Eng5022: Control M – Lab Report

Name: **Sarthak Ahuja**
Student Number: **2707209A**

I. Objective

To design and evaluate a controller which would regulate the position of the output shaft of a servo motor. Initially, a linear model would be used to assess the performance; followed by a realistic model of the system (real system). A comparison among the results of the performance using the linear simulation and the real system is done in conclusion.

II. Tools and Methods Used

For the initial lab MATLAB R2021b and Simulink software was used to model the linear system and draw observations. For the lab including realistic model of the system, a Quanser Qube Servo 2 system – Rotary servo motor was used, along with Quanser QUARC toolbox. MATLAB and Simulink were used as usual, apart from now also allowing to interface with the connected hardware. Git Version Control was used to maintain the different updates over the MATLAB scripts and command logs.

III. Procedure

3.1 Simulation - Open Loop system and continuous time controller

3.1.1 Initially, the electrical and mechanical components of the servo motor system are modelled, helping to derive an analytical model of the open loop system.

3.1.2 The Laplace transform of equation 1 is performed, followed by transfer function derivation. This is simultaneously implemented in MATLAB and converted into its monic form. The poles and zeros are calculated by equating the denominator and numerator to zero. *[Observation 4.1.1]*

3.1.3 An open-loop system with the simulated plant is constructed in Simulink with an LTI system and scope block. Two different input frequencies of 1 rad/s and 5 rad/s and multiple observations are noted down. *[Observation 4.1.2]*

3.1.4 A continuous time controller is designed consisting of a Proportional Derivate structure. The controller frequency response is formulated and made realizable, followed by the resultant compensator transfer function calculation. The same is implemented in MATLAB too using relevant commands.

3.1.5 The bode plot of plant, controller and loop gain is plotted in MATLAB, while their phase, magnitude is analyzed. The effect of the controller on loop gain is also noted. Using minimum stability margins, the cross-over frequencies, phase margins and gain margins of plant and loop are observed. *[Observation 4.1.3]*

3.1.6 Using MATLAB, the root locus of the feedback loop is calculated for two scenarios, while also noting down the locations of the poles at these feedback gains. *[Observation 4.1.4]*

3.1.7 In Simulink, a similar model as last is constructed, though consisting of a closed-loop feedback controller C now. The values of the output from plant and the reference are compared now. *[Observation 4.1.5]*

3.1.8 Nyquist plot for the two gain values is generated and analyzed. The phase margin, delay margin and frequency are noted down for both gains. Bode plot of same gain values is generated and phase margin is determined, while also compared from Nyquist plot. *[Observation 4.1.6]*

3.1.9 In Simulink, add a transport delay to previous model, and use the second gain value. Now, the time delay value of the Transport Delay block is changed to a value slightly smaller and larger than the delay margin previously calculated. The plant output and reference plots are generated, while system behavior is noted. *[Observation 4.1.7]*

3.2 Simulation – Digital Control

3.2.1 With plant function P, the discrete equivalent is calculated with the help of transformation tables. The result is verified using an c2d MATLAB command. Now continuous controller (in previous section) is converted to a digital one using emulation – pole zero matching. The updated gain and transfer function are noted down, both being updated in MATLAB too. [*Observation 4.2]*

3.2.2 The bode plot of continuous and digital compensators is generated. The property at multiple frequencies is noted, and phase at 200 rad/s is calculated for both compensators. *[Observation 4.2]*

3.2.3 Using Simulink, a third model is constructed with digital controller and zero order hold plant. The time histories of plant output are compared for The continuous and discrete plant. Performances in terms of overshooting and settle time are noted. *[Observation 4.2]*

3.2.4 For the discrete equivalent of the plant, compensator, and its gain Kd is re-computed for a sample time of 0.04s. Output of plant is observed, and stability is observed. *[Observation 4.2]*

3.3 Real Time Evaluation

3.3.1 Now, the Quanser QUARC toolbox is used along the hardware and a template hardware Simulink model is introduced. The real time simulation is run with different frequencies of the sine wave, noting down their result. *[Observation 4.3]*

3.3.2 This time, a feedback loop is implemented along with interfacing with the real servo system. With a nil transport delay, the closed loop system is compared with the previous output.

3.3.3 With a lesser gain value the experiment is repeated along with smaller delay margins. Behavior of real servo systems in all scenarios is noted down. *[Observation 4.3]*

3.3.4 The servo disk is removed, and behavior of closed loop system is analyzed. *[Observation 4.3]*

3.3.5 The digital controller of the same is tested with an addition of digital time compensator to the model in Simulink. Time histories of continuous and digital controller are compared in terms of overshooting and settle time. *[Observation 4.3]*

3.3.6 The time period is increased and the effect on stability of the real plant is discussed. *[Observation 4.3]*

IV. OBSERVATIONS

4.1.1 Here, putting the values provided, in monic form – we get:

$$G(s) = \frac{k_t/(J_{eq}*R_m)}{s^2+(k_t*k_m)/(J_{eq}*R_m)} = \frac{238.6}{s^2+10.02s}$$

Poles: 0, -10.02 *Zeros: Nil*

4.1.2 For input signal (1 rad/s), the output signal follows similar sinusoidal shape, but the amplitude of output is much larger than input reference.
For input signal (5 rad/s), there are a lot more oscillations, but the amplitude has reduced approximately by 5 times (as much as frequency was increased). The phase shift has increased a lot too, with signals close to being out of phase now. *[Appendix Figure 1,2]*

4.1.3 Analyzing the bode plot, The plant(P) and loop(L) gain, both start with infinite gains, but drift apart approaching the 0 dB line, due to different poles. The phase of both starts at -90° going close to -180°, but not crossing it. For the compensator(C), the gain starts on the 0dB line but shows a positive slope increase by 20dB in magnitude, due to which we observe a positive increase in the phase from 0° to 45° because of the presence of a zero. Though the pole present caused the magnitude to level at 20dB and the phase to come down to 0°. Hence, C affected P and L, by increasing phase margin and crossover frequency, thereby increased the stability margins of the plant, making the system more robust. *[Appendix Figure 3]*

The gain and phase cross-over frequencies (Wcg, Wcp) phase margins (Pm) and gain margins (Gm) for P and L are shown in table 1below:

	Gm	Pm	Wcg	Wcp
P(s)	Infinity	35.7645	Infinity	13.9156
L(s)	Infinity	76.9667	Infinity	23.2368

Table 1 – Comparing P(s) and L(s)

4.1.4 The feedback gains for which
A. the damping is 1.0 and the frequency of the slowest pole is 5 rad/s: **0.2**
B. the overshoot is 5%: **2.2**
The location of poles in both conditions is shown in table 2 below: *[Appendix Figure 4]*

	Gain K	Pole 1	Pole 2	Pole 3
A	0.2	-5	-10	-95
B	2.2	-50-52.4i	-50+52.4i	-10

Table 2 – Root locus gains and poles

4.1.5 Both the gain plots closely follow the reference, though there is slight damping for Ka, but Kb is slight underdamped. Kb also has an overshoot compared to Ka, due to complex poles and hence yields a fast response and lower damping in comparison. *[Appendix Figure 5,6]*

4.1.6 The phase margin (Pm), delay margin (Dm) is shown below in the table:

	Pm	Frequency	Dm	Stable
A	87.3	4.76	0.32	Yes
B	64.7	47.4	0.0238	Yes

Table 3 – Margins for GainA, GainB

Both the Nyquist and Bode plots gave similar results as expected.
Here L1 = Ka*L, L2= Kb*L *[Appendix Figure 7,8,9]*

4.1.7 Performance at Transport delay of
0.0238s – The signal is stable
0.015s – The signal decays faster as less than delay margin
0.025s–The signal oscillates exponentially & is unstable as more than delay margin
[Appendix Figure 10, 11a,11b]

4.2 Calculating P_zoh, Dd and Kd analytically, we get using pole zero matching:

$$P\_zoh(z) = \frac{0.1154z+0.01116}{z^2-1.905z+0.9046} \qquad Kd = 14.6704$$

$$Dd(z) = \frac{z-0.9048}{z-0.3679}$$

Observing from Bode plot for Kb*D, Kd*Dd *[Appendix Figure 12]*, At low frequencies both remain same, but as the value increases the analog part has higher values. Similarly, the analog tends to 0° phase later than digital.

At 200 rad/s the phase for:
Digital part = 14.5° Analog part= 23.6°

Implementing this in Simulink Model 4, we can observe by comparing it by analogue implementation in Simulink Model 2. Here, Overshooting is slightly higher in digital case in comparison to continuous system and the settle time is same both cases, continuous and digital. *[Appendix Figure 13]*

Now using T = 0.04s,

$$P_{zoh}(z) = \frac{0.1678z+0.1468}{z^2-1.67z+0.6697}$$

$$K_d = 6.5509 \quad \text{and } Dd = \frac{z-0.6703}{z-0.01832}$$

Running Simulink model 4, we can see that the output is marginally stable now. *[Appendix Figure 14]*

4.3 In both frequencies 1 rad/s and 5 rad/s, the simulation and real system have similar results though, the amplitude of real system has a minimal decay with time. This may be due to unmodelled friction, or time latency for the real system to process signals from the computer.
[Appendix 2, Figure 15, 16]

From the continuous feedback control, it can be observed that, there are changes in real system and simulated system with gains 0.2, 2.2 due to a small real system steady state offset and smaller overshoot. The control signal is not zero and exceeds the plant capacity. *[Appendix 2, Figure 17, 18]*

When the delay margin of 0.001 and 0.02 is added for gain 0.2, the output y shifts accordingly. *[Appendix 2, Figure 19, 20]*

When the load disk is removed, the system with gain 0.2, 2.23 is observed. This causes changes in closed loop system as expected due changing of load moment of inertia. *[Appendix 2, Figure 21, 22]*

Constructing the real time digital implementation, in Simulink Model 7, for T = 0.01s, the system Output oscillates and then fades down in the end. It is still marginally stable. Overshooting is higher in the case of digital, though settle time is similar in both cases. Though for T as 0.04s, the system output oscillates too much to become close to instability. *[Appendix 2, Figure 23, 24]*

V. CONCLUSION AND IMPROVEMENTS

This control lab allowed to understand and appreciate the difference between system simulation and the behaviour of the real system. Both cases of continuous feedback control and digital control were considered. The results of the real system can be made more accurate by considering unmodelled friction, loss of improper current within the electric circuit, and the time latency for the real system to process the input signals from the computer, if any. It can be said that though the stability of system might be affected by adding feedback, the simulation, mirrored the behaviour of the real system in major cases, with very less deviations

VI. APPENDIX

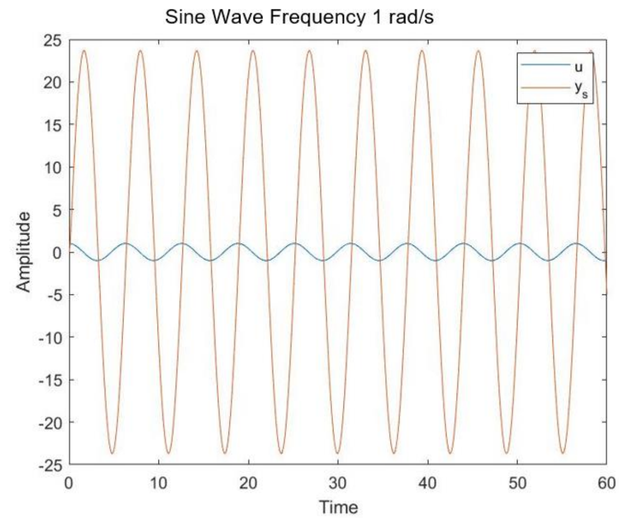

Figure 1 – Sine wave at frequency of 1 rad/s

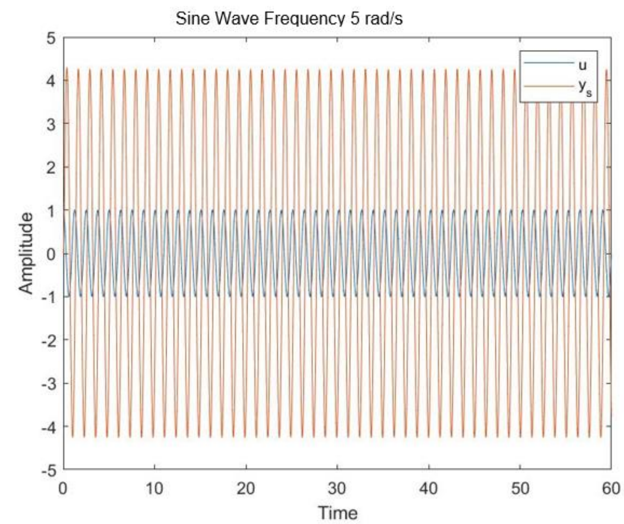

Figure 2 – Sine wave at frequency of 5 rad/s

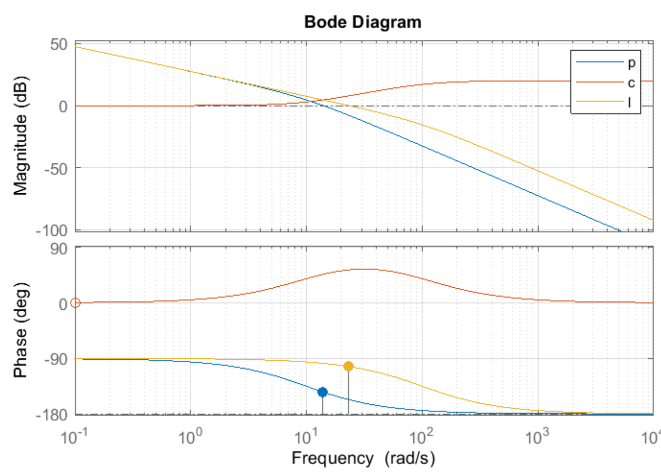

Figure 3 – Bode plot for P, C, L

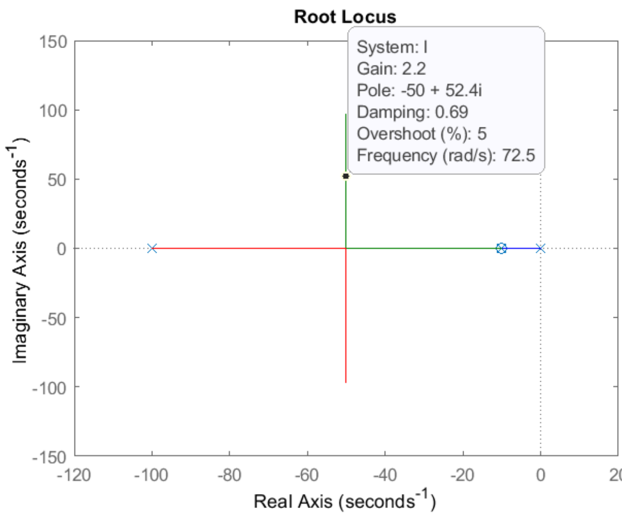

Figure 4 – Root locus plot with Gain 2.2

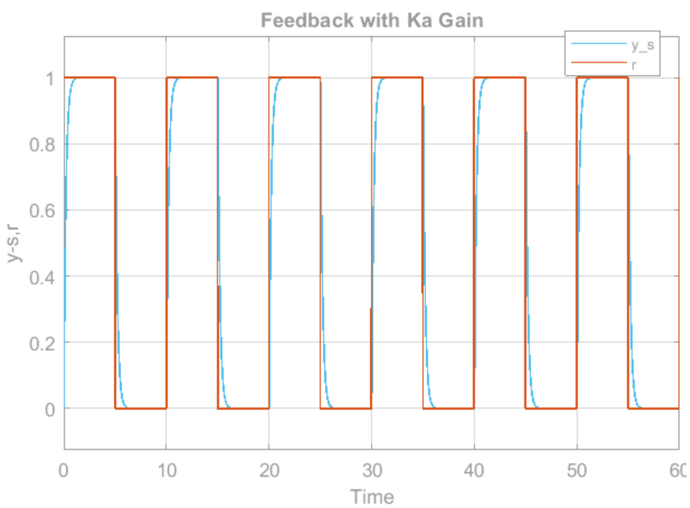

Figure 5 – Feedback of system with Ka Gain

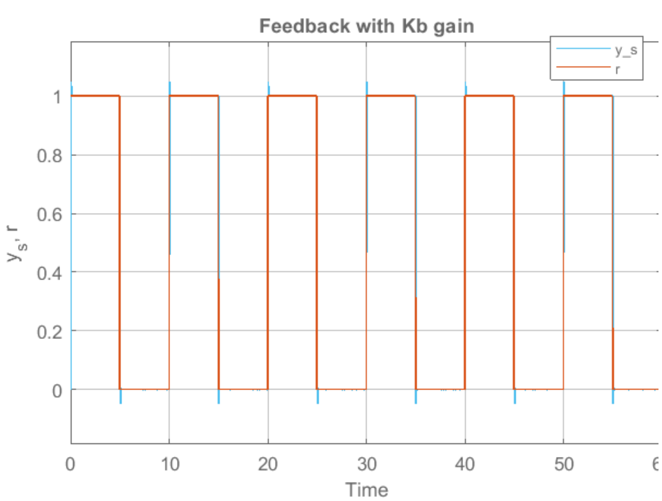

Figure 6 – Feedback of system with Kb Gain

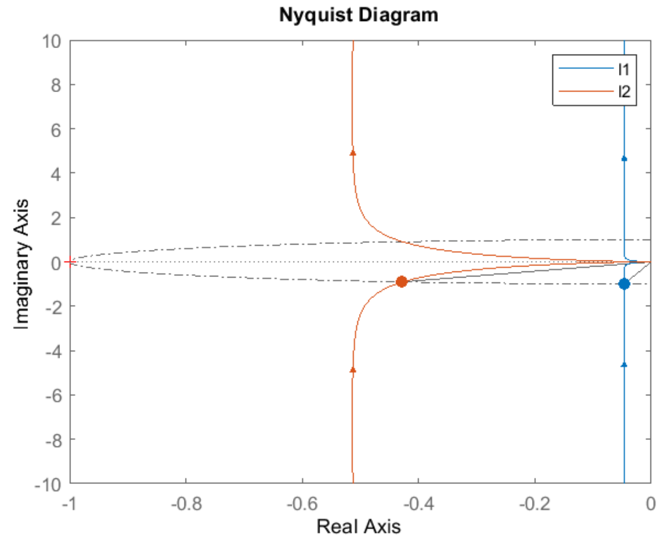

Figure 7 – Nyquist plot with Ki*L (i=a,b)

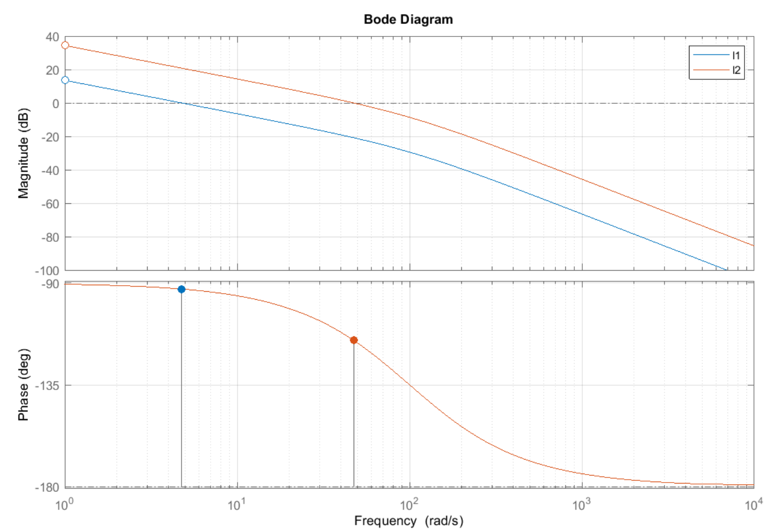

Figure 8 – Bode Plot with Ki*L (i=a,b)

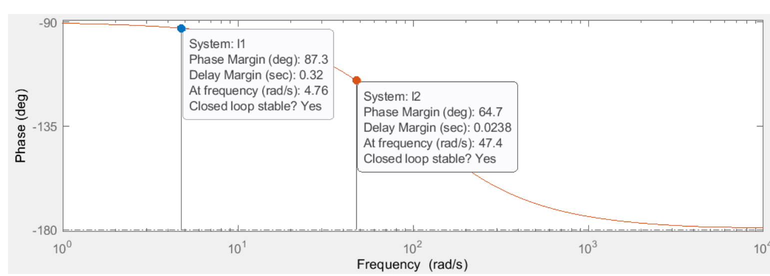

Figure 9 – Bode Plot with stability characteristics

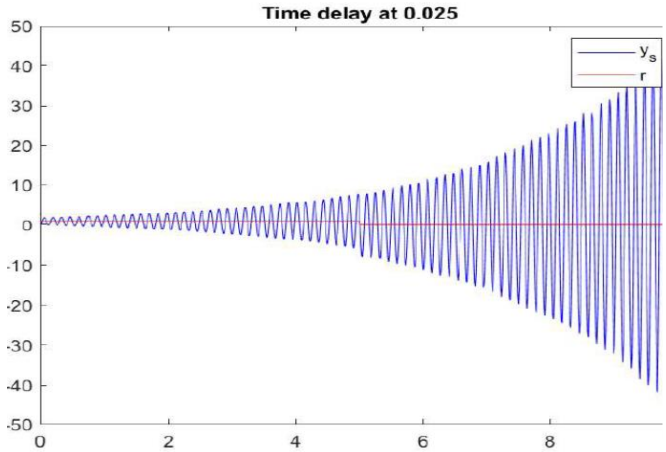

Figure 10 – Time delay at 0.025s

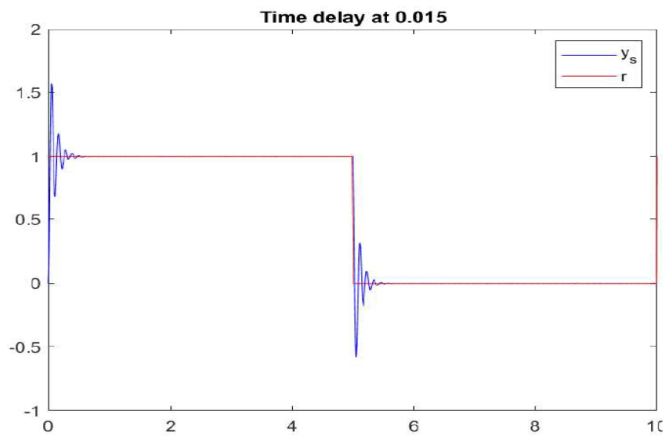

Figure 11a – Time delay at 0.015 s

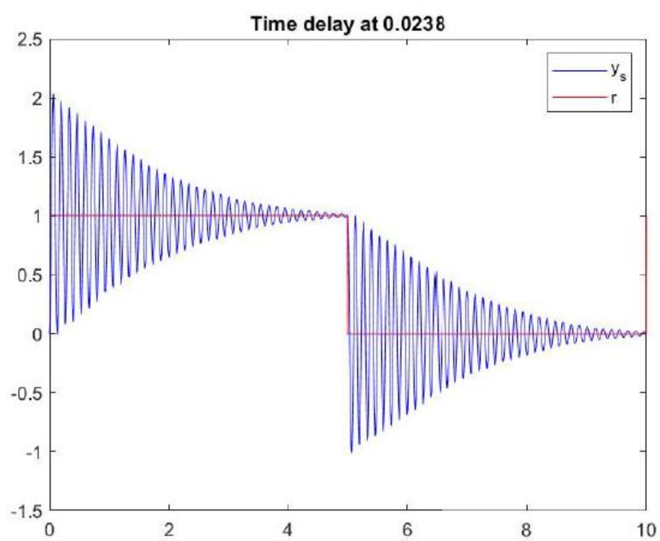

Figure 11b – Time delay at 0.0238s

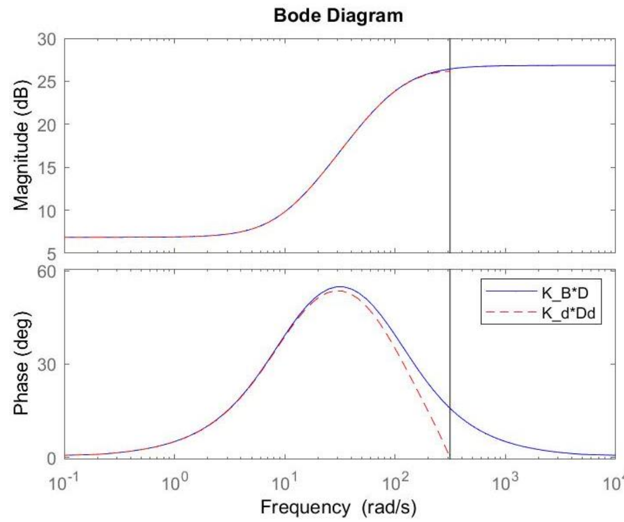

Figure 12 – Bode plot

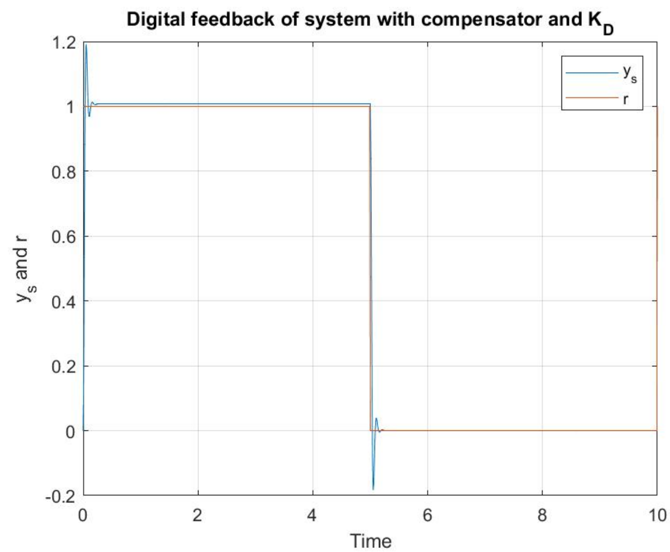

Figure 13 – Digital feedback with Kd

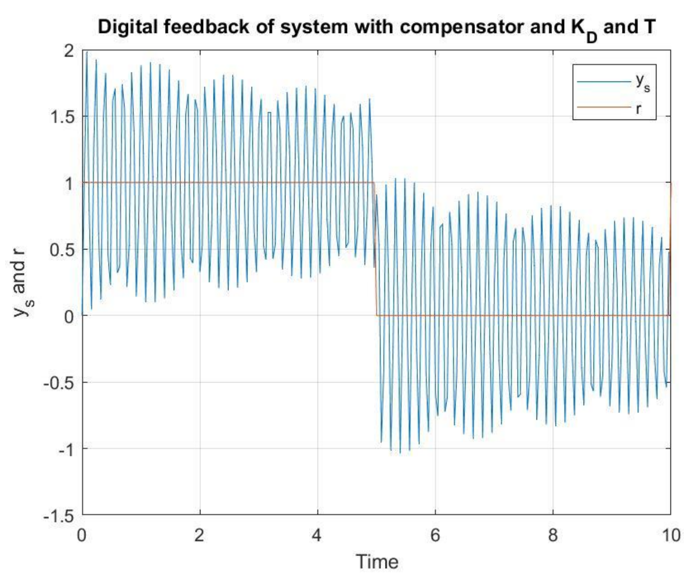

Figure 14 – Digital feedback with Kd (T=0.04s)

VII. Appendix 2

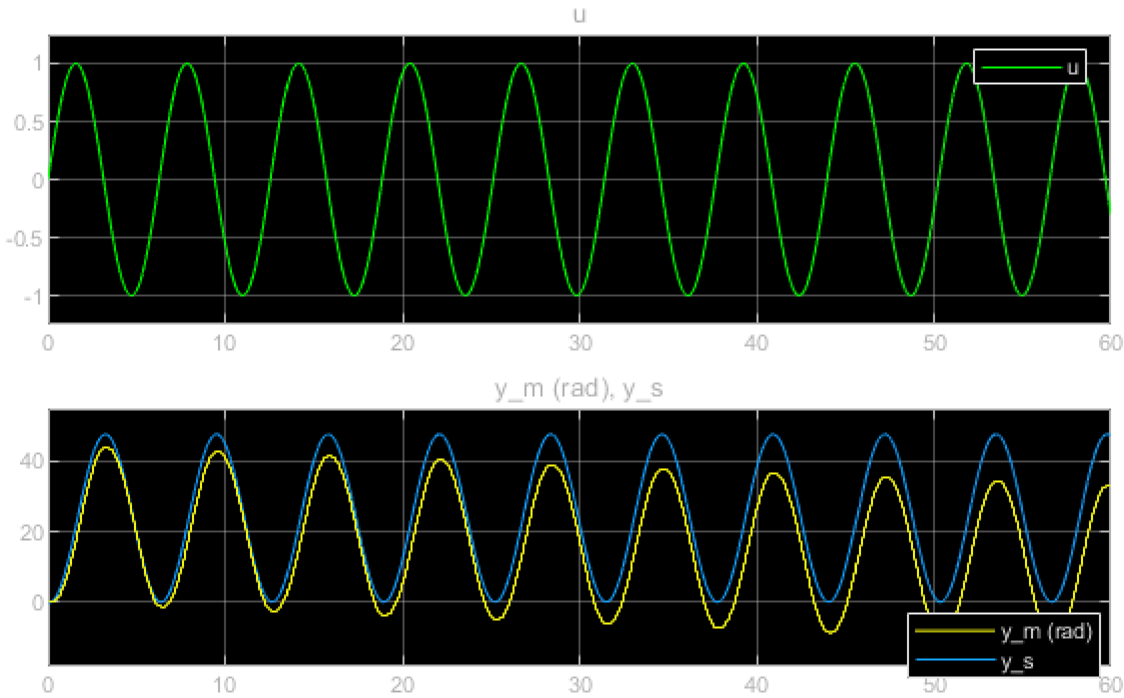

Figure 15 – Sine Wave at frequency of 1rad/s

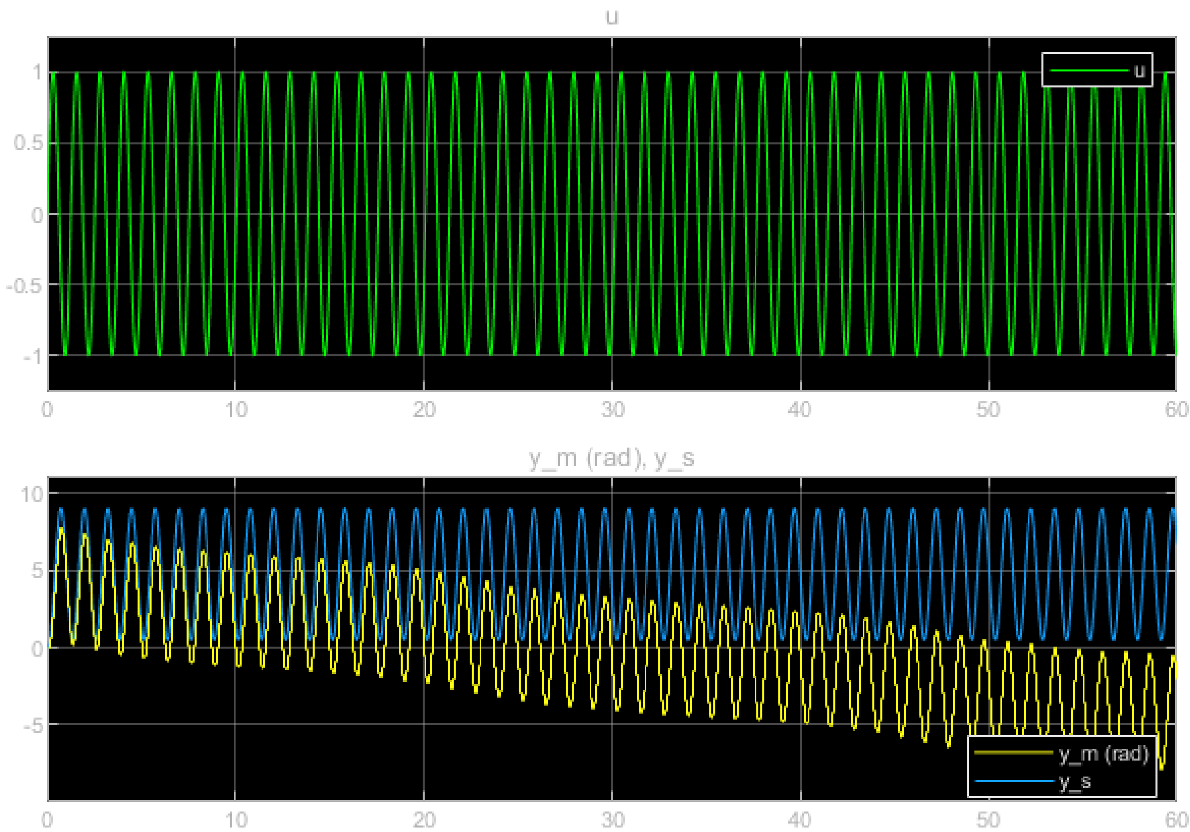

Figure 16 – Sine wave at Frequency of 5rad/s

VII. Appendix 2

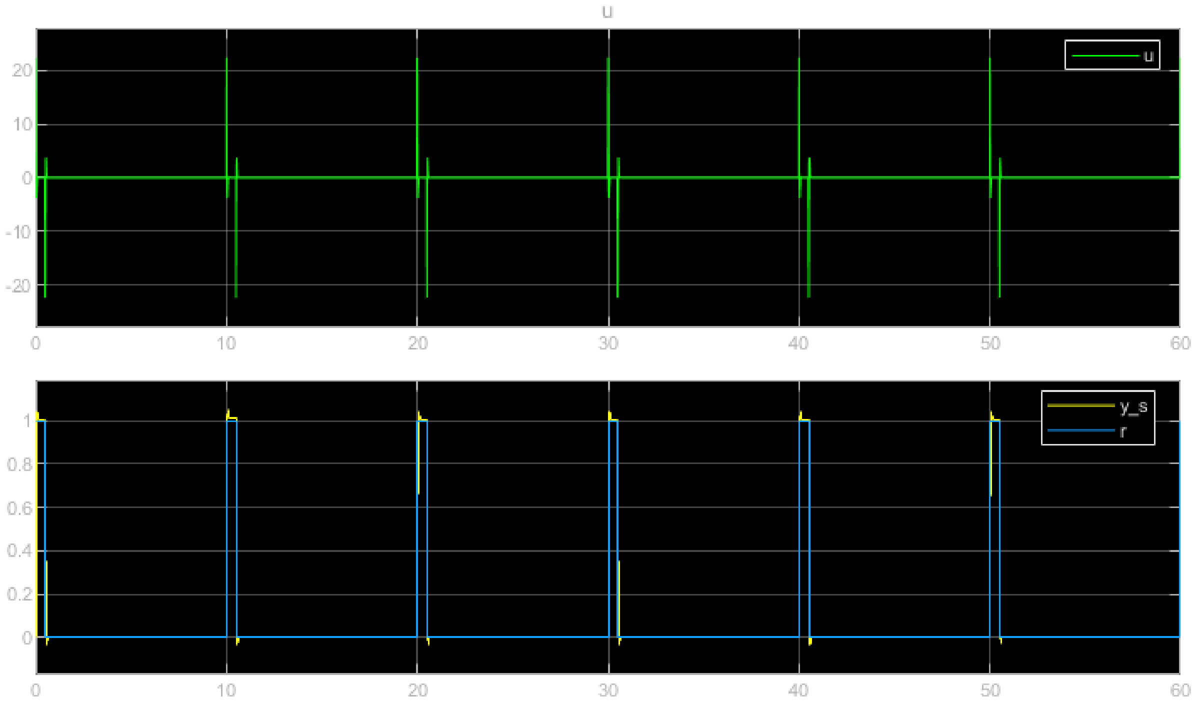

Figure 17 – Continuous Feedback control with Gain 2.23

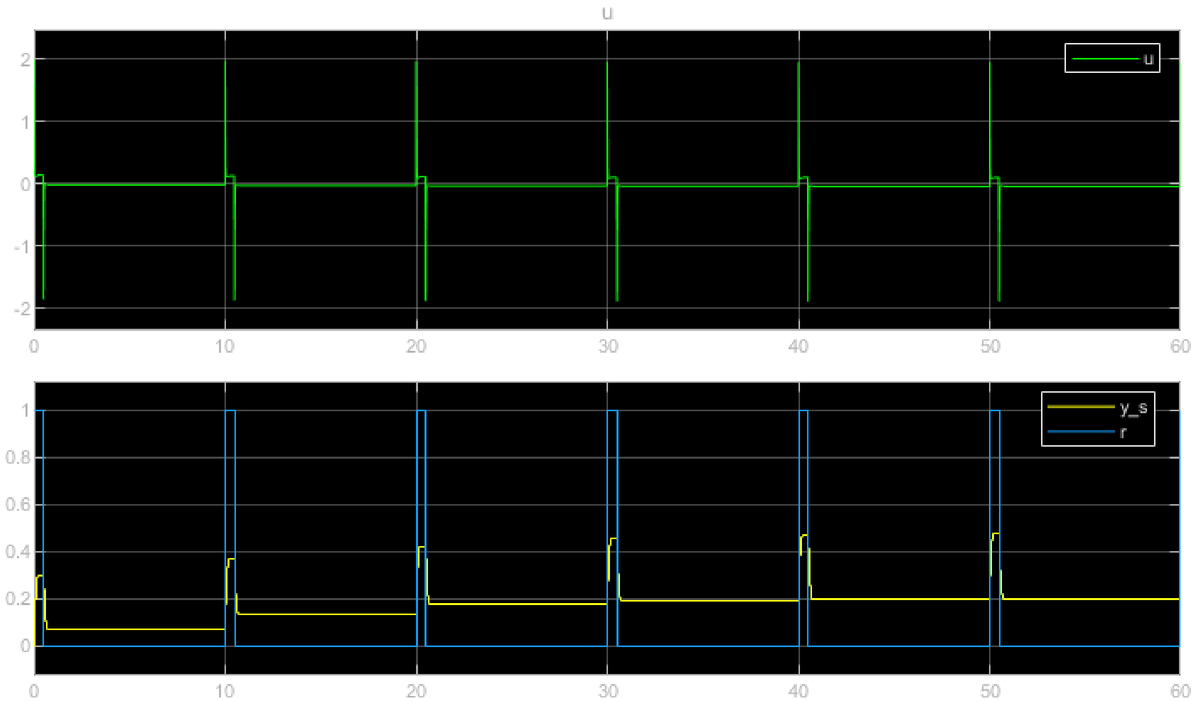

Figure 18 – Continuous Feedback control with Gain 0.2

VII. Appendix 2

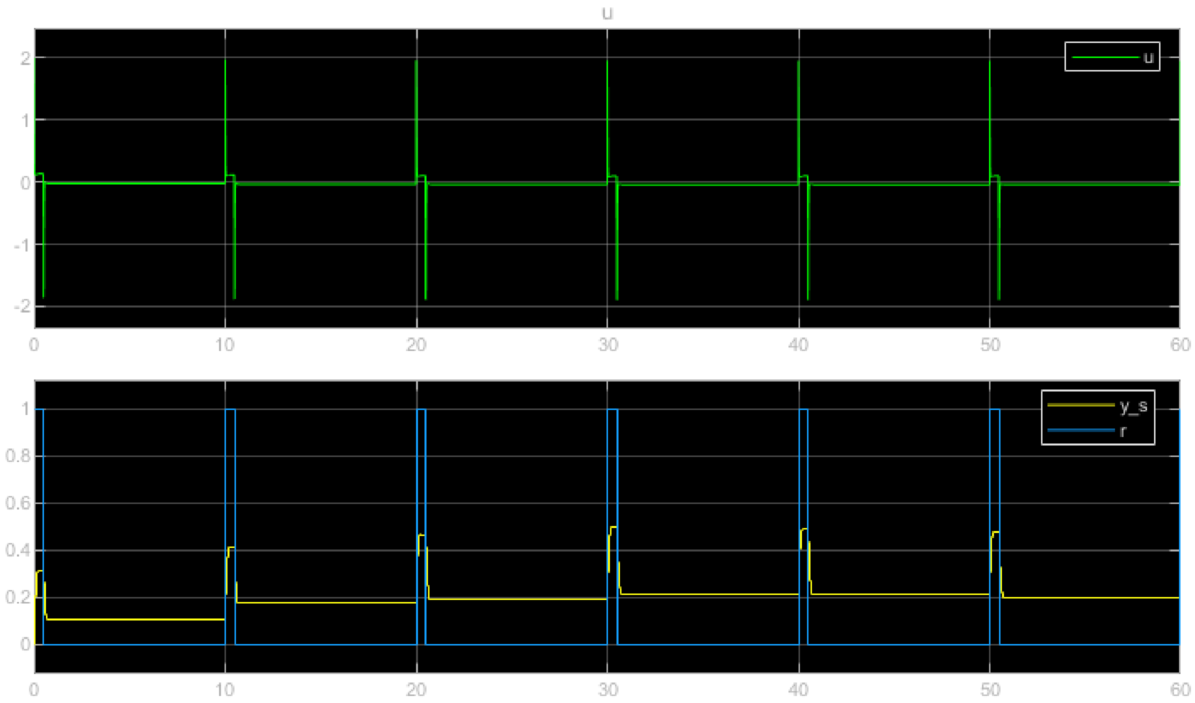

Figure 19 – Continuous Feedback control with Gain 0.2 (delay margin 0.001s)

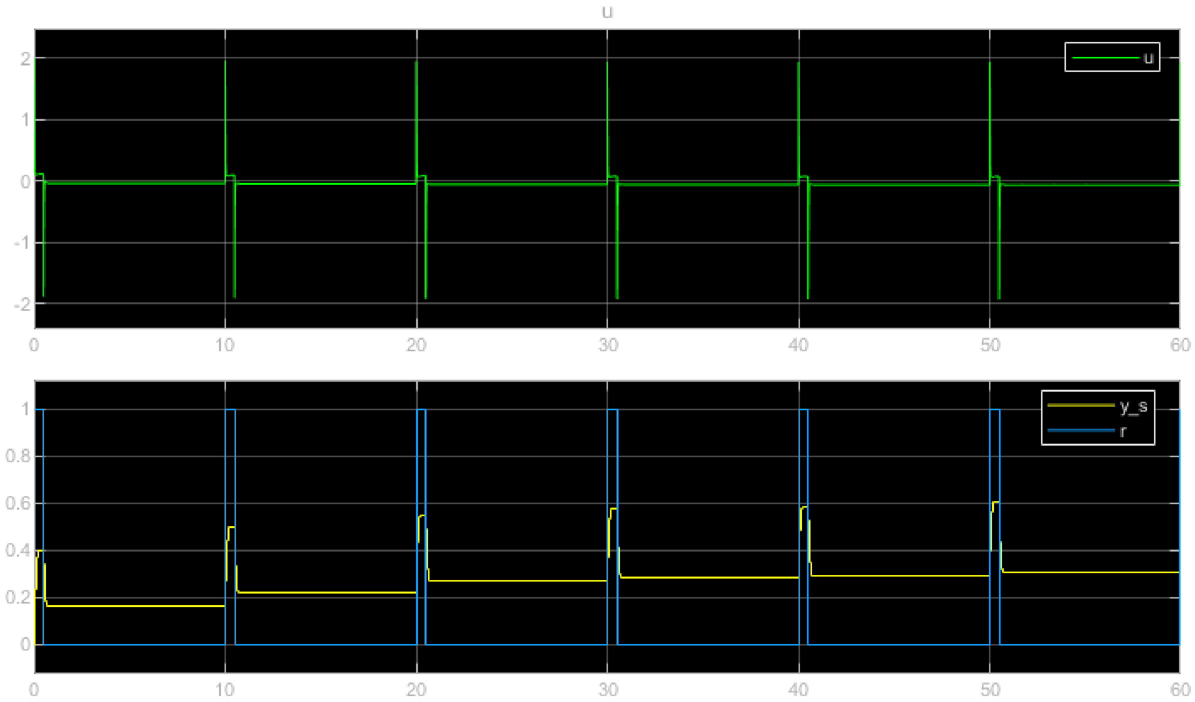

Figure 20 – Continuous Feedback control with Gain 0.2 (delay margin 0.02s)

VII. Appendix 2

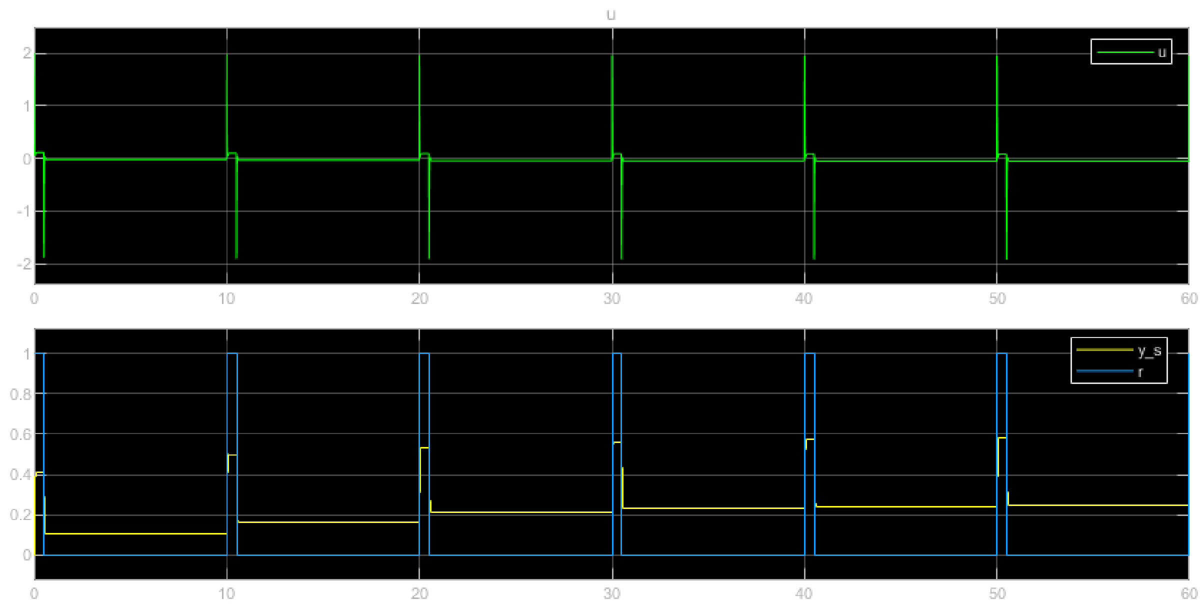

Figure 21 – Continuous Feedback control with Gain 0.2 (Disk removed)

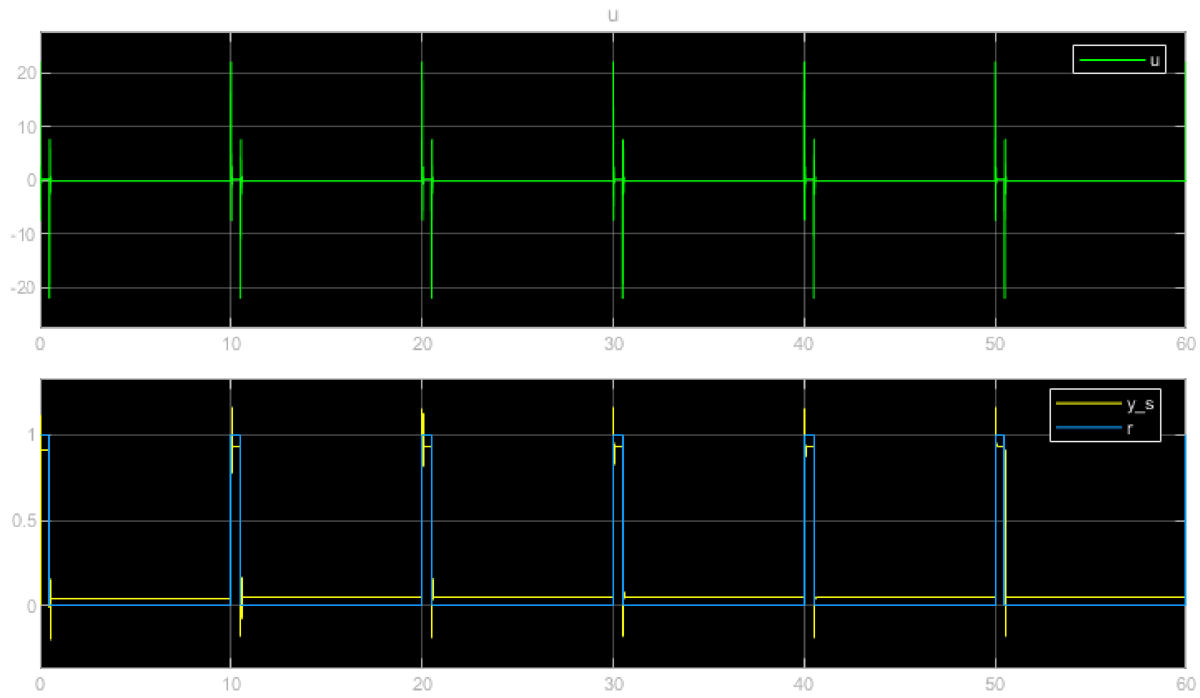

Figure 22 – Continuous Feedback control with Gain 2.23 (Disk removed)

VII. Appendix 2

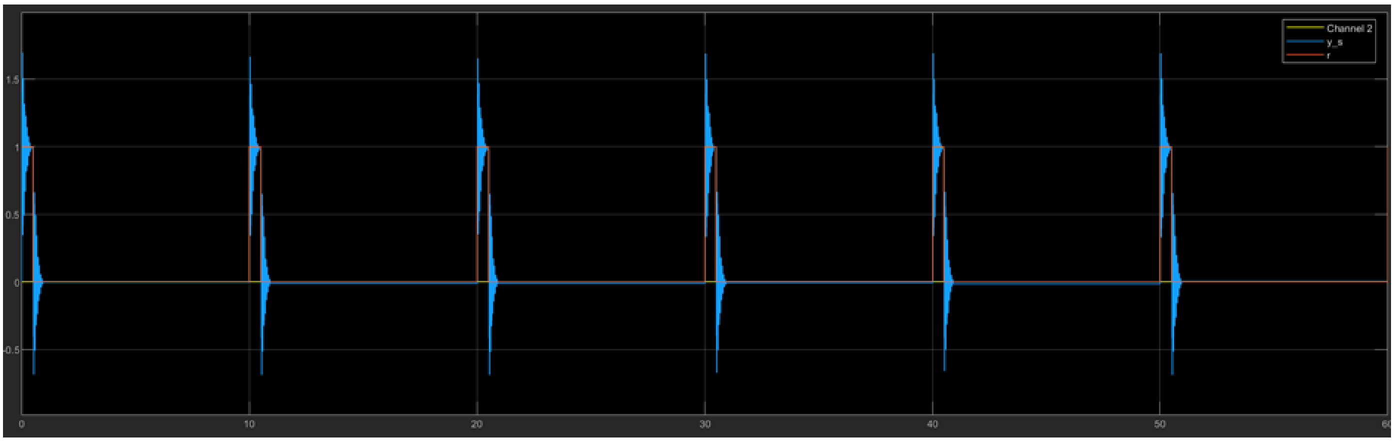

Figure 23 – Digital Feedback control with Gain Kd (T = 0.01s)

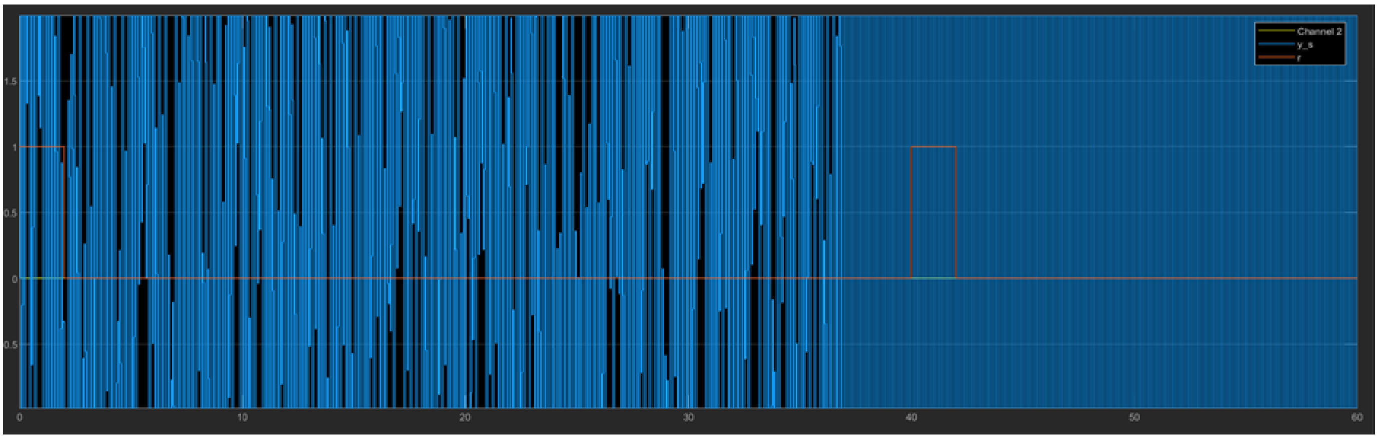

Figure 24 – Digital Feedback control with Gain Kd (T = 0.04s)