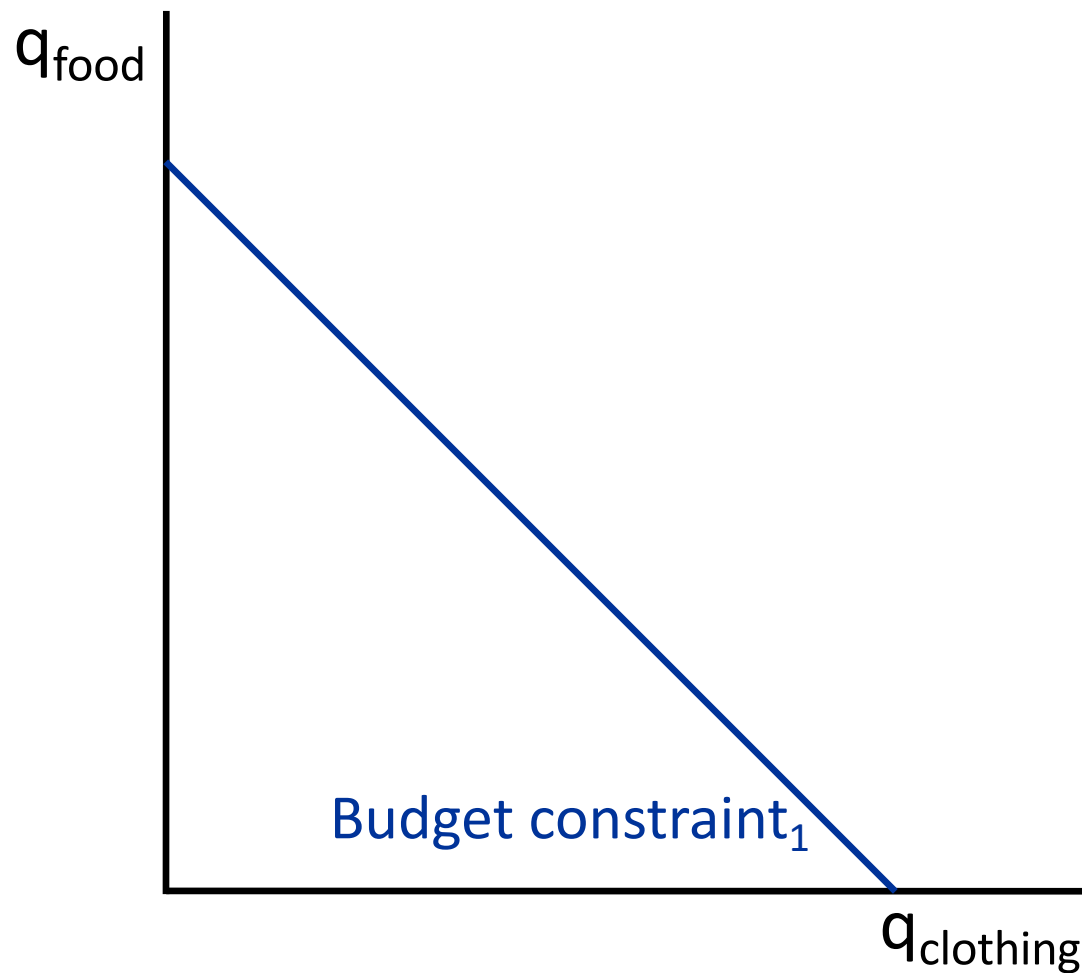$$P_{food} \cdot q_{food} + P_{clothing} \cdot q_{clothing} = Income$$
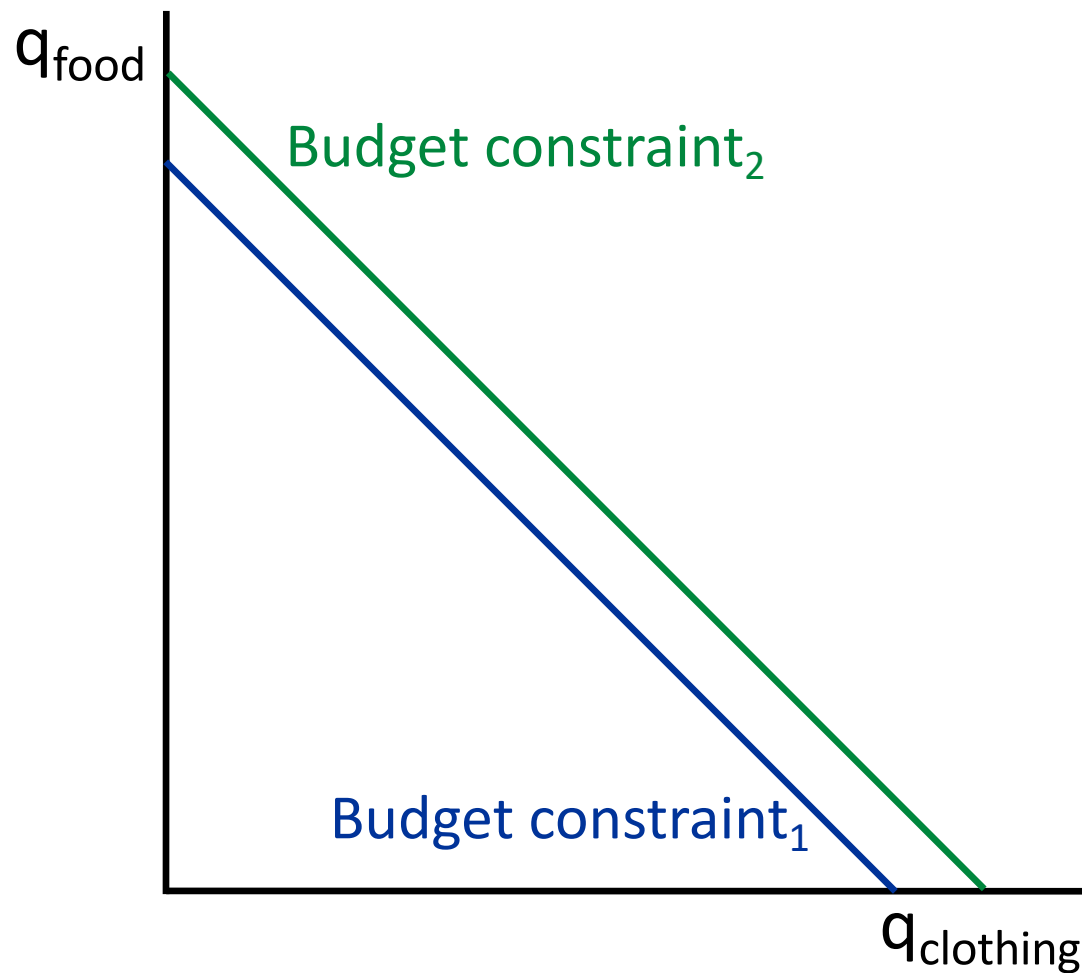
Budget Constraint for the Case of Two Goods

$$P_{food} \cdot q_{food} + P_{clothing} \cdot q_{clothing} = Income$$

q_{food}

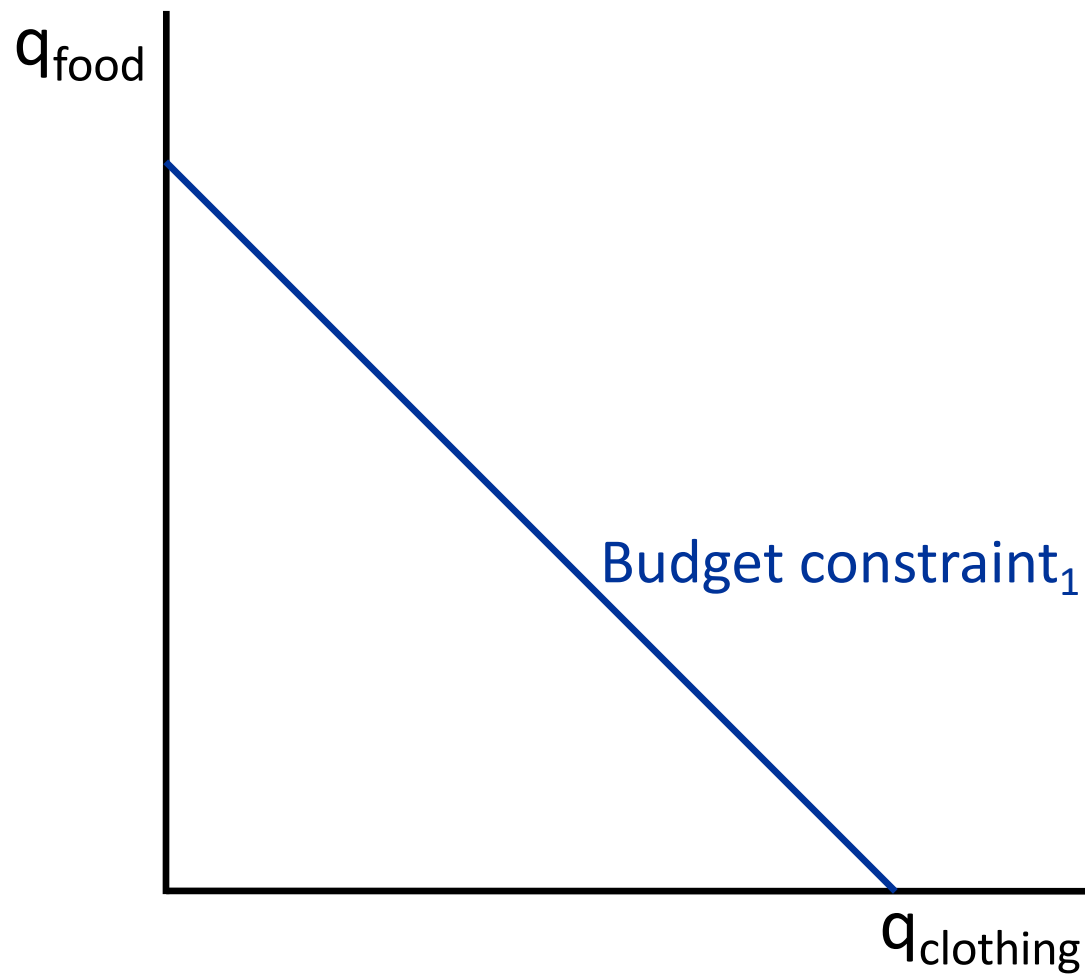$\text{Intercept} = \dfrac{Income}{P_{food}}$

Budget constraint

$\text{Slope} = -\dfrac{P_{clothing}}{P_{food}}$

$\text{Intercept} = \dfrac{Income}{P_{clothing}}$

$q_{clothing}$

A Rise in the Household's Income

q_{food}

$q_{clothing}$

Budget constraint$_1$

A Rise in the Household's Income

A Rise in the Price of Clothing

A Rise in the Price of Clothing

q_{food}

Budget constraint$_1$

Budget constraint$_2$
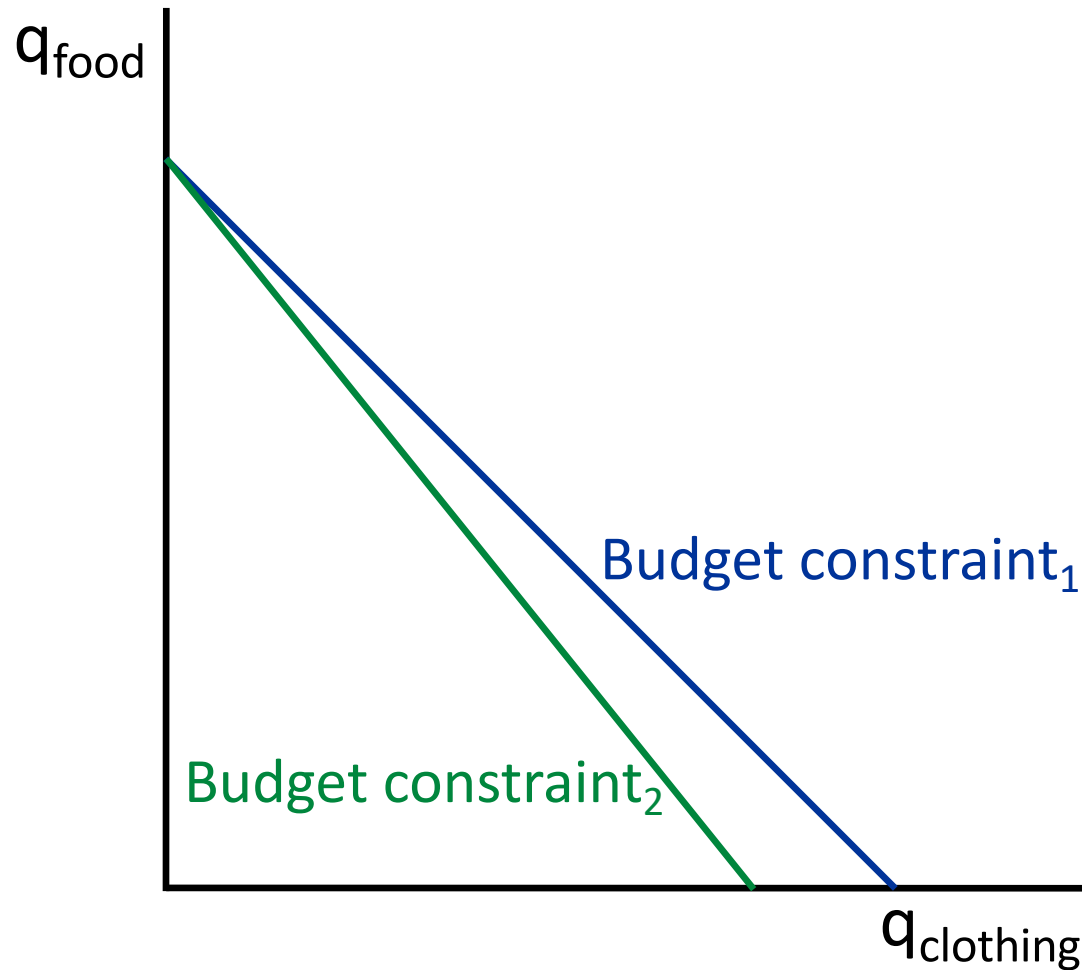
$q_{clothing}$

Recall that the slope of the budget constraint is $-P_{clothing}/P_{food}$.

What point does the consumer choose on the budget constraint?
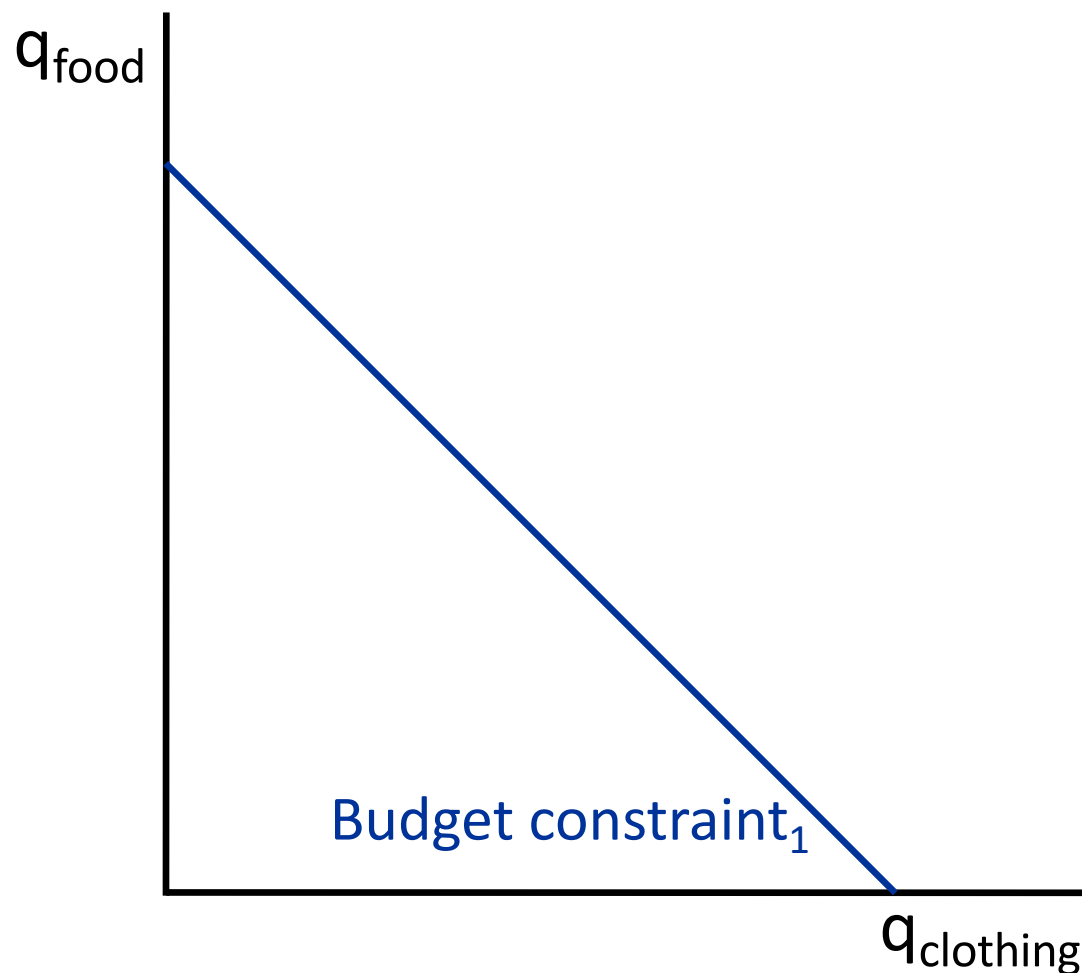
III. UTILITY MAXIMIZATION

What do we think consumers maximize?

What do we think consumers maximize?

- Happiness, satisfaction, utility.

- Economists don't make judgments about *what* gives people happiness.

- Preferences of individuals are "sacred" for economists and are revealed by their choices
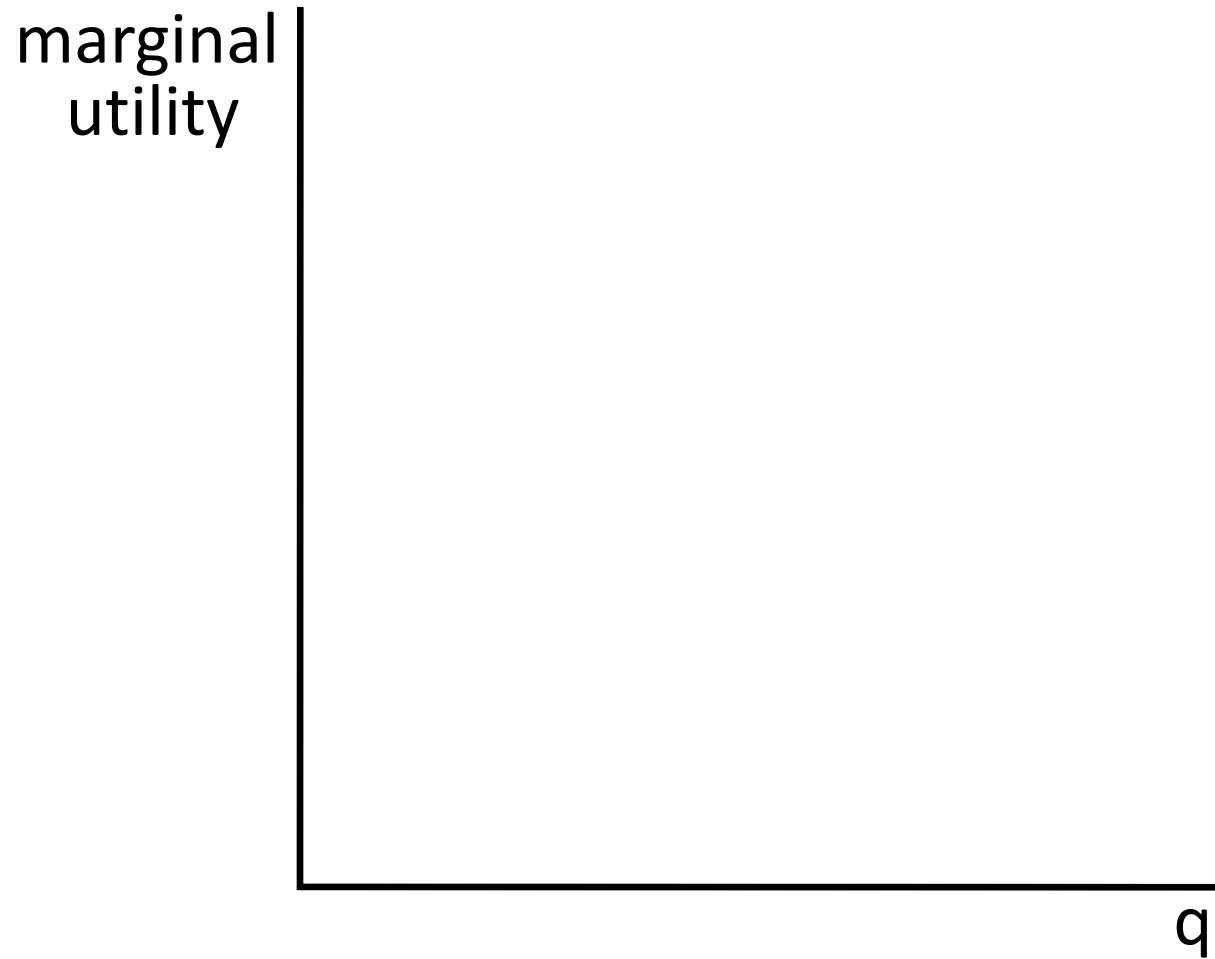
Utility

- **Total utility:** The total happiness one gets from consuming a given amount of a good.

- **Marginal utility:** The extra utility derived from consuming one more unit of a good.

Diminishing Marginal Utility

- As a household consumes more of a given good, the marginal utility of the good declines.

- Canonical example is food:

  - when hungry, an extra unit of food gives you a lot of utility

  - when satiated, an extra unit of good gives you a lot less utility, eventually negative

Diminishing Marginal Utility

marginal
utility

q

Diminishing Marginal Utility
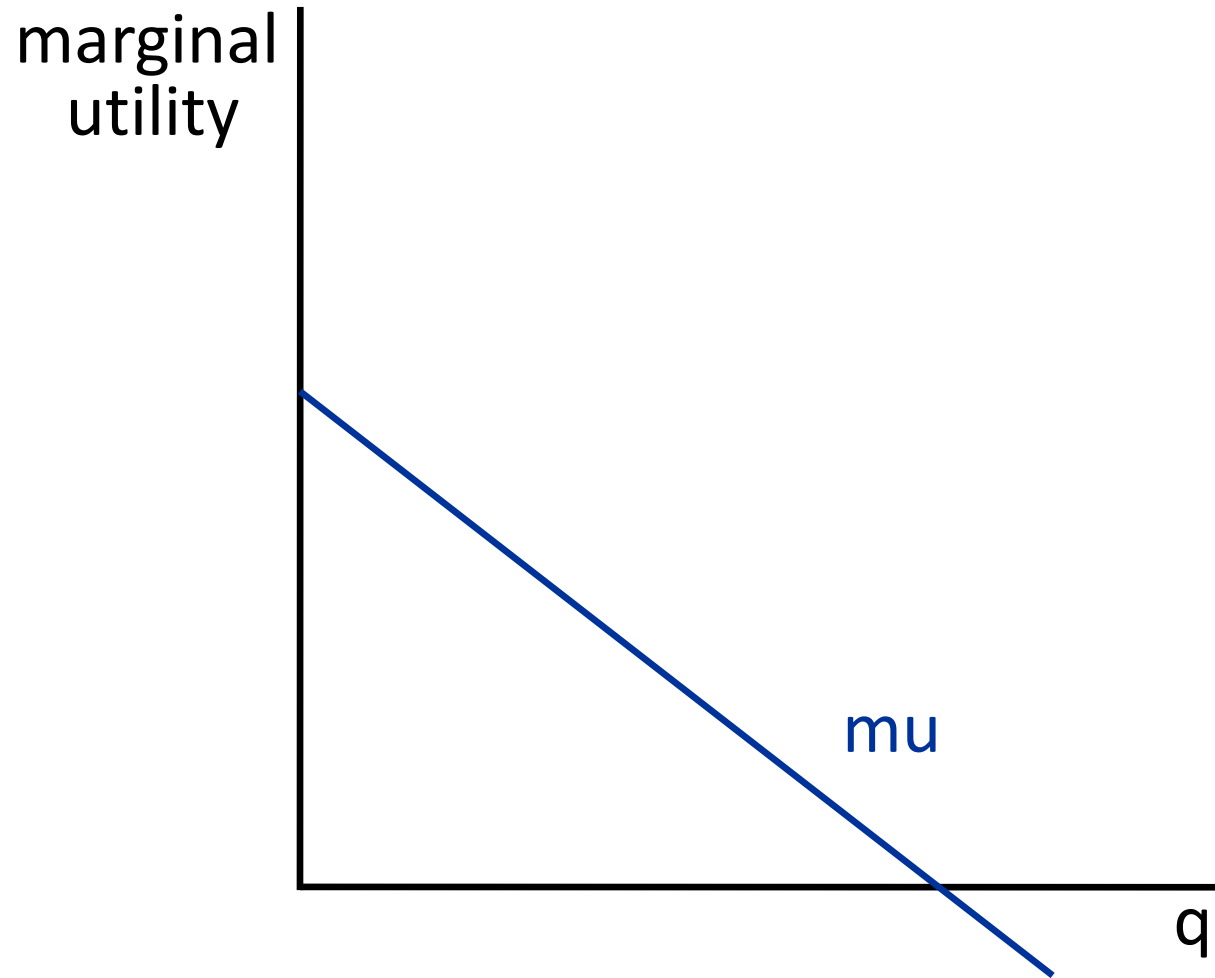
Relationship between
Total Utility and Marginal Utility

- Suppose

$$u = f(q)$$

where q is the quantity of some good a household consumes, and u is the total utility the household gets from consuming the good.

- Then

$$mu = f'(q)$$

where mu is marginal utility

$f'(q)$ is derivative of function f at point q

Recall about derivatives

- Function f(x)

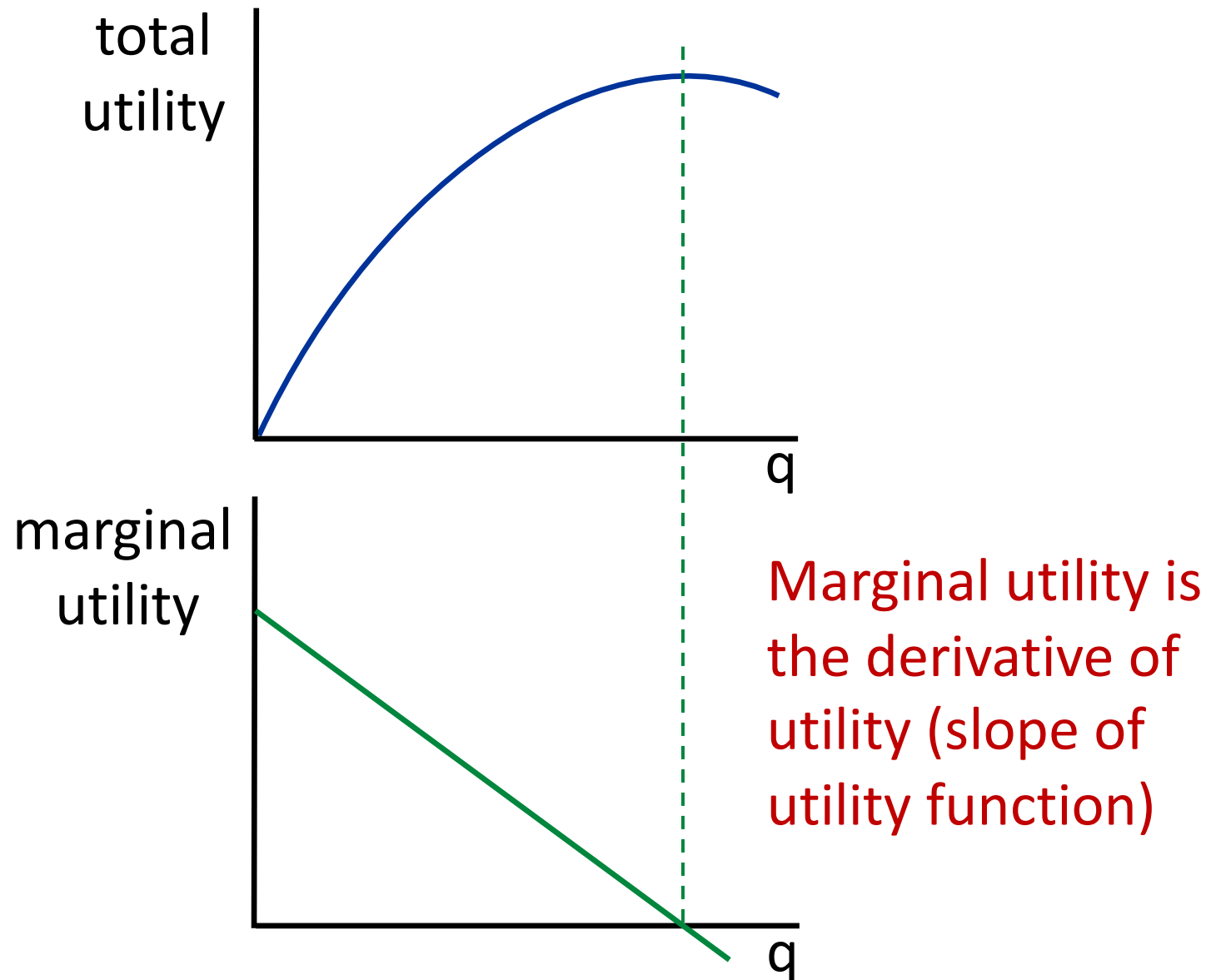- Derivative f'(x) is the slope of function at x

$$f'(x)=(f(x+1)-f(x))/1$$

assuming that 1 extra unit is small

- Formal mathematical definition is:

f'(x) = limit of (f(x+e)-f(x))/e when e goes to zero

- In economics: marginal always means derivative (adding one extra small unit) so that we can use calculus
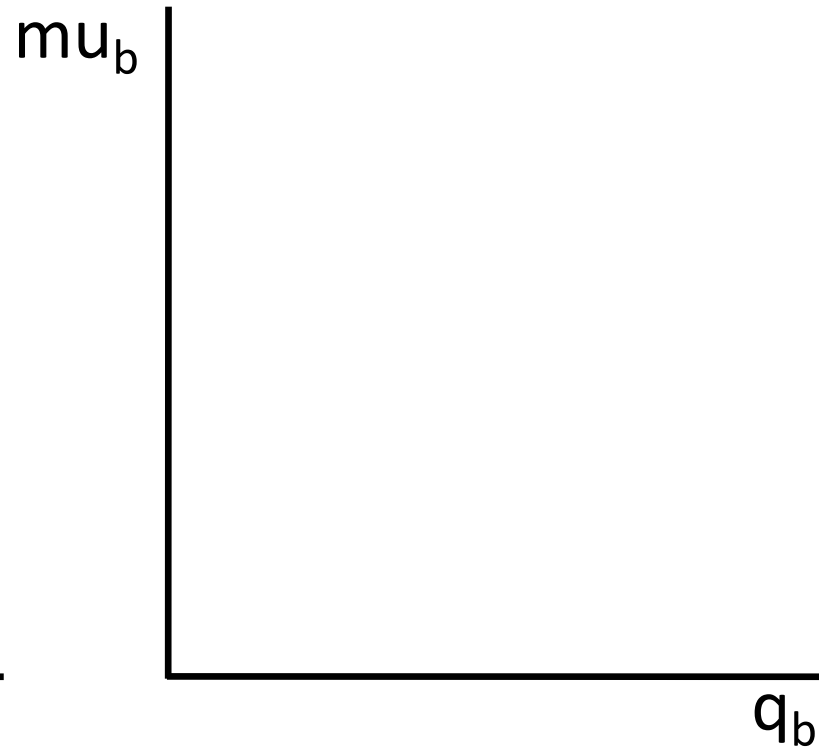
Relationship between Total and Marginal Utility

total
utility

q

marginal
utility

Marginal utility is
the derivative of
utility (slope of
utility function)

q

Marginal Utility Likely Declines at Different Rates for Different Goods

Good a

mu_a

q_a

Good b

mu_b
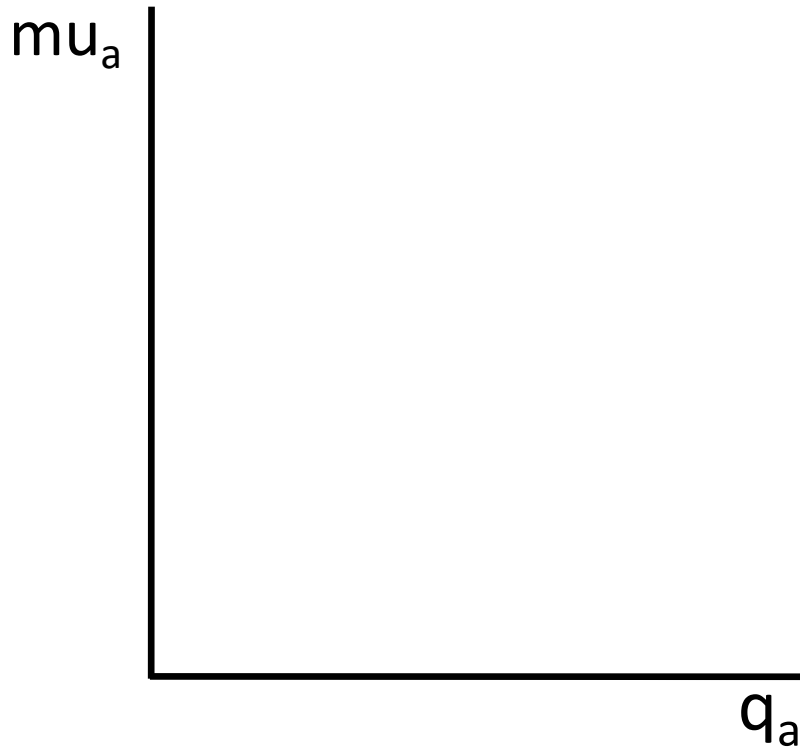
q_b

Marginal Utility Likely Declines at Different Rates for Different Goods

Good a (food)

Good b (clothes)

mu_a

q_a

mu_a

mu_b

q_b

mu_b
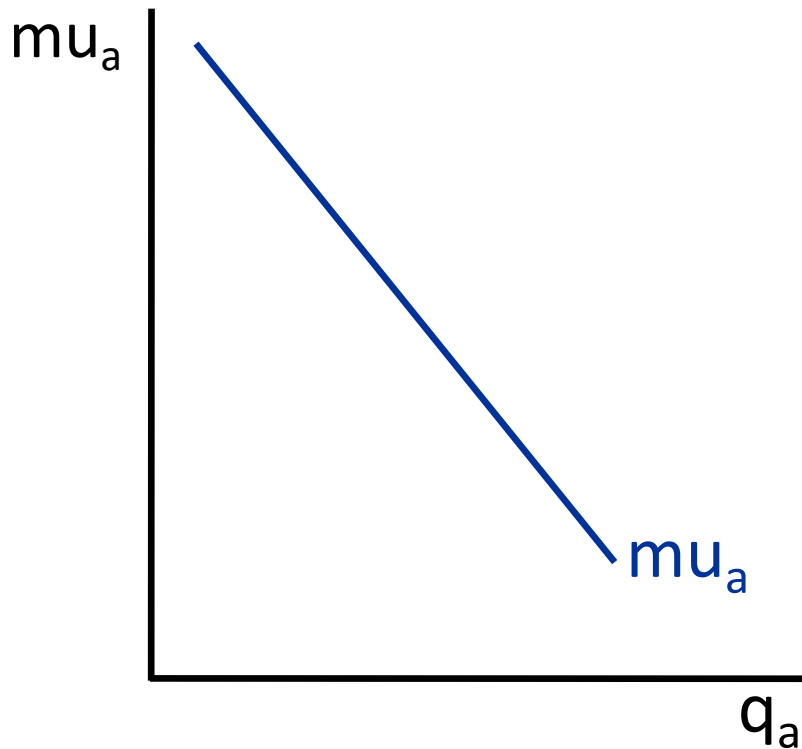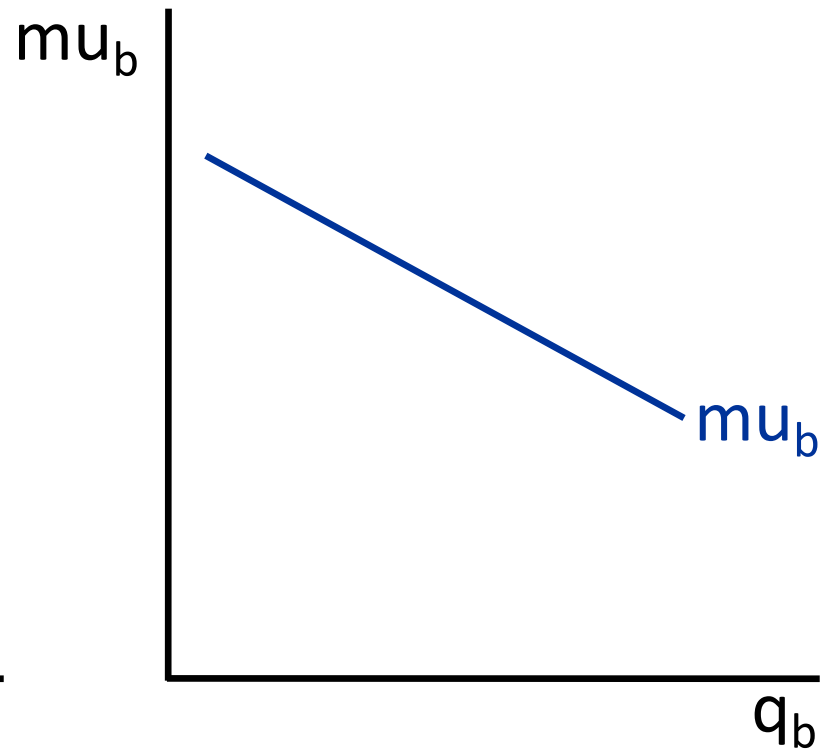
The Condition for Utility Maximization (the Rational Spending Rule)

- A household is doing the best that it can—that is, it is maximizing its utility—if:

  The marginal utility derived from spending one more dollar on a good is the same for all goods.

  Technical (but not substantial) assumption: people can fine tune the exact quantity they buy → we can apply mathematical calculus

The Condition for Utility Maximization with Just Two Goods (Food and Clothing)

$$\frac{\$1}{P_c} \, mu_c = \frac{\$1}{P_f} \, mu_f$$

Where the *P*'s are the market prices of the two goods and the *mu*'s are the marginal utilities of an additional unit of the two goods for the household.

This is the same as:

$$\frac{mu_c}{P_c} = \frac{mu_f}{P_f}$$

The General Condition for Utility Maximization (the Rational Spending Rule)

$$\frac{mu_a}{P_a} = \frac{mu_b}{P_b} = \ldots = \frac{mu_z}{P_z},$$

where the *P*'s are the market prices of the different goods, and the *mu*'s are the marginal utilities of an additional unit of the different goods for the household.

IV. CONSUMER OPTIMIZATION AND THE DEMAND CURVE
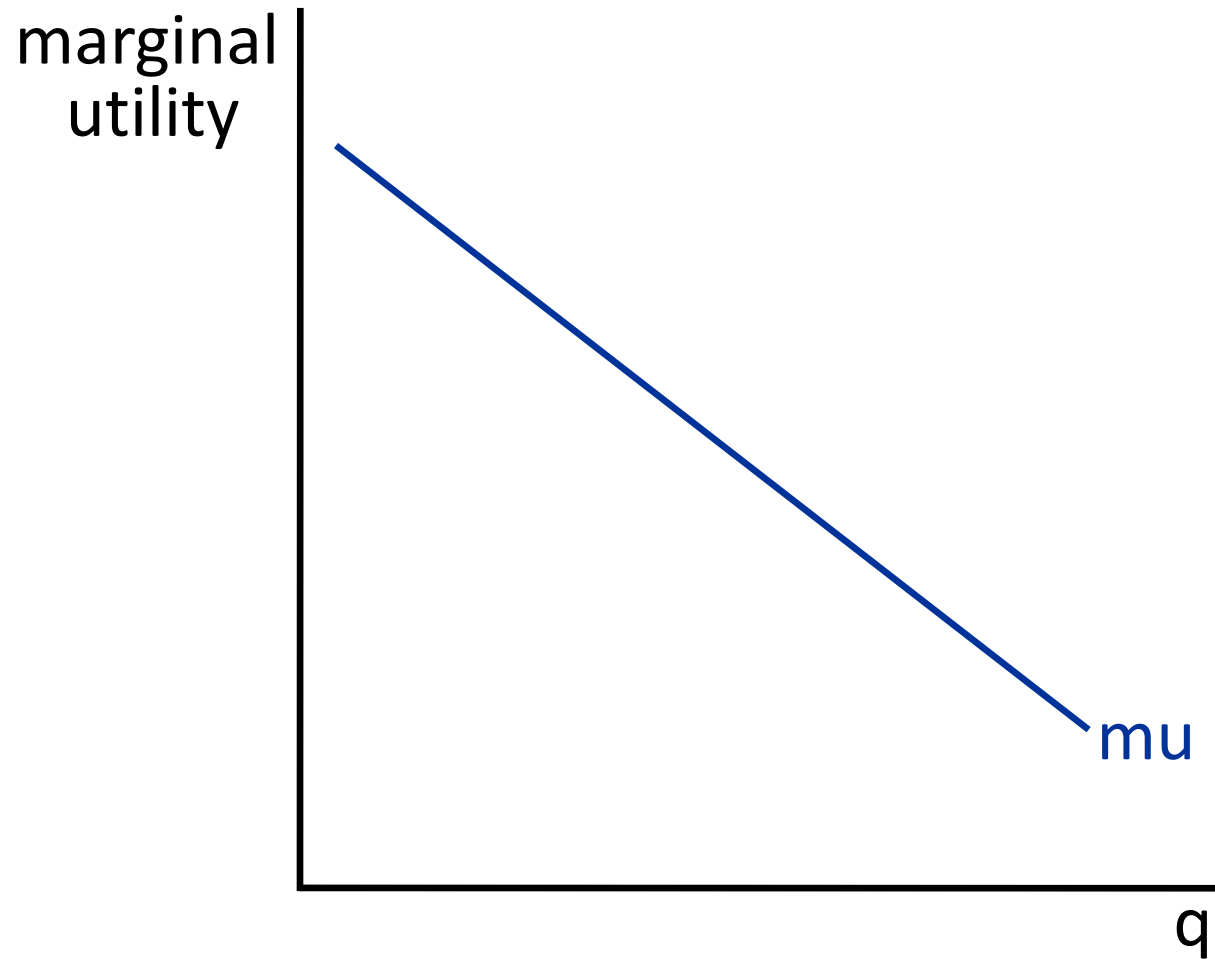
A Rise in the Price of Clothing

- Suppose the household starts with:

$$\frac{mu_c}{P_c} = \frac{mu_f}{P_f}$$

- If P_c rises, and the household didn't change its purchases, then:

$$\frac{mu_c}{P_c} < \frac{mu_f}{P_f}$$

Diminishing Marginal Utility

A Rise in the Price of Clothing

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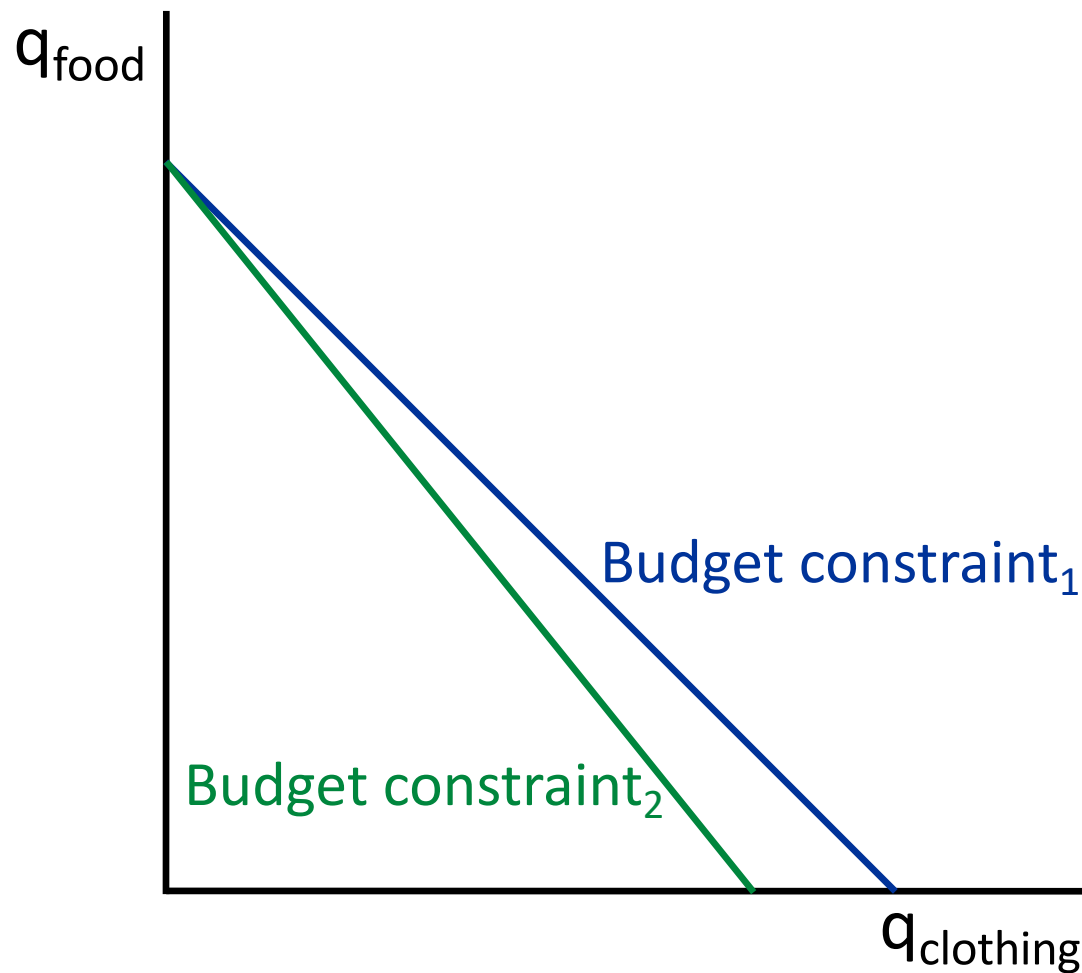
$$\frac{mu_c}{P_c} < \frac{mu_f}{P_f}$$

- The household will need to buy less clothing (and more food) until:

$$\frac{mu_c}{P_c} = \frac{mu_f}{P_f}$$

Why Demand Curves Slope Down

- Substitution effect: When the price of a good rises, a household wants less of the good and more of other goods, because the good becomes relatively more expensive.

A Rise in the Price of Clothing

q_{food}

Budget constraint$_1$

Budget constraint$_2$

$q_{clothing}$

Why Demand Curves Slope Down

- Substitution effect: When the price of a good rises, a household wants less of the good and more of other goods, because the good is relatively more expensive.
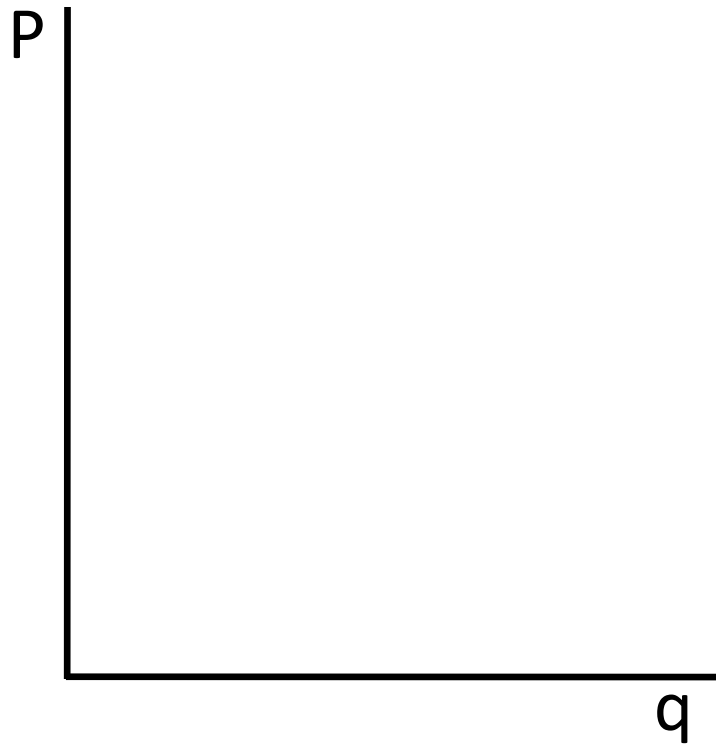
  "if price increases, I buy something else"

- Income effect: When the price of a good rises, a household wants less of all goods, because its budget constraint has changed for the worse.

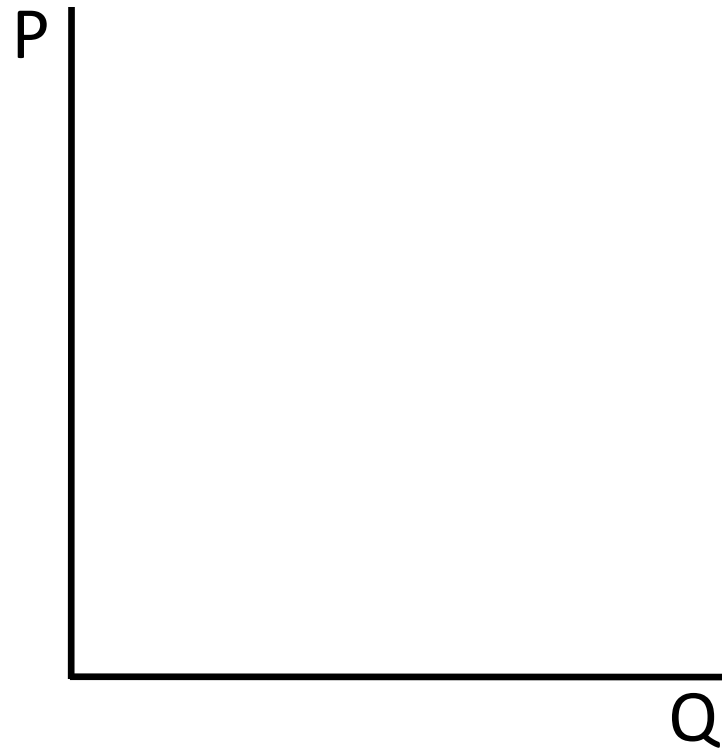 "if price increases, I can't buy as much"
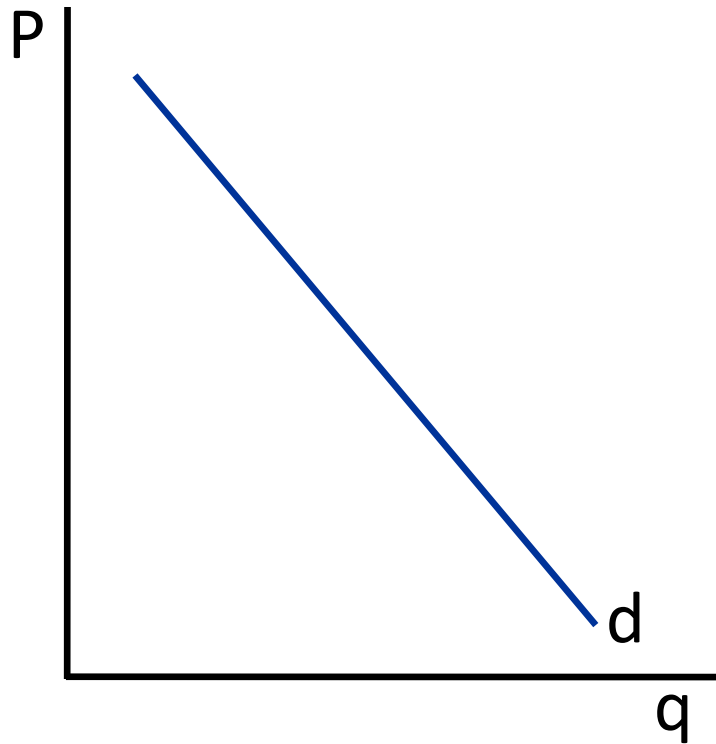
Demand Curves

Individual Household
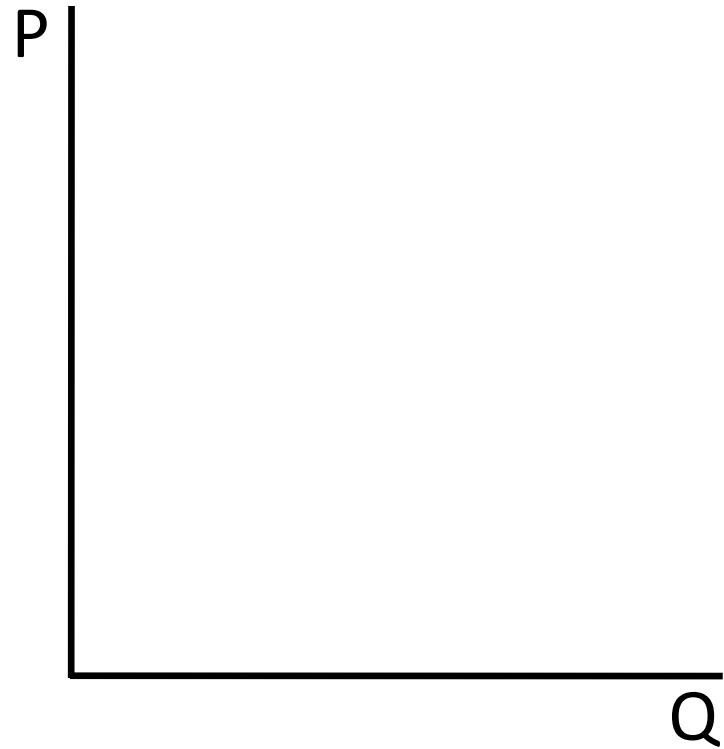
P |

| q

Market

P |

| Q

Demand Curves

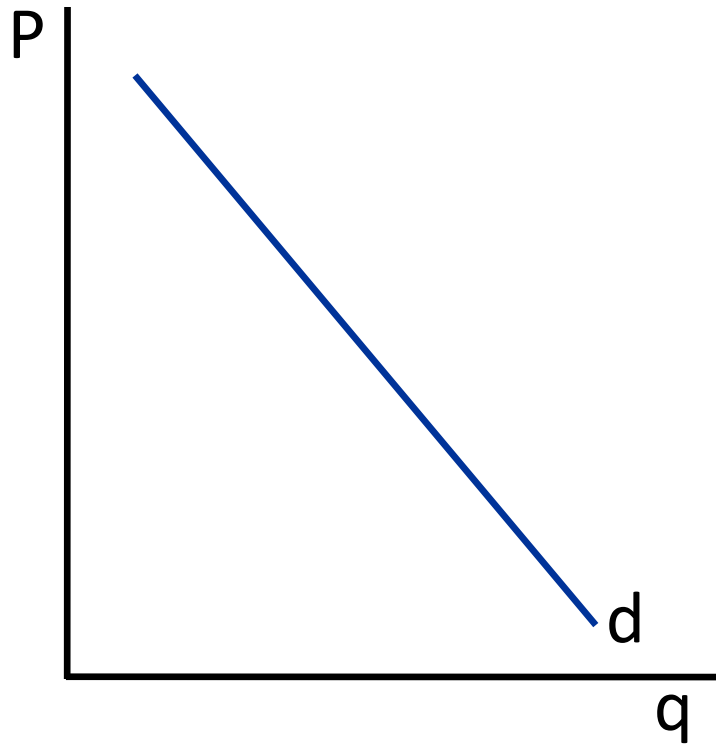## Individual Household

P

d

q
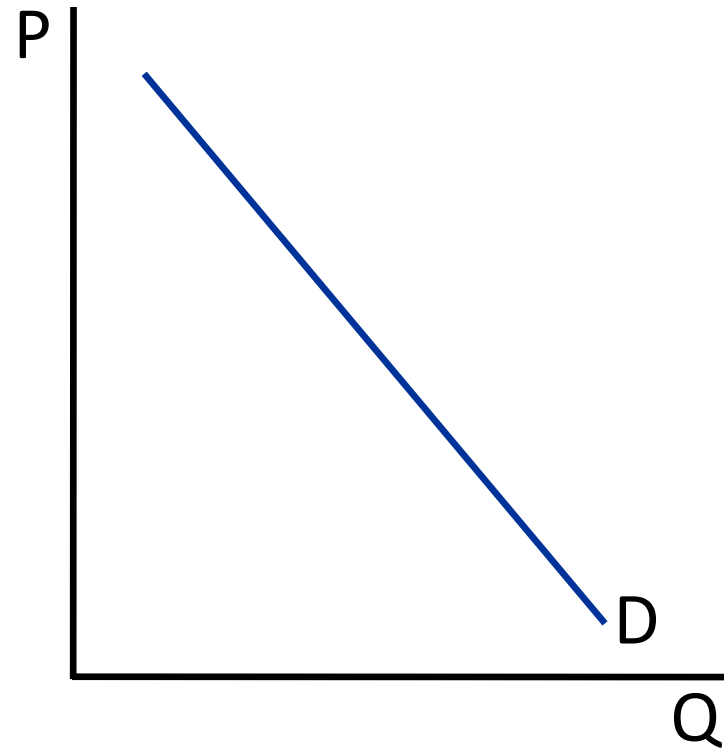
Market

P

Q

Household and Market Demand Curves

- The market demand curve is the **horizontal sum** of each individual household's demand curve.

- Because each household's demand curve (d) slopes down, the market demand curve (D) slopes down.

Demand Curves

Household and Market Demand Curves

- The market demand curve is the **horizontal sum** of each individual household's demand curve.

- Because each household's demand curve (d) slopes down, the market demand curve (D) slopes down.

- Because each household's demand curve is derived from optimizing behavior, the market demand curve is as well.

Positive News about Blueberries
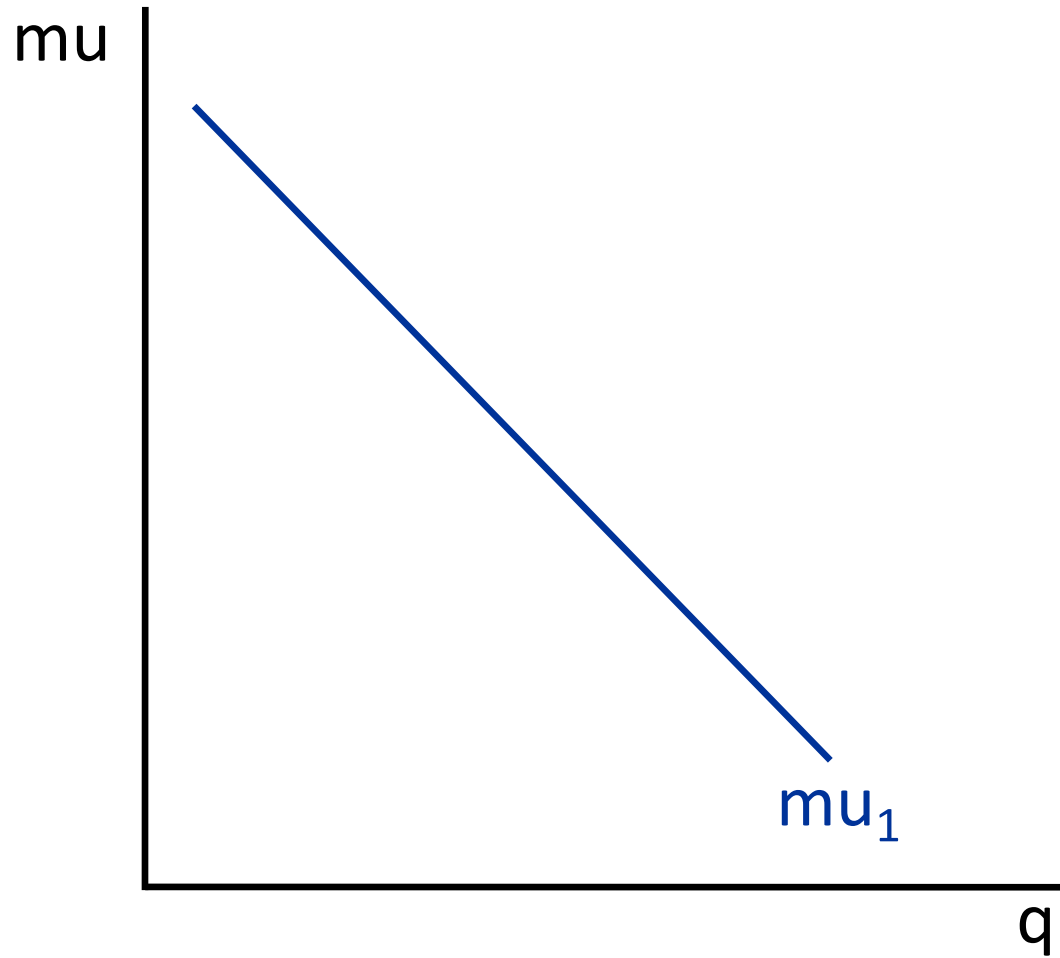
Positive News about Blueberries
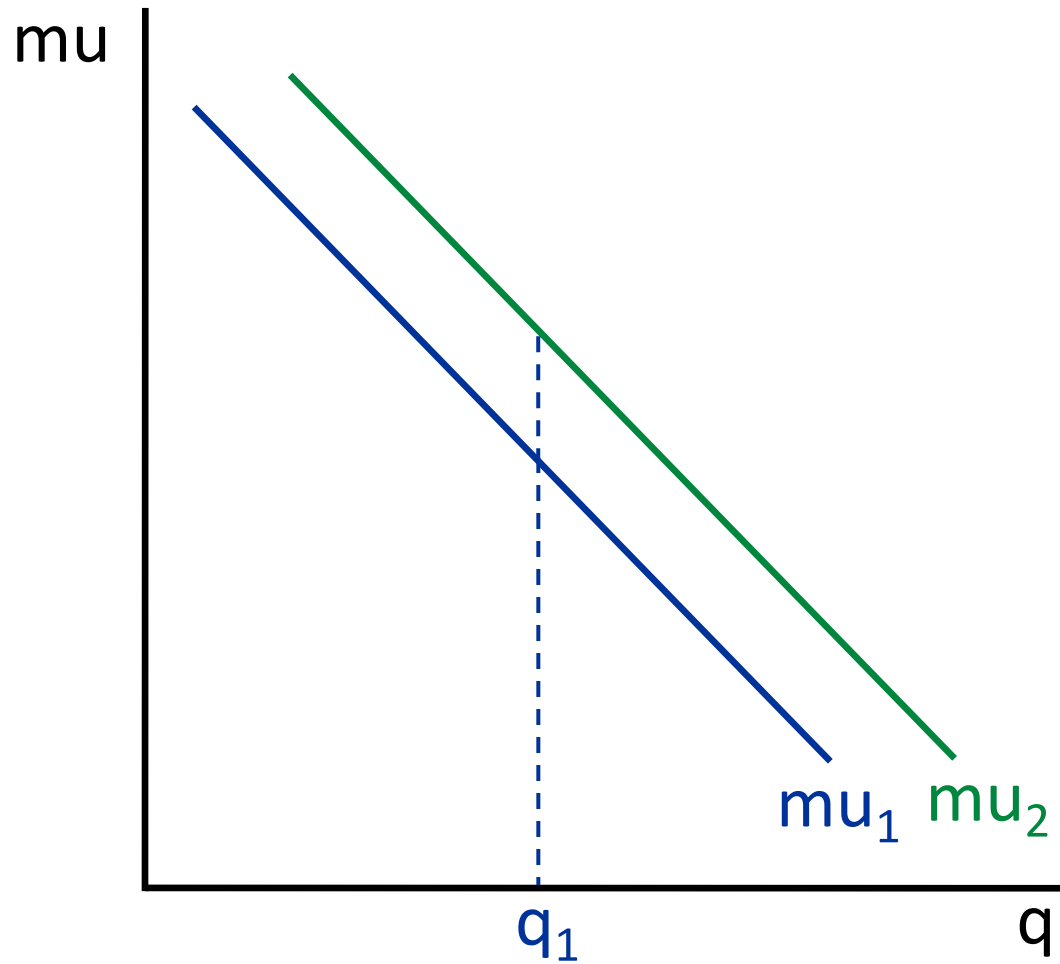
Positive News about Blueberries
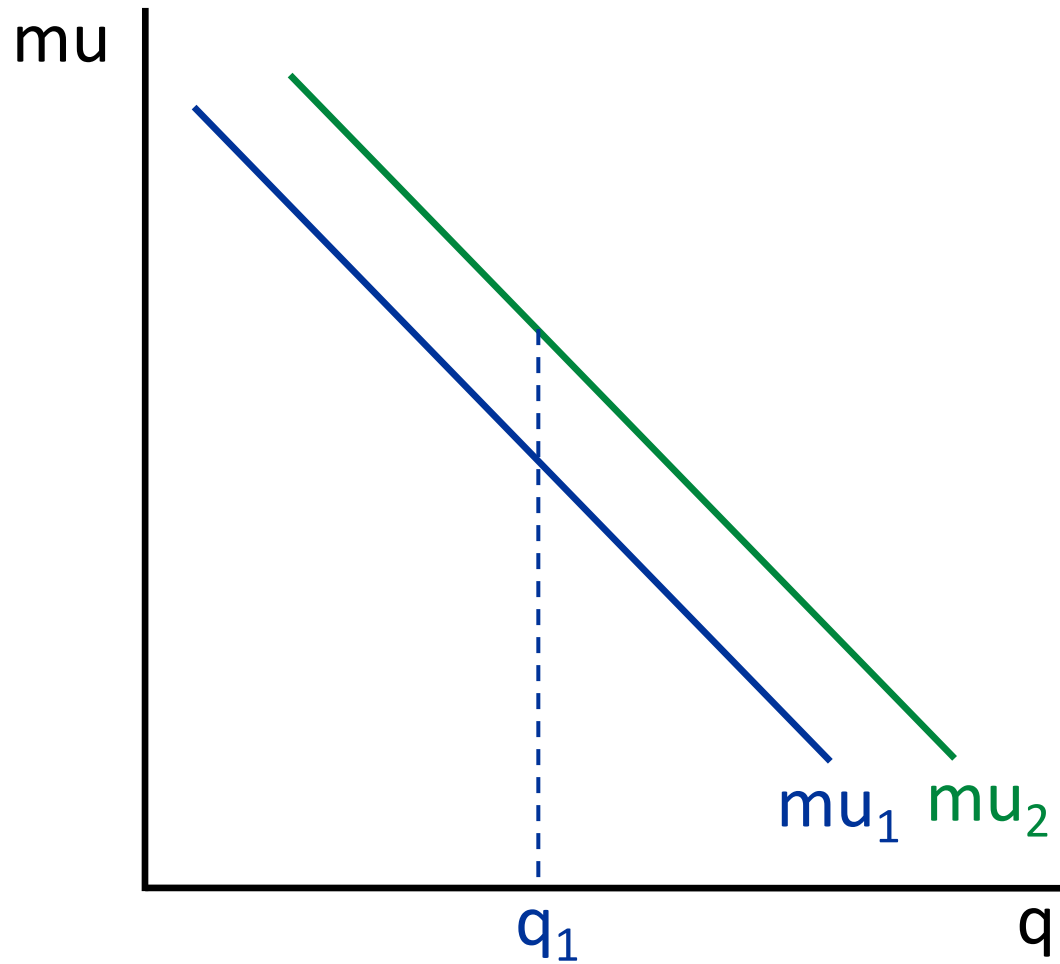
- An optimizing consumer sets:

$$\frac{mu_{blueberries}}{P_{blueberries}} = \frac{mu_{everything\ else}}{P_{everything\ else}}$$

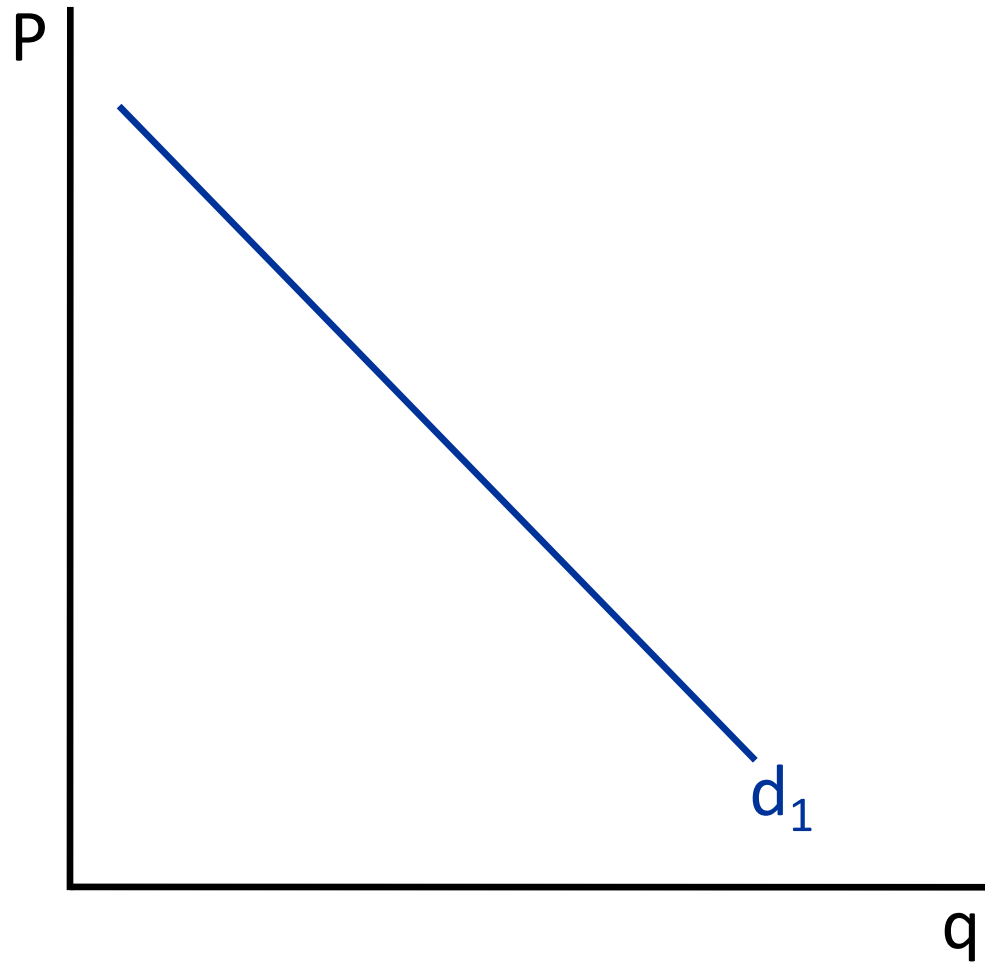- A rise in the $mu_{blueberries}$ causes:

$$\frac{mu_{blueberries}}{P_{blueberries}} > \frac{mu_{everything\ else}}{P_{everything\ else}}$$

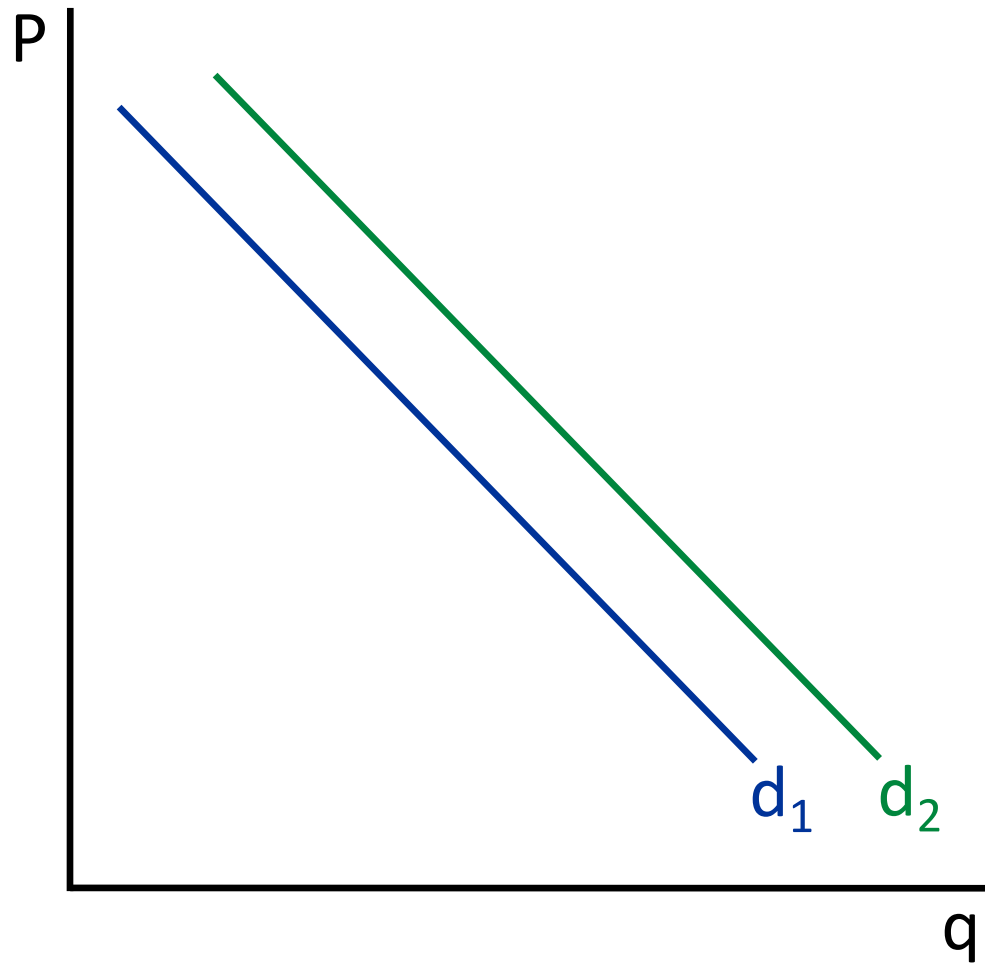- The optimizing consumer will want to consume more blueberries at the same $P_{blueberries}$.

Positive News about Blueberries

Effect of Positive News on the Demand Curve

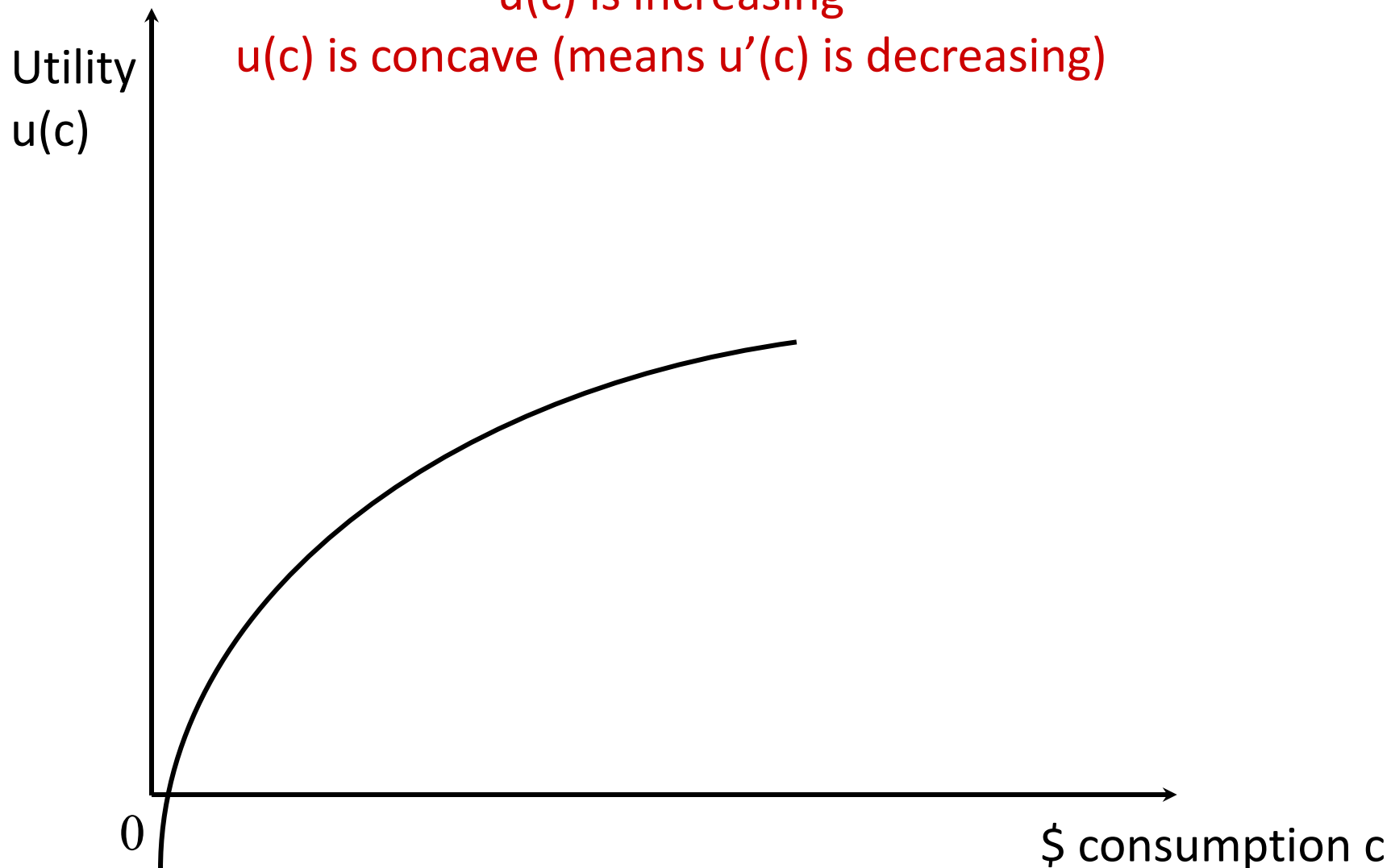# Effect of Positive News on the Demand Curve

Utility of Income

- Consumer has income c that is used to buy all sorts of goods: $P_a \cdot q_a + P_b \cdot q_b + P_c \cdot q_c + \dots + P_z \cdot q_z = c$ where c is total amount spent on consumption.

- This consumer optimization generates utility $u(c)$ with $u(.)$ an increasing function of c.

- The marginal utility of income is $u'(c)$, the slope of utility function $u(c)$ = extra utility from having +\$1

- We expect $u'(c)$ to **decrease** with c which means that $u(c)$ is **concave** in c.

Utility Function

u(c) is increasing

u(c) is concave (means u'(c) is decreasing)

Utility
u(c)

0

$ consumption c

Utility Function

u(c) is increasing
u(c) is concave (means u'(c) is decreasing)

Utility
u(c)

Example:
u(c)=log(c)
u'(c)=1/c
10% increase in c gives
same utility whether c is
high or low

0

$ consumption c