COE 202: Digital Logic Design Combinational Logic Part 2

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Objectives

- Standard Forms
- Deriving Boolean functions from truth tables
  - Sum of Min Terms
  - Product of Max Terms
- SOP vs POS
- Mapping of these expressions into logic circuit implementations

Standard Forms

- A Boolean function can be written algebraically in a variety of ways
- Standard form: is a standard algebraic expression of the function:
  - Help simplification procedures and frequently results in more desirable logic circuits (e.g. less number of gates)
- Standard form: contains product terms and sum terms
  - Product term: $X'Y'Z$ (logical product with AND)
  - Sum term: $X + Y + Z'$ (logical sum with OR)

Minterms and Maxterms

- A minterm: a product term in which all variables (or literals) of the function appear exactly once (complemented or not complemented)
- A maxterm: a sum term in which all the variables (or literals) of the function appear exactly once (complemented or not complemented)
- A function of n variables – have 2^n possible minterms and 2^n possible maxterms

- Example: for the function F(X,Y,Z),
  - the term X'Y is not a minterm, but XYZ' is a minterm
  - The term X'+Z is not a maxterm, but X+Y'+Z' is maxterm

Minterms

Minterms for two variables F(X,Y)

X	Y	Product Terms	Symbol	m_0	m_1	m_2	m_3
0	0	X'Y'	m_0	1	0	0	0
0	1	X'Y	m_1	0	1	0	0
1	0	XY'	m_2	0	0	1	0
1	1	XY	m_3	0	0	0	1

Variable complemented if 0
Variable uncomplemented if 1

m_i indicated the i^{th} minterm
For each binary combination of X and Y there is a minterm
The index of the minterm is specified by the binary combination
m_i is equal to 1 for ONLY THAT combination

Maxterms

Maxterms for two variables F(X,Y)

X	Y	Sum Terms	Symbol	M_0	M_1	M_2	M_3
0	0	X+Y	M_0	0	1	1	1
0	1	X+Y′	M_1	1	0	1	1
1	0	X′+Y	M_2	1	1	0	1
1	1	X′+Y′	M_3	1	1	1	0

Variable complemented if 1
Variable not complemented if 0

M_i indicated the i^{th} maxterm
For each binary combination of X and Y there is a maxterm
The index of the maxterm is specified by the binary combination
M_i is equal to 0 for ONLY THAT combination

Minterms and Maxterms

- In general, a function of n variables has
  - 2^n minterms: $m_0, m_1, \ldots, m_{2^n-1}$
  - 2^n maxterms: $M_0, M_1, \ldots, M_{2^n-1}$

- $m_i' = M_i$ or $M_i' = m_i$

Example: for F(X,Y):

$m_2 = XY' \rightarrow m_2' = X'+Y = M_2$

Example

- A Boolean function can be expressed algebraically from a give truth table by forming the logical sum of ALL the minterms that produce 1 in the function

Example:

Consider the function defined by the truth table

F(X,Y,Z) ➔ 3 variables ➔ 8 minterms

F can be written as

$F = X'Y'Z' + X'YZ' + XY'Z + XYZ$, or

$= m_0 + m_2 + m_5 + m_7$

$= \Sigma m(0,2,5,7)$

X	Y	Z	m	F
0	0	0	m_0	1
0	0	1	m_1	0
0	1	0	m_2	1
0	1	1	m_3	0
1	0	0	m_4	0
1	0	1	m_5	1
1	1	0	m_6	0
1	1	1	m_7	1

Example (Cont.)

- A Boolean function can be expressed algebraically from a give truth table by forming the logical product of ALL the maxterms that produce 0 in the function
- **Example**:

Consider the function defined by the truth table

F(X,Y,Z) ➔ in a manner similar to the previous example, F′ can be written as

$F' = m_1 + m_3 + m_4 + m_6$

$= \Sigma m(1,3,4,6)$

Now apply DeMorgan's rule

$F = F'' = [m_1 + m_3 + m_4 + m_6]'$

$= m_1'.m_3'.m_4'.m_6'$

$= M_1.M_3.M_4.M_6$

$= \Pi\ M(1,3,4,6)$

X	Y	Z	M	F	F′
0	0	0	M_0	1	0
0	0	1	M_1	0	1
0	1	0	M_2	1	0
0	1	1	M_3	0	1
1	0	0	M4	0	1
1	0	1	M_5	1	0
1	1	0	M_6	0	1
1	1	1	M_7	1	0

Note the indices in this list are those that are missing from the previous list in $\Sigma m(0,2,5,7)$

Expressing Functions with Minterms and Maxterms

F = X . (Y' + Z)				
X	Y	Z		F=X. (Y'+Z)
0	0	0		0
0	0	1		0
0	1	0		0
0	1	1		0
1	0	0		1
1	0	1		1
1	1	0		0
1	1	1		1

F $= XY'Z' + XY'Z + XYZ$
$= m_4 + m_5 + m_7$
$= \Sigma m(4,5,7)$

F $= \Pi\ M(0,1,2,3,6)$

Summary

- A Boolean function can be expressed algebraically as:
  - The logical sum of minterms
  - The logical product of maxterms
- Given the truth table, writing F as
  - Σm_i – for all minterms that produce 1 in the table, or
  - ΠM_i – for all maxterms that produce 0 in the table
- Another way to obtain the Σm_i or ΠM_i is to use ALGEBRA – see next example

Example

- Write E = Y′ + X′Z′ in the form of Σm_i and ΠM_i?

- Solution: **<u>Method1</u>**

First construct the Truth Table as shown

Second:

E = $\Sigma m(0,1,2,4,5)$, and

E = $\Pi M(3,6,7)$

X	Y	Z	m	M	E
0	0	0	m_0	M_0	1
0	0	1	m_1	M_1	1
0	1	0	m_2	M_2	1
0	1	1	m_3	M_3	0
1	0	0	m_4	M4	1
1	0	1	m_5	M_5	1
1	1	0	m_6	M_6	0
1	1	1	m_7	M_7	0

Example (Cont.)

Solution: **<u>Method2_a</u>**

$E = Y' + X'Z'$

$= Y'(X+X')(Z+Z') + X'Z'(Y+Y')$

$= (XY'+X'Y')(Z+Z') + X'YZ'+X'Z'Y'$

$= XY'Z+X'Y'Z+XY'Z'+X'Y'Z'+$
$X'YZ'+X'Z'Y'$

$= m_5 + m_1 + m_4 + m_0 + m_2 + m_0$

$= m_0 + m_1 + m_2 + m_4 + m_5$

$= \Sigma m(0,1,2,4,5)$

To find the form ΠMi, consider the remaining indices

$E = \Pi M(3,6,7)$

Solution: **<u>Method2_b</u>**

$E = Y' + X'Z'$

$E' = Y(X+Z)$

$= YX + YZ$

$= YX(Z+Z') + YZ(X+X')$

$= XYZ+XYZ'+X'YZ$

$E = (X'+Y'+Z')(X'+Y'+Z)(X+Y'+Z')$

$= M_7 \,.\, M_6 \,.\, M_3$

$= \Pi M(3,6,7)$

To find the form Σm_i, consider the remaining indices

$E = \Sigma m(0,1,2,4,5)$

SOP vs POS

- Boolean function can be represented as Sum of Products (SOP) or as Product of Sums (POS) of their input variables
  - $AB+CD = (A+C)(B+C)(A+D)(B+D)$
- The sum of minterms is a special case of the SOP form, where all product terms are minterms
- The product of maxterms is a special case of the POS form, where all sum terms are maxterms
- SOP and POS are referred to as the standard forms for representing Boolean functions

SOP vs POS

SOP → POS

$F = AB + CD$

$= (AB+C)(AB+D)$

$= (A+C)(B+C)(AB+D)$

$= (A+C)(B+C)(A+D)(B+D)$

Hint 1: Use id15: $X+YZ=(X+Y)(X+Z)$

Hint 2: Factor

POS → SOP

$F = (A'+B)(A'+C)(C+D)$

$= (A'+BC)(C+D)$

$= A'C+A'D+BCC+BCD$

$= A'C+A'D+BC+BCD$

$= A'C+A'D+BC$

Hint 1: Use id15 $(X+Y)(X+Z)=X+YZ$

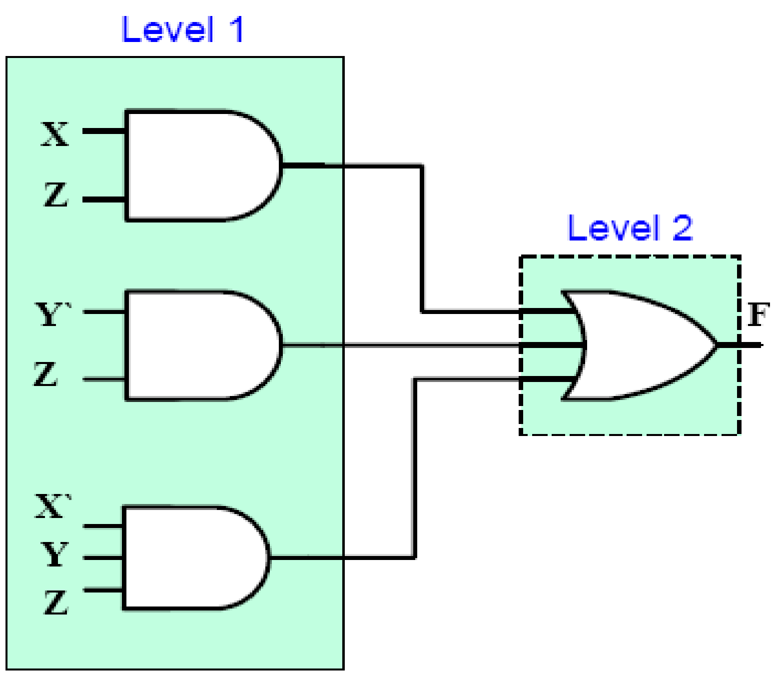
Hint 2: Multiply

Implementation of SOP

Any SOP expression can be implemented using 2-levels of gates

The 1st level consists of AND gates, and the 2nd level consists of a single OR gate

Also called 2-level Circuit



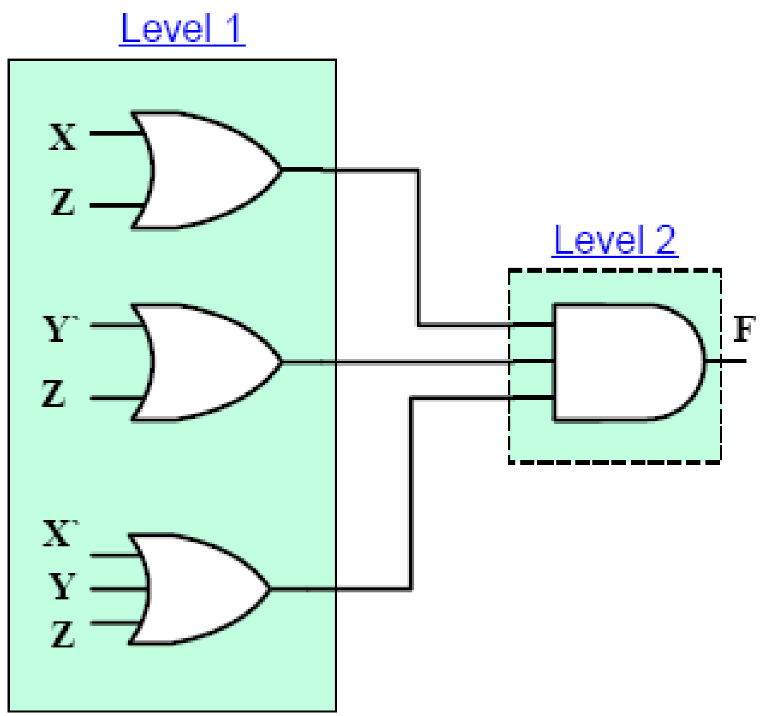
Two-Level Implementation ($F = XZ + Y`Z + X`YZ$)
Level-1: AND-Gates ; Level-2: One OR-Gate

Implementation of POS

Any POS expression can be implemented using 2-levels of gates

The 1st level consists of OR gates, and the 2nd level consists of a single AND gate

Also called 2-level Circuit

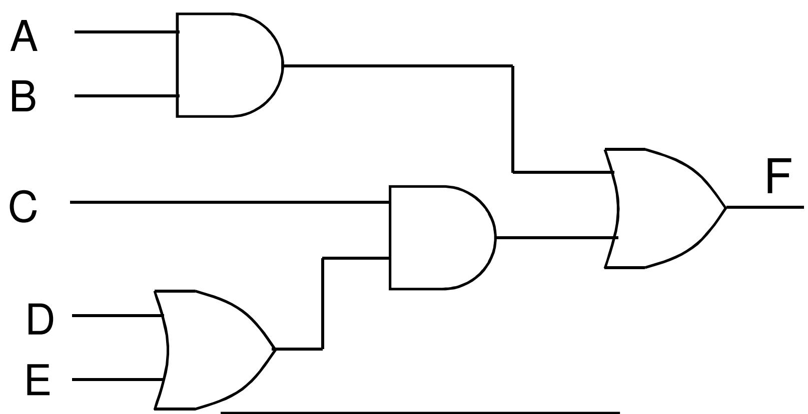


Two-Level Implementation {F = (X+Z)(Y`+Z)(X`+Y+Z)}

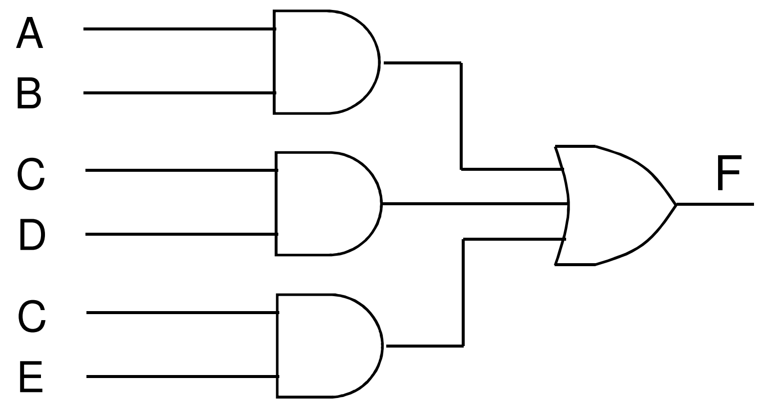
Level-1: OR-Gates ; Level-2: One AND-Gate

Implementation of SOP

- Consider F = AB + C(D+E)
  - This expression is NOT in the sum-of-products form
  - Use the identities/algebraic manipulation to convert to a standard form (sum of products), as in F = AB + CD + CE
- Logic Diagrams:



3-level circuit



2-level circuit