Modeling the Temporary Market Impact Function gt(x)

Problem Overview

In algorithmic trading, a key challenge is minimizing **market impact**, the cost induced by one's own trades. The **temporary impact function** gt(x) captures the price deviation caused by executing a trade of size x at time t, typically due to liquidity depletion in the limit order book.

This function is essential when designing optimal execution strategies that break large orders into smaller parts across time intervals. Getting the form of gt(x) right is critical ,overly simplistic models lead to underestimation of trading costs and poor execution.

Traditional Modeling Approach: Linear Approximation

A common approach is to model temporary impact linearly:

gt(x)=βtx

This model assumes that:

- Slippage is proportional to trade size
- The order book has uniform depth at all price levels
- There's no significant cost penalty for larger trades

While this approximation is convenient , particularly for deriving closed-form solutions it severely underestimates real trading costs when:

- Order sizes are large
- The order book is shallow
- The trader walks multiple levels of the book

Empirically, we find this linear model to be too naive, especially for shares beyond top-of-book liquidity.

My Proposed Model: Convex Quadratic Slippage

Instead, we model the slippage as:

$g_t(x)=\alpha_t x+\beta_t x^2$
Where:

- α_t represents the baseline spread cost (e.g., half of bid-ask spread)
- β_t models the convex cost as the trade consumes deeper price levels

This formulation captures two realities:

1. **Initial slippage** is linear for small trades (within top-of-book)
2. **Convex growth** occurs when trades walk deeper into the LOB

This model is analytically simple yet rich enough to capture key features of real execution cost.

Empirical Analysis Using 3 Tickers

We validated this model using granular LOB data from 3 tickers (e.g., SOUN) across many minutes of the trading day.

Method:

- For each minute, we simulated market buy orders of varying sizes (50–300 shares)
- At each size, we calculated realized slippage:

  slippage = average execution price − mid-price

- We fit both linear and quadratic regressions to g□(x)
- Extracted coefficients α□, β□ and computed R^2 values

Results:

- The quadratic model consistently outperformed the linear one across all tickers

- For many minutes, linear fits underpredicted impact by over 30% at larger sizes
- Quadratic fits had R^2 values above 0.95, confirming convexity

These results empirically support the presence of nonlinear impact, justifying the inclusion of the $\beta_{\square} \cdot x^2$ term.

Practical Use

In real-world execution:

- We update $\alpha_{\square}$, $\beta_{\square}$ every minute using live LOB snapshots
- Then use convex optimization (e.g., quadratic programming) to decide the $x_{\square}$ that minimizes total cost:

  $$\text{minimize } \sum g_{\square}(x_{\square})$$
  $$\text{subject to } \sum x_{\square} = S$$

This provides a data-driven, mathematically grounded approach to optimal trade scheduling.

Conclusion

Linear models of temporary impact are too simplistic for modern trading environments. Our analysis shows that a **quadratic slippage model** better captures the nonlinear cost structure of real limit order books. It enables more realistic, adaptive execution strategies that minimize cost while respecting liquidity constraints.