Birla Institute of Technology & Science, Pilani

Department of Mathematics

Second Semester 2021-2022

MATH F243 **ProblemSheet-2**

Q.1 Let T be a tree:

a) Use the fact that there is a unique path between any pair of vertices in a tree to show that if v is an internal vertex of T then T - v is not connected.

b) If v is a leaf in a tree T show that T - v is connected.

Q.2 Let T, T' be two spanning trees of a connected graph G. Prove that there is an edge e' $\in E(T')\backslash E(T)$ such that T'+e−e' and T+e'−e are both spanning trees of G ?

Q.3 Consider a graph G = (V, E), where V = $v_1, v_2, \ldots, v_{100}$, E = (v_i, v_j) $1 \leq i \leq j \leq 100$, and weight of the edge (v_i, v_j) is $|i - j|$. The weight of the minimum spanning tree of G is ?

Q.4 Let G be a complete undirected graph on 4 vertices, having 6 edges with weights being 1, 2, 3, 4, 5, and 6. The maximum possible weight that a minimum weight spanning tree of G can have is ?

Q.5 Let G = (V,E) be an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Is the following statement about the minimum spanning trees (MSTs) of G TRUE?

I. If e is the lightest edge of some cycle in G, then every MST of G includes e

Q.6 Let G be a connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes?

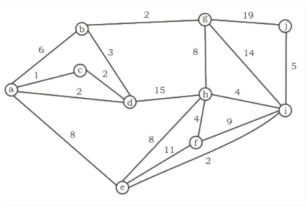
Q.7 Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry W_{ij} in the matrix W below is the weight of the edge {i, j}.

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T?

Q.8 For each degree sequence below, decide whether it must always, must never, or could possibly be a degree sequence for a tree. Remember, a degree sequence lists out the degrees (number of edges incident to the vertex) of all the vertices in a graph in non-increasing order.

a. (4,1,1,1,1)
b. (3,3,2,1,1)
c. (2,2,2,1,1)
d. (4,4,3,3,3,2,2,1,1,1,1,1,1,1)



Q.9 What is the weight of a minimum spanning tree of the given graph ?

Q.10 For $2 \leq k \leq n-1$, the n-vertex graph formed by adding one vertex adjacent to every vertex of P_{n-1} has a spanning tree with diameter k.

Q.11 If $n \geq 2$ and $d_1, \ldots, d_n$ are positive integers, then there exists a tree with these as its vertex degrees if and only if $d_n = 1$ and $\sum_{i=1}^{n} d_i = 2(n-1)$.