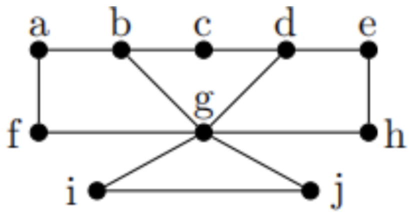
Birla Institute of Technology & Science, Pilani

Department of Mathematics

Second Semester 2021-2022

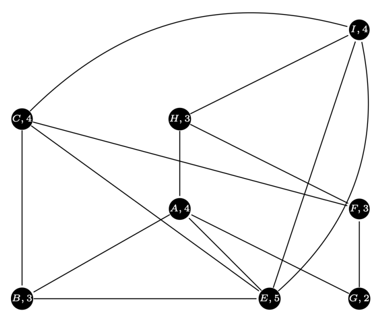
MATH F243 **ProblemSheet-3**



Q.1
a)Is the above graph contains an euler path or an euler cicruit?
b)Is the above graph Hamiltonian?

Q.2 Let T be a depth first search tree in an undirected graph G. Vertices u and n are leaves of this tree T. The degrees of both u and n in G are at least 2. which one of the following statements is true?
(A) There must exist a vertex w adjacent to both u and n in G
(B) There must exist a vertex w whose removal disconnects u and n in G
(C) There must exist a cycle in G containing u and n
(D) There must exist a cycle in G containing u and all its neighbours in G.

Q.3 Let G be a simple undirected graph. Let T_D be a depth first search tree of G. Let T_B be a breadth first search tree of G. Consider the following statements.(I) No edge of G is a cross edge with respect to T_D. (A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D.) (II) For every edge (u,v) of G, if u is at depth i and v is at depth j in T_B, then $|i-j| = 1$.
Which of the statements above must necessarily be true?



Q.4 (a) Label the degree of each vertex in Graph 1.
(b) Determine if Graph 1 has an Euler path or an Euler cycle. If it does, then find one. If not, then explain why not.

	Urbana	Gotham	Metropolis	Bedrock	Quahog
Urbana		1	2	3	3
Gotham	1		4	2	5
Metropolis	2	4		7	4
Bedrock	3	2	7		13
Quahog	3	5	4	13	

Q.5
The distances between various cities are given in the table below.
(a) Draw the corresponding graph to the table.
(b) How many Hamiltonian cycles exist in the graph?
Use Kruskal's algorithm to find the minimal spanning tree of the graph from part (a). Draw the minimal spanning tree below.

Q.6 (Ex 6.12) Let G be a 3-regular graph of order 12 and H a 4-regular graph of order 11.
a)Is G + H Eulerian?
b)Is G + H Hamiltonian?

Q.7 Prove that The $4 \times n$ chessboard has no knight's tour. A knight can move from one square to another that differs by 1 in one coordinate and by 2 in other coordinate.

Q.8 The edges of a connected graph with 2k odd vertices can be partitioned into k trails if k > 0. (Assume graph is connected, because the conclusion is not true for G=H1+H2 when H1 has some odd vertices and H2 is Eulerian).

Q.9 Let G=(V_n,E_n) such that: G's vertices are words over $\sigma = \{a, b, c, d\}$ with length n, such that there aren't two adjacent equal chars. An edge is defined to be between two vertices that are different by only one char.
Does the graph contain an Euler cycle?