

Distribution Network Design Project

Part A

To present a mathematical solution for this problem, we decided to consider the following assumptions first, as considering all variables would represent highly complex computational and formulation capabilities:

Assumptions:

1. The distance between each node is 15 minutes.
2. Stop time is 1/10 of the demand served.
3. Demand is constant and not variable.
4. Cost of opening a hub is 0.
5. Cost of operating vans is 0.
6. Unlimited van capacity.

To solve this, we aimed to generate a multi-objective model where we seek to maximize customer coverage while minimizing the maximum distance traveled. In other words, we aim to minimize the maximum distance D_{max} plus the sum of customer demand multiplied by the distance between the location and the customer. The model is presented in the below.

Sets:

$i \in \{1,2,3,\dots,49\} \rightarrow \text{Libraries/Customers}$

$j \in \{1,2,3,\dots,49\} \rightarrow \text{Possible Hub Locations}$

$k \in \{1,2\} \rightarrow \text{No of Vans}$

$W \in \{0, 0.1, 0.2, 0.3, \dots, 1\} \rightarrow \text{Weight of OF}$

Parameters:

d_{ij} : A binary parameter indicating if location j covers customer i

h_i : Represents the demand of each customer i

Decision variables:

x_{ij}^k : A binary variable indicating if customer i is assigned to location j if van k cover.

y_j^k : A binary variable indicating if location j is open or not.

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Model:

$$OF: Min W * \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} h_i * d_{ij} * x_{ij}^k + (1 - W) * MaxDist$$

S.T.

1. Coverage Constraint in time.

$$15 * \sum_{j \in J} y_j^k + 0.1 \sum_{i \in I} \sum_{j \in J} x_{ij}^k * h_i \leq 345 \quad \forall k \in K$$

2. Each customer is assigned to one hub.

$$\sum_{j \in J} \sum_{k \in K} x_{ij}^k = 1 \quad \forall i \in I$$

3. Each customer is assigned to an open facility.

$$x_{ij}^k \leq y_j^k \quad \forall i \in I, j \in J, k \in K$$

4. $MaxDist$ must be greater than or equal to the distance between each customer and their assigned location.

$$MaxDist \geq d_{ij} * x_{ij}^k \quad \forall i \in I, j \in J, k \in K$$

5. Binary Variables

$$x_{ij}^k: Binary\{0,1\} \quad \forall i \in I, j \in J, k \in K$$

$$y_j^k: Binary\{0,1\} \quad \forall i \in I, j \in J, k \in K$$

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For this part we are only considering **P-Median** part of the mathematical model. The weight used is $W=1$. The results are shown in the figure below.

Parameters, Decision Variables, & Sets



```
table1 = pd.read_excel('/Users/ichang/Desktop/city.xlsx', sheet_name='matrix')
table2 = pd.read_excel('/Users/ichang/Desktop/city.xlsx', sheet_name='demand')
traveltime=table1.iloc[3:,3:].values
demand=table2.iloc[1:,5].values
cities=table2.iloc[1:,1].values
city=range(len(cities))
hub=range(len(cities))
vehicle=[1,2]
m = Model("facility")
scenarios=[0,0.2,0.4,0.6,0.8,1]
scenario=range(len(scenarios))
x = m.addVars(city,hub,vehicle, vtype='B', name="transport amount")
y= m.addVars(hub,vehicle, vtype='B', name='facility location')
max=m.addVar(vtype='C',name='minimax')
len(traveltime)
demand
objective1=quicksum(demand[i]*traveltime[i][j]*x[i,j,k] for i in city for j in hub for k in vehicle)
objective2=max
```

Formulation



```
m.setObjective(objective1, GRB.MINIMIZE)
for k in vehicle:
    m.addConstr(quicksum(15*y[j,k] for j in hub)+15+quicksum(1/10*x[i,j,k]*demand[i] for i in city for j in hub) <=360,'p-constraint')
for i in city:
    m.addConstr(quicksum(x[i,j,k] for j in hub for k in vehicle)==1)
for i in city:
    for j in hub:
        for k in vehicle:
            m.addConstr(x[i,j,k]<=y[j,k])
for i in city:
    for j in hub:
        for k in vehicle:
            m.addConstr(max>=traveltime[i,j]*x[i,j,k])
m.optimize()
print(f"objective is: {m.ObjVal}")
for k in vehicle:
    for j in hub:
        if y[j,k].X > 0.99:
            print(f"Hub is opened at {cities[j]} serviced by vehicle {k}")
            for i in city:
                if x[i,j,k].X> 0:
                    print(f"city {cities[i]} is assigned to this hub")
```

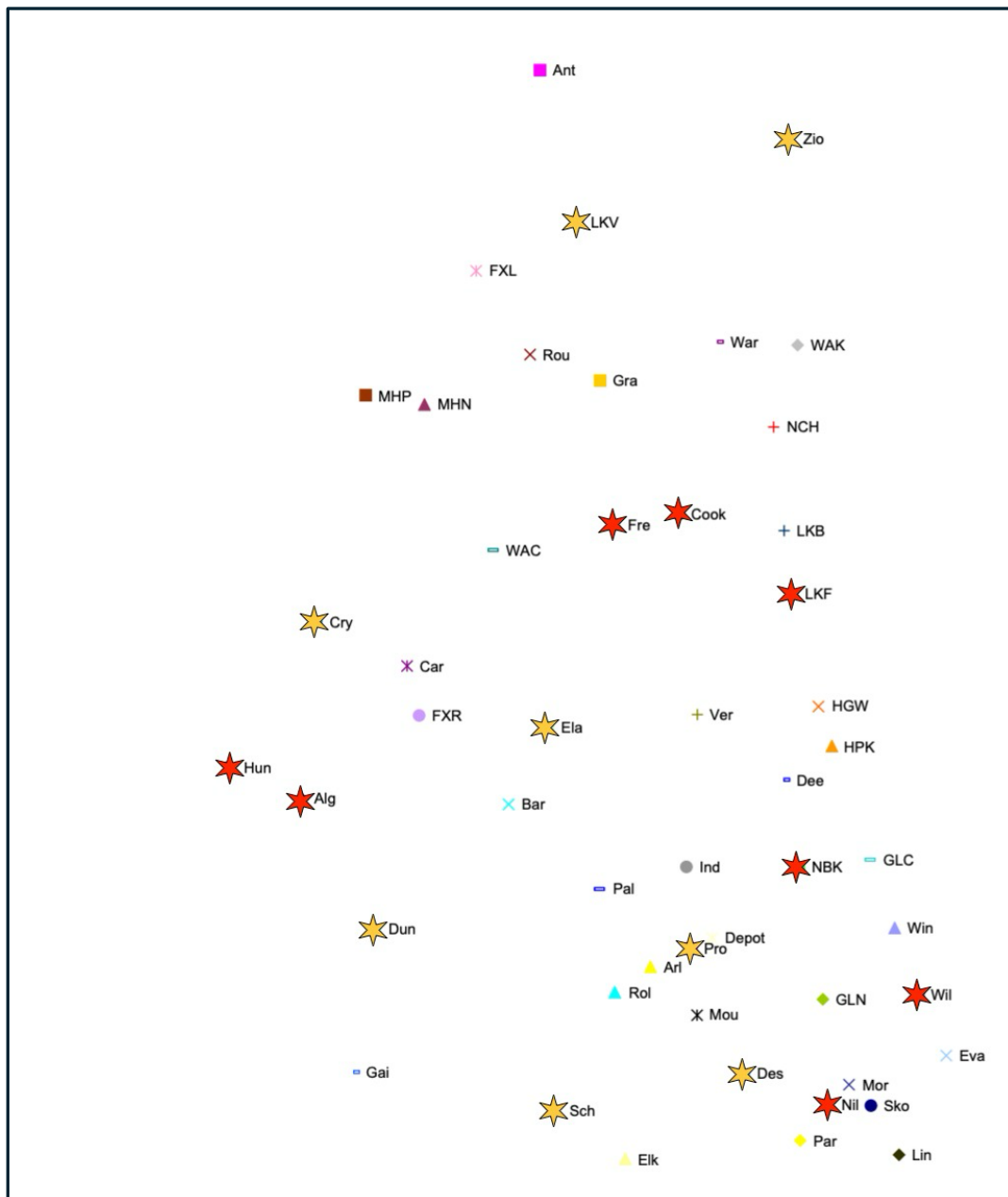
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Results



Total number of hubs to be opened is 16.

- ★ Van 1 will be assigned to these hubs.
- ★ Van 2 will be assigned to these hubs.



MAP of North Suburban Library Systems

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Results



Hub is opened at Wil serviced by vehicle 1
city Eva is assigned to this hub
city Lin is assigned to this hub
city Wil is assigned to this hub
city Win is assigned to this hub
Hub is opened at Cry serviced by vehicle 2
city Car is assigned to this hub
city Cry is assigned to this hub
city MHP is assigned to this hub
city MHN is assigned to this hub
Hub is opened at Des serviced by vehicle 2
city Des is assigned to this hub
Hub is opened at Dun serviced by vehicle 2
city Dun is assigned to this hub
city FXR is assigned to this hub
city Gai is assigned to this hub
Hub is opened at Ela serviced by vehicle 2
city Bar is assigned to this hub
city Ela is assigned to this hub
city Pal is assigned to this hub
city WAC is assigned to this hub
Hub is opened at LKV serviced by vehicle 2
city Ant is assigned to this hub
city FXL is assigned to this hub
city LKV is assigned to this hub
city Rou is assigned to this hub

Hub is opened at Pro serviced by vehicle 2
city Arl is assigned to this hub
city Ind is assigned to this hub
city Mou is assigned to this hub
city Pro is assigned to this hub
Hub is opened at Sch serviced by vehicle 2
city Elk is assigned to this hub
city Rol is assigned to this hub
city Sch is assigned to this hub
Hub is opened at Zio serviced by vehicle 2
city WAK is assigned to this hub
city Zio is assigned to this hub

objective is: 8472.312591220523
Hub is opened at Alg serviced by vehicle 1
city Alg is assigned to this hub
Hub is opened at Cook serviced by vehicle 1
city Cook is assigned to this hub
city Ver is assigned to this hub
city War is assigned to this hub
Hub is opened at Fre serviced by vehicle 1
city Fre is assigned to this hub
city Gra is assigned to this hub
Hub is opened at Hun serviced by vehicle 1
city Hun is assigned to this hub
Hub is opened at LKF serviced by vehicle 1
city HGW is assigned to this hub
city LKB is assigned to this hub
city LKF is assigned to this hub
city NCH is assigned to this hub
Hub is opened at Nil serviced by vehicle 1
city GLN is assigned to this hub
city Mor is assigned to this hub
city Nil is assigned to this hub
city Par is assigned to this hub
city Sko is assigned to this hub
Hub is opened at NBK serviced by vehicle 1
city Dee is assigned to this hub
city GLC is assigned to this hub
city HPK is assigned to this hub
city NBK is assigned to this hub

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Recommendations to better serve communities in greater need:

Being a multi objective model, we can put different weights on both the objectives depending on what management's goals are. If the weighted sum of libraries served is the goal, the weight should be closer to 1. If the goal is to minimize the maximum distance to provide accessibility to remote communities, then the weight shall be put closer to 0. The weight parameter can be changed in parameter section of the code. The current model assumes that the travel time between hubs is 15minutes. however, this may or may not be accurate and must be accessed using actual travel time to get a performance measurement. if management decides to change other parameters based on newfound data such as demand, demand parameters shall be changed in the parameter section of the code.

We can do **Reassessment and Redesign** in following things:

1. **Regular Data Review:**

By establishing a regular review process for demand patterns, service effectiveness, and community feedback. This could be monthly or quarterly. We can adjust the hub locations and routes based on data-driven insights from these reviews.

2. **Community Feedback Mechanism:**

We can create a feedback mechanism for library patrons and staff to report issues with the hub system and suggest improvements.

3. **Performance Metrics:**

We can set a clear performance metrics such as delivery times, patron satisfaction, and service coverage for continuous monitoring. By using these metrics, we can identify when the system needs to be reassessed or redesigned.

4. **Budget Considerations:**

We can continuously monitor the budget impact of the hub system and look for ways to optimize operations without compromising service quality.

5. **Adaptation to Change:**

We can develop a plan for quick adaptation in response to changes in funding, community needs, or other external factors. Also, we can have contingency plans in place to address unexpected challenges or opportunities.

6. **Technology and Innovation:**

Staying abreast of technological advancements that can improve logistics and service delivery and be prepared to integrate such technologies into the system can also work.

7. **Training and Development:**

We can always ensure that NSLS staff are trained to use whatever system or technology is in place for reassessment and redesign, making the transition as seamless as possible.

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Some Recommendations to better serve communities in greater need:

1. We can use a mathematical model to determine the optimal location of hubs and library assignments. This might involve a location-allocation model where libraries are assigned to the nearest hub based on travel time and demand. The model should account for the stopping time at each hub based on the demand, which affects the total route time.
2. We can also prioritize libraries serving marginalized communities in the hub selection process, possibly by assigning a higher weight to their demand and by considering the accessibility of hubs for patrons without personal transportation, possibly by including a term in the model that penalizes hub locations that are far from public transportation or community centers.
3. Our solution approach must be adaptable for when the demand volumes and other parameters change. We could include a parameter that represents community need.
4. We need to verify the solution against the constraints and objectives once we determine the hubs, create vehicle routes using the actual distances also we need to ensure that the routes comply with the driver availability constraint.
5. We also need to check how changes in parameters affect our solution and one thing to remember is to our model should account for the stopping time at each hub based on the demand, which affects the total route time because the driver's total travel time plus stopping time should not exceed 12 hours in aggregate for both vans.

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Reference:

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p – Median Problem Parameters

Input Data

P: number of facilities to locate
I: set of customers (demand nodes) to **serve**
J: set of potential facility locations
 h_i : demand at customer i
 d_{ij} : distance from customer i to facility j

Decision Variables

Y_j : opening a facility at location j
= 1 if facility j is opened
= 0 otherwise
 X_{ij} : assignment variable
= 1 if customer node i is **assigned** to facility j
= 0 otherwise

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p-Median Problem Formulation

$$\text{minimize. } \sum_{\{i \in I, j \in J\}} h_i d_{ij} X_{ij}.$$

Subject to:

$$\sum_{j \in J} Y_j \leq P \quad \text{Open at most P facilities}$$

$$\sum_{j \in J} X_{ij} = 1 \quad \forall i \in I \quad \text{Each customer is assigned to one facility}$$

$$X_{ij} \leq Y_j \quad \forall i \in I, j \in J \quad \text{If a facility is not opened (Y}_j=0\text{) then no customers can be assigned to it}$$

$$X_{ij} \in \{0,1\}. \quad \forall i \in I, j \in J$$
$$Y_j \in \{0,1\}. \quad \forall j \in J \quad \text{All variables are binary}$$

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P-center

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minimize maximum distance

subject to number of facilities

discrete location choices

p – Center Problem Parameters

Input Data

- P: number of facilities to locate
- I: set of customers (demand nodes) to serve
- J: set of potential facility locations
- d_{ij} : distance from customer i to facility j
- If i cannot be reached from location j , then set d_{ij} to a very large number

Decision Variables

- Y_j : opening a facility at location j
 - = 1 if facility j is opened
 - = 0 otherwise
- X_{ij} : assignment variable
 - = 1 if customer node i is **assigned** to facility j
 - = 0 otherwise
- D_{Max} : Maximum distance to any customer

P-center

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minimize maximum distance

subject to number of facilities

discrete location choices

p-Center Problem Formulation

Minimize D_{Max}

Subject to:

$\sum_{j \in J} Y_j \leq P$	$\sum_{j \in J} X_{ij} = 1 \quad \forall i \in I$	Open at most P facilities
$X_{ij} \leq Y_j \quad \forall i \in I, j \in J$		Each customer is assigned to one facility
$D_{Max} \geq \sum_{j \in J} d_{ij} X_{ij} \quad \forall i \in I$		If a facility is not opened ($Y_j=0$) then no customers can be assigned to it
$X_{ij} \in \{0,1\}. \quad \forall i \in I, j \in J$	$Y_j \in \{0,1\}. \quad \forall j \in J$	The maximum distance is greater than or equal to the distance to any customer
		All variables are binary