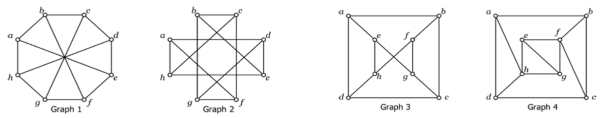
Birla Institute of Technology & Science, Pilani

Department of Mathematics

Second Semester 2021-2022

MATH F243 **ProblemSheet-1**

Q.1 Which of the following graphs are isomorphic (write all pairs)



Q.2 How many different total number of connected undirected simple graphs are possible on the 6 labeled vertices such that degree of the vertices 1, 3, and 5 is one?

Q.3 Explain why in every social gathering there are at least two persons who are friends with the same number of persons.

Q.4 Let G be a simple graph where the vertices correspond to each of the squares of an 8×8 chess board and where two squares are adjacent if, and only if, a knight can go from one square to the other in one move. What is/are the possible degree(s) of a vertex in G? How many vertices have each degree? How many edges does G have?

Q.5 If a simple graph with no isolated vertices has no induced subgraph with exactly two edges, then prove that it is a complete graph.

Q.6 Prove that there exist a graph G with degree sequence $d = (d_1, . . . , d_{2k})$ $d_{2i} = {}_{2i\text{-}1} = i$ $1 < i < k$.

Q.7 Give a counterexample to the following statement and add a hypothesis to correct it:
If e is a cut-edge in G, then at least one endpoint of e is a cut vertex.

Q.8 Prove that if a connected graph G remains connected after removing an edge e from G if and only if e is part of some cycle/circuit in G.

Q.9 Let G be a connected graph with at least three vertices. Form G' from G by adding an edge between every pair of vertices which are distance 2 apart in G. Show that G' formed in this way has no cut-vertices.

Q.10 Compute the number of non-isomorphic (labelled) simple graphs of order 5 and size 4 such that in each graph there exists a vertex v with deg v > 2.