Boosting Blackjack Returns with Machine Learned Betting Criteria

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Abstract

This paper investigates a new method to boost Blackjack betting returns. We show betting criteria identified by genetic algorithm significantly outperform standard game theoretic criteria on ten different professional counting systems.

1. Introduction

Blackjack or "21" is a simple card game [20]. It is the most popular table game in traditional casino venues and increasingly prevalent on-line [18]. Blackjack is furthermore ubiquitous on mobile consumer electronic devices including organizers [15]; cell phones [13]; PDAs [16]; and key ring-size card counting computers. [2] According to game theorists, however, the "correct" way to play Blackjack is the Basic Strategy which generally has negative expectation for players. [12] Players have theoretically positive expectation if, with the Basic Strategy, they also use card counting to predict favorable decks and vary bet amounts in accordance with Kelly's principle. [12] While there are numerous ways to count cards [1,3,10], standard game theoretic betting criteria for single-parameter counting strategies assumes the player's edge is approximately 0.50% of the "true count" after discounting the House's advantage of 0.50% for the Basic Strategy. [12,21]

In this paper, we investigate a new approach to betting that does not assume standard criteria. Indeed, the data suggests opportunities to significantly boost player betting returns using alternative criteria learned by artificial intelligence means, specifically, genetic algorithm (GA) [9].

Consider as an example the Hi-Lo [3], a professional counting system for which the GA evolves non-standard betting criteria. The data indicates the Hi-Lo improves in 283 of 400 ($p<10^{-15}$) Bernoulli trials of 100,000 games (or 40 million games total) using the new criteria compared to standard criteria. The GA-inspired criteria also reduce the number of trials with losses from 98 to 44 ($p<10^{-7}$) and furthermore, improve the Hi-Lo in relation to other systems. For instance, the Hi-Lo outperforms the Canfield Expert [3] in 307 trials when both use standard criteria. The Hi-Lo using GA criteria outperforms the Canfield in 354 trials—47 trials ($p<10^{-5}$) above the reference Hi-Lo performance.

We investigate ten different counting systems and report statistically significant results consistent with the ones mentioned above. Our primary motivations for studying these particular systems include the case that they are among the most widely known, frequently referenced systems on record in the public domain. [1,3,10] They are also representative of classes of balanced, low betting level systems which, as we explain below, are not only psychologically tractable for human players [10]. These same characteristics lend themselves to computer modeling. While this study is not intended to be an exhaustive analysis, we hypothesize nevertheless that other systems not examined here may similarly respond positively to GA treatments such as those we explore.

2. Related work

Initial machine learning efforts for Blackjack were carried out by Gryk [11]. This work was followed by Yun [22], Caverlee [4], Coleman [5], and Coleman [6]. They each used evolutionary search to learn play strategies with fixed bets and no card counting. Coleman [7] focused on learning to count cards with a fixed play strategy and variable bets that depended on standard criteria given in Humble and Cooper [12] which in turn was informed by results from Griffin [10] and other game theorists beginning with Thorp [17].

The present paper focuses on learning to bet which is new since prior machine learning efforts concentrated on learning to play or learning to count with standard criteria. For a given counting system, we use a fixed play strategy, namely, an optimized version of the Basic Strategy for our benchmark setting. However, we evolve new betting criteria for the system under consideration rather than use the standard criteria. The new criteria has a non-linear form similar to the version in Humble and Cooper [12]. The

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difference is the count multiplier, α, and the House discount, β, are not fixed at 0.00515 and 0.00540, respectively. Rather, α and β vary, and potentially uniquely, for each counting system. Learning new criteria as such would appear to readily lend itself to heuristic analysis. Unfortunately, the high standard error of Blackjack, which players experience as wide fluctuations in returns, is an impediment that requires billions of game evaluations to obtain statistically reliable solutions. [7] As observed by others [4,5,6,7], the barriers to search by A.I., particularly population-based approaches, are challenging primarily because brute force simulation is computationally impractical. However, Coleman [7] first introduced and applied a memory-bound workaround called traces that speeds up game simulation by more than two orders of magnitude. Although the search method in that case employed scatter search [14], traces do not depend on the search method. We use this feature to speed up the search by GA and thus, from a strictly practical point of view, traces play a key role in our study.

3. Betting systems

3.1 Benchmark setting

The game rules of Blackjack frequently depend on the venue. We use the following Humble and Cooper [12] benchmark setting: four decks; dealer stands on soft 17; no double down after splits. The play strategy is optimized for four decks per the Basic Strategy given in Humble and Cooper [12]. The shoe is reshuffled when ½ or fewer decks remain.

3.2 Bet modeling

Card counting systems generally assign weights to the "count" of observed cards that have been dealt from the deck. The single-parameter, running count, C_{Run}, is the dot product of the weights vector, **w**, and the observed count vector, **v**, namely,

$$C_{Run} = \mathbf{w} \cdot \mathbf{v} = \Sigma\, w_i\, v_i \qquad (1)$$

where i is the index, {i: 0→ Ace, 1→"2",…,12→"K"}. The betting level, λ, is broadly defined as the largest absolute integer weight in the system. λ=0 implies no card counting since C_{Run}=0.

The true count, C_{True}, is the running count calibrated to deck size and for comparison purposes, normalized to level, namely,

$$C_{True} = C_{Run} / (|«D»|\, \lambda) \qquad (2)$$

where «D» is the set of cards to be dealt from the shoe during the next game. The betting criteria function, BCF, is a real value function that uses the non-linear operator, max[•], to predict the player's advantage as follows:

$$BCF(C_{True}, \alpha, \beta) = \max[\, \alpha \times C_{True} - \beta, 0\,] \qquad (3)$$

where α and β are the betting criteria. For standard criteria, α=0.00515 and β=0.00540.

The bet amount, K_j, which the player places *before* the start of game j, relies on Kelly's principle [12] and the minimum House bet, K_{min}, namely,

$$K_j = \max[BCF(C_{True}, \alpha, \beta) \times F_j,\ K_{min}\,] \qquad (5)$$

where F_j is the player's bankroll at the start of game j. The net profit and loss for game j is PL_j given by,

$$PL_j = \Sigma_{i=0\ldots h-1}\, H_{i,j}(K_j\,, «D») \qquad (6)$$

where $H_{i,j}(\bullet)$ is the earnings on hand i of h hands of game j given a bet amount, K_j, and the shoe, «D». Following Coleman [7], we rewrite Equation 6 as,

$$PL_j = K_j \times \Sigma_{i=0\ldots h-1}\, H_{i,j}(1, «D») \qquad (7)$$

which is suitable for traces. At the end of game j, the player has realized a new bankroll, F_{j+1},

$$F_{j+1} = F_j + PL_j \qquad (8)$$

3.3 Statistical methods

If F_0 is the initial bankroll, the mean hand return is

$$฿ = (F_{n-1} - F_0\,) / n \qquad (9)$$

To compare different BCFs robustly, we use 400 Bernoulli trials. Each trial consists of n=100,000 heads up (player v. dealer) games. There are a total of 40 million games in the 400 trials. A single trial is based on a random seed that uniquely shuffles approximately 12,000 decks in a trial. $฿^{GA}$ is the mean return based on GA identified criteria. $฿^{Control}$ is the mean return for the control, that is, the professional system using standard criteria. s^{GA} is the number of successes (i.e., trials) where $฿^{GA}$ > $฿^{Control}$. The null hypothesis is H_0:s^{GA}≤200; the alternative is H_1:s^{GA}>200. That is, we expect the system under GA criteria to match or under perform the same system using standard criteria. We assess the statistical significance of s^{GA} non-parametrically using the Binomial test where the probability of success is q=0.50. [8]

The loss analysis uses a variation of the non-parametric Binomial test. [8] That is, a trial is a success if ฿>0 and a failure otherwise. The number of failures for the GA is f^{GA} and $f^{Control}$ for the control. The null hypothesis is H_0:f^{GA}-$f^{Control}$> 0 where as the alternative is H_1:f^{GA}-$f^{Control}$ ≤0. In other words, we expect GA criteria to generate the same or more negative trials compared to standard criteria. To assess the statistical significance of f^{GA} – $f^{Control}$, we use the normal distribution to approximate the Binomial distribution. [8] Specifically, we have mean μ=0 and standard deviation $\sigma=(400\cdot q\cdot(1-q))^{1/2}$ =10 where q=0.50.

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Similarly, we evaluate the excess benefit using the Binomial test. Let A be a reference system, that is, neither a GA or control system. Let $s^{Control}$ be the number of successes where, $฿^{Control} > ฿^{A}$. The excess improvement is $Y=s^{GA} - s^{Control}$. The null hypothesis is H_0:Y ≤0. The alternative is H_1:Y>0. Namely, we expect the system under GA criteria to have the same or worse relative performance compared to the same system under standard criteria. Again we use the normal distribution to approximate the Binomial distribution with the mean and standard deviation the same as above.

3.4 Professional systems

We study ten professional systems as indicated by the table below. These systems and their game theoretic features can be found in Griffin (1999) and online.[1,3]

Table 1. Professional systems.

#	Trade Name	Label	λ
0	(No counting)	Null	0
1	Hi-Opt I	HO1	1
2	Canfield Expert	CE	1
3	Hi-Lo	HL	1
4	Revere Advanced Plus-Minus	RAPM	1
5	Silver Fox	SF	1
6	Uston Advanced Plus-Minus	UAPM	1
7	Griffin-1	G1	1
8	Hi-Opt II	HO2	2
9	Omega II	OM2	2
10	Revere Point Count	RPC	2

These systems have two characteristics that are relevant for our purposes. Firstly, they are λ=1 and λ=2 betting levels. (The null or "no counting" system is not optimized by the GA. We use it only as an experimental control.) Secondly, they are balanced, that is, the sum of weights is zero as the models in the table below indicate.

Table 2. Weight vector, **w**, by counting system.

#	Sys.	A	2	3	4	5	6	7	8	9	T
0	Null	0	0	0	0	0	0	0	0	0	0
1	HL	-1	1	1	1	1	1	0	0	0	-1
2	UAPM	-1	0	1	1	1	1	1	0	0	-1
3	SF	-1	1	1	1	1	1	1	0	-1	-1
4	HO1	0	0	1	1	1	1	0	0	0	-1
5	RAPM	0	1	1	1	1	1	0	0	-1	-1
6	CE	0	0	1	1	1	1	1	0	-1	-1
7	G1	0	0	0	1	1	1	1	0	0	-1
8	HO2	0	1	1	2	2	1	1	0	0	-2
9	OM2	0	1	1	2	2	2	1	0	-1	-2
10	RPC	-2	1	2	2	2	2	1	0	0	-2

Balance suggests players can readily detect when the count is favorable. Low betting level implies simpler mental arithmetic. Griffin [10] noted that aside from the theoretical opportunities afforded by card counting, these features, which characterize the above systems, are particularly attractive for human application.

4. Learning approach

The genetic algorithm uses Blackjack memory traces which we summarize below. For more details, the reader can consult Coleman [7].

The GA objective function maximizes mean hand return (Eq. 9). In other words, the fittest genome identifies α and β of the BCF which best predicts the player's advantage given C_{True}.

4.1 Traces

Coleman [7] gives a detailed discussion on traces. In the simplest terms, a trace, T_j, is a memory-bound instance of a game. It has three properties: (i) **v**, the vector of observed card counts after game j-1, (ii) |«D»|, the number of decks remaining after game j-1, and (iii) the *pro forma* outcome of game j, namely, PL_j^0, assuming λ=0 (i.e., no card counting). The *pro forma* outcome is easily converted to a game outcome, namely, PL_j, as follows:

$$PL_j = PL_j^0 \times K_j \qquad (10)$$

where G_j is the bet amount varies according to the bankroll, F_j, **v**, and **w**. A trace can be loaded into memory once and Eq. 10 evaluated many times without playing full games. This significantly speeds up Blackjack simulation.

4.2 Genetic algorithm

We developed the GA configuration mainly through trial and error analysis using the "simple" genetic algorithm class from Wall [19]. We start with a population of 200 genomes, each with two real parameters, α and β. They are initially randomized on the interval, [0, 0.02] and [-0.02, 0] respectively. However, α and β are not otherwise constrained by the GA as such since the genetic mutation function is

$$param^{new} = param^{old} \cdot (1 + r - 0.50) \qquad (11)$$

where $r \in (0,1)$ is a uniform random deviate. In other words, $\alpha \in (0,\infty)$ and $\beta \in (-\infty,0)$. The mutation rate and crossover probability are 0.10 and 0.95, respectively. The GA is programmed to run for a maximum of 100 generations—it never converges. The objective function, Ω, is

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$$\Omega(\alpha,\beta)=\text{argmax}(\alpha,\beta)\ (F_{n-1} - F_0) / n \qquad (12)$$

where argmax(α,β) gives the values of α and β which maximizes the mean hand return for n=500,000 traces. The value n directly affects the generality and statistical quality of the fittest solution. We learned from trial and error that n<200,000 produce statistically unreliable or overfit solutions. The value of n also determines the total number of games the GA evaluates which in this case are 10^{10} games for each system.

5. Results

5.1 GA-identified criteria

The table below gives GA-inspired criteria for each system.

Table 3. Fittest GA-inspired criteria for each system.

#	Sys.	α	β	τ
1	HO1	0.00336	-0.00108	2.41
2	CE	0.00351	-0.00246	1.79
3	HL	0.00356	-0.00115	2.67
4	RAPM	0.00332	-0.00109	2.36
5	SF	0.00347	-0.00095	2.65
6	UAPM	0.00341	-0.00087	2.60
7	G1	0.00324	-0.00083	2.39
8	HO2	0.00370	-0.00071	3.23
9	OM2	0.00367	-0.00085	3.07
10	RPC	0.00368	-0.00095	3.03

We note the GA-inspired criteria and standard criteria diverge as expected. In fact, our BCF appears consistently flatter compared to standard criteria, namely, we have $\alpha < 0.00515$ and the intercept, $\beta > -0.00540$. The above table also includes the derived value, τ, which is the C_{True} intersection of standard and GA functions.

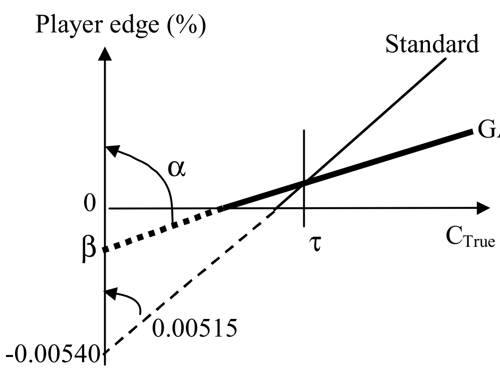

Figure 1. Canonical BCF divergence.

The figure below depicts this diverging relationship in canonical form. (Note that the dotted parts of each function represent the non-linear cut-offs of the max operator.)

5.2 Trial comparisons

The table below gives the trial comparisons. That is, we are comparing trial-by-trial the control versus the same system using GA-inspired criteria given in Table 3 above. The null hypothesis is H_0:$s^{GA} \leq 200$.

Table 4. Trial comparison successes in 400 trials.

#	Sys.	s^{GA}	p
1	HO1	298	$<10^{-15}$
2	CE	348	$<10^{-15}$
3	HL	283	$<10^{-15}$
4	RAPM	332	$<10^{-15}$
5	SF	329	$<10^{-15}$
6	UAPM	288	$<10^{-15}$
7	G1	311	$<10^{-6}$
8	HO2	264	$<10^{-10}$
9	OM2	287	$<10^{-15}$
10	RPC	270	$<10^{-12}$

As the table suggests, the GA is able to improve each system significantly on a trial-by-trial basis.

5.3 Aggregate comparisons

The table below gives aggregate comparisons on negative trials for the control versus the same system using GA-inspired criteria. The null hypothesis is H_0:f^{GA}-$f^{Control} > 0$ is the failures in which ฿$^{GA} < 0$ compared to ฿$^{Control} < 0$.

Table 5. Negative return trials.

#	Sys.	f^{GA}	$f^{Control}$	$f^{GA} - f^{Control}$	p
1	HO1	71	123	-52	$<10^{-7}$
2	CE	87	171	-84	$<10^{-15}$
3	HL	44	98	-54	$<10^{-8}$
4	RAPM	84	168	-84	$<10^{-15}$
5	SF	62	128	-66	$<10^{-11}$
6	UAPM	37	97	-60	$<10^{-9}$
7	G1	70	130	-60	$<10^{-9}$
8	HO2	67	93	-26	0.005
9	OM2	71	109	-38	$<10^{-4}$
10	RPC	39	79	-40	$<10^{-4}$

In each case, GA-inspired criteria significantly reduces the number of negative trials.

Finally, the graph below shows the relative improvement due to GA-inspired criteria in relation to standard criteria when compared with a reference system of the same betting level, λ. On the x-axis, there are the ten counting systems corresponding to the

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ten in our study. Each data point, "+" represents $Y=s^{GA}-s^{Control}$ where s^{GA} and $s^{Control}$ are the number of Bernoulli successes in relation to a reference system. The y-axis is the range of Y. The cut-off for statistical significance is Y=14 (p=0.0488).

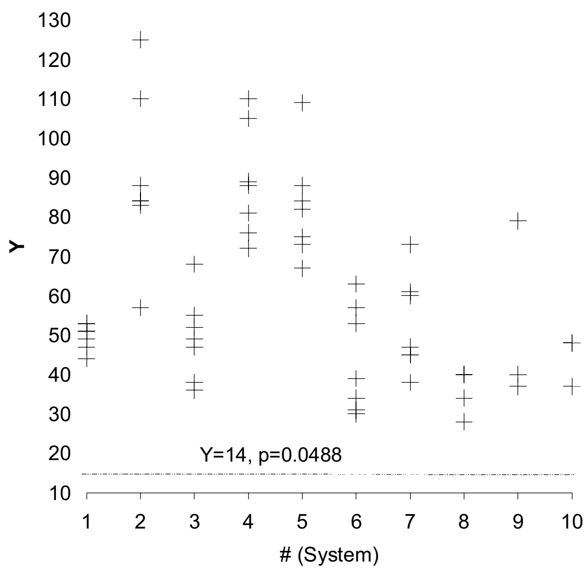

Figure 2. Relative (excess) performance.

As an example, consider counting system #1, the Hi-Opt I. We compare it with systems #2 - #7 since they are all λ=1. The vertical filament of "+" data points under column #1 in the chart gives the relative performance in each case including the Null system. The excess improvement, Y, for the Hi-Opt I ranges from 44 for the CE (#3) to 53 for the UAPM (#6). The p-values range respectively from $<10^{-5}$ to $<10^{-7}$.

Note that some data points are not shown for some systems. Some data points are overlaid while others are out of range high, that is Y>130. In general, however, the chart shows GA-inspired criteria significantly improves the relative performance of the control in relation to other systems as no data point, "+", is below the Y=14 cut-off of statistical significance.

6. Conclusions

We believe these statistically significant improvements stem from the GA tendency to evolve criteria that are optimized for consistent rather than volatile returns. In other words, the flatter GA criteria suggested by Figure 1 is more conservative from a betting perspective. Future research might explore this hypothesis by analyzing the standard errors of returns. Future work might also explore not only higher betting levels, λ>2, which seem to us promising. There is also the potential of "hybrid" criteria using τ to non-linearly combine GA and standard criteria.

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