AS1209: MATRIX COMPUTATION

Report on
Project: Solving Electrical Circuit using SVD based Matrix Computation

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Title:

Solving Electrical Circuit Using SVD Based on Matrix Computation
[$b=Ax \rightarrow V=IR$]

Introduction:

Efficient numerical techniques are frequently needed to solve large circuits with numerous nodes and components in contemporary electrical engineering problems.
This project uses Singular Value Decomposition (SVD) to implement a circuit solver that can handle both consistent and inconsistent systems and analyse resistive networks.

The standard Ohm's Law serves as the foundation for the system:

$V = I \times R$

Where:

V: Vector of voltage (known at nodes)

I: Current vector (problems to be solved)

R: Circuit connectivity resistance matrix

The project shows how SVD can calculate least-squares solutions for inconsistent systems and exact solutions for consistent systems

Methodology:

• Forming the Circuit Matrix:
Each node in the circuit is modeled, and the resistance matrix R is constructed.
Each equation corresponds to KCL/KVL constraints, forming the linear system:

$$A \cdot I = V$$

Where $A = R$, I = currents, V = applied voltages

• Applying SVD:

- Decompose the matrix:

  $A=U\Sigma V^T$

- U: Orthonormal basis of voltage space (nodes)
- V: Orthonormal basis of current space
- Σ: Singular values representing the stiffness of the circuit

• Solving the System:

- Consistent systems: $I=A^+V$ gives exact currents

- Inconsistent systems: $I=A^{+}V$ gives least-squares currents, minimizing the residual $|V-AI|$

• Residual and Consistency Check:

- Residual vector: $r=V-A\cdot I$
- Norm of residual: $\|r\|_2$determines if the system is consistent or inconsistent

• Interpretation of Singular Values:

- Large singular values → stiff circuit (strong constraints, well-defined currents)
- Small singular values → near short-circuit or rank-deficient network (ill-conditioned)

Language Used:

- Python

Why SVD?

Typical linear solvers break when:

- Matrix is singular
- Equations are dependent
- Circuit is overdetermined ($m > n$)
- Circuit is underdetermined ($m < n$)
- System is inconsistent (no exact solution)

But SVD always works, because every matrix has a decomposition:

- $A=U\Sigma V^{T}$

The pseudo inverse is:

- $A^{+}=V\Sigma^{+}U^{T}$

This gives a correct:

- Exact solution when consistent
- Least-squares solution when inconsistent
- Minimum-norm solution when underdetermined

This makes SVD the most stable and general-purpose method for solving circuits

Derivation:

Consider a resistive network
Using KCL at each node:

- $\sum I_{incoming} = \sum I_{outgoing}$

Using Ohm's Law:

- $I = (V_i - V_j)/R_{ij}$

Substituting currents into KCL gives a linear equation for each node:

- $\sum (V_i - V_j)/R_{ij} = I_{inj}$

This produces a conductance matrix, which is your matrix **A**

Thus the equation becomes:

- $A.I = V$

So in my code:

- A → Resistances / conductances
- x → Currents (I)
- b → Applied voltages (V)

SVD Mathematical Interpretation:

SVD splits the matrix into three parts:

- $A = U\Sigma V^T$

U (Voltage Modes): Columns represent voltage distribution patterns across the network.

V (Current Modes): Columns represent independent current flow patterns.

Σ (Singular Values): Represent the strength of each electrical mode:

- Small σ → Open Circuit (Low Current)
- Large σ → Short Circuit (High Current)
- Zero σ → Dependent equations → Infinite solutions

Pseudo Inverse Formula:

- $A^{+}=V\Sigma^{+}U^{T}$

  And therefore:

  $$I=x=A^{+}b$$

  A^{+} is the mathematical inverse of resistance matrix when A is not invertible

Code:

```
#importing module numpy as we need matrix operations
import numpy as np
#defining function to solve linear system Ax=b using manual SVD
def solve_system_manual_svd(A,b,tol=1e-10):
    #Converting A and b to numpy arrays
    A=np.array(A,dtype=float)
    b=np.array(b,dtype=float)
    #getting dimensions of A
    m,n=A.shape
    print("\n================================")
    #printing A and b
    print("A=\n",A)
    print("b=\n",b)
    #1)Compute A^T.A to caluclate V matrix
    AtA=A.T@A
    print("\nA^T.A=\n",AtA)
    #2)Eigen-decomposition of ATA => V
    #eigenvalues and eigenvectors
    evalsA,evecsA=np.linalg.eig(AtA)
    #sorting the eigenvalues and eigenvectors in descending order
    idx=np.argsort(evalsA)[::-1]
    evalsA=evalsA[idx]
    evecsA=evecsA[:,idx]
    #we have V from eigenvectors of A^T A
    V=evecsA
    print("\nV from eigenvectors of A^T.A:\n",V)
    print("Eigenvalues λ_i=\n",evalsA)
    #3)Singular values
    sig=np.sqrt(np.maximum(evalsA,0))
    print("\nSingular values σ_i=\n",sig)
    #Constructing Sigma matrix
    Sigma=np.zeros((m,n))
    #Filling diagonal of Sigma with singular values
    for i in range(min(m,n)):
        Sigma[i,i]=sig[i]
    #printing Sigma
    print("\nΣ=\n",Sigma)
    #Sigma pseudoinverse
    Sigma_pinv=np.zeros((n,m))
    #Filling diagonal of Sigma_pinv with reciprocals of non-zero singular values
    for i in range(min(m,n)):
        if sig[i]>tol:
            Sigma_pinv[i,i]=1/sig[i]
    #printing Sigma_pinv
    print("\nΣ⁺=\n",Sigma_pinv)
    #4)Compute U using AA^T eigenvectors
    AAt=A@A.T
    print("\nA.A^T=\n",AAt)
```

```python
    #eigen-decomposition of AAt => U
    evalsU,evecsU=np.linalg.eig(AAt)
    #sorting the eigenvalues and eigenvectors in descending order
    idx2=np.argsort(evalsU)[::-1]
    evalsU=evalsU[idx2]
    evecsU=evecsU[:,idx2]
    #we have U from eigenvectors of A A^T
    U=evecsU
    print("\nU from eigenvectors of A.A^T:\n",U)
    print("Eigenvalues λ_i=\n",evalsU)
    #A⁺=V Σ⁺ U^T
    A_pinv=V@Sigma_pinv@U.T
    print("\nA⁺=V.Σ⁺.U^T=\n",A_pinv)
    #x=A⁺b
    x=A_pinv@b
    print("\nx=A⁺.b=\n",x)
    #7)Residual
    r=b-A@x
    normr=np.linalg.norm(r)
    print("\nResidual (r=b-Ax)=\n",r)
    print("||r||₂=",normr)
    #Checking consistency
    if normr<1e-8:
        print("\nSYSTEM CONSISTENT")
    else:
        print("\nSYSTEM INCONSISTENT")

    print("==============================\n")
    return x,normr
#for loading UF sparse matrix files
def load_uf_matrix(A_path, b_path):
    print("\nLoading UF Sparse Matrix Files...")
    print("A file:", A_path)
    print("b file:", b_path)
    # Reading the matrix files using mmread
    A = mmread(A_path)
    b = mmread(b_path)
    # Convert to dense only if matrix is sparse
    if hasattr(A, "toarray"):
        A = A.toarray()
    if hasattr(b, "toarray"):
        b = b.toarray()
    b = np.array(b).flatten()
    #print shapes
    print("Loaded UF matrix shapes -> A:", A.shape, ", b:", b.shape)
    return A, b
# ================================
# LOAD CIRCUIT_1 FROM UF DATABASE
# ================================
UF_A_PATH = r"C:\All Projects\Matrix Computation (5th Sem)\circuit_1_x.mtx"
UF_B_PATH = r"C:\All Projects\Matrix Computation (5th Sem)\circuit_1_b.mtx"
A_uf, b_uf = load_uf_matrix(UF_A_PATH, UF_B_PATH)
print("\nSOLVING UF CIRCUIT_1 USING MANUAL SVD...\n")
solve_system_manual_svd(A_uf, b_uf)
print("\nCONSISTENT (m=n)")
solve_system_manual_svd(A1, b1)
# m = n INCONSISTENT
A2 = [
    [1,2,3],
    [2,4,6],
    [1,1,1]
]
b2 = [6,12,5]  # inconsistent because eq1 & eq2 proportional but b not proportional

print("\nINCONSISTENT (m=n)")
solve_system_manual_svd(A2, b2)
# m > n CONSISTENT (overdetermined but consistent)
A3 = [
    [1,2],
    [2,4],
    [3,6]
]
b3 = [5,10,15]  # perfectly proportional → consistent

print("\nCONSISTENT (m>n)")
solve_system_manual_svd(A3, b3)
# m > n INCONSISTENT
A4 = [
    [1,2],
    [2,4],
    [3,6]
]
b4 = [5,10,20]   # breaks proportionality → inconsistent

print("\nINCONSISTENT (m>n)")
solve_system_manual_svd(A4, b4)
```

```python
# m < n CONSISTENT (infinite solutions)
A5 = [
    [1,2,0,0],
    [0,1,1,1]
]
b5 = [5,3]  # always solvable → consistent

print("\nCONSISTENT (m<n)")
solve_system_manual_svd(A5, b5)
# m < n INCONSISTENT
A6 = [
    [1,2,0,0],
    [0,1,1,1]
]
b6 = [5,300]   # second equation cannot satisfy → inconsistent

print("\nINCONSISTENT (m<n)")
solve_system_manual_svd(A6, b6)
# 50x50 CONSISTENT MATRIX
np.random.seed(0)
R = np.random.randint(1,5,(50,50))
A7 = (R + R.T) + 50*np.eye(50)     # SPD → invertible → consistent
b7 = np.random.randint(1,10,50)

print("\nCONSISTENT (50x50)")
solve_system_manual_svd(A7, b7)
# 50x50 INCONSISTENT MATRIX
np.random.seed(1)
A8 = np.random.randint(1, 10, (50, 50))
A8[0] = A8[1].copy()
b8 = np.random.randint(1, 10, 50)
b8[0] = 1
b8[1] = 9
print("\nINCONSISTENT (50x50)")
solve_system_manual_svd(A8, b8)
```

Output:

The output for the Matrix taken from University of Florida:

```
Loading UF Sparse Matrix Files...
A file: C:\All Projects\Matrix Computation (5th Sem)\circuit_1_x.mtx
b file: C:\All Projects\Matrix Computation (5th Sem)\circuit_1_b.mtx
Loaded UF matrix shapes -> A: (2624, 1) , b: (2624,)

SOLVING UF CIRCUIT_1 USING MANUAL SVD...


==============================
A=
 [[ 1.        ]
 [ 0.        ]
 [ 0.        ]
 ...
 [ 0.00968308]
 [ 1.00914698]
 [-0.00982259]]
b=
 [ 0.00000000e+00  1.00000000e+00  0.00000000e+00 ... -2.40658842e-17
  4.90343172e-02  1.00914698e+00]

A^T.A=
 [[360.98256495]]

V from eigenvectors of A^T.A:
 [[1.]]
Eigenvalues λ_i=
 [360.98256495]

Singular values σ_i=
 [18.99954118]

Σ=
 [[18.99954118]
 [ 0.        ]
 [ 0.        ]
 ...
 [ 0.        ]
 [ 0.        ]
 [ 0.        ]]

Σ⁺=
 [[0.05263285 0.         0.         ... 0.         0.         0.        ]]

A.A^T=
 [[ 1.00000000e+00  0.00000000e+00  0.00000000e+00 ...  9.68307968e-03
   1.00914698e+00 -9.82258733e-03]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 ...  0.00000000e+00
   0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 ...  0.00000000e+00
   0.00000000e+00  0.00000000e+00]
 ...
 [ 9.68307968e-03  0.00000000e+00  0.00000000e+00 ...  9.37620321e-05
   9.77165066e-03 -9.51128958e-05]
 [ 1.00914698e+00  0.00000000e+00  0.00000000e+00 ...  9.77165066e-03
   1.01837764e+00 -9.91243438e-03]
 [-9.82258733e-03  0.00000000e+00  0.00000000e+00 ... -9.51128958e-05
  -9.91243438e-03  9.64832218e-05]]

U from eigenvectors of A.A^T:
 [[ 5.26328500e-02+0.00000000e+00j  1.73633380e-02+0.00000000e+00j
   3.56516900e-03-7.49553413e-03j ...  3.50554304e-03+3.00990985e-03j
  -1.62312828e-02-2.14649938e-03j -1.62312828e-02+2.14649938e-03j]
 [ 0.00000000e+00+0.00000000e+00j  0.00000000e+00+0.00000000e+00j
   0.00000000e+00+0.00000000e+00j ...  0.00000000e+00-0.00000000e+00j
   0.00000000e+00+0.00000000e+00j  0.00000000e+00-0.00000000e+00j]
 [ 0.00000000e+00+0.00000000e+00j  0.00000000e+00+0.00000000e+00j
   0.00000000e+00+0.00000000e+00j ...  0.00000000e+00-0.00000000e+00j
   0.00000000e+00+0.00000000e+00j  0.00000000e+00-0.00000000e+00j]
 ...
 [ 5.09648080e-04+0.00000000e+00j  8.83534786e-05+0.00000000e+00j
   7.88008611e-05-4.89236189e-04j ... -4.15844100e-04-1.03319406e-03j
   1.02837543e-03-1.68118721e-04j  1.02837543e-03+1.68118721e-04j]
 [ 5.31142818e-02+0.00000000e+00j -4.74804255e-02+0.00000000e+00j
   7.21246808e-02+3.07708889e-02j ... -8.54088275e-02+8.47756221e-03j
   7.51260289e-02+3.90987881e-03j  7.51260289e-02-3.90987881e-03j]
 [-5.16990765e-04+0.00000000e+00j  1.54356892e-04+0.00000000e+00j
   2.07652756e-04-3.61441883e-05j ...  2.90075754e-04-3.59992681e-04j
   2.99087380e-04-8.43424644e-05j  2.99087380e-04+8.43424644e-05j]]
Eigenvalues λ_i=
 [ 3.60982565e+02+0.00000000e+00j  4.34501980e-14+0.00000000e+00j
  3.29367744e-14+1.61545562e-14j ... -3.26854397e-14-2.49643210e-14j
 -3.86186852e-14+3.85756292e-15j -3.86186852e-14-3.85756292e-15j]

A⁺=V.Σ⁺.U^T=
 [[ 2.77021689e-03+0.j  0.00000000e+00+0.j  0.00000000e+00+0.j ...
   2.68242309e-05+0.j  2.79555603e-03+0.j -2.72106974e-05+0.j]]


A⁺=V.Σ⁺.U^T=
 [[ 2.77021689e-03+0.j  0.00000000e+00+0.j  0.00000000e+00+0.j ...
   2.68242309e-05+0.j  2.79555603e-03+0.j -2.72106974e-05+0.j]]

x=A⁺.b=
 [0.00011545+0.j]

Residual (r=b-Ax)=
 [-1.15446537e-04+0.j  1.00000000e+00+0.j  0.00000000e+00+0.j ...
 -1.11787802e-06+0.j  4.89178147e-02+0.j  1.00914812e+00+0.j]
||r||₂= 3.189131862990722

SYSTEM INCONSISTENT
==============================
```

The output for the Matrix hardcoded in the code:

```
CONSISTENT (m=n)

==============================
A=
 [[ 5. -1.  0.  0.  0.]
 [-1.  4. -1.  0.  0.]
 [ 0. -1.  3. -1.  0.]
 [ 0.  0. -1.  2. -1.]
 [ 0.  0.  0. -1.  1.]]
b=
 [10.  5.  0.  0.  0.]

A^T.A=
 [[26. -9.  1.  0.  0.]
 [-9. 18. -7.  1.  0.]
 [ 1. -7. 11. -5.  1.]
 [ 0.  1. -5.  6. -3.]
 [ 0.  0.  1. -3.  2.]]

V from eigenvectors of A^T.A:
 [[-0.77704718  0.54249497 -0.30151134 -0.10387993  0.0140272 ]
 [ 0.57979962  0.42980092 -0.60302269 -0.33321906  0.06657528]
 [-0.23537429 -0.63177876 -0.30151134 -0.63177876  0.23537429]
 [ 0.06657528  0.33321906  0.60302269 -0.42980092  0.57979962]
 [-0.0140272  -0.10387993 -0.30151134  0.54249497  0.77704718]]
Eigenvalues λ_i=
 [33.01832654 17.70501608  9.          3.21222139  0.06443599]

Singular values σ_i=
 [5.74615755 4.20773289 3.         1.79226711 0.25384245]

Σ=
 [[5.74615755 0.         0.         0.         0.        ]
 [0.         4.20773289 0.         0.         0.        ]
 [0.         0.         3.         0.         0.        ]
 [0.         0.         0.         1.79226711 0.        ]
 [0.         0.         0.         0.         0.25384245]]

Σ⁺=
 [[0.17402934 0.         0.         0.         0.        ]
 [0.         0.23765767 0.         0.         0.        ]
 [0.         0.         0.33333333 0.         0.        ]
 [0.         0.         0.         0.55795255 0.        ]
 [0.         0.         0.         0.         3.93945135]]

A.A^T=
 [[26. -9.  1.  0.  0.]
 [-9. 18. -7.  1.  0.]
 [ 1. -7. 11. -5.  1.]
 [ 0.  1. -5.  6. -3.]
 [ 0.  0.  1. -3.  2.]]

U from eigenvectors of A.A^T:
 [[-0.77704718  0.54249497 -0.30151134 -0.10387993  0.0140272 ]
 [ 0.57979962  0.42980092 -0.60302269 -0.33321906  0.06657528]
 [-0.23537429 -0.63177876 -0.30151134 -0.63177876  0.23537429]
 [ 0.06657528  0.33321906  0.60302269 -0.42980092  0.57979962]
 [-0.0140272  -0.10387993 -0.30151134  0.54249497  0.77704718]]
Eigenvalues λ_i=
 [33.01832654 17.70501608  9.          3.21222139  0.06443599]

A⁺=V.Σ⁺.U^T=
 [[0.21212121 0.06060606 0.03030303 0.03030303 0.03030303]
 [0.06060606 0.3030303  0.15151515 0.15151515 0.15151515]
 [0.03030303 0.15151515 0.57575758 0.57575758 0.57575758]
 [0.03030303 0.15151515 0.57575758 1.57575758 1.57575758]
 [0.03030303 0.15151515 0.57575758 1.57575758 2.57575758]]

x=A⁺.b=
 [2.42424242 2.12121212 1.06060606 1.06060606 1.06060606]

Residual (r=b-Ax)=
 [ 3.55271368e-15  0.00000000e+00  5.32907052e-15 -1.99840144e-15
 -1.33226763e-15]
||r||₂= 6.840271371877088e-15

SYSTEM CONSISTENT
==============================


INCONSISTENT (m=n)

==============================
A=
 [[1. 2. 3.]
 [2. 4. 6.]
 [1. 1. 1.]]
b=
 [ 6. 12.  5.]
```

```
A^T.A=
 [[ 6. 11. 16.]
 [11. 21. 31.]
 [16. 31. 46.]]

V from eigenvectors of A^T.A:
 [[-0.27993046 -0.8688914   0.40824829]
 [-0.5376403  -0.21041921 -0.81649658]
 [-0.79535014  0.44805299  0.40824829]]
Eigenvalues λ_i=
 [7.25867012e+01 4.13298848e-01 3.87065493e-15]

Singular values σ_i=
 [8.51978293e+00 6.42883231e-01 6.22145877e-08]

Σ=
 [[8.51978293e+00 0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 6.42883231e-01 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 6.22145877e-08]]

Σ⁺=
 [[1.17373882e-01 0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 1.55549243e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 1.60734007e+07]]

A.A^T=
 [[14. 28.  6.]
 [28. 56. 12.]
 [ 6. 12.  3.]]

U from eigenvectors of A.A^T:
 [[ 4.39126385e-01 -8.46641469e-02  8.94427191e-01]
 [ 8.78252771e-01 -1.69328294e-01 -4.47213595e-01]
 [ 1.89314788e-01  9.81916448e-01  1.35192587e-15]]
Eigenvalues λ_i=
 [7.25867012e+01 4.13298848e-01 5.63150368e-15]

A⁺=V.Σ⁺.U^T=
 [[ 5.86917620e+06 -2.93458785e+06 -1.33333332e+00]
 [-1.17383522e+07  5.86917610e+06 -3.33333351e-01]
 [ 5.86917600e+06 -2.93458825e+06  6.66666676e-01]]

x=A⁺.b=
 [-3.66666663 -1.66666674  0.33333336]

Residual (r=b-Ax)=
 [12.00000002 24.00000004 10.00000001]
||r||₂= 28.635642172735633

SYSTEM INCONSISTENT
==============================


CONSISTENT (m>n)

==============================
A=
 [[1. 2.]
 [2. 4.]
 [3. 6.]]
b=
 [ 5. 10. 15.]

A^T.A=
 [[14. 28.]
 [28. 56.]]

V from eigenvectors of A^T.A:
 [[-0.4472136  -0.89442719]
 [-0.89442719  0.4472136 ]]
Eigenvalues λ_i=
 [70.  0.]

Singular values σ_i=
 [8.36660027 0.        ]

Σ=
 [[8.36660027 0.        ]
 [0.         0.        ]
 [0.         0.        ]]

Σ⁺=
 [[0.11952286 0.         0.        ]
 [0.         0.         0.        ]]

A.A^T=
 [[ 5. 10. 15.]
 [10. 20. 30.]
 [15. 30. 45.]]
```

```
U from eigenvectors of A.A^T:
 [[-0.26726124 -0.78958632  0.13876693]
 [-0.53452248 -0.35548382 -0.84472653]
 [-0.80178373  0.50018466  0.51689538]]
Eigenvalues λ_i=
 [7.00000000e+01 3.69982769e-15 7.08998356e-16]

A⁺=V.Σ⁺.U^T=
 [[0.01428571 0.02857143 0.04285714]
 [0.02857143 0.05714286 0.08571429]]

x=A⁺.b=
 [1. 2.]

Residual (r=b-Ax)=
 [8.88178420e-16 1.77635684e-15 1.77635684e-15]
||r||₂= 2.6645352591003757e-15

SYSTEM CONSISTENT
==============================


INCONSISTENT (m>n)

==============================
A=
 [[1. 2.]
 [2. 4.]
 [3. 6.]]
b=
 [ 5. 10. 20.]

A^T.A=
 [[14. 28.]
 [28. 56.]]

V from eigenvectors of A^T.A:
 [[-0.4472136  -0.89442719]
 [-0.89442719  0.4472136 ]]
Eigenvalues λ_i=
 [70.  0.]

Singular values σ_i=
 [8.36660027 0.        ]

Σ=
 [[8.36660027 0.        ]
 [0.         0.        ]
 [0.         0.        ]]

Σ⁺=
 [[0.11952286 0.         0.        ]
 [0.         0.         0.        ]]

A.A^T=
 [[ 5. 10. 15.]
 [10. 20. 30.]
 [15. 30. 45.]]

U from eigenvectors of A.A^T:
 [[-0.26726124 -0.78958632  0.13876693]
 [-0.53452248 -0.35548382 -0.84472653]
 [-0.80178373  0.50018466  0.51689538]]
Eigenvalues λ_i=
 [7.00000000e+01 3.69982769e-15 7.08998356e-16]

A⁺=V.Σ⁺.U^T=
 [[0.01428571 0.02857143 0.04285714]
 [0.02857143 0.05714286 0.08571429]]

x=A⁺.b=
 [1.21428571 2.42857143]

Residual (r=b-Ax)=
 [-1.07142857 -2.14285714  1.78571429]
||r||₂= 2.988071523335985

SYSTEM INCONSISTENT
==============================


CONSISTENT (m<n)

==============================
A=
 [[1. 2. 0. 0.]
 [0. 1. 1. 1.]]
b=
 [5. 3.]
```

```
A^T.A=
 [[1. 2. 0. 0.]
 [2. 5. 1. 1.]
 [0. 1. 1. 1.]
 [0. 1. 1. 1.]]

V from eigenvectors of A^T.A:
 [[-0.3406402   0.39584281  0.85280287  0.        ]
 [-0.89180762  0.1511985  -0.42640143  0.        ]
 [-0.21052722 -0.64048711  0.21320072 -0.70710678]
 [-0.21052722 -0.64048711  0.21320072  0.70710678]]
Eigenvalues λ_i=
 [6.23606798e+00 1.76393202e+00 5.43456339e-17 0.00000000e+00]

Singular values σ_i=
 [2.49721204e+00 1.32813103e+00 7.37194912e-09 0.00000000e+00]

Σ=
 [[2.49721204 0.         0.         0.        ]
 [0.         1.32813103 0.         0.        ]]

Σ⁺=
 [[0.40044657 0.        ]
 [0.         0.75293776]
 [0.         0.        ]
 [0.         0.        ]]

A.A^T=
 [[5. 2.]
 [2. 3.]]

U from eigenvectors of A.A^T:
 [[ 0.85065081 -0.52573111]
 [ 0.52573111  0.85065081]]
Eigenvalues λ_i=
 [6.23606798 1.76393202]

A⁺=V.Σ⁺.U^T=
 [[-0.27272727  0.18181818]
 [-0.36363636 -0.09090909]
 [ 0.18181818 -0.45454545]
 [ 0.18181818 -0.45454545]]

x=A⁺.b=
 [-0.81818182 -2.09090909 -0.45454545 -0.45454545]
```

```
Residual (r=b-Ax)=
 [10.  6.]
||r||₂= 11.661903789690601

SYSTEM INCONSISTENT
==============================


INCONSISTENT (m<n)

==============================
A=
 [[1. 2. 0. 0.]
 [0. 1. 1. 1.]]
b=
 [  5. 300.]

A^T.A=
 [[1. 2. 0. 0.]
 [2. 5. 1. 1.]
 [0. 1. 1. 1.]
 [0. 1. 1. 1.]]

V from eigenvectors of A^T.A:
 [[-0.3406402   0.39584281  0.85280287  0.        ]
 [-0.89180762  0.1511985  -0.42640143  0.        ]
 [-0.21052722 -0.64048711  0.21320072 -0.70710678]
 [-0.21052722 -0.64048711  0.21320072  0.70710678]]
Eigenvalues λ_i=
 [6.23606798e+00 1.76393202e+00 5.43456339e-17 0.00000000e+00]

Singular values σ_i=
 [2.49721204e+00 1.32813103e+00 7.37194912e-09 0.00000000e+00]

Σ=
 [[2.49721204 0.         0.         0.        ]
 [0.         1.32813103 0.         0.        ]]

Σ⁺=
 [[0.40044657 0.        ]
 [0.         0.75293776]
 [0.         0.        ]
 [0.         0.        ]]

A.A^T=
 [[5. 2.]
 [2. 3.]]
```

```
U from eigenvectors of A.A^T:
 [[ 0.85065081 -0.52573111]
 [ 0.52573111  0.85065081]]
Eigenvalues λ_i=
 [6.23606798 1.76393202]

A⁺=V.Σ⁺.U^T=
 [[-0.27272727  0.18181818]
 [-0.36363636 -0.09090909]
 [ 0.18181818 -0.45454545]
 [ 0.18181818 -0.45454545]]

x=A⁺.b=
 [  53.18181818  -29.09090909 -135.45454545 -135.45454545]

Residual (r=b-Ax)=
 [ 10. 600.]
||r||₂= 600.0833275470999

SYSTEM INCONSISTENT
==============================


CONSISTENT (50x50)

==============================
A=
 [[52.  5.  4. ...  6.  7.  8.]
 [ 5. 54.  5. ...  6.  4.  2.]
 [ 4.  5. 52. ...  5.  6.  7.]
 ...
 [ 6.  6.  5. ... 52.  6.  7.]
 [ 7.  4.  6. ...  6. 52.  3.]
 [ 8.  2.  7. ...  7.  3. 56.]]
b=
 [7. 4. 4. 3. 8. 4. 6. 9. 4. 9. 6. 1. 8. 1. 2. 7. 7. 5. 6. 8. 7. 1. 2. 7.
 7. 5. 3. 3. 5. 5. 4. 2. 1. 7. 7. 1. 9. 6. 3. 9. 5. 5. 1. 3. 6. 8. 8. 2.
 4. 3.]

A^T.A=
 [[4074. 1774. 1663. ... 1820. 1935. 2060.]
 [1774. 4338. 1750. ... 1852. 1688. 1490.]
 [1663. 1750. 4098. ... 1753. 1847. 1951.]
 ...
 [1820. 1852. 1753. ... 4037. 1814. 1944.]
 [1935. 1688. 1847. ... 1814. 4042. 1527.]
 [2060. 1490. 1951. ... 1944. 1527. 4489.]]

V from eigenvectors of A^T.A:
 [[ 1.40246981e-01  2.22546041e-02  2.54106053e-01 ... -1.14377167e-01
   1.50796320e-01 -1.54035224e-01]
 [ 1.44358561e-01  1.01967763e-01  5.11392358e-02 ... -1.39909437e-01
  -2.11441108e-04  2.48995853e-01]
 [ 1.42183918e-01 -1.73875324e-01 -7.01278436e-02 ...  1.94817927e-01
  -1.65949794e-01 -3.08100357e-01]
 ...
 [ 1.39137725e-01 -1.31623904e-01  6.08050007e-02 ...  5.52191652e-02
   7.09285556e-02 -7.06078863e-02]
 [ 1.39945639e-01  2.76911165e-02  4.64065705e-02 ...  2.98520324e-01
  -4.35944348e-02  2.36471916e-01]
 [ 1.41345053e-01 -1.27793762e-01  1.77425398e-01 ... -8.06764052e-02
  -5.79866968e-02  5.59690909e-02]]
Eigenvalues λ_i=
 [90410.65106727  5019.95754827  4755.00700994  4565.11042935
  4404.32675942  4303.08276968  4107.30344506  4059.82575878
  3942.45642878  3808.47921837  3781.0728849   3707.22266244
  3542.56755711  3460.67707029  3383.50120938  3276.98037824
  3229.94371936  3087.22123284  2999.15816162  2958.00270553
  2866.58757111  2718.48186998  2652.35512875  2581.92614024
  2513.61918798  2503.96884131  2420.21392498  2346.62196514
  2302.87803184  2245.30467334  2128.90917366  2121.83487031
  2010.1877566   1953.77434379  1888.99850578  1844.98922851
  1807.83559275  1731.50588594  1650.7421473   1568.6476912
  1542.86776452  1461.36020471  1449.38789514  1294.1697933
  1238.14614317  1136.0234536   1117.38835949  1084.45950688
  1010.61665986   911.64967219]

Singular values σ_i=
 [300.68363951  70.85165875  68.95655886  67.56560093  66.36510197
  65.59788693  64.08824732  63.71676199  62.78898334  61.71287725
  61.49042921  60.88696628  59.51947208  58.82751967  58.16787094
  57.24491574  56.83259381  55.56276841  54.76457031  54.38752344
  53.5405227   52.1390628   51.50102066  50.81265728  50.1360069
  50.03967267  49.19566978  48.44194427  47.98831141  47.38464597
  46.14010375  46.06337884  44.83511745  44.2015197   43.46261043
  42.9533378   42.51864994  41.61136727  40.6293262   39.60615724
  39.27935545  38.2277413   38.07082735  35.97457148  35.18730088
  33.70494702  33.42735945  32.93113279  31.79019754  30.19353693]
```

```
Σ=
 [[300.68363951   0.           0.         ...   0.           0.
    0.        ]
 [  0.          70.85165875   0.         ...   0.           0.
    0.        ]
 [  0.           0.          68.95655886 ...   0.           0.
    0.        ]
 ...
 [  0.           0.           0.         ...  32.93113279   0.
    0.        ]
 [  0.           0.           0.         ...   0.          31.79019754
    0.        ]
 [  0.           0.           0.         ...   0.           0.
   30.19353693]]

Σ⁺=
 [[0.00332575 0.         0.         ... 0.         0.         0.        ]
 [0.         0.014114   0.         ... 0.         0.         0.        ]
 [0.         0.         0.01450188 ... 0.         0.         0.        ]
 ...
 [0.         0.         0.         ... 0.0303664  0.         0.        ]
 [0.         0.         0.         ... 0.         0.03145624 0.        ]
 [0.         0.         0.         ... 0.         0.         0.03311967]]

A.A^T=
 [[4074. 1774. 1663. ... 1820. 1935. 2060.]
 [1774. 4338. 1750. ... 1852. 1688. 1490.]
 [1663. 1750. 4098. ... 1753. 1847. 1951.]
 ...
 [1820. 1852. 1753. ... 4037. 1814. 1944.]
 [1935. 1688. 1847. ... 1814. 4042. 1527.]
 [2060. 1490. 1951. ... 1944. 1527. 4489.]]

x=A⁺.b=
 [ 0.07091553  0.00478269 -0.01706541 -0.01364695  0.07962159 -0.01310685
  0.03943787  0.11899821 -0.00843355  0.09380799  0.01450241 -0.0830788
  0.09777619 -0.06456748 -0.02110077  0.0575876   0.08219393  0.01887682
  0.04057963  0.08114608  0.05382814 -0.07591544 -0.05115631  0.05353218
  0.06795374  0.01153288 -0.01982704 -0.0253335   0.02958243  0.02014383
 -0.00401379 -0.04763043 -0.06440405  0.06020098  0.04361868 -0.05302514
  0.09586331  0.02830483 -0.01487443  0.08699143  0.01505441  0.04045922
 -0.0696485  -0.01474396  0.03119047  0.07123915  0.0753993  -0.04411138
 -0.01435539 -0.00811576]

Residual (r=b-Ax)=
 [ 1.07469589e-13  3.55271368e-15  3.81916720e-14  2.53130850e-14
  5.15143483e-14 -8.61533067e-14 -7.19424520e-14 -1.20792265e-13
 -1.42108547e-14  1.06581410e-14 -1.28785871e-13 -1.33226763e-15
 -2.48689958e-14 -9.76996262e-15 -4.17443857e-14 -9.76996262e-15
  1.19904087e-13 -4.44089210e-14 -1.59872116e-14  2.75335310e-14
 -6.03961325e-14 -7.77156117e-15  4.39648318e-14 -3.01980663e-14
  6.75015599e-14  2.84217094e-14  1.68753900e-14 -1.37667655e-14
  8.52651283e-14  3.64153152e-14  3.68594044e-14 -1.25677246e-13
 -6.66133815e-14 -7.99360578e-15  1.02140518e-13  2.55351296e-14
 -1.24344979e-14 -7.81597009e-14 -2.97539771e-14  1.01252340e-13
 -2.13162821e-14  1.59872116e-14 -4.66293670e-15  5.24025268e-14
  2.57571742e-14 -1.59872116e-14 -1.59872116e-14  4.97379915e-14
 -5.32907052e-15 -8.26005930e-14]
||r||₂= 4.067279587084137e-13

SYSTEM CONSISTENT
==============================


INCONSISTENT (50x50)

==============================
A=
 [[4. 8. 8. ... 3. 4. 2.]
 [4. 8. 8. ... 3. 4. 2.]
 [3. 8. 3. ... 2. 2. 9.]
 ...
 [7. 7. 2. ... 4. 3. 3.]
 [5. 7. 7. ... 3. 6. 9.]
 [2. 9. 5. ... 6. 5. 9.]]
b=
 [1. 9. 3. 1. 8. 3. 1. 8. 5. 9. 8. 9. 6. 8. 2. 6. 2. 9. 4. 6. 9. 2. 1. 1.
 2. 6. 9. 6. 3. 9. 3. 6. 5. 4. 3. 6. 8. 2. 2. 3. 1. 7. 5. 3. 9. 7. 6. 9.
 8. 8.]
A^T.A=
 [[1588. 1439. 1288. ... 1146. 1250. 1168.]
 [1439. 1793. 1380. ... 1258. 1241. 1337.]
 [1288. 1380. 1609. ... 1165. 1232. 1301.]
 ...
 [1146. 1258. 1165. ... 1389. 1122. 1086.]
 [1250. 1241. 1232. ... 1122. 1471. 1162.]
 [1168. 1337. 1301. ... 1086. 1162. 1565.]]

V from eigenvectors of A^T.A:
 [[-0.1452384  -0.03267876  0.00811729 ... -0.06002813 -0.05978448
  -0.12065197]
 [-0.15394056 -0.02451524  0.18172843 ...  0.22949747 -0.09235957
   0.01775836]
 [-0.14277447  0.10167616  0.28097454 ... -0.05147246  0.12765693
   0.15201798]
 ...
 [-0.12845628  0.05691043  0.07117377 ...  0.09653019  0.05282432
  -0.12976095]
 [-0.1324054   0.01766821  0.18010757 ...  0.03951527 -0.04481679
   0.19014096]
 [-0.13657655 -0.25428755  0.14502106 ...  0.15107127 -0.05127584
  -0.17115911]]
Eigenvalues λ_i=
 [6.31598035e+04 1.31199842e+03 1.06953893e+03 9.49881854e+02
 9.19415639e+02 9.03593162e+02 8.54720795e+02 7.65353685e+02
 7.05818686e+02 6.84272734e+02 6.68234785e+02 6.07589200e+02
 5.55677471e+02 4.93374344e+02 4.49309761e+02 4.25105302e+02
 4.09329703e+02 3.90930998e+02 3.69084883e+02 3.41162512e+02
 3.12840067e+02 2.93797943e+02 2.84126486e+02 2.45561056e+02
 2.26535331e+02 1.97927557e+02 1.87968093e+02 1.77275201e+02
 1.64186199e+02 1.47677790e+02 1.33543910e+02 1.14049138e+02
 1.01810397e+02 9.01527668e+01 7.91892060e+01 7.48899709e+01
 6.86926072e+01 5.55703665e+01 4.84882237e+01 4.17256623e+01
 3.79097416e+01 3.09984092e+01 1.92204186e+01 8.95863173e+00
 8.17411077e+00 3.15689680e+00 1.79979601e+00 9.72829298e-01
 6.04848453e-01 1.18610639e-14]
```

```
Singular values σ_i=
 [2.51316142e+02 3.62215188e+01 3.27038060e+01 3.08201534e+01
 3.03218673e+01 3.00598264e+01 2.92356083e+01 2.76650264e+01
 2.65672484e+01 2.61586073e+01 2.58502376e+01 2.46493245e+01
 2.35728121e+01 2.22120315e+01 2.11969281e+01 2.06180819e+01
 2.02318982e+01 1.97719751e+01 1.92115820e+01 1.84705850e+01
 1.76872855e+01 1.71405351e+01 1.68560519e+01 1.56703879e+01
 1.50510907e+01 1.40686729e+01 1.37101456e+01 1.33144734e+01
 1.28135163e+01 1.21522751e+01 1.15561200e+01 1.06793791e+01
 1.00901138e+01 9.49488108e+00 8.89883172e+00 8.65389917e+00
 8.28810033e+00 7.45455340e+00 6.96334860e+00 6.45954041e+00
 6.15708873e+00 5.56762150e+00 4.38410978e+00 2.99309735e+00
 2.85904018e+00 1.77676583e+00 1.34156476e+00 9.86321093e-01
 7.77720035e-01 1.08908512e-07]

Σ=
 [[2.51316142e+02 0.00000000e+00 0.00000000e+00 ... 0.00000000e+00
  0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 3.62215188e+01 0.00000000e+00 ... 0.00000000e+00
  0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 3.27038060e+01 ... 0.00000000e+00
  0.00000000e+00 0.00000000e+00]
 ...
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 ... 9.86321093e-01
  0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 ... 0.00000000e+00
  7.77720035e-01 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 ... 0.00000000e+00
  0.00000000e+00 1.08908512e-07]]

Σ'=
 [[3.97905200e-03 0.00000000e+00 0.00000000e+00 ... 0.00000000e+00
  0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 2.76078981e-02 0.00000000e+00 ... 0.00000000e+00
  0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 3.05774808e-02 ... 0.00000000e+00
  0.00000000e+00 0.00000000e+00]
 ...
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 ... 1.01386861e+00
  0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 ... 0.00000000e+00
  1.28580974e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 ... 0.00000000e+00
  0.00000000e+00 9.18201878e+06]]
A.A^T=
 [[1801. 1801. 1392. ... 1486. 1353. 1204.]
 [1801. 1801. 1392. ... 1486. 1353. 1204.]
 [1392. 1392. 1725. ... 1489. 1366. 1303.]
 ...
 [1486. 1486. 1489. ... 1909. 1392. 1330.]
 [1353. 1353. 1366. ... 1392. 1535. 1250.]
 [1204. 1204. 1303. ... 1330. 1250. 1517.]]

U from eigenvectors of A.A^T:
 [[-1.52544430e-01  4.19165695e-01  3.66317437e-02 ...  6.13597149e-02
  -3.64437611e-02 -7.07106781e-01]
 [-1.52544430e-01  4.19165695e-01  3.66317437e-02 ...  6.13597149e-02
  -3.64437611e-02  7.07106781e-01]
 [-1.49403557e-01 -9.31637944e-02 -5.02727530e-02 ... -2.69227376e-01
  -6.17314670e-02 -8.85172461e-14]
 ...
 [-1.59304983e-01 -1.05122958e-01 -1.85907376e-01 ... -8.60254604e-02
  -2.34633893e-01  1.99982532e-13]
 [-1.43061969e-01 -9.54762883e-02  2.47305033e-02 ... -6.17404468e-02
  -2.02626738e-01  8.87304187e-14]
 [-1.36992805e-01 -2.11888944e-01  1.11102731e-01 ... -1.68001172e-02
  -1.77422785e-01 -4.85120126e-14]]
Eigenvalues λ_i=
 [6.31598035e+04 1.31199842e+03 1.06953893e+03 9.49881854e+02
 9.19415639e+02 9.03593162e+02 8.54720795e+02 7.65353685e+02
 7.05818686e+02 6.84272734e+02 6.68234785e+02 6.07589200e+02
 5.55677471e+02 4.93374344e+02 4.49309761e+02 4.25105302e+02
 4.09329703e+02 3.90930998e+02 3.69084883e+02 3.41162512e+02
 3.12840067e+02 2.93797943e+02 2.84126486e+02 2.45561056e+02
 2.26535331e+02 1.97927557e+02 1.87968093e+02 1.77275201e+02
 1.64186199e+02 1.47677790e+02 1.33543910e+02 1.14049138e+02
 1.01810397e+02 9.01527668e+01 7.91892060e+01 7.48899709e+01
 6.86926072e+01 5.55703665e+01 4.84882237e+01 4.17256623e+01
 3.79097416e+01 3.09984092e+01 1.92204186e+01 8.95863173e+00
 8.17411077e+00 3.15689680e+00 1.79979601e+00 9.72829298e-01
 6.04848453e-01 5.04516406e-13]
```

```
A'=V.Σ'.U^T=
 [[ 7.83353121e+05 -7.83353149e+05  6.02616801e-03 ...  4.66297730e-02
   3.11379319e-02  1.79068646e-02]
 [-1.15299098e+05  1.15299125e+05 -3.29428673e-02 ...  6.79315424e-03
   6.57051381e-04  1.84972413e-02]
 [-9.87002280e+05  9.87002243e+05  1.86076210e-03 ... -3.63106303e-02
  -4.34776709e-02 -4.07422140e-02]
 ...
 [ 8.42494721e+05 -8.42494701e+05  4.15760106e-03 ... -4.24424622e-02
  -7.14235138e-02  1.71930796e-02]
 [-1.23452206e+06  1.23452205e+06  2.15122182e-02 ...  5.63121777e-03
  -1.04012628e-03  2.81551645e-02]
 [ 1.11127927e+06 -1.11127925e+06 -2.61494639e-02 ...  1.36968889e-03
   8.59788609e-03  2.23370261e-02]]

x=A'.b=
 [-6.26682496e+06  9.22393346e+05  7.89601793e+06 -1.48714825e+06
 -1.18903606e+06  5.78689065e+06 -7.56748020e+06 -8.81812726e+06
  5.21262166e+06 -1.70314493e+07  1.15599211e+07 -1.19206475e+06
 -1.31358135e+07 -2.13072482e+06 -6.55469842e+05  6.05286605e+05
  4.24846572e+06 -4.91019145e+06  1.72097243e+07 -3.25364727e+06
  5.87544414e+06  6.77074870e+06  8.10809838e+06  2.02737938e+06
 -2.61801383e+06 -1.38628389e+07 -8.20800423e+05 -7.05689896e+06
 -9.08792832e+05 -1.44079070e+07 -5.65611633e+06 -7.98931800e+06
 -5.14643320e+05  3.93357497e+06  4.08200308e+06  3.94855748e+06
  1.20398372e+07  4.52146825e+06 -1.87520007e+06  8.07181136e+06
  1.34320419e+06  7.54906533e+06  1.18511398e+07 -1.12868040e+03
  4.99727378e+05 -5.82126918e+06  1.67076693e+06 -6.73995845e+06
  9.87617643e+06 -8.89023387e+06]

Residual (r=b-Ax)=
 [-4.38894521  3.61105479 -6.98785694 -5.88294822  4.1245558  -0.78650054
 -4.10411366  5.82156932 -0.92290471  5.99606073  2.64304697  1.65217496
  6.37730951 -2.78937773 -8.90477402  2.93766461 -0.85245734  5.54878049
  1.9582376   1.15603851  3.75728791 -6.40266149 -0.04947075 -6.82990839
 -6.17255449  6.24594003 -2.55999213  4.07762431 -0.0622599   4.97997013
 -0.86371732  2.04538915  2.03512572 -2.10373649 -2.20135657  3.64986238
  5.66305974 -3.22992302 -3.00335537 -2.25261132 -7.11603095  3.14903611
 -2.38517462 -3.77575873  4.27829618 -1.51866845 -1.43158498  1.90048547
  0.31480549  3.44344305]
||r||₂= 28.940219055568598

SYSTEM INCONSISTENT
==============================
```

Verification if the Answer Obtained by Code is Correct or Not:

Manual solution for m > n Inconsistent Matrix (A4 Matrix in code)

Q. $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 10 \\ 20 \end{bmatrix}$

Sol → Check if the system is Inconsistent:

$$Ax = b$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 20 \end{bmatrix}$$

$x_1 + 2x_2 = 5, \quad 2x_1 + 4x_2 = 10, \quad 3x_1 + 6x_2 = 20$

$x_1 = 5 - 2x_2, \quad 2(5-2x_2) + 4x_2 = 10$

$10 - 4x_2 + 4x_2 = 10$

$\{10 = 10\}$ → Inconsistent

as x_1 can take any value.

→ Now we will solve this $Ax = b$ using SVD.

$$A = U \Sigma V^T$$

For V: $A^T \cdot A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}_{2\times3} \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}_{3\times2}$

$$A^T \cdot A = \begin{bmatrix} 14 & 28 \\ 28 & 56 \end{bmatrix}_{2\times2}$$

Eigen values of $A^T A$ = 70, 0

Eigen vectors, $\lambda = 70$ → $\begin{bmatrix} -0.4472 \\ -0.8944 \end{bmatrix}$

Eigen vectors, $\lambda = 0$ → $\begin{bmatrix} -0.8944 \\ 0.4472 \end{bmatrix}$

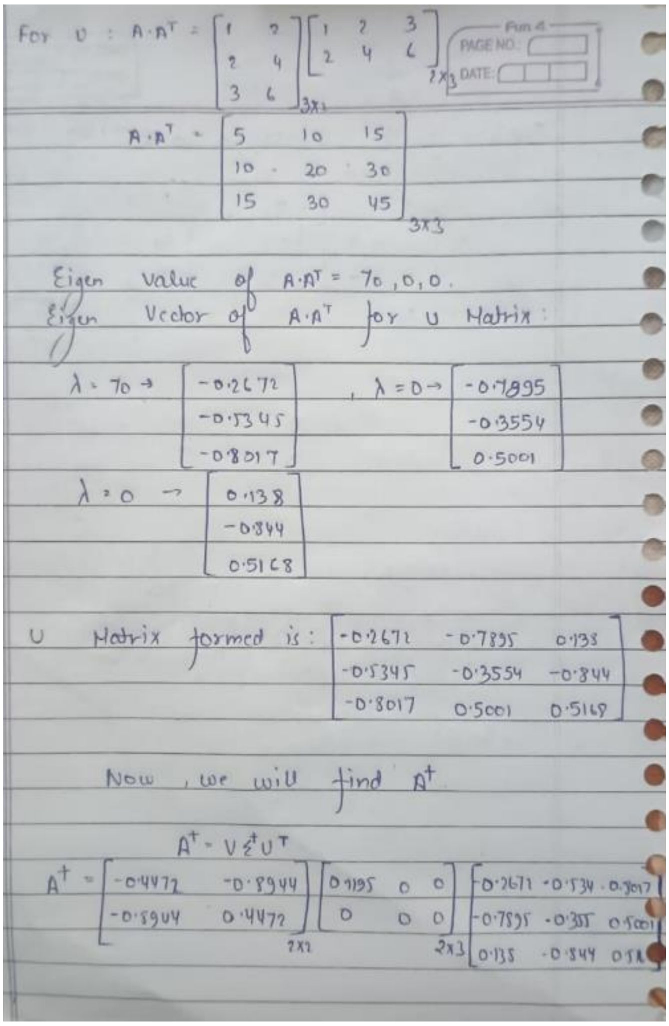
V matrix formed is: $\begin{bmatrix} -0.4472 & -0.8944 \\ -0.8944 & 0.4472 \end{bmatrix}$

$$\sigma = \sqrt{\lambda_i}$$

$\sigma_1 = \sqrt{70}, \sqrt{0} = 8.366, 0.$

$\Sigma = \begin{bmatrix} 8.366 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Sigma^+ = \begin{bmatrix} 8.366 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\Sigma^+ = \begin{bmatrix} 0.1195 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



For U : $A \cdot A^T = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}_{3\times 2} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3}$

$$A \cdot A^T = \begin{bmatrix} 5 & 10 & 15 \\ 10 & 20 & 30 \\ 15 & 30 & 45 \end{bmatrix}_{3\times 3}$$

Eigen value of $A \cdot A^T = 70, 0, 0$.

Eigen Vector of $A \cdot A^T$ for U Matrix:

$$\lambda = 70 \rightarrow \begin{bmatrix} -0.2672 \\ -0.5345 \\ -0.8017 \end{bmatrix}, \quad \lambda = 0 \rightarrow \begin{bmatrix} -0.7895 \\ -0.3554 \\ 0.5001 \end{bmatrix}$$

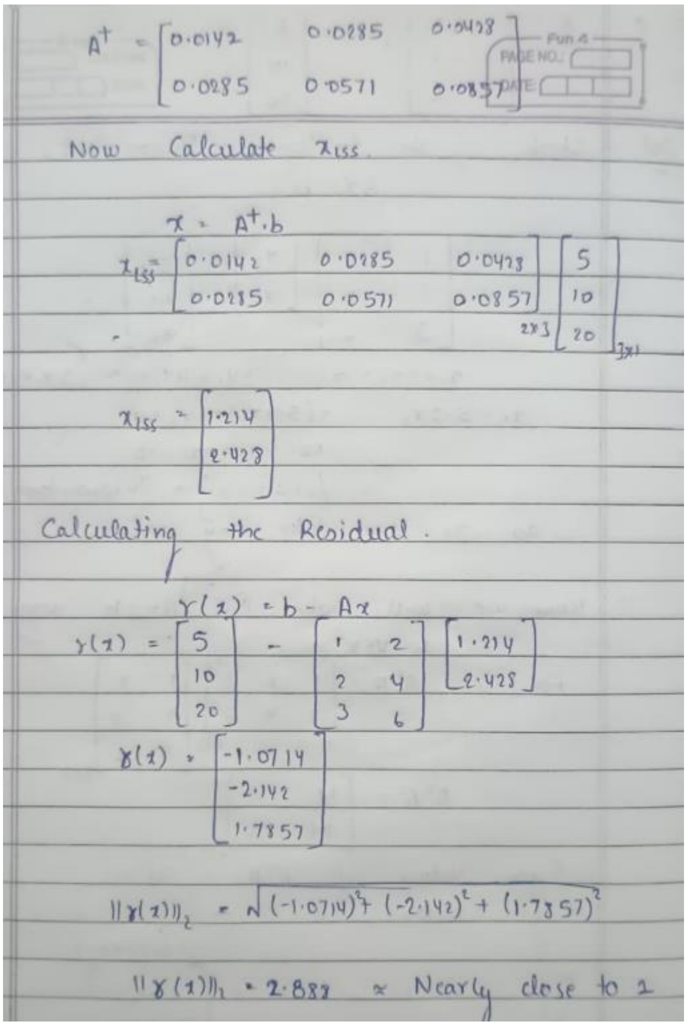
$$\lambda = 0 \rightarrow \begin{bmatrix} 0.138 \\ -0.844 \\ 0.5168 \end{bmatrix}$$

U Matrix formed is : $\begin{bmatrix} -0.2672 & -0.7895 & 0.138 \\ -0.5345 & -0.3554 & -0.844 \\ -0.8017 & 0.5001 & 0.5168 \end{bmatrix}$

Now, we will find A^+

$$A^+ = V \Sigma^+ U^T$$

$$A^+ = \begin{bmatrix} -0.4472 & -0.8944 \\ -0.8944 & 0.4472 \end{bmatrix}_{2\times 2} \begin{bmatrix} 0.1195 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3} \begin{bmatrix} -0.2671 & -0.534 & -0.8017 \\ -0.7895 & -0.355 & 0.5001 \\ 0.135 & -0.844 & 0.5[illegible] \end{bmatrix}$$



$$A^+ = \begin{bmatrix} 0.0142 & 0.0285 & 0.0428 \\ 0.0285 & 0.0571 & 0.0857 \end{bmatrix}$$

Now Calculate x_{LSS}.

$$x = A^+ \cdot b$$

$$x_{LSS} = \begin{bmatrix} 0.0142 & 0.0285 & 0.0428 \\ 0.0285 & 0.0571 & 0.0857 \end{bmatrix}_{2\times 3} \begin{bmatrix} 5 \\ 10 \\ 20 \end{bmatrix}_{3\times 1}$$

$$x_{LSS} = \begin{bmatrix} 1.214 \\ 2.428 \end{bmatrix}$$

Calculating the Residual.

$$r(x) = b - Ax$$

$$r(x) = \begin{bmatrix} 5 \\ 10 \\ 20 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1.214 \\ 2.428 \end{bmatrix}$$

$$r(x) = \begin{bmatrix} -1.0714 \\ -2.142 \\ 1.7857 \end{bmatrix}$$

$$\|r(x)\|_2 = \sqrt{(-1.0714)^2 + (-2.142)^2 + (1.7857)^2}$$

$\|r(x)\|_2 = 2.888 \approx$ Nearly close to 2

=== END OF REPORT ===