Table 4.2

1	*Primal feasible* Maximize $z = 2x_1 + x_2$, subject to: $x_1 + x_2 \leq 4$, $x_1 - x_2 \leq 2$, $x_1 \geq 0, \quad x_2 \geq 0$.	*Dual feasible* Minimize $v = 4y_1 + 2y_2$, subject to: $y_1 + y_2 \geq 2$, $y_1 - y_2 \geq 1$, $y_1 \geq 0, \quad y_2 \geq 0$.
2	*Primal feasible and unbounded* Maximize $z = 2x_1 + x_2$, subject to: $x_1 - x_2 \leq 4$, $x_1 - x_2 \leq 2$, $x_1 \geq 0, \quad x_2 \geq 0$.	*Dual infeasible* Minimize $v = 4y_1 + 2y_2$, subject to: $y_1 + y_2 \geq 2$, $-y_1 - y_2 \geq 1$, $y_1 \geq 0, \quad y_2 \geq 0$.
3	*Primal infeasible* Maximize $z = 2x_1 + x_2$, subject to: $-x_1 - x_2 \leq -4$, $x_1 + x_2 \leq 2$, $x_1 \geq 0, \quad x_2 \geq 0$.	*Dual feasible and unbounded* Minimize $v = -4y_1 + 2y_2$, subject to: $-y_1 + y_2 \geq 2$, $-y_1 + y_2 \geq 1$, $y_1 \geq 0, \quad y_2 \geq 0$.
4	*Primal infeasible* Maximize $z = 2x_1 + x_2$, subject to: $-x_1 + x_2 \leq -4$, $x_1 - x_2 \leq 2$, $x_1 \geq 0, \quad x_2 \geq 0$.	*Dual infeasible* Minimize $v = -4y_1 + 2y_2$, subject to: $-y_1 + y_2 \geq 2$, $y_1 - y_2 \geq 1$, $y_1 \geq 0, \quad y_2 \geq 0$.

We can show that the complementary-slackness conditions follow directly from the strong duality property just presented. Recall that, in demonstrating the weak duality property, we used the fact that:

$$\sum_{j=1}^{n} c_j \hat{x}_j \leq \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \hat{x}_j \hat{y}_i \leq \sum_{i=1}^{m} b_i \hat{y}_i \tag{18}$$

for any $\hat{x}_j$, $j = 1, 2, \ldots, n$, and $\hat{y}_i$, $i = 1, 2, \ldots, m$, feasible to the primal and dual problems, respectively. Now, since these solutions are not only feasible but optimal to these problems, equality must hold throughout. Hence, considering the righthand relationship in (18), we have:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \hat{x}_j \hat{y}_i = \sum_{i=1}^{m} b_i \hat{y}_i,$$

which implies:

$$\sum_{i=1}^{m} \left[\sum_{j=1}^{n} a_{ij} \hat{x}_j - b_i \right] \hat{y}_i = 0.$$