MDL Assignment 1 Report

Team 48:

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Task 1:

LinearRegression().fit()

Given test data, LinearRegression().fit() minimises the square difference between the model and the data to find the weights and bias of a regression that yield the most accurate results. In the case of a simple regression, this happens to be the 'line of best fit'. It uses gradient descent.

$$y = b + \sum_{i=1}^{n} w_1 x_1$$

To calculate the aforementioned 'accuracy' of a set of weights and bias, the MSE (Mean Squared Error) is used. LinearRegression().fit() aims to reduce this MSE as much as possible to result in the most accurate set of weights and bias that it can achieve.

Task 2:

Bias

Bias is one of two ways of measuring the correctness of an ML model. It represents the difference between the 'expected value' or 'average value' of the model and the actual value we are trying to predict, i.e. the accuracy of the model.

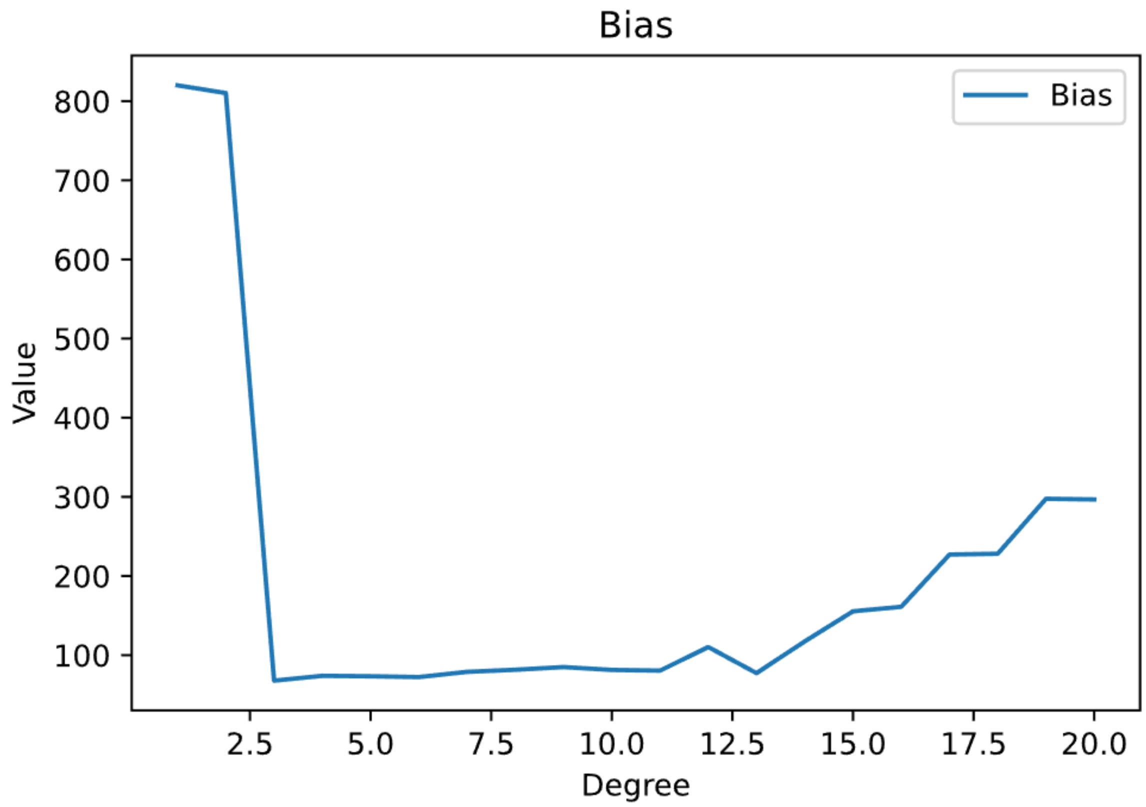
Bias is calculated by using the absolute value function in place of the square in the Bias^2 formula:

$$Bias^2 = (E[\hat{f}(x)] - f(x))^2$$

Code:

```
bias2 = np.mean( (np.mean(predMatrix, axis = 0) - testData[:, 1] ) ** 2 )
bias = np.mean( abs(np.mean(predMatrix, axis = 0) - testData[:, 1] ))
```

where predMatrix is the collection of y_predict from the trained models for each degree



Variance

Variance is the other way in which the correctness of an ML model is measured. It represents the 'variability' of the model's prediction, i.e. how precise the model is.

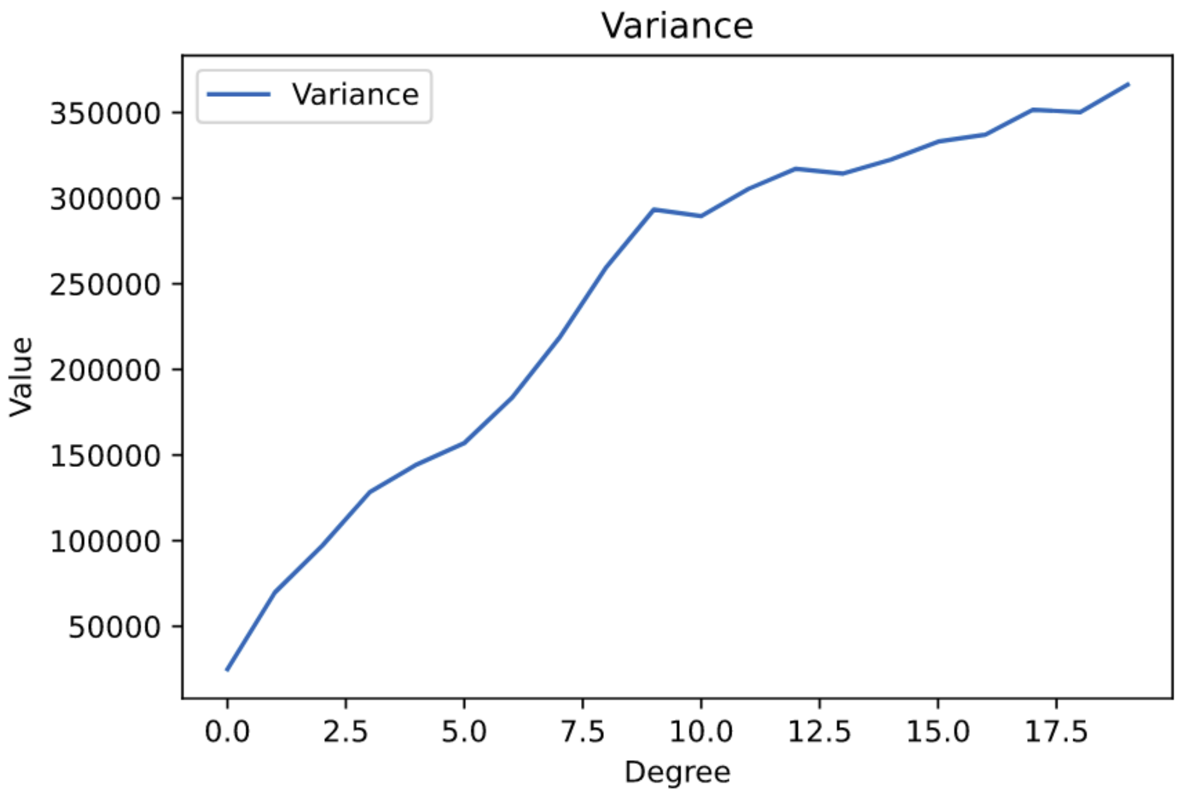
Variance is calculated using the formula:

$$Variance = E\left[(\hat{f}(x) - E[\hat{f}(x)])^2\right]$$

Code:

```
variance = np.mean(np.var(predMatrix, axis = 0))
```

where predMatrix is the collection of y_predict from the trained models for each degree
Where np.var is numpy's builtin to function compute variance



Task 3:

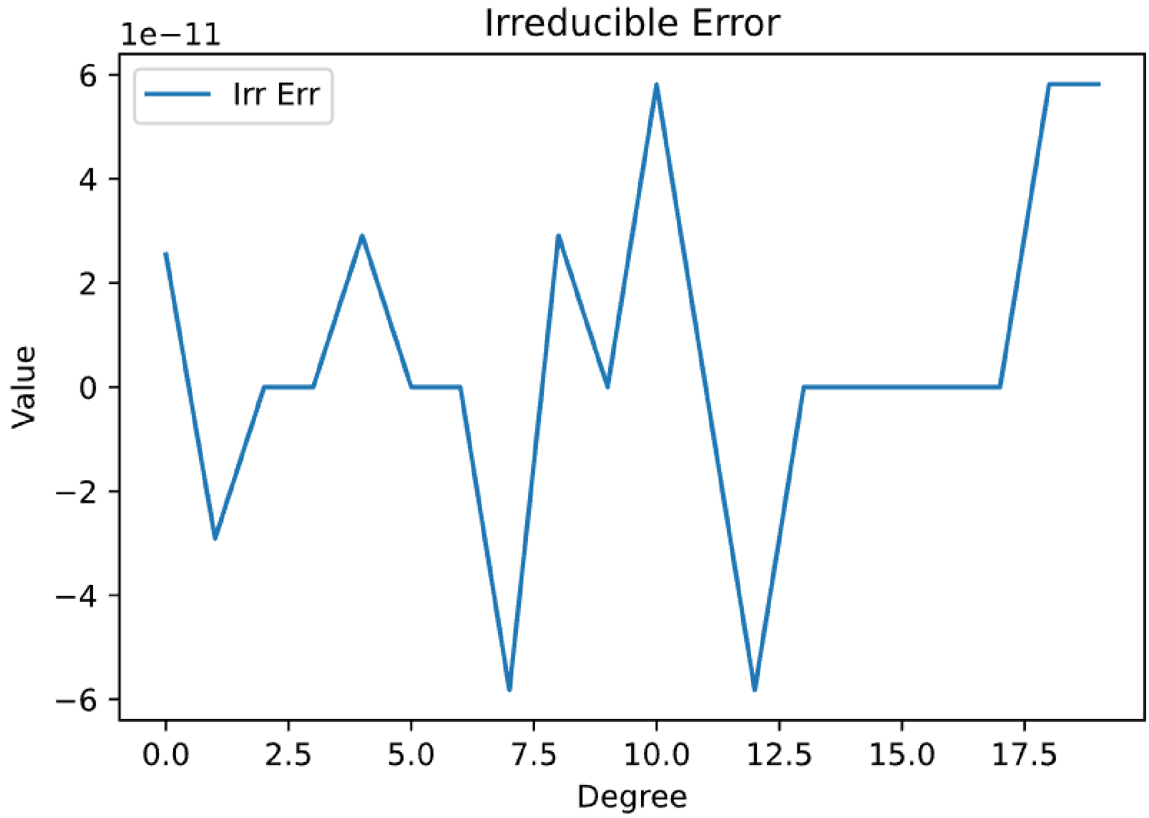
Calculating Irreducible Error

Irreducible error is a measure of the 'noise' in the supplied data. It is referred to as 'irreducible' as this error arises from the data and not the model and thus cannot be reduced no matter how good the created model is.

To calculate irreducible error, we used the formula:

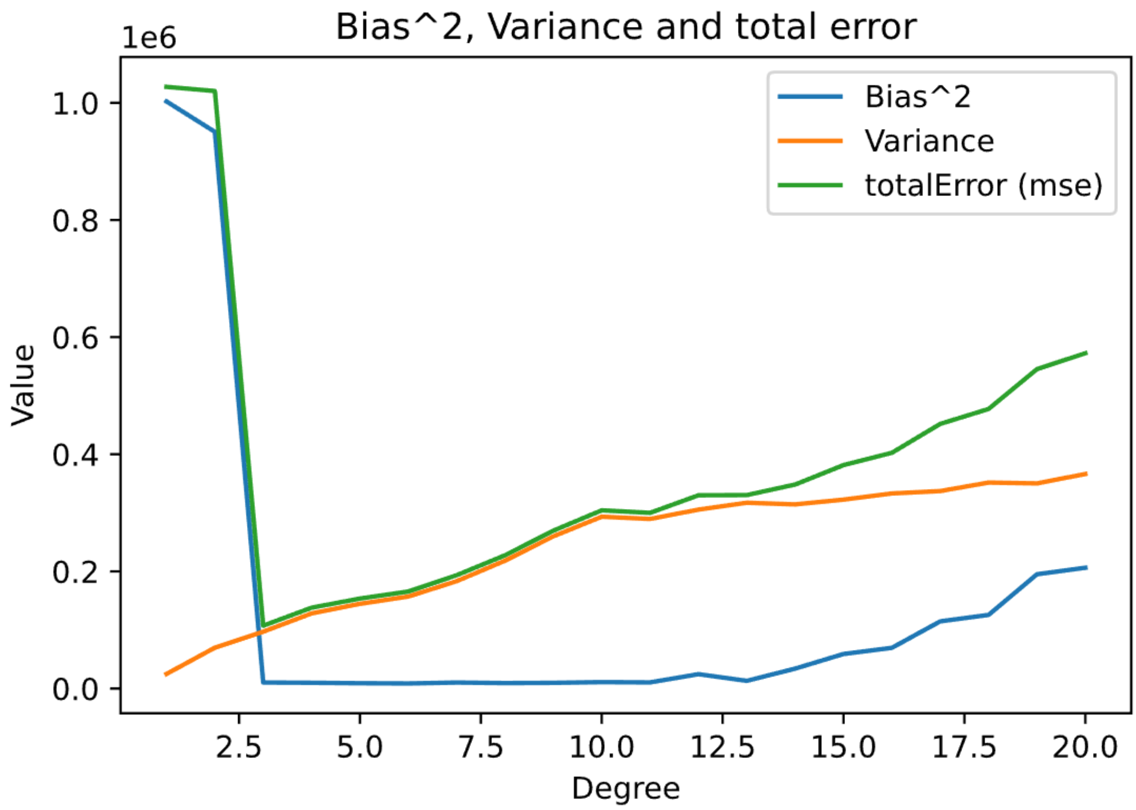
$$E[(f(x) - \hat{f}(x))^2] = Bias^2 + \sigma^2 + Variance$$
$$\sigma^2 = E[(f(x) - \hat{f}(x))^2] - (Bias^2 + Variance)$$

Plotting irreducible error



Task 4:

Plotting Bias^2 - Variance graph



Understanding the graph:

In this graph we display three different values: $Bias^2$, Variance and Total Error. Total error, as the name suggests, represents the total of $Bias^2$, Variance, and Irreducible error. This is what we aim to minimise when optimising our model. We observe that as the 'degree' or the complexity of our model increases, the variance increases and the $bias^2$ decreases initially and increases at the end.

At lower degrees, the model fails to accurately represent the training data and the test data. This is due to the lower number of features not being able to fully represent the data being provided. Due to this we observe a high bias. The low observed variance is due to the model being consistent but inaccurate which leads to a lower variance but a higher bias.

At higher degrees the model 'over-fits', i.e. the model represents the training data very accurately but with the loss of generality. This results in the model performing poorly on test data or any data other than the data it was trained with. This leads to a higher bias (than the middle degrees) as it conforms too much to the training data but not the test data, and a high variance as the predictions of the models become less precise and more spread out.

Observing the graph, we find that the total error reaches a minimum at degree=3. Hence, the data is best represented by a 3^{rd} degree function.

Understanding the type of data:

The bias is high for models that represent polynomials of lower degrees as it under-fits the data (is not complex enough to represent the data).

The variance is higher for more complex models as they over-fit. Variance tells you the degree of spread in your data set, and as over-fitting occurs, it leads to the predictions of the models to be increasingly chaotic and spread out.

Tabulating the results:

degree	bias	variance
1	820.007	38675.6
2	810.233	55083.9
3	67.7832	58210.8
4	73.8759	71113.8
5	73.316	91106.6
6	72.2846	108828
7	78.8863	116725
8	81.5498	144484
9	84.9207	158619
10	81.3285	159168
11	80.4445	170230
12	110.248	173786
13	77.2448	166664
14	117.41	163289
15	155.396	169819
16	161.111	182661
17	227.113	186223
18	228.15	197952
19	297.603	198454
20	296.723	208133