

Q1.

Jobs

	1	2	3	4	5
I	8	4	20	7	1
II	0	9	5	5	4
III	3	8	9	2	6
IV	4	3	1	0	3
V	9	5	8	9	5

Mens

Conditions

1. NO. of Rows = NO. of columns
2. Matrix is minimization matrix

STEP 1: ROW MINIMA

	J ₁	J ₂	J ₃	J ₄	J ₅
M ₁	7	3	1	6	0
M ₂	0	9	5	5	4
M ₃	1	6	7	0	4
M ₄	4	3	1	0	3
M ₅	4	0	3	4	0

STEP 2: COLUMN MINIMA

	J ₁	J ₂	J ₃	J ₄	J ₅
M ₁	7	3	0	6	0
M ₂	0	9	4	5	4
M ₃	1	6	6	0	4
M ₄	4	3	0	0	3
M ₅	4	0	2	4	0

STEP 3: OPTIMALITY TEST

	J ₁	J ₂	J ₃	J ₄	J ₅
M ₁	7	3	0	6	0 ①
M ₂	0	9	4	5	4
M ₃	1	6	6	0	4
M ₄	4	3	0	0	3 ②
M ₅	4	0	2	4	0 ③
	④		⑤		

STEP 4: ASSIGNMENT

	J ₁	J ₂	J ₃	J ₄	J ₅
M ₁	7	3	0	6	0
M ₂	0	9	4	5	4
M ₃	1	6	6	0	4
M ₄	4	3	0	0	3
M ₅	4	0	2	4	0

Optimal Solution

No. of vertical and
Horizontal line = $\frac{\text{Order of matrix}}{2}$

$$5 = 5$$

STEP 5: CALCULATION OF MINIMUM TIME

MEN	JOB	TIME
M ₁	J ₅	1
M ₂	J ₁	0
M ₃	J ₄	2
M ₄	J ₃	1
M ₅	J ₂	5

TOTAL = 9 hrs.

Q2.

Jobs

	1	2	3	4	5	6
I	5	0	6	8	7	4
II	5	2	3	0	6	7
III	3	4	4	3	5	2
IV	3	9	7	2	7	6
V	9	8	7	8	4	5
VI	1	8	7	4	2	3

Mens

Conditions

1. No. of rows and No. of columns
2. Matrix is minimize matrix

STEP 1: ROW MINIMA

	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆
M ₁	5	0	6	8	7	4
M ₂	5	2	3	0	6	7
M ₃	1	2	2	1	3	0
M ₄	1	7	5	0	5	4
M ₅	5	4	3	4	0	1
M ₆	0	7	6	3	1	2

STEP 2: COLUMN MINIMA

	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆
M ₁	5	0	4	8	7	4
M ₂	5	2	1	0	6	7
M ₃	1	2	0	1	3	0
M ₄	1	7	3	0	5	4
M ₅	5	4	1	4	0	1
M ₆	0	7	4	3	1	2

STEP 3: OPTIMALITY TEST

	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆
M ₁	5	0	4	8	7	4 ⁽³⁾
M ₂	5	2	1	0	6	7
M ₃	1	2	0	1	3	0 ⁽¹⁾
M ₄	1	7	3	0	5	4
M ₅	5	4	1	4	0	1 ⁽⁴⁾
M ₆	0	7	4	3	1	2 ⁽⁵⁾

(2)

STEP 4: OPTIMALITY TEST

	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆
M ₁	5	0	4	9	7	4
M ₂	4	1	0	0	6	6 ⁽¹⁾
M ₃	1	2	0	2	3	0 ⁽²⁾
M ₄	0	6	2	0	4	3 ⁽³⁾
M ₅	5	4	1	5	0	1
M ₆	0	7	4	4	1	2

(4) (5)

(6)

Optimal solution

STEP 5: ASSIGNMENT

	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆
M ₁	5	0	4	9	7	4
M ₂	4	1	0	0	6	6
M ₃	1	2	0	2	3	0
M ₄	0	6	2	0	4	3
M ₅	5	4	1	5	0	1
M ₆	0	7	4	4	1	2

Non-optimal solution

STEP 6: CALCULATION OF MINIMUM TIME

Mens	Job	Time
M ₁	J ₂	0
M ₂	J ₃	3
M ₃	J ₆	2
M ₄	J ₄	2
M ₅	J ₅	4
M ₆	J ₁	1

TOTAL 12 Hrs

Row & Column = Order of matrix

Q3

	D1	D2	D3	D4	Av.
O1	23	27	16	18	30
O2	12	17	20	51	40
O3	22	28	12	32	53
Req.	22	35	25	41	-

Conditions

1. It is a minimum matrix
2. Total Availability = Total Requirement
 $123 = 123$

PART I: VOGEL'S METHOD

	D1	D2	D3	D4	Av.	P1	P2	P3	P4	P5
O1	23	27	16	18	30	2	-	-	-	-
O2	12	17	20	51	40	5	5	5	8	12
O3	22	28	12	32	53	10	10	10	10	22
Req.	22	35	25	41						

P1 10 10 4 14↑

P2 10 11 8 19↑

P3 10 11↑ 8 -

P4 10 - 8 -

P5 10 - - -

Feasibility Test

$$M+N-1 = \text{No. of allocation}$$

$$3+4-1 = 6$$

Above solution is feasible.

PART II: OPTIMATIVITY TEST (MODI APPROACH)

Table 1: $U_i + V_j$ Table

	D1	D2	D3	D4	U_i
O1	(8)	(13)	(-2)	18	14
O2	12	17	(2)	(22)	-10
O3	22	(27)	12	32	0
V_j	22	27	12	32	

Table 2: Δ_{ij} Table

	D1	D2	D3	D4
O1	15	14	8	
O2			18	29
O3		1		

optimal solution

CALCULATION OF MIN. TRAN. COST

O	D	Unit	Cost	Total
O1	D4	30	18	540
O2	D1	5	12	60
O2	D2	35	17	595
O3	D1	17	22	374
O3	D3	25	12	300
O3	D4	11	32	352
TOTAL				2221

Q5.

	W ₁	W ₂	W ₃	Requirement	Conditions
A	5	7	8	70	1 Requirement = Availability
B	4	4	6	30	2 Matrix should be minimize
C	6	7	7	50	

Availability 65 42 43

Part I: Vogel's Method

	W ₁	W ₂	W ₃	Req.	P ₁	P ₂	P ₃	P ₄	P ₅
A	5 ⁶⁵	7 ⁵	8	70	2	2	1		
B	4	4 ³⁰	6	30	0	-	-		
C	6	7	7 ⁴³	50	1	1	0		

Feasibility Test

$$m+n-1 = \text{No. of Allocation}$$

$$5 = 5$$

Ava. 65 42 43

P₁ 1 3↑ 1

P₂ 1 0 1

P₃ - 0 1↑

P₄

Above solution is feasible

Part II Optimality Test (MODI APPROACH)

Table 1: $U_i + V_j$ Table

	W ₁	W ₂	W ₃	U_i
A	5	7	(7)	7
B	(2)	4	(4)	4
C	(5)	7	7	7

V_j^0 -2 0 0

CALCULATION OF MIN. TRANSPORT COST

CUSTOMER	WAREHOUSE	COST	q_j	TOTAL
A	W ₁	5	65	325
A	W ₂	7	5	35
B	W ₂	4	30	120
C	W ₂	7	7	49
C	W ₃	7	43	301
TOTAL				830

Table 2: Δ_{ij}^0 Table

	W ₁	W ₂	W ₃
A			1
B	2		2
C	1		

Optimal solution

Q6.

	A	B	Max.	
M ₁	2	6	24	$Z = 5x + 2y$
M ₂	6	2	24	STC
Profit	5	2		$2x + 6y \leq 24$
No. of unit	x	y		$6x + 2y \leq 24$

Converting Inequalities into equalities

$$\text{Max } Z = 5x + 2y + 0s_1 + 0s_2$$

$$2x + 6y + s_1 = 24$$

$$6x + 2y + s_2 = 24$$

For initial solution

$$x = y = 0 \quad s_1 = 24 \quad s_2 = 24$$

I Simplex Method Table

			C _j	5	2	0	0	Min.
C _B	V _B	S _V		x	y	s_1	s_2	Ratio
0	s_1	24		2	6	1	0	12
0	s_2	24		6	2	0	1	4 \leftarrow Key
	Z _j			0	0	0	0	
	N _E R			5 \uparrow Key	2	0	0	

II Simplex Table

			C _j	5	2	0	0	Min.
C _B	V _B	S _V		x	y	s_1	s_2	Ratio
0	s_1	16		0	$1/3$ Key	1	$-1/3$	3 \leftarrow Key
5	x	4		1	$1/3$	0	$1/6$	12
	Z _j			5	$5/3$	0	$-5/6$	
	N _E R			0	$1/3 \uparrow$ Key	0	$-5/6$	

III Simplex Table							
			C _j	5	2	0	0
C _B	V _B	S _V	x	y	S ₁	S ₂	
2	y	3	0	1	3/16	-1/16	
5	x	3	1	0	-1/16	7/48	
Z _j			5	2	1/6	29/48	
N _{ER}			0	0	-1/6	-29/48	

Since all the value of N_{ER} are zero or negative
so above solution is optimal where $x=3$ $y=3$

$$\begin{aligned}
 \text{Max } Z &= 5x + 2y \\
 &= 5 \times 3 + 2 \times 3 \\
 &= 21
 \end{aligned}$$

Q7 Purchase Price = Rs 500

PVF @ 5%

Resale value = Nil

Running cost each year increases by 100

YEAR	RUNNING COST	PVF @ 5%	DISCOUNTED RUNNING COST	CUMULATIVE DISC. RUNNING COST	TOTAL COST	CUMULATIVE PVF	WEIGHTED AVERAGE
1	0	1	0	0	500	1	500
2	100	0.952	95.2	95.2	595.2	1.952	304.91
3	200	0.907	181.4	276.6	776.6	2.859	951.8 271.6
4	300	0.863	258.9	535.6	1036.6	3.722	278.5
5	400	0.822	328.8	864.4	1364.4	4.544	300.2
6	500	0.784	391.5	1255.9	1755.9	5.328	329.5
7	600	0.746	447.6	1703.5	2203.5	6.074	362.7

Q8

Month	Survival	Failure	Probability
0	100	0	0
1	97	3	0.03
2	90	10	0.10
3	70	30	0.30
4	30	70	0.70
5	15	85	0.85
6	0	100	1.00

STEP 1: PROBABILITY OF FAILURE OF BULB IN i^{th} week

$$P_0 = 0$$

$$P_1 = 0 - 0.03 = 0.03$$

$$P_2 = 0.10 - 0.03 = 0.07$$

$$P_3 = 0.30 - 0.10 = 0.20$$

$$P_4 = 0.70 - 0.30 = 0.40$$

$$P_5 = 0.85 - 0.70 = 0.15$$

$$P_6 = 1 - 0.85 = 0.15$$

STEP 2: NO. OF BULB REPLACED IN i^{th} week

$$N_0 = 10000$$

$$N_1 = (10000 \times 0.03) = 300$$

$$N_2 = (10000 \times 0.07) + (300 \times 0.03) = 709$$

$$N_3 = (10000 \times 0.20) + (300 \times 0.07) + (709 \times 0.03) = 2042$$

$$N_4 = (10000 \times 0.40) + (300 \times 0.20) + (709 \times 0.07) + (2042 \times 0.03) = 4171$$

$$N_5 = (10000 \times 0.15) + (300 \times 0.40) + (709 \times 0.20) + (2042 \times 0.07) + (4171 \times 0.03) = 2030$$

$$N_6 = (10000 \times 0.15) + (300 \times 0.15) + (709 \times 0.40) + (2042 \times 0.20) + (4171 \times 0.07) + (2030 \times 0.03) = 2590$$

STEP 3: EXPECTED LIFE

$$\Rightarrow 1 \times 0.03 + 2 \times 0.07 + 3 \times 0.20 + 4 \times 0.40 + 5 \times 0.15 + 6 \times 0.15 \\ = 4.02 \text{ week}$$

STEP 4: EXPECTED NO. OF RESISTOR REPLACED

$$= \frac{10000}{4.02} = 248.8$$

STEP 5: AVERAGE COST OF INDIVIDUAL REPLACEMENT

$$= 248.8 \times 1 = 248.8$$

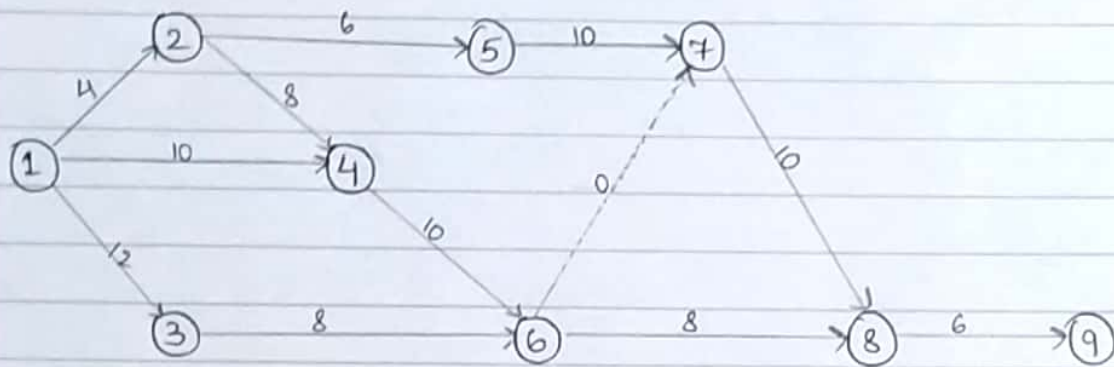
STEP 6: GROUP REPLACEMENT PERIOD

Month	No. of Replaced	Cumulative	Individual	G.P Cost	Total	Average
1	300	300	300	3500	3800	3800
2	709	1009	1009	3500	4509	2254.5
3	2042	3051	3051	3500	6551	2183.6
4	4171	7222	7222	3500	10722	2680.5
5	2030	9252	9252	3500	12752	2550.4
6	2590	11842	11842	3500	15342	2557

Resistor should be replaced after 3rd week

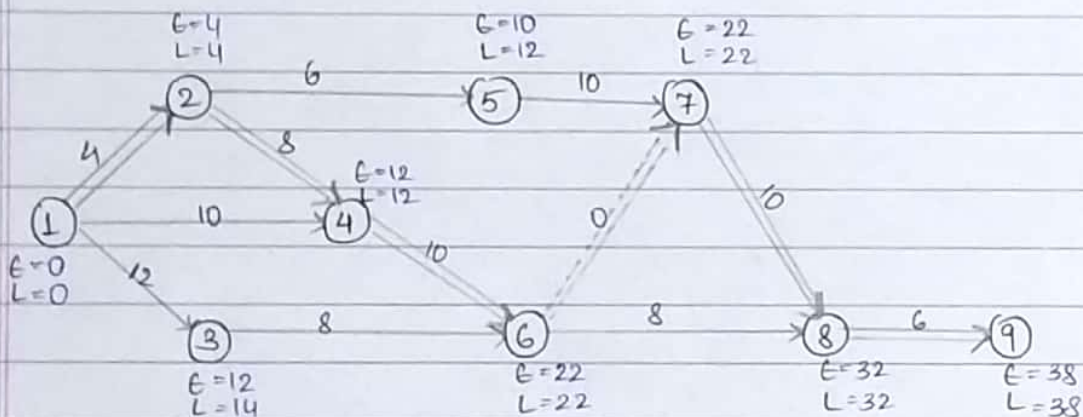
Q9

ACTIVITY	DURATION	ACTIVITY	DURATION
1-2	4	4-6	10
1-3	12	5-7	10
1-4	10	6-7	0
2-4	8	6-8	8
2-5	6	7-8	10
3-6	8	8-9	6



Critical path = 1-2-4-6-7-8-9

Project length = 38 days



CALCULATION

ACTIVITY	DURATION	ES	EF	LS	LF	TF	FF	IF
1-2	4	0	4	0	4	0	0	0
1-3	12	0	12	2	14	2	0	0
1-4	10	0	10	2 2	12	2	2	2
2-4	8	4	12	4 4	12	0	0	0
2-5	6	4	10 10	12 6	12 12	2	0	0
3-6	8	12	20	12 4	22	2	2	0
4-6	10	12	22	12	22	0	0	0
5-7	10	10	20	12 2	22	2	2	0
6-7	0	22	22	22 4	22 2	0	0	0
6-8	8	22	30	22 4	32	2	2	2
7-8	10	22	32	22	32	0	0	0
8-9	6	32	38	32	38	0	0	0

ES = Earliest Start time

EF = Earliest Finish time

LS = Latest start time

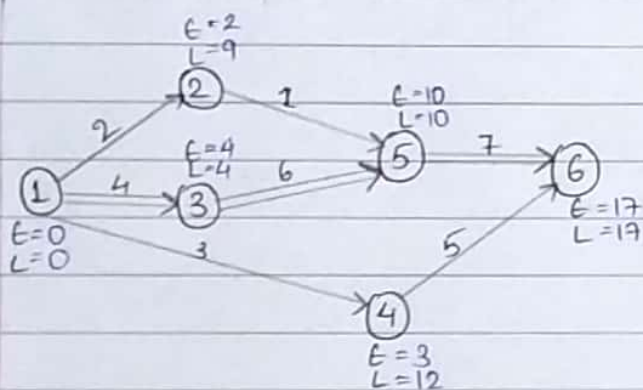
LF = Latest Finish time

TF = Total Float

FF = Free Float

IF = Independent Float

Q10.	ACTIVITY	ESTIMATED DURATION IN WEEK			Expected Time
		OPTIMISTIC	MOST LIKELY	PESSIMISTIC	
	1-2	1	1	7	2
	1-3	1	4	7	4
	1-4	2	2	8	3
	2-5	1	1	1	1
	3-5	2	5	14	6
	4-6	2	5	8	5
	5-6	3	6	15	7



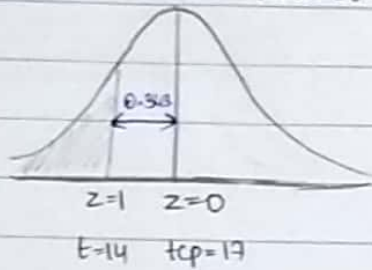
Critical path
= 1-3-5-6

Project length
= 17 weeks.

Activity	Variance	Standard Deviation	Standard Deviation $= \sqrt{V}$ $= \sqrt{9} = 3$
1-2	1		
1-3	1	1	
1-4	1		
2-5	0		
3-5	4	4	
4-6	1		
5-6	4	$\frac{4}{9}$	

(i) At least 3 week earlier than expected

$$t_{cp} = 17 \quad t = 14 \quad \sigma_{cp} = 3$$

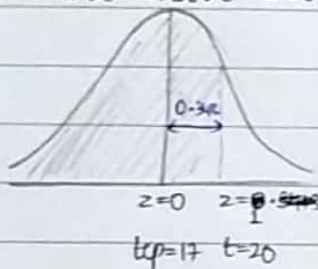


$$|z| = \frac{t - t_{cp}}{\sigma_{cp}} = \frac{3}{3} = 1$$

$$\text{Probability} = 0.50 - 0.2420 = 0.2580$$

(ii) No more than 3 week later than expected

$$t_{cp} = 17 \quad t = 20 \quad \sigma_{cp} = 3$$

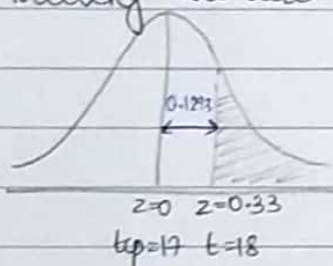


$$|z| = \frac{20 - 17}{3} = 1$$

$$\text{Probability} = 0.50 + 0.2420 = 0.7420$$

(iii) If the project's due date is 18 week, what is probability of not meeting the due date

$$t_{cp} = 17 \quad t = 18 \quad \sigma_{cp} = 3$$



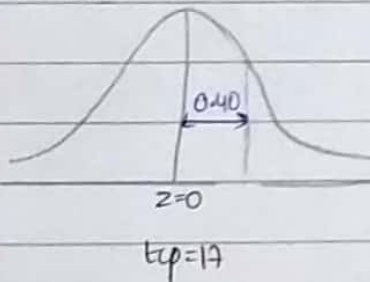
$$|z| = \frac{18 - 17}{3} = 0.33$$

Required Probability

$$= 0.50 - 0.1293 = 0.3707$$

(iv) What due date has about 90% chance of being met.

$$t_{cp} = 17 \quad \sigma_{cp} = 3$$



$$0.40 \text{ nearest to } 0.3997 \quad z = 1.28$$

$$1.28 = \frac{x - 17}{3} \quad x = 20.84$$

within 20.84 week the project complete 90%.