Problem Statement 1

Process $1 \to 2$ $\quad (T_1, P_1, V_1) \to (T_2, P_2, V_2)$ [adiabatic process]

$(70^\circ C, 1\,bar, V_1) \to (150^\circ C, P_2, V_2)$

Assume: $n = 1$;

$T_1 = 273.15 + 70 = 343.15\,K$

$T_2 = 273.15 + 150 = 423.15\,K$

$$V_1 = \frac{nRT_1}{P_1} = \frac{1 \times 8.314 \frac{Pa\text{-}m^3}{mol\,K} \times 343.15}{100 \times 10^3\,Pa}$$

$$V_1 = 0.0285\,m^3$$

$PV^\gamma = \text{const.}$ [for adiabatic process]

$\gamma = \frac{C_p}{C_v} = \frac{5}{3} = 1.667$

$$\therefore \quad P\left[\frac{nRT}{P}\right]^\gamma = \text{const.}$$

$$\rightarrow \quad P^{1-\gamma}(k)\,T^\gamma = \text{const.}$$

$$\Rightarrow \quad P^{1-\gamma}T^\gamma = \text{const.}$$

$$\therefore \quad P_2 = P_1\left[\frac{T_1}{T_2}\right]^{\frac{\gamma}{1-\gamma}}$$

$$\therefore\ P_2 = (10^5 Pa)\left[\frac{343.15}{423.15}\right]^{\frac{1.667}{1-1.667}}$$

$$P_2 = 1.6886\times10^5\ Pa$$

$$V_2 = \frac{nRT_2}{P_2} = \frac{1\times 8.314\ \frac{Pa-m^3}{mol\,K}\times 423.15\,K}{1.6886\times10^5\ Pa}$$

$$\boxed{V_2 = 0.0208\ m^3}$$

Process $2\rightarrow3$: Constant Pressure Process.

$T_3 = T_1 = 343.15\,K$ [Constant Temp. Process $3\rightarrow1$]

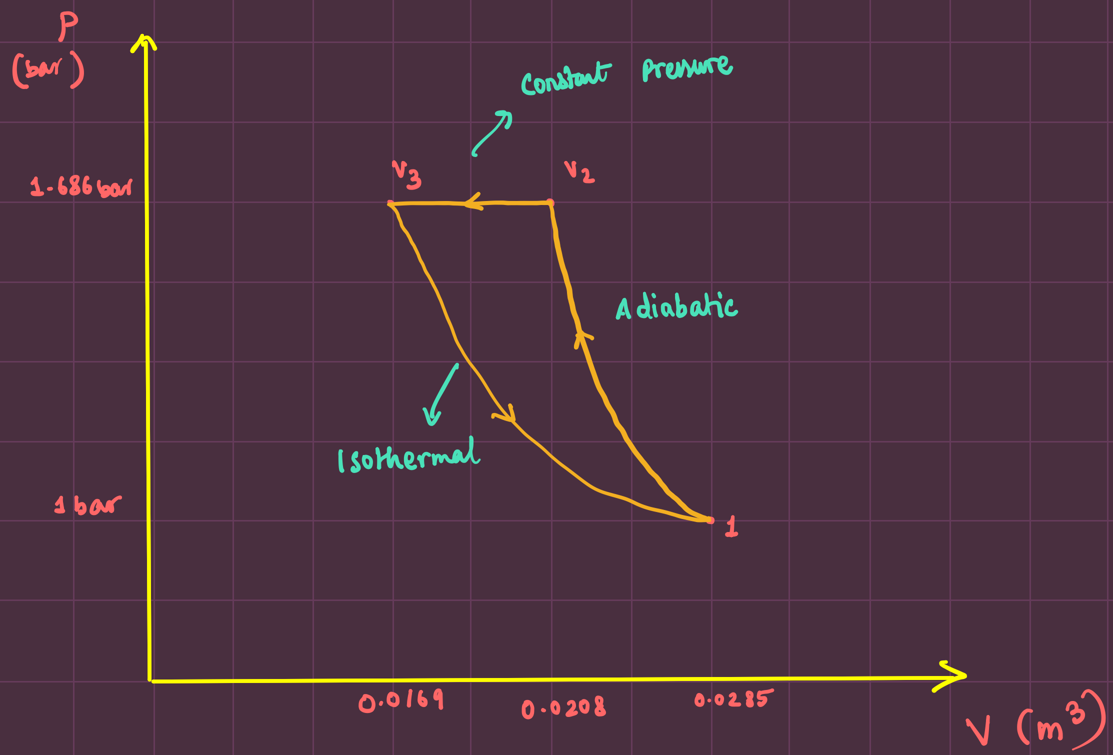
$P_3 = P_2 = 1.6886\times10^5\,Pa$ [Constant Pressure Process]

$$\therefore\ V_3 = \frac{nRT_3}{P_3} = \frac{1\times 8.314\times 343.15}{1.6886\times10^5}$$

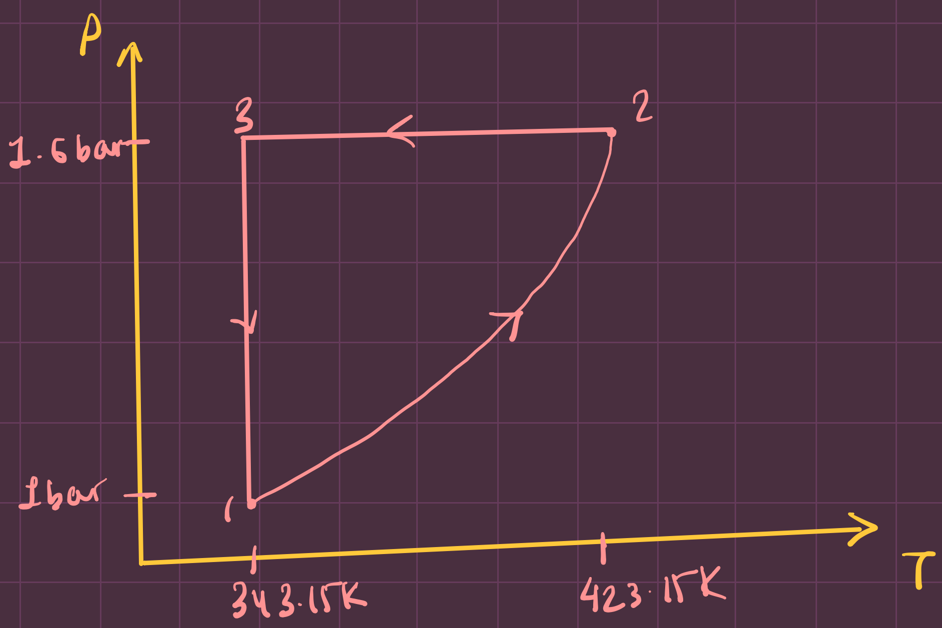
$$= 0.0169\ m^3$$

Plot :→ Reversible process :→

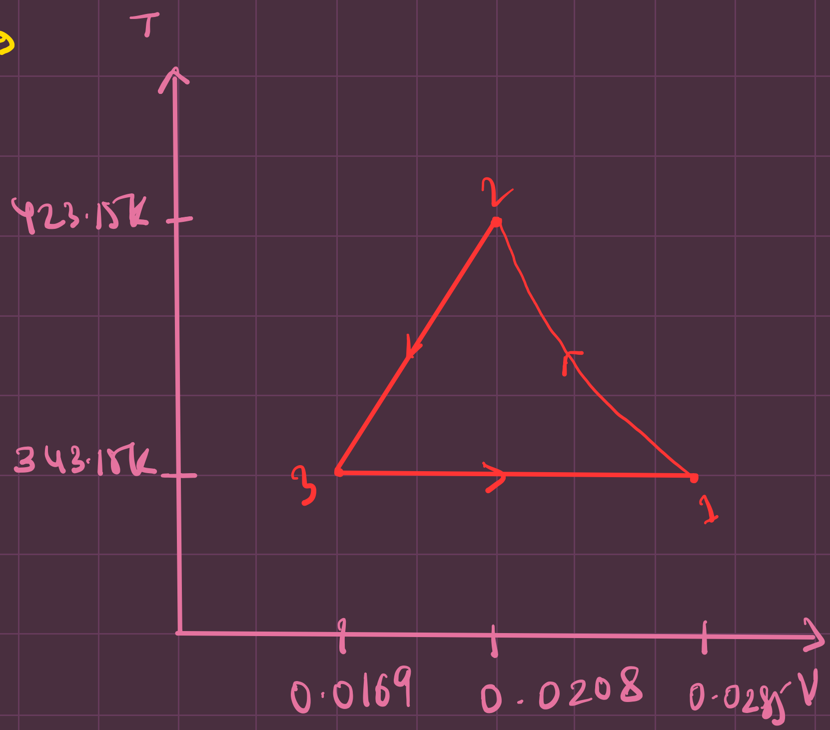
① PV :→



② PT :→



③ TV diagram :→



Calculation of work and heat

$$* \quad W_{1\to2} = \frac{P_2V_2 - P_1V_1}{1-\gamma} = \frac{(1.6886\times10^5\times0.0208) - (10^5\times0.0285)}{1-5/3}$$

Adiabatic

$$= \underline{\underline{-997.68\ J}}$$

$$* \quad W_{2\to3} = P_2(V_3 - V_2) = 1.6886\times10^5\left[0.0169 - 0.0208\right]$$

Isobaric

$$= -665.12\ J$$

* $W_{3\to1} = nRT_1 \log\left[\frac{V_1}{V_3}\right]$

$$= 1 \times 8.314 \times (343.15) \log\left[\frac{0.0285}{0.0169}\right]$$

$$= \underline{\underline{1497\ J}}$$

* for Process $1\to2$: Adiabatic;

$$Q_{1\to2} = 0$$

* for process $2\to3$: Isobaric;

$$Q_{2\to3} = nC_p(T_3 - T_2)$$

$$= 1 \times \frac{5}{2} \times 8.314 \times [343.15 - 423.15]$$

$$Q_{2\to3} = \underline{\underline{-1662.8 J}}$$

* for process $3\to1$: Isothermal;

$$Q_{3\to1} = W_{3\to1} = 1497 J.$$

$\therefore$ * $W_{total, rev} = W_{1\to 2} + W_{2\to 3} + W_{3\to 1}$

$= \underline{-168.15\ J}$

* $Q_{total, rev} = Q_{1\to 2} + Q_{2\to 3} + Q_{3\to 1}$

$= +168.15\ J$

Calculation of internal energy & enthalpy

* $U_{1\to 2} = -W_{1\to 2}$ $[\because Q_{1\to 2} = 0]$

$= \underline{\underline{997.68\ J}}$

* $U_{2\to 3} = nC_v\,(T_3 - T_2)$

$= 1 \times \frac{3}{2} \times 8.314\ [343.15 - 423.15]$

$= \underline{\underline{-997.68\ J}}$

* $U_{3\to 1} = 0$ $[\because T_2 = T_3]$

$\therefore U_{total} = U_{1\to 2} + U_{2\to 3} + U_{3\to 1}$

$= 997.68 + (-997.68) + 0$

$= \underline{\underline{0\ J}}$

$$U_{1\to 2} = nC_p\,(T_2 - T_1)$$

$$= 1 \times \frac{5}{2} \times 8.314 \times [423.15 - 343.15]$$

$$= 1662.8\ J$$

$$U_{2\to 3} = nC_p\,(T_3 - T_2)$$

$$= 1 \times \frac{5}{2} \times 8.314 \times (343.15 - 423.15)$$

$$= -1662.8\ J$$

$$U_{3\to 1} = 0 \qquad [\because\ T_3 = T_1]$$

$$\therefore\ U_{total} = U_{1\to 2} + U_{2\to 3} + U_{3\to 1}$$

$$= 1662.8 + (-1662.8) + 0$$

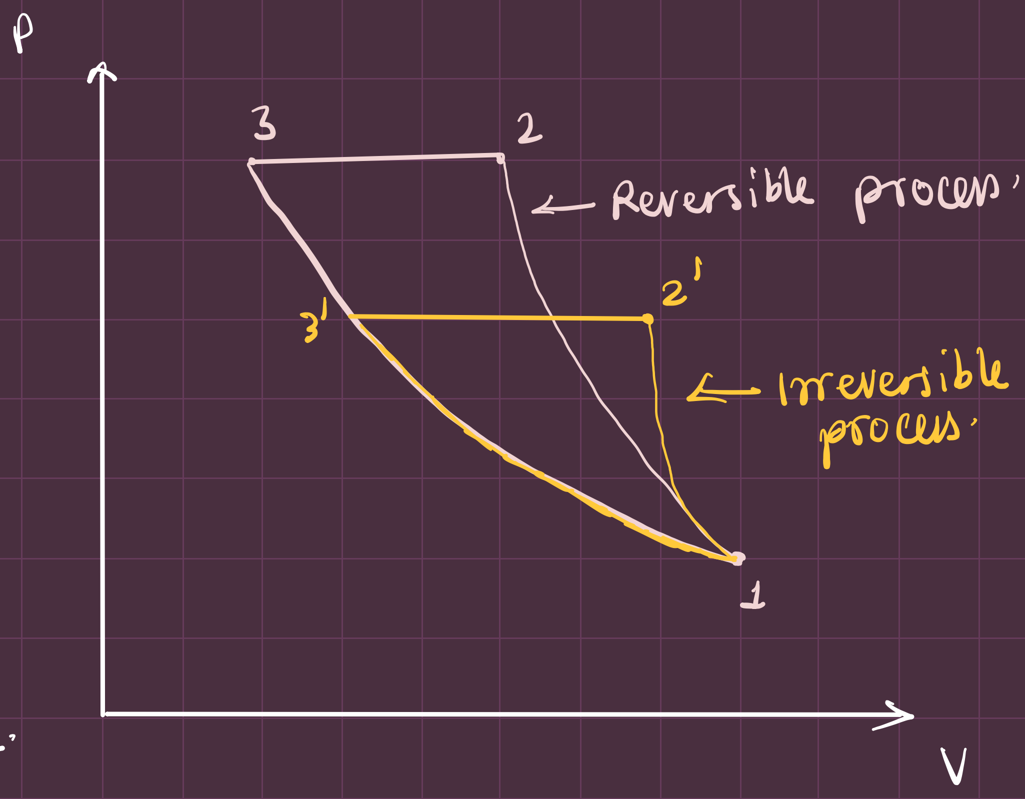
$$= 0\ J$$

Irreversible process

* work efficiency (η) = 0.75.

* Irreversible work is less than reversible work, as some amount of work is not transferred into heat.

(ROUGH DIAGRAM)



IRR. PROCESS WORK.

$$W_{1\to2,irr} = 0.75\, W_{1\to2,rev} = -748.26\,J$$

$$W_{2\to3,irr} = 0.75\, W_{2\to3,rev} = -498.84\,J$$

$$W_{3\to1,irr} = 0.75\, W_{3\to1,rev} = 1210\,J$$

$$\therefore W_{irr,total} = -126.11\,J$$

★ Now, 1^{st} Law of Thermo^{d} should be satisfied even in irrev. process,

★ Also; U & H remains same as they are state dependent and not path dependent.

$$\therefore \quad U_{1\to2} = 997.68 J \qquad H_{1\to2} = 1662.8 J$$
$$U_{2\to3} = -997.68 J \qquad H_{2\to3} = -1662.8 J$$
$$U_{3\to1} = 0 J \qquad H_{3\to1} = 0 J$$

Now; ① $Q_{1\to2} - W_{1\to2} = U_{1\to2}$

$\rightarrow Q_{1\to2} = 997.68 + (-748.26)$

$= \underline{249.42 J}$

② $Q_{2\to3} - W_{2\to3} = U_{2\to3}$

$\rightarrow Q_{2\to3} = -997.68 + [-498.84]$

$= \underline{-1496.52 J}$

③ $Q_{3\to1} - W_{3\to1} = \underbrace{U_{3\to1}}_{=0}$

$\therefore Q_{3\to1} = \underline{1210 J}$

Process 1→2

* $V_{1,irr} = \frac{nRT_1}{P_1} = \frac{1 \times 8.314 \times 343.15}{10^5\ Pa} = \underline{\underline{0.0285\ m^3}}$

$P_{1,irr} = 10^5\ Pa$

$T_{1,irr} = 343.15K$

Process 2→3

*

$$P_{2,irr} = P_{1,irr} + 0.75\left[P_2 - P_{1,irr}\right]$$

$$= \underline{\underline{1.5164 \times 10^5\ Pa}}$$

$T_{2,irr} = 423.15K$

$$V_{2,irr} = \frac{nRT_{2,irr}}{P_{2,irr}} = \frac{1 \times 8.314 \times 423.15}{1.52 \times 10^5}$$

$$= \underline{\underline{0.0232\ m^3}}$$

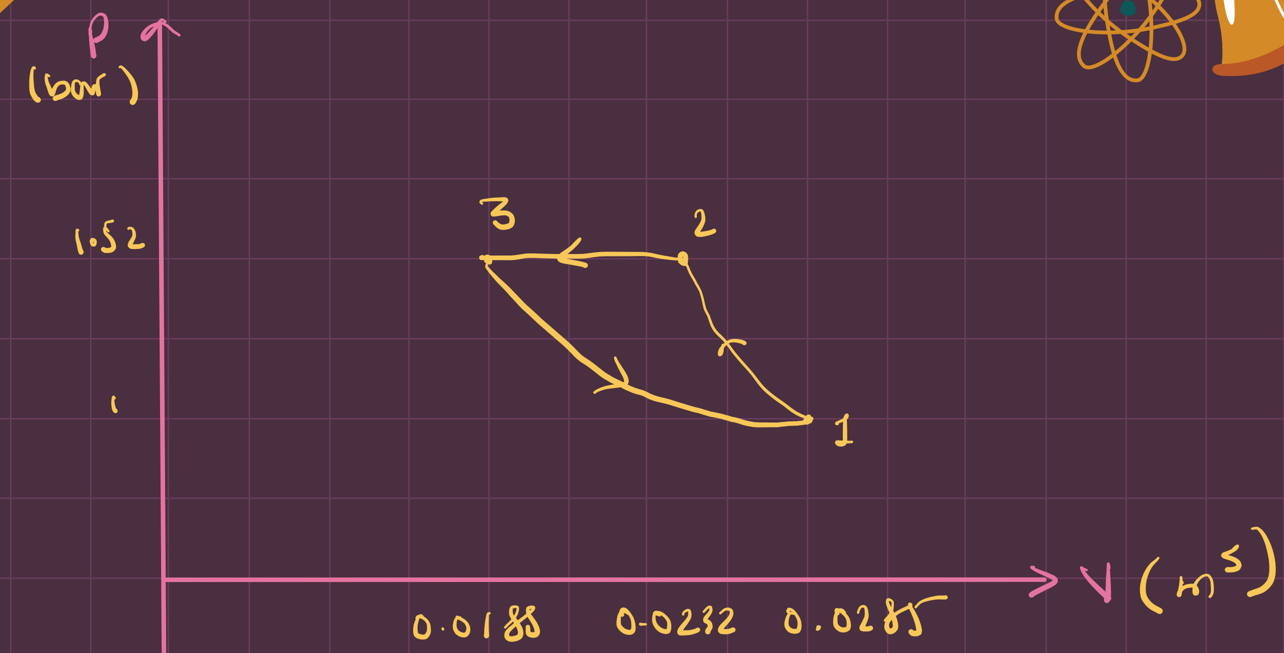
Process 3→1

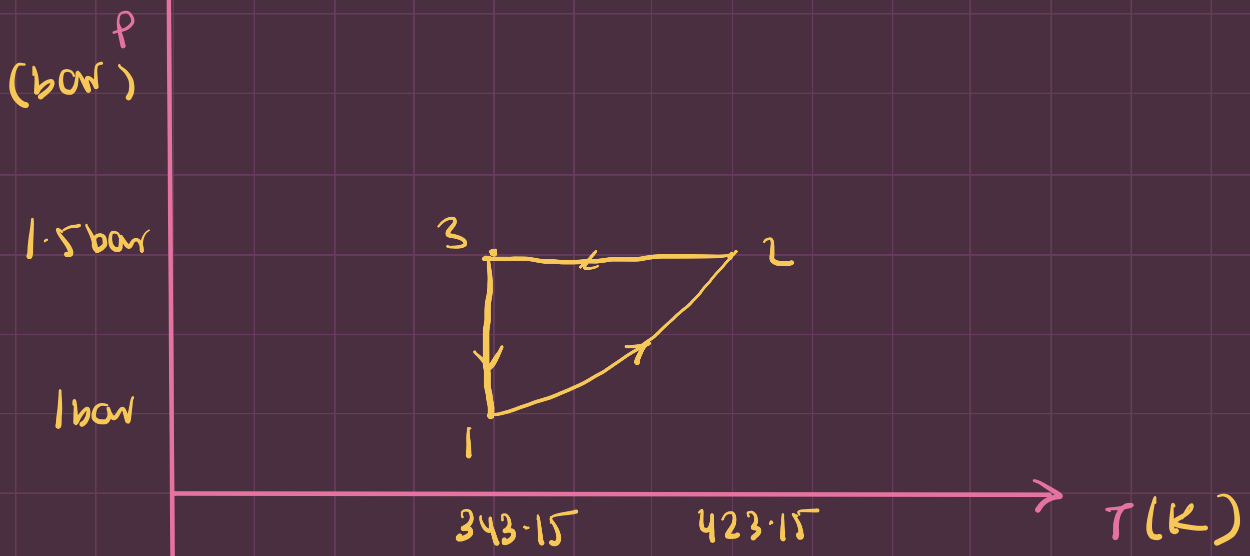
*

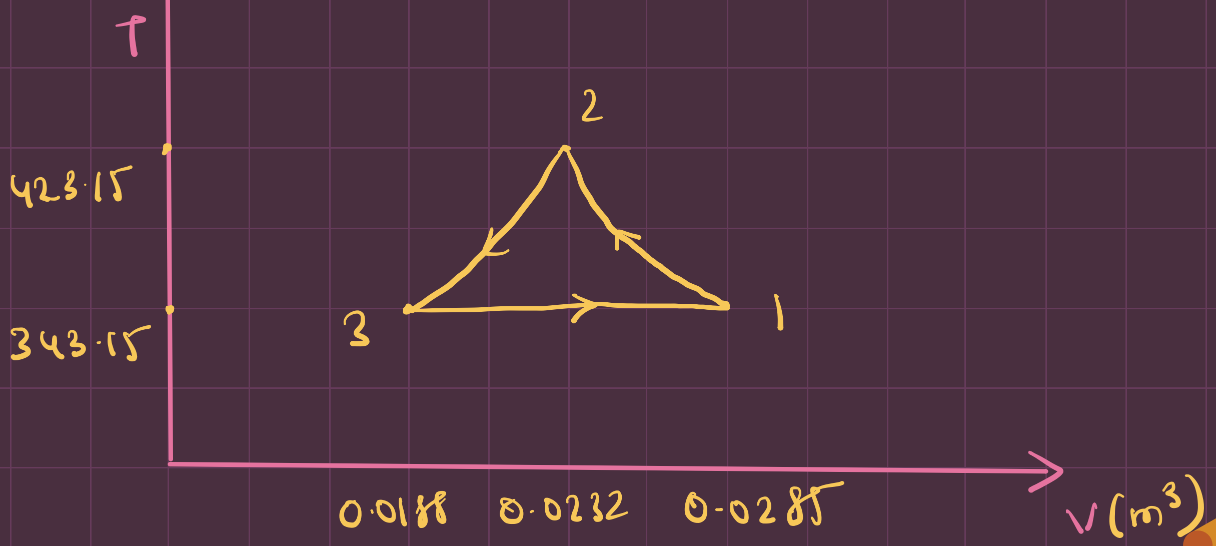
$P_{3,irr} = P_{2,irr}$

$T_{3,irr} = T_{1,irr}$

$$V_{3,irr} = \frac{nRT_{3,irr}}{P_{2,irr}} = \underline{\underline{0.0188\ m^3}}$$


P
(bar)
1.52
1
3
2
1
V (m³)
0.0188
0.0232
0.0285


P
(bar)
1.5 bar
1 bar
3
2
1
T (K)
343.15
423.15


T
423.15
343.15
2
3
1
V (m³)
0.0188
0.0232
0.0285