Collision Probability of Space Objects

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Student's Declaration

I hereby declare that the work presented in the report entitled **"Collision Probability of Space Objects"** submitted by me for the partial fulfillment of the requirements for the degree of *Bachelor of Technology* in *Electronics & Communication Engineering* at Indraprastha Institute of Information Technology, Delhi, is an authentic record of my work carried out under guidance of **Prof. Sanat K Biswas**. Due acknowledgements have been given in the report to all material used. This work has not been submitted anywhere else for the reward of any other degree.

Sarthak Kumar **Place & Date: IIITD, 9 December,2023**
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Certificate

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Prof. Sanat K Biswas **Place & Date: IIITD, 9 December, 2023**
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Abstract

The primary objective of this research project was to conduct a comprehensive analysis of state-of-the-art algorithms designed for the calculation of Collision Probability between satellites and other space objects, particularly space debris. Our investigation revealed a categorization of space object encounters into three main types: instantaneous, short-term, and long-term encounters. Notably, we identified that collision probability assessment for each encounter type could be further subcategorized as cumulative and instantaneous.

To evaluate the accuracy of collision probability calculations, we focused on short-term encounters and implemented two widely utilized methods: Patera's method and Alfano's method. These methods were rigorously tested against a reference collision probability derived through the Monte Carlo Approach. Our findings shed light on the effectiveness and precision of these methodologies in predicting collision probabilities for short-term space object encounters.

In conclusion, this research contributes valuable insights into the categorization of space object encounters and the efficacy of specific algorithms in predicting collision probabilities. The comparative analysis of Patera's and Alfano's methods provides a nuanced understanding of their performance against a benchmark Monte Carlo Approach, offering implications for the enhancement of collision probability calculations in space debris management.

Keywords: Collision Probability, Space Situational Awareness

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Contents

Chapter 1

Introduction

The computation of collision probability among space objects is a crucial aspect of space situational awareness, specifically in the realms of conjunction assessment and collision avoidance. In the initial stages, the predominant approach involved the use of Monte Carlo simulations for predicting collision probabilities. While these simulations demonstrate accuracy with a substantial number of samples, their computational intensity poses practical limitations. In response to this challenge, numerous approximation methods have been devised over the last three decades to provide more computationally efficient alternatives.

The increasingly crowded space environment poses a significant threat to space assets, with the heightened risk of collisions. The impact of one Resident Space Object (RSO) on another, such as a satellite or space station, has the potential to lead to mission failures or, worse, fragmentation into numerous fragments of debris.

Various methods have been proposed to evaluate the collision risk associated with RSOs. Notably, NASA has employed the exclusion volume method as a preventive measure against potential conjunctions. This approach mandates spacecraft to execute avoidance maneuvers whenever a tracked piece of debris is projected to traverse a cubic area measuring 5 km $\times$ 2 km $\times$ 2 km centered on the spacecraft. While effective, this method provides a conservative estimate for RSO conjunction assessment, lacking the ability to quantify risk and potentially triggering unnecessary collision avoidance maneuvers due to false alarms(3).

Since 1981, collision probability has been employed to gauge the collision risk between two RSOs, offering a more nuanced and quantifiable approach to collision risk assessment.

After analysing various algorithms we found that spacecraft encounter can be modelled into two categories:
1) Short Term Encounter
2) Long Term Encounter

1.1 Short Term Encounter

Relative motion velocities reach several kilometers per second, and the duration of time spent in the encounter region is only a fraction of a second.(3)

For short-term encounters, the following assumptions are considered:

- The relative position between two objects remains rectilinear throughout the encounter.

- The relative velocity between the two objects remains constant throughout the encounter.
- There is no uncertainty in velocity during the encounter.
- The position uncertainty remains constant during the encounter, equivalent to the value at the closest approach point.

1.2 Long Term Encounter

In the context of long-term encounters, the relative velocity between Resident Space Objects (RSOs) tends to be relatively low. As this relative velocity diminishes, the reliability of the rectilinear motion assumptions made in the short-term encounter model also wanes. This becomes especially apparent when the duration of time spent in the encounter region becomes substantial, leading to deviations in the relative velocity from its initially assumed constancy. Consequently, the combined error covariance matrices cease to be constants, and the trajectory between the two objects evolves into a curved path.

In practical terms, short-term encounters receive more emphasis than their long-term counterparts, primarily due to the frequent incidence of RSO encounters characterized by significantly high relative velocities, often on the order of thousands of meters per second. This emphasizes the need to account for dynamic variations in relative velocity when modeling encounters, particularly in scenarios where the initially assumed rectilinear motion may no longer accurately represent the encounter dynamics.(3)

1.3 Calculating Collision Probability

1.3.1 General Method

The depiction of position uncertainty arising from orbit determination is captured within a three-dimensional Probability Density Function (PDF). The calculation of collision probability entails the trivariate integration, where the RSO's position uncertainty PDF intersects within a specified control volume. Fundamentally, the majority of methodologies employed for computing collision probabilities pivot around this integral process, underscoring the critical role of integrating the three-dimensional probability distribution in evaluating the likelihood of collisions.

1.3.2 Optimization

Let rp and rs represent the radii of circumscribed spheres centered on the primary and secondary Resident Space Objects (RSOs), respectively. The combined sphere, often referred to as a hardball or joint sphere, is precisely defined as a sphere centered on the secondary RSO, with a radius equal to the sum of the radii of the two RSOs . In mathematical terms, this is expressed as rc=rp+rs

When the primary RSO traverses within the confines of the combined sphere, a collision is imminent, and the collision probability is quantified as the likelihood of this specific event occurring.

The uncertainty of primary RSO is integrated in the region of intersection with the hard body sphere to get the collision probability. This approach to calculating collision probability is made computationally efficient by taking into consideration some fundamental assumptions, as we will see in upcoming sections(3).

The collision probability can also be categorized into two categories:
1) Instantaneous Collision Probability
2) Cumulative Collision Probability

1.3.3 Instantaneous Collision Probability

The instantaneous collision probability represents the likelihood of a collision occurring at a specific moment. Its probability integration occurs across the volume of the combined sphere $r_c = r_p + r_s$ within the physical three-dimensional space. The integral function involves the joint Probability Density Function (PDF) at the time of the closest approach (TCA), with its origin centered on the primary RSO. Denoting the position covariance matrices of the primary and secondary RSOs as C_p and C_s respectively, the combined covariance matrix C_c is derived as $C_c = C_p + C_s$ due to the uncorrelated distributions of the two RSOs.

The variability in the instantaneous collision probability is contingent upon the location of the combined sphere. Mathematically, this probability can be computed by integrating the combined PDF at the specified time t over the volume of the combined sphere.(3)

1.3.4 Cumulative Collision Probability

The cumulative collision probability characterizes the overall likelihood of collision between two Resident Space Objects (RSOs) over a specified period. Its probability integration occurs across the volume swept out by the motion of the combined sphere. In short-term encounters, this integration volume takes the form of an infinitely long cylinder. On the contrary, in long-term encounters, the volume can be exceptionally intricate and demands case-specific analysis.(3)

Theoretically, the cumulative collision probability P_c can be determined by integrating the instantaneous collision probability $P_{c,i}(t)$ over the entire encounter duration $[t_1, t_2]$:

$$P_c = \int_{t_1}^{t_2} P_{c,i}(t)\, dt$$

Given that the majority of encounters are short-term in nature, our analysis has focused on two state-of-the-art algorithms for computing collision probability—specifically, Alfano's and Patera's Method. Subsequently, we rigorously assessed the accuracy of these methods by conducting verification through Monte Carlo simulations.

Chapter 2

Patera's Method

In the realm of space situational awareness and collision avoidance, accurate assessment of collision probabilities between objects is paramount. Patera's method offers an insightful mathematical approach to address this challenge. Developed for scenarios where objects move past each other at constant velocities, the method streamlines the computation of collision probabilities by transforming the problem into a more manageable form.

A novel formulation was crafted to simplify a previously complex two-dimensional integral into a more manageable one-dimensional integral, featuring a straightforward exponential function in the integrand. Departing from traditional approaches, this method involves integration around the perimeter of the area, a strategy that effectively reduces the number of integrand evaluations and significantly enhances computational speed.

What sets this formulation apart is its departure from the conventional requirement to assume a spherical shape for space objects. This distinctive feature allows for the handling of space objects with highly irregular shapes, as defining the perimeter for integration becomes an intuitively straightforward task.

The assessment of collision risk for a specific space object follows a methodical process. Initially, cataloged objects are screened based on their orbital elements, eliminating those with no possibility of collision. The remaining objects undergo propagation, and their proximity to the object of interest is meticulously examined against a critical distance, guiding the decision on whether a probability calculation is warranted.

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Introduction

Patera's method is a robust mathematical framework designed to address the intricate task of assessing collision probabilities between two orbiting objects. In scenarios where objects move past each other at constant velocities, Patera's method transforms the complex three-dimensional probability density function (PDF) into a more manageable two-dimensional distribution. This comprehensive analysis outlines the key steps, equations, and numerical implementation of Patera's method.

Key Steps in Patera's Method:

1. Probability Density Transformation

The foundation of Patera's method lies in the transformation of a three-dimensional Gaussian probability density function $\rho(\mathbf{x})$, representing the uncertainty in relative positions between two objects, to a two-dimensional Gaussian $h(\mathbf{x})$. This transformation is accomplished by considering the encounter frame.(4)

$$\rho(\mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}\sigma_x\sigma_y\sigma_z} \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right]$$

$$h(\mathbf{x}) = \frac{1}{2^{\frac{3}{2}}\pi\sigma_x\sigma_y\sigma_z\sqrt{a}} \exp\left[-\alpha x^2 - \beta y^2\right]$$

2. Collision Probability Density

The collision probability density per unit area is encapsulated in the function $h(\mathbf{x})$, which is depicted by constant contour lines in Figure 1. This function characterizes the uncertainty in relative positions and serves as a crucial element in collision probability calculations.

$$prob = \frac{-1}{2\pi} \oint_{ellipse} \exp\left(-\alpha r^2\right) \, d\theta$$

3. Integration Over Hard-Body Area

Rather than directly integrating over the hard-body area, Patera simplifies the calculation by transforming it into an integration about a contour enclosing the hard-body area. This strategic simplification enhances computational efficiency.

4. Coordinate Rotation

A coordinate rotation is employed to eliminate cross terms in the probability density function, resulting in a further simplified expression. This rotation transforms the relative displacement vector $\mathbf{q}$ to $\mathbf{q}_r$, facilitating subsequent calculations.

Equations for Coordinate Rotation:

$$\mathbf{q}_r = T\mathbf{q}, \quad T = \cos(\phi)\sin(\phi) - \sin(\phi)\cos(\phi)$$

$$\cos(\phi) = \sqrt{\frac{1}{2}\left[1 - \frac{(f-e)}{\sqrt{g^2 + (f-e)^2}}\right]}, \quad \sin(\phi) = \pm\sqrt{\frac{1}{2}\left[1 + \frac{(f-e)}{\sqrt{g^2 + (f-e)^2}}\right]}$$

5. Scale Change

To achieve additional symmetry in the probability density, a scale change is implemented. This scale change involves multiplying components of $\mathbf{q}_r$ by suitable scale factors, contributing to a more streamlined analytical form.

Equation for Scale Change:

$$y = \sqrt{\frac{\alpha}{\beta}} y', \quad q_{rs}(2) = \sqrt{\frac{\beta}{\alpha}} q_r(2)$$

6. Contour Integration

The integration over the hard-body area is further simplified through contour integration. The collision probability is expressed as a closed-path integral around the hard-body ellipse, reducing the complexity of the computation.

Equation for Closed-Path Integral:

$$prob = \frac{-1}{4\sqrt{2}\pi\sigma_x\sigma_y\sigma_z\sqrt{a\beta\alpha}} \oint_{ellipse} \exp\left(-\alpha r^2\right) \, d\theta$$

7. Numerical Implementation

The numerical implementation involves efficient computational procedures, including a point-wise evaluation of the integrand around a unit circle and subsequent summation to obtain the closed-path integral. The computational efficiency of Patera's method is demonstrated through its reduced function calls and faster execution times compared to alternative models.

Chapter 3

Alfano's Method

Salvatore Alfano's method specializes in analyzing collision probabilities among spherical objects featuring linear relative motion during brief encounters. The methodology intricately integrates both positional uncertainties and the physical dimensions of the objects at their closest approach. The ensuing covariance ellipsoid and hardbody are strategically projected onto the encounter plane, positioned perpendicular to the relative velocity.

To streamline the computational efficiency of the algorithm, the initially complex 2-D integral is transformed into a more manageable 1-D integral, employing error functions. This reduction facilitates a more efficient numerical computation. Furthermore, to enhance computational efficiency even further, the method leverages Simpson's one-third rule for numerical integration. Simpson's rule divides the integration interval into subintervals, utilizing quadratic polynomials to approximate the function within each subinterval. This integration technique significantly contributes to the method's computational efficacy, making it a robust tool for collision probability analysis in space scenarios with spherical objects exhibiting linear relative motion(2).

3.1 Formulation

3.1.1 Assumptions and Definitions

Objects in space are modeled as spheres, and the point of closest approach is crucial for collision probability analysis. Linear relative motion is assumed, and uncertainties are represented by covariance ellipsoids.

3.1.2 Double Integral and Error Function

The central mathematical formulation involves a double integral that combines covariance ellipsoids and physical object dimensions. The double integral is expressed in terms of the error function (erf), a common mathematical function used in probability calculations.

P = $1\frac{}{2\pi\sigma_x\sigma_y{}_{-OBJ}^{OBJ}\int_{-\sqrt{OBJ^2-x^2}}^{\sqrt{OBJ^2-x^2}}\exp\left[-\frac{1}{2}\left(\frac{(x+x_m)^2}{\sigma_x^2}+\frac{(y+y_m)^2}{\sigma_y^2}\right)\right]dy\,dx}$

This double integral is then reduced to a more manageable single integral using the error function:

$$P=\frac{1}{\sqrt{8\pi}\sigma_x}\int_{-OBJ}^{OBJ}\left[erf\left[\frac{(y_m+\sqrt{OBJ^2-x^2})}{\sigma_y\sqrt{2}}\right]+erf\left[\frac{(-y_m+\sqrt{OBJ^2-x^2})}{\sigma_y\sqrt{2}}\right]\right]\exp\left[-\frac{(x+x_m)^2}{2\sigma_x^2}\right]dx$$

3.1.3 Numerical Integration with Simpson's One-Third Rule

To efficiently compute the above integral numerically, Simpson's one-third rule is employed. The integral is discretized into steps, and the error function is approximated using a numerical series. The number of terms in the series, denoted by m, is determined based on the parameters of the problem.

$$P = \frac{\mathrm{d}x}{3\sqrt{8\pi}\sigma_x}\left(m_{-0} + m_{even} + m_{odd}\right)$$

Calculation of m_{-0}

$$m_{-0} = 2\left[erf\left[\frac{(y_m + y(x))}{(\sigma_y\sqrt{2})}\right] - erf\left[\frac{(y_m - y(x))}{(\sigma_y\sqrt{2})}\right]\right]\left[\exp\left[-\frac{(x_m + x)^2}{2\sigma_x^2}\right] + \exp\left[-\frac{(x_m - x)^2}{2\sigma_x^2}\right]\right]$$

Calculation of m_{even}

$$\mathrm{m}_{even} = 2\sum_{i=1}^{m-1}\left[\left[erf\left[\frac{(y_m+y(x))}{(\sigma_y\sqrt{2})}\right] - erf\left[\frac{(y_m-y(x))}{(\sigma_y\sqrt{2})}\right]\right]\left[\exp\left[-\frac{(x_m+x)^2}{2\sigma_x^2}\right] + \exp\left[-\frac{(x_m-x)^2}{2\sigma_x^2}\right]\right]\right]$$
$$+ 2\left[erf\left[\frac{(y_m+OBJ)}{(\sigma_y\sqrt{2})}\right] - erf\left[\frac{(y_m-OBJ)}{(\sigma_y\sqrt{2})}\right]\right]\left[\exp\left[-\frac{(x_m)^2}{2\sigma_x^2}\right]\right]$$

Calculation of m_{odd}

$$m_{odd} = 4\sum_{i=1}^{m}\left[\left[erf\left[\frac{(y_m + y(x))}{(\sigma_y\sqrt{2})}\right] - erf\left[\frac{(y_m - y(x))}{(\sigma_y\sqrt{2})}\right]\right]\left[\exp\left[-\frac{(x_m + x)^2}{2\sigma_x^2}\right] + \exp\left[-\frac{(x_m - x)^2}{2\sigma_x^2}\right]\right]\right]$$

Calculation of x **and** $\mathrm{d}x$

$$x = \frac{OBJ(2i - n)}{n} \quad \mathrm{d}x = \frac{OBJ \cdot 2}{n}$$

Determining the Number of Terms m

$$m = int\left(\frac{5 \cdot OBJ}{\min\left(\sigma_x, \sigma_y, \sqrt{x_m^2 + y_m^2}\right)}\right)$$

3.2 Numerical Testing

The proposed method, with the detailed calculation of m_{-0}, m_{even}, and m_{odd}, is validated through extensive numerical testing. The results consistently demonstrate the accuracy and reliability of the approach across

Chapter 4

Implementation

Two Resident Space Objects (RSOs) were characterized based on their distinct location and velocity attributes. Subsequently, the time of closest approach (Tcpa) was computed, guiding the subsequent steps in the collision probability assessment process. Both RSOs were then propagated into the future by the Tcpa time, and their individual error covariance matrices were amalgamated to derive the combined error covariance. This process, tailored for short-term encounters, involves summing the error covariances of each RSO.

The resultant velocity and location of the propagated objects were determined. To facilitate collision probability calculations using two distinct methods, the position vectors of each RSO were projected onto the 2-D encounter plane.

Prior to projecting the position and miss vectors onto the encounter plane, a transformation matrix U was computed. This matrix played a pivotal role in the subsequent steps of collision probability assessment.

The collision probability was initially computed using the Monte Carlo Approach, establishing a benchmark against which the results obtained from the other two algorithms could be compared. This rigorous approach ensured a comprehensive and reliable evaluation of the effectiveness and accuracy of the alternative algorithms.

4.1 Time of Closest Approach(Tcpa)

The Time of Closest Approach (TCPA) is a critical parameter in collision risk assessment, representing the instance when two space objects are closest during their orbital paths. Calculated by

$$t_{cpa} = -\frac{(\tilde{\rho}_o \cdot v_r)}{(v_r \cdot v_r)}$$

, where $\tilde{\rho}_o = \overline{\rho}_o + e_d - e_s + v_r t$, $\overline{\rho}_o$ is the vector from the origin to the closest approach, e_d and e_s are position vectors, v_r is the relative velocity vector, and t is time. Numerical optimization, like scipy's minimize function, is often used to find the minimum distance and determine the precise TCPA. This parameter is pivotal in collision prediction, offering insights into the temporal proximity of potential orbital collisions.(1)

4.2 Transformation to Encounter Plane

In the context of conjunction analysis, the transformation matrix C is derived to project the uncertainty from the three-dimensional space into a two-dimensional conjunction plane. This plane is defined by the relative velocity vector $v_r = v_d - v_s$ and its orthogonal vectors.(**?**)

Let's define the unit vectors in the new coordinate system:

$$\hat{\imath} = \frac{v_r}{\|v_r\|}, \quad \hat{\jmath} = \frac{v_d \times v_s}{\|v_d \times v_s\|}, \quad \hat{k} = \hat{\imath} \times \hat{\jmath}$$

Here, $\times$ represents the cross product, and $\|\cdot\|$ denotes the Euclidean norm.

The matrix C can then be formed by arranging these unit vectors as its columns:

$$C = [\hat{\imath}\hat{\jmath}\hat{k}]$$

The resulting C matrix, known as the transformation matrix, provides a basis for the conjunction plane. It allows projecting the uncertainty along these new unit vectors, simplifying the analysis of conjunction scenarios.

This process is crucial for reducing the three-dimensional problem to a two-dimensional one, making the analysis more manageable while retaining essential information for collision probability calculations.

4.3 Monte Carlo

Random samples were systematically generated for the propagated objects, adhering to their respective covariance matrices. The collision probability was determined by assessing the number of instances where the distance between each sample fell below the radius of the hard body circle. The ratio of the total number of collisions to the overall number of generated samples provided the collision probability using the Monte Carlo method. Despite its computational expense, this approach stands out as the most reliable method for accurately estimating collision probabilities(3).

$$P_c = P(DCA < r_c) = \frac{N_c}{N_t}$$

4.4 Patera's Method

Utilizing the values of σ_x, σ_y, and σ_z, along with the transformation matrix U, we compute the collision parameters a, c, d, e, f, α, β, T, and qr. Here, qr represents the scaled location in the encounter plane, and T corresponds to the rotation matrix.(4)

4.4.1 Coordinate Transformation:

- Perform a coordinate rotation to eliminate cross terms in the probability density function.

$$X_e = TX'_m + \begin{bmatrix} q_r(1) \\ q_{rs}(2) \end{bmatrix}$$

- Use a scale change to make the probability density function symmetric.

$$X_m = MX', \quad M = \begin{pmatrix} s & 0 \\ 0 & s\sqrt{\beta/\alpha} \end{pmatrix}$$

4.4.2 Contour Integration:

- Define a unit circle point as $X = (1, 0)$. - Obtain a second point by multiplying X by an infinitesimal rotation matrix R.

$$X' = RX, \quad R = \begin{bmatrix} \cos(\varepsilon) & -\sin(\varepsilon) \\ \sin(\varepsilon) & \cos(\varepsilon) \end{bmatrix}$$

- Calculate the angle between the two vectors using the cross product:

$$\mathrm{d}\theta = \sin^{-1}\left(\frac{X_e \times X'_e}{|X_e|\,|X'_e|}\right)$$

- Evaluate the integrand at the midpoint of the vectors:

$$int = \exp\left\{-\alpha\left[\left(X_e + X'_e\right)/2\right]^2\right\}$$

- Sum the values of the integrand multiplied by $\mathrm{d}\theta$ for each pair of points around the ellipse:

$$sum = sum + (int)(d\theta)$$

4.4.3 Calculation of Probability:

- Determine the collision probability when the origin is excluded from the hard body:

$$prob = -\frac{sum}{2\pi}$$

- If the origin is included in the hard-body ellipse:

$$prob = 1 - \frac{sum}{2\pi}$$

This numerical implementation simplifies the collision probability calculation by reducing it to a contour integral, offering computational efficiency and general applicability, even for non-spherical objects. The closed-path integral is efficiently approximated by summing values of the integrand around the ellipse.

4.5 Alfano's Method

The projected miss vectors and relative velocity vectors are derived from the propagated position vectors and velocity vectors. Subsequently, these computed values, along with the σ_x and σ_y parameters, are employed in the collision probability calculation.

The step-by-step implementation is detailed below.

4.5.1 Calculate the Optimal Number of Steps (m):

- Determine the optimal number of steps (m) based on the formula:

$$m = int\left(\frac{5 \cdot OBJ}{\min(\sigma_x, \sigma_y, \sqrt{xm^2 + ym^2})}\right)$$

- Ensure m lies within the range of 10 to 50.

4.5.2 Calculate the Step Size ($\mathrm{d}x$):

- Compute the step size ($\mathrm{d}x$) using:

$$\mathrm{d}x = \frac{2 \cdot OBJ}{m}$$

4.5.3 Calculate m_{odd} and m_{even}:

- Utilize the following expressions for m_{odd} and m_{even}:

$$m_{odd} = 4\sum_{i=1}^{m}\left[erf\left(\frac{ym + y(x)}{\sigma_y\sqrt{2}}\right) - erf\left(\frac{ym - y(x)}{\sigma_y\sqrt{2}}\right)\right]\left[\exp\left(\frac{-(xm + x)^2}{2\sigma_x^2}\right) + \exp\left(\frac{-(xm - x)^2}{2\sigma_x^2}\right)\right]$$

where $x = (2i - 1) \cdot \mathrm{d}x - OBJ$

$$m_{even} = 2\sum_{i=1}^{m-1}\left[erf\left(\frac{ym + y(x)}{\sigma_y\sqrt{2}}\right) - erf\left(\frac{ym - y(x)}{\sigma_y\sqrt{2}}\right)\right]\left[\exp\left(\frac{-(xm + x)^2}{2\sigma_x^2}\right) + \exp\left(\frac{-(xm - x)^2}{2\sigma_x^2}\right)\right] \quad +2\left[erf\left(\right.\right.$$

where $x = 2i \cdot \mathrm{d}x - OBJ$

4.5.4 Calculate m_0:

- Calculate m_0 using the expression:

$$m_0 = 2\left[erf\left(\frac{ym + y(x)}{\sigma_y\sqrt{2}}\right) - erf\left(\frac{ym - y(x)}{\sigma_y\sqrt{2}}\right)\right]\left[\exp\left(\frac{-(xm + x)^2}{2\sigma_x^2}\right) + \exp\left(\frac{-(xm - x)^2}{2\sigma_x^2}\right)\right]$$

where $x = 0.015 \cdot \mathrm{d}x - OBJ$

4.5.5 Compute Collision Probability (P):

- Finally, the collision probability (P) is obtained as:

$$P = \frac{\mathrm{d}x}{3\sqrt{8\pi}\sigma_x}\left(m_0 + m_{even} + m_{odd}\right)$$

Chapter 5

Results

The simulation of the two objects was conducted using the Astropy library, incorporating their position and velocity vectors. Specifically tailored velocity vectors were chosen to emulate conditions conducive to a short-term encounter. The position vector of the secondary object was defined in relation to the position vector of the primary object, with the following expressions:

$$r_{primary} = [x, y, z]$$

$$r_{secondary} = [x + \delta R, y + \delta R, z + \delta R]$$

Various scenarios were generated by systematically adjusting the value of δR across different cases.

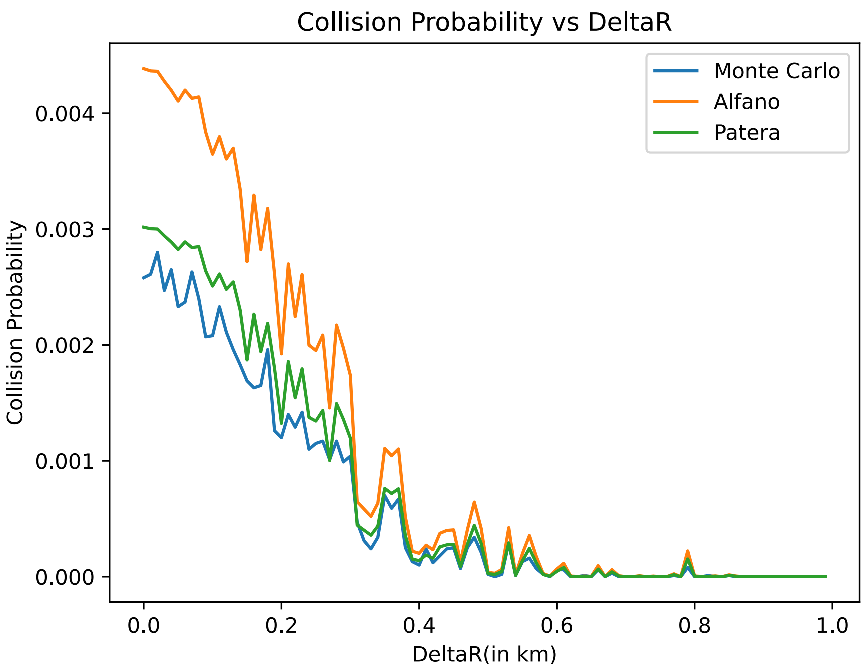

Figure 5.1: Collision Probability Analysis for Larger Range of deltaR

Both algorithms demonstrate accurate calculations of collision probability. However, it was noted that Alfano's method, while exhibiting higher computational efficiency, tends to overestimate the collision probability for close approaches.

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