Question 1:

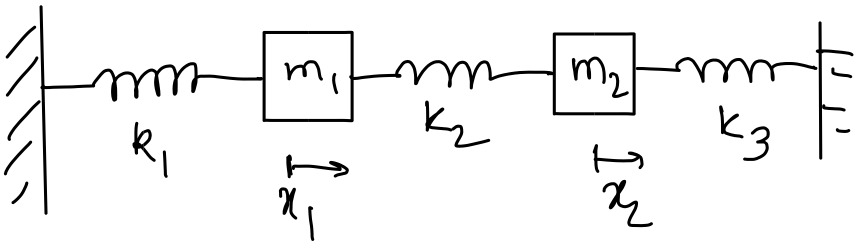
A 2-DOF (degree of freedom) mechanical system consists of two masses, $m_1 = 2\,\text{kg}$ and $m_2 = 1\,\text{kg}$, connected by springs with stiffness values $k_1 = 50\,\text{N/m}$, $k_2 = 100\,\text{N/m}$, and $k_3 = 50\,\text{N/m}$. The masses are arranged such that:

- Mass m_1 is connected to a wall with spring k_1,
- Mass m_1 and m_2 are connected by spring k_2,
- Mass m_2 is connected to a wall with spring k_3.

(a) Formulate the equations of motion for the system.

(b) Use MATLAB to determine the natural frequencies and mode shapes of the system.

(a)



for m_1: $\quad m_1\ddot{x}_1 = -k_1x_1 + k_2(x_2 - x_1)$

$\rightarrow m_1\ddot{x}_1 = -(k_1+k_2)x_1 + k_2x_2$

OR $\quad m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = 0$ ——— (1)

for m_2: $\quad m_2\ddot{x}_2 = -k_3x_2 - k_2(x_2 - x_1)$

$\rightarrow m_2\ddot{x}_2 = -(k_2+k_3)x_2 + k_2x_1$

OR $\quad m_2\ddot{x}_2 + (k_2+k_3)x_2 - k_2x_1 = 0$ ——— (2)

$\therefore$ Mass Matrix:

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

Stiffness Matrix:

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}$$

Such that $M\ddot{x} = -Kx$

where, $\ddot{x} = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Now, putting values of $\begin{cases} m_1 = 2kg \\ m_2 = 1kg \end{cases}$ $\quad k_1 = 50\ N/m$, $k_2 = 100\ N/m$, $k_3 = 50\ N/m$

$\therefore$ Eq^n of Motion :-

$$\begin{cases} 2\ddot{x}_1 = -150x_1 + 100x_2 \\ \ddot{x}_2 = -150x_2 + 100x_1 \end{cases}$$

(b)

CODE

```
%Given values
m1 = 2; %kg
m2 = 1; %kg
k1 = 50; %N/m
k2 = 100; %N/m
k3 = 50; %N/m

% Mass and stiffness matrices
M = [m1 0; 0 m2];

K = [k1+k2 -k2;-k2 k2+k3];

% Solve the generalized eigenvalue problem
[phi, omega2] = eig(K, M);

% Natural frequencies (rad/s)
omega = sqrt(diag(omega2));

% Convert to Hz
frequencies_Hz = omega / (2*pi);

% Display results
disp('Natural frequencies in Hz:');
disp(frequencies_Hz);

disp('Mode shapes (normalized):');
disp(phi);
```

OUTPUT:

Natural frequencies in Hz:

0.9068

2.2084

Mode shapes (normalized):

-0.6059 -0.3645

-0.5155 0.8569

1. *For mode 1, both the masses move in the same direction, with m_1 moving faster than m_2.*
2. *For mode 2, the masses move in the opposite direction.*