Birla Institute of Technology & Science, Pilani

Department of Mathematics

Second Semester 2021-2022

MATH F243 **ProblemSheet-4**

Q.1 Show that a connected graph is k-edge-connected if and only if each of its blocks is k-edge-connected.

Q.2 For a connected graph G with at least three vertices, the following are equivalent.
A) G is 2-edge-connected.
B) Every edge of G appears in a cycle.
C) G has a closed trail containing any specified pair of edges.
D) G has a closed trail containing any specified pair of vertices.

Q.3 If G is a 2-connected graph and v ∈ V(G), then v has a neighbor u such that G – u – v is connected.

Q.4 Let G be a 5-connected graph, show that between any 3 distinct vertices u, v and w there are 2 cycles which have C and C' which have only the points u and v in common and do not go through w.

Q.5 G is a graph on n vertices and $2n-2$ edges. The edges of G can be partitioned into two edge-disjoint spanning trees. Which of the following is NOT true for G?

A. For every subset of k vertices, the induced subgraph has at most $2k-2$ edges.
B. The minimum cut in G has at least 2 edges.
C. There are at least 2 edge-disjoint paths between every pair of vertices.
D. There are at least 2 vertex-disjoint paths between every pair of vertices.

Q.6 Consider the following problem. You are given a flow network with unit-capacity edges: it consists of a directed graph G = (V, E), a source s and a sink t. You are also given a parameter k. The goal is delete k edges so as to reduce the maximum s – t flow in G as much as possible. In other words, you should find a set of edges F so that $|F|$ = k and the maximum s – t flow in the graph G0 = (V, E \F) is as small as possible.

Q.7 Let G be a k-connected graph. Show using the definitions that if G_0 is obtained from G by adding a new vertex V adjacent to at least k vertices of G, then G_0 is k-connected.

Q.8 Let G be a connected graph with all degrees even. Show that G is 2-edge-connected.

Q.9 Prove that G is 2-connected if and only if for any three vertices x, y, z there is a path in G from x to z containing y.

Q.10 Prove that a graph G on at least k + 1 vertices is k-connected if and only if G – X is connected for every vertex set X of size k – 1.