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Q2) Mathematical framework that gives us x_i when we are at time t_i

$$\Sigma x_i = S.$$

Using Lagrange Multipliers,

For eg:
We need to by 300 Shares over 3 intervals, morning, midday, afternoon.

x_1, x_2, x_3 : Number of shares bought at each interval.

Using equation from Q_1, The slippage at each interval is modelled as:

$$g_t(x_i) = \alpha_i x_i + \beta_i x_i^2$$

Goal $\longrightarrow$ Minimize the slippage subject to constraint

$$x_1 + x_2 + x_3 = 300$$

Minimize:

$$C = \sum_{i=1}^{3} \alpha_i x_i + \beta_i x_i^2$$

Subject to: $x_1 + x_2 + x_3 = 300$

$$L(x_1, x_2, x_3, \lambda) = \sum_{i=1}^{3} (\alpha_i x_i + \beta_i x_i^2) - \lambda(x_1 + x_2 + x_3 - 300)$$

Estimated using order book :

Interval	α_i	β_i
Morning	0.01	0.0001
Midday	0.015	0.00005
Afternoon	0.008	0.00015

To find minimum, we take partial derivations & set them to 0

$$\frac{\partial L}{\partial x_1} = \alpha_1 + 2\beta_1 x_1 - \lambda = 0$$

$$\frac{\partial L}{\partial x_2} = \alpha_2 + 2\beta_2 x_2 - \lambda = 0$$

$$\frac{\partial L}{\partial x_3} = \alpha_3 + 2\beta_3 x_3 - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = x_1 + x_2 + x_3 - 300 = 0$$

$0.01 + 2 \times 0.0001 \cdot x_1 = \lambda$

$0.015 + 2 \times 0.00005 \cdot x_2 = \lambda$

$0.008 + 2 \cdot 0.00015 \; x_3 = \lambda$

Equating & solving :

$x_1 = 93.64, \quad x_2 = 137.28 \quad x_3 = 69.08$

Using lagrange multipliers, we find optimal allocation

Morning: 94
Midday: 137
Afternoon: 69.