

STAT 333

Day 1

Basic Review

Conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$P(A \cap B) = P(A|B)P(B)$$

Law of total probability:

$$P(A) = \sum_{i=1}^n P(A|B_i)P(B_i)$$

Baye's Rule:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

Example 1.2

Show that when n is large and p is small, the $BIN(n, p)$ distribution may be approximated by a $POI(\lambda)$ distribution where $\lambda = np$.

$$\begin{aligned} \lim_{n \rightarrow \infty} \lim_{p \rightarrow 0} BIN(n, p) &= \lim_{n \rightarrow \infty} \lim_{p \rightarrow 0} \binom{n}{x} (p)^x (1-p)^{n-x} \\ &= \lim_{n \rightarrow \infty} \lim_{p \rightarrow 0} \frac{n(n-1)\dots(n-x+1)}{x!} \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x} \\ &= \lim_{n \rightarrow \infty} \lim_{p \rightarrow 0} \frac{n(n-1)\dots(n-x+1)}{n^x} \frac{\lambda^x}{x!} \frac{\left(1 - \frac{\lambda}{n}\right)^n}{\left(1 - \frac{\lambda}{n}\right)^x} \\ &= \frac{\lambda^x}{x!} \frac{e^{-\lambda}}{e^x} \\ &= \frac{\lambda^x}{x!} e^{-\lambda} \\ \lambda = np &\implies \lim_{n \rightarrow \infty} \lim_{p \rightarrow 0} BIN(n, p) = POI(\lambda) \end{aligned}$$

Day 2
