CBSE Maths 10, 2019

Contents

1 Discrete maths

1.1. If in an A.P., $a = 15$, d= -3, and $a_n = 0$. Find n value.
Solution: We know that $a_n = a + (n-1)d$,

$$\begin{aligned} 0 &= 15 + (n-1)(-3) \\ 3(n-1) &= 15 \\ n &= 6 \end{aligned}$$

1.2. If S_n, the sum of first n terms of an A.P.,is given by $S_n = 2n^2 + n$ then find its n^{th} term
Solution: We know that $S_n - S_{n-1} = n^{th}$ term
Given $S_n = 2n^2 + n$,

$$\begin{aligned} S_n &= 2n^2 + n \\ S_{n-1} &= 2(n-1)^2 + (n-1) \\ S_n - S_{n-1} &= (2n^2 + n) - (2(n-1)^2 + (n-1)) \\ n^{th} term &= 4n - 1 \end{aligned}$$

1.3. If the 17$^{\text{th}}$ of an A.P., exceeds 10$^{\text{th}}$ term by 7, then fine the common difference.
Solution: We know that $a_n = a + (n-1)d$
Given $a_{17} - a_{10} = 7$

$$\begin{aligned} a_{17} &= a + 16d \\ a_{10} &= a + 9d \\ a_{17} - a10 &= 7d = 7 \\ d &= 1 \end{aligned}$$

1.4. If in an A.P., the n^{th} term is $\frac{1}{m}$ and m^{th} is given by $\frac{1}{n}$. Find
(i) $(mn)^{th}$ term
(ii) Sum of the first (mn) terms
Solution: We know that $a_n = a + (n-1)d$, $a_m = \frac{1}{n}$, $a_n = \frac{1}{m}$

$$a + (m-1)d = \frac{1}{n} \tag{1.4.1}$$

$$a + (n-1)d = \frac{1}{m} \tag{1.4.2}$$

$$\begin{pmatrix} 1 & m-1 \\ 1 & n-1 \end{pmatrix} \begin{pmatrix} a \\ d \end{pmatrix} = \begin{pmatrix} \frac{1}{n} \\ \frac{1}{m} \end{pmatrix}$$

$$\begin{pmatrix} a + (m-1)d \\ a + (n-1)d \end{pmatrix} = \begin{pmatrix} \frac{1}{n} \\ \frac{1}{m} \end{pmatrix}$$

Considering the augmented matrix

$$\begin{pmatrix} a + (m-1)d & \frac{1}{n} \\ a + (n-1)d & \frac{1}{m} \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow R_1 - R_2} \begin{pmatrix} a + (m-1)d & \frac{1}{n} \\ (m-n)d & \frac{m-n}{mn} \end{pmatrix} \tag{1.4.3}$$

$d = \frac{1}{mn}$
Substituting in (3.7.1) we get
$a = \frac{1}{mn}$

a)

$$\begin{aligned} a_{mn} &= a + (mn-1)d \\ a_{mn} &= (\frac{1}{mn}) + (mn-1)(\frac{1}{mn}) \\ a_{mn} &= 1 \end{aligned}$$

b)

$$S_n = n(\frac{a+l}{2})$$
$$S_{mn} = mn(\frac{a+l}{2})$$
$$S_{mn} = \frac{1+mn}{2}$$

1.5. Find the HCF of 612 and 1314
Solution:

```
612)1314(2
    1224
    ----
      90)612(6
         540
         ---
          72)90(1
             18
             --
             18)72(4
                72
                --
                 ×
```

The HCF of 612 & 1314 is 18

1.6. Show that $5 - 3\sqrt{2}$ is an irrational number, where $\sqrt{2}$ is given to be an irrational number.

1.7. Show that any positive integer is of the form $6m + 1$ or $6m + 3$ or $6m + 5$, where m is some integer.

2 Probability and Statistics

2.1. A die has 6 faces having the letters A,B,C,A,A,B. The die is thrown once. What is the probability of getting (i) A (ii) B.
Solution:
Total no. of faces on the die =6
No. of faces having A on it = 3
No. of faces having B on it = 2
Therefore,

$$P(A)= \frac{\text{No. of faces having A}}{\text{Total no. of faces}} = \frac{3}{6} = \frac{1}{2}$$

$$P(B)= \frac{\text{No. of faces having B}}{\text{Total no. of faces}} = \frac{2}{6} = \frac{1}{3}$$

2.2. Find the probability that a number selected at random from the numbers 3,4,4,4,5,5,6,6,6,7 will be their mean.
Solution:

$$\text{Mean} = \frac{3+4+4+4+5+5+6+6+6+7}{10}$$
$$= 5$$
$$P(5) = \frac{2}{10} = \frac{1}{5}$$

2.3. Find the mode of the following frequency distribution:

Class:	10-14	14-18	18-22	22-26	26-30	30-34	34-38
Frequency:	8	6	11	20	25	22	10

TABLE 2.3

Solution: Here, maximum frequency is 25.
Modal class is 26-30.
So, $l = 26, f_0 = 20, f_1 = 25, f_2 = 22, h = 4$

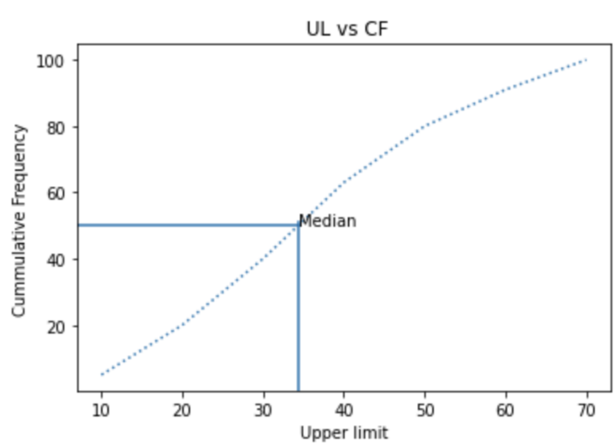
$$\text{Mode} = l + (\frac{f_1 - f_0}{2f_1 - f_0 - f_2})h$$
$$\text{Mode} = 28.5$$

2.4. For the following frequency distribution, obtain frequency distribution curve and hence obtain the median.

Class:	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency:	5	15	20	23	17	11	9

TABLE 2.4

Solution:



Median occurs as 34.3

2.5. Find the values of frequencies x and y in the following frequency distribution table if

N=100, and median = 32.

Marks:	0-10	10-20	20-30	30-40	40-50	50-60	Total
No. of students:	10	x	25	30	y	10	100

TABLE 2.5

Solution:

Marks:	0-10	10-20	20-30	30-40	40-50	50-60	Total
No. of students:	10	x	25	**30**	y	10	100
Cumm Freq:	10	10+x	35+x	65+x	65+x+y	75+x+y	75+x+y = 100

TABLE 2.5

$$75 + x + y = 100$$
$$x + y = 25$$
$$f = 30, h = 10, c_f = 35 + x, \frac{N}{2} = 50$$
$$\text{Median} = l + \frac{(N/2) - c_f}{f} h$$
$$32 = 30 + (\frac{50 - 35 - x}{30})10$$
$$x = 9$$
$$y = 16$$

3 Linear Algebra

3.1. The area of two similar triangles are 25 sq.cm and 121 sq.cm. Find the ratio of their corresponding sides.

Solution:

We know that,the ratio of area of two similar triangles is equal to the squares of the ratio of two corresponding sides.

Let, 'a' cm and 'b' cm be the lengths of the corresponding sides of the two triangles with area 25 sq. cm and 121 sq. cm respectively.

Then, we must have

$$\frac{25}{121} = \frac{a^2}{b^2}$$
$$\frac{5}{11} = \frac{a}{b}$$

The ratio of their corresponding sides is 5:11.

3.2. Find the values of for which the distance between the point A $\begin{pmatrix} x \\ 2 \end{pmatrix}$ and B $\begin{pmatrix} 9 \\ 8 \end{pmatrix}$ is 10.

Solution:

In 2D space the distance derived from L^2

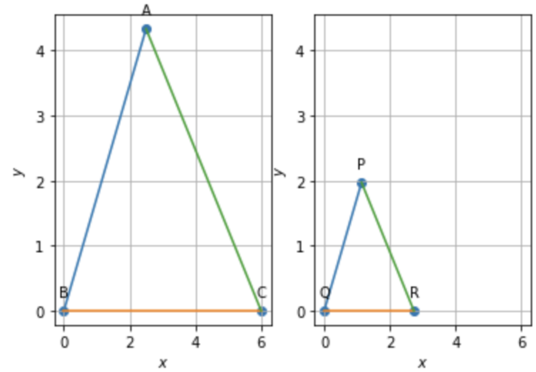


Fig. 3.1

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(x - 9)**2 + 36
(1, 17)
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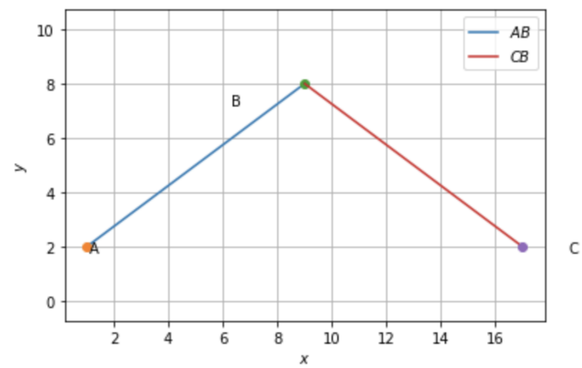

Fig. 3.2

norm.

$$\|A - B\| = \sqrt{\left(A - B\right)^T \left(A - B\right)}$$
$$(A - B) = \begin{pmatrix} x - 9 \\ -6 \end{pmatrix}$$
$$\|A - B\| = \sqrt{\begin{pmatrix} x - 9 \\ -6 \end{pmatrix} \begin{pmatrix} x - 9 & -6 \end{pmatrix}}$$
$$\|A - B\| = \sqrt{(x - 9)^2 + 36}$$
$$10 = \sqrt{x^2 - 18x + 117}$$
$$x^2 - 18x + 17 = 0$$
$$(x - 17)(x - 1) = 0$$

Thus, the values of x are 17,1.

3.3. A juice seller was serving his customers using glass. The inner diameter of the cylindrical glass was 5 cm , but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of the glass was 10 cm, find what the apparent capacity of the glass was and what the actual capacity was ?.

Solution: Given,that the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass.

Symbol	Description	Value
D	inner diameter	5 cm
h	height of glass	10 cm
r	radius	$\frac{D}{2}$ = 2.5 cm
V_{app}	apparent volume	?
V_{actual}	actual volume	?

TABLE 3.3

Actual capacity of Glass= Volume of cylinder - Volume of hemisphere.
$V_{app} = \pi r^2 h$
$V_{app} = \pi(2.5^2)(10)$
$V_{app} = 196.25cm^3$
Volume of hemisphere = $\frac{2}{3}\pi r^3$
Volume of hemisphere = $\frac{2}{3}\pi(2.5)^3$
Volume of hemisphere = $32.7cm^3$
$V_{actual} = 196.25cm^3 - 32.7cm^3$
$V_{actual} = 163.55cm^3$

3.4. A girl empties a cylindrical bucket, full of sand, of base radius 18 cm and height 32 cm on the floor to form a conical heap of sand. if the height of this conical heap is 24 cm, then find its slant height correct up to one place of decimal.
Solution: Given,

Symbol	Description	Value
r_1	base radius	18 cm
h_1	height of bucket	32 cm
h_2	height of cone	24 cm
l	slant height of cone	$\sqrt{h_2^2 + r_2^2}$=?

TABLE 3.4

Volume of sand in Cylindrical bullet = Volume of conical head of sand
$\pi r_1^2 h_1 = \frac{1}{3}\pi r_2^2 h_2$

$r_2^2 = \dfrac{18 * 18 * 32 * 3}{24}$
$r_2 = 36$
Slant height = $\sqrt{h_2^2 + r_2^2}$
Slant height = $\sqrt{24^2 + 36^2}$
Slant height = 43.2cm

3.5. An open metal bucket is in the shape of frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet. The diameter of the two circular end of buckets are 45 cm and 25 cm, the total vertical height of the bucket is 24 cm, find the area of metallic sheet used to make the bucket. Also find the volume of the water it can hold. **Solution:** Given,

Symbol	Description	Value
r_1	First radius	22.5 cm
r_2	Second radius	12.5 cm
h	height of frustum cone	24 cm

TABLE 3.5

l = Slant height of the frustum cone.

Slant height = $\sqrt{h^2 + (r_1 - r_2)^2}$
Slant height = $\sqrt{24^2 + (22.5 - 12.5)^2}$
Slant height = 26cm

Area of metallic sheet used=Curved surface area of the frustum of cone+area of circular base+curved surface area of cylinder.

Area of metallic sheet used = $\pi(r_1 - r_2)l + \pi r_2^2$
Area of metallic sheet used = $1307.7cm^2$

Volume of water that the bucket can hold = $\frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2)h$

Volume of water that the bucket can hold = $23719.02cm^3$

3.6. The mid-point of the line segment joining A $\begin{pmatrix} 2a \\ 4 \end{pmatrix}$ and B $\begin{pmatrix} -2 \\ 3b \end{pmatrix}$ is P $\begin{pmatrix} 1 \\ 2a+1 \end{pmatrix}$. The values of a and b are .
Solution: : Given, P as mid point which is $\dfrac{A+B}{2}$

$A + B = \begin{pmatrix} 2a \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 3b \end{pmatrix}$

$\dfrac{A+B}{2} = \begin{pmatrix} a-1 \\ 1.5b+2 \end{pmatrix} = P$

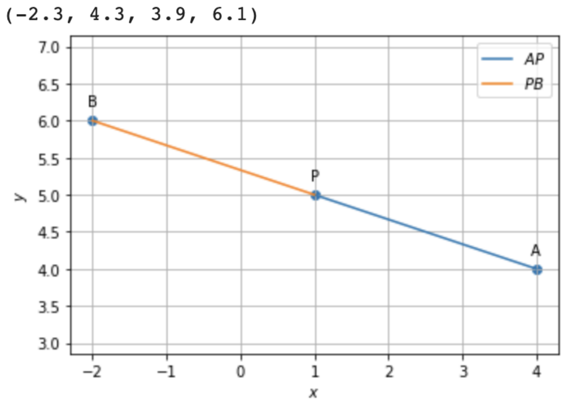

Fig. 3.6: Line segment P as mid point

$$\begin{pmatrix} a-1 \\ 1.5b+2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2a+1 \end{pmatrix}$$
$a - 1 = 1$;
$a = 2$
$1.5b + 2 = 2a + 1; 1.5b + 2 = 5;$
$b = 2$

3.7. For what value of p, are the points A $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, B $\begin{pmatrix} p \\ -1 \end{pmatrix}$, P $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ are collinear.
Solution: The equation of a line passing through a point is given by

$$\mathbf{n}^T\mathbf{x} = c \tag{3.7.1}$$

From (3.7.1) The equation of the line passing through points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ is given by:

$$\mathbf{n}^T\mathbf{A} = c \tag{3.7.2}$$
$$\mathbf{n}^T\mathbf{B} = c \tag{3.7.3}$$
$$\mathbf{n}^T\mathbf{C} = c \tag{3.7.4}$$

if the points are collinear then

$$\mathbf{B} - \mathbf{A}^T\mathbf{n} = 0 \tag{3.7.5}$$
$$\mathbf{C} - \mathbf{B}^T\mathbf{n} = 0 \tag{3.7.6}$$
$$\begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{B} \end{pmatrix}^T \mathbf{n} = 0 \tag{3.7.7}$$

Considering (3.7.7)

$$\begin{pmatrix} p-2 & -(1+p) \\ -2 & 4 \end{pmatrix}^T \mathbf{n} = 0$$
$$\begin{pmatrix} p-2 & -2 \\ -(1+p) & 4 \end{pmatrix}^T \mathbf{n} = 0 \tag{3.7.8}$$

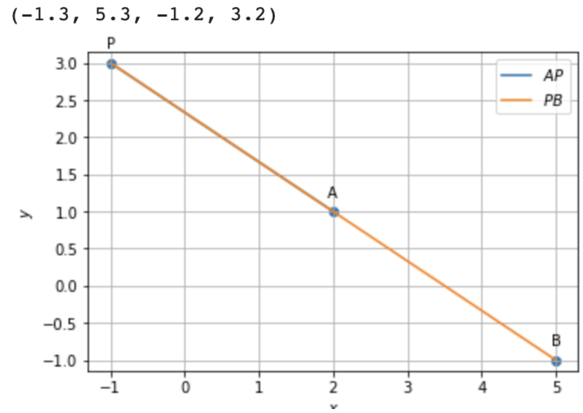

Fig. 3.7: Collinear points

Since, $\mathbf{n}$ is non zero, (3.7.8) must be

$$\begin{vmatrix} p-2 & -2 \\ -(1+p) & 4 \end{vmatrix} = 0$$

$[(p-2) * 4] - [2 * (1+p)] = 0$
Solving this we get **p=5**

3.8. Point P divides the line segment joining the points A $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ B $\begin{pmatrix} 5 \\ -8 \end{pmatrix}$ such that $\frac{AP}{AB} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$, find the value of k.
Solution: Given $\frac{AP}{AB} = \frac{1}{3}$
$\frac{AP}{AP+PB} = \frac{1}{3}$

From this we get,
$\frac{AP}{PB} = \frac{1}{2}$

Consider the line

$$\begin{pmatrix} -2 & 1 \end{pmatrix} \mathbf{x} = k \tag{3.8.1}$$

which is in the form

$$\mathbf{n}^T\mathbf{x} = c \tag{3.8.2}$$

divides the line segment $\mathbf{A}$ and $\mathbf{B}$ in 1 : 2 ratio, which means $s = 2$. $\mathbf{P}$ is point of intersection of two lines.
From the section formula we can write,

$$\mathbf{P} = \frac{1}{s+1}[\mathbf{B} + s\mathbf{A}] \tag{3.8.3}$$

$$\mathbf{P} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

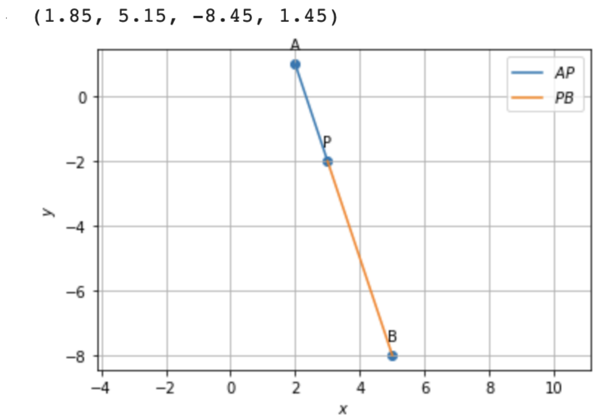

Fig. 3.8

The point $\mathbf{P}$ passes through the line $\mathbf{n}^T\mathbf{x} = c$, therefore,

$$\mathbf{n}^T\mathbf{P} = c$$

$$\mathbf{n}^T\left(\frac{\mathbf{B} + s\mathbf{A}}{s+1}\right) = k \tag{3.8.4}$$

Solving for k from (3.8.1) and (3.8.2), we get,

$$\mathbf{n}^T = \begin{pmatrix} -2 & 1 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ -8 \end{pmatrix}$$

Substituting the above values in (3.8.4) we get,

$$k = \begin{pmatrix} 2 & -1 \end{pmatrix} \frac{\begin{pmatrix} 5 \\ -8 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 1 \end{pmatrix}}{3}$$

$$k = 8$$

Therefore, the value of k is 8.

3.9. In the figure $\angle AOB = 60$ °, $\angle EOF = 80$ °, $\angle COD = 40°$and radius of the circle is 7cm. Find the area of the sectors for which angles are given.

Input Parameters	Output Parameters
r=7cm	$\theta_3 = 90°$
O = $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\mathbf{A} = \begin{pmatrix} r \\ 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} r\cos 60 \\ -r\sin 60 \end{pmatrix}$
$\theta_1 = 30°$	$\mathbf{C} = \begin{pmatrix} r\cos(60+\theta_1) \\ -r\sin(60+\theta_1) \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} r\cos(60+40+\theta_1) \\ -r\sin(60+40+\theta_1) \end{pmatrix}$
$\theta_2 = 60°$	$\mathbf{E} = \begin{pmatrix} r\cos(80+\theta_3) \\ r\sin(80+\theta_3) \end{pmatrix}$, $\mathbf{F} = \begin{pmatrix} 0 \\ r \end{pmatrix}$

TABLE 3.9

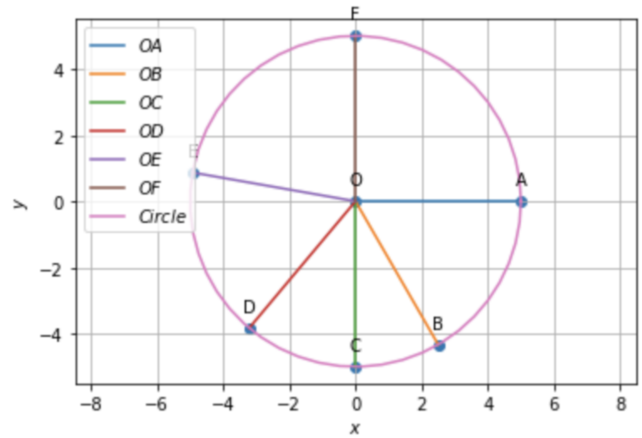

Fig. 3.9: Circles and Sectors

Solution: : Given angles are $\angle AOB = 60$ °, $\angle EOF = 80$ °, $\angle COD = 40°$, sum of the angles = 180 °.

Area of sector = $\frac{\theta°}{360°} * \pi r^2$

Area of the sectors for which angles are given is $\frac{\pi}{2} * r^2$

3.10. Construct a pair of tangents to a circle of radius 4 cm which are inclined to each other at 60°.

Solution: :

Symbol	Value	Description
r	4	radius of the circle
d	8	Distance of **P** from the origin
$\sin\theta$	$\frac{r}{d}$	Angle between tangent from **P** and d
P	**0**	Origin
O	$\begin{pmatrix} d \\ 0 \end{pmatrix}$	Centre of the circle
$\mathbf{Q_i}$	$r\cot\theta\begin{pmatrix} \cos\theta \\ \pm\sin\theta \end{pmatrix}$	Points of contact

TABLE 3.10

3.11. In ΔABC, ∠ B = 90° and D is the mid-point of BC. Prove that $AC^2 = AD^2 + 3CD^2$.

3.12. In Figure, E is a point on CB produced of an isosceles Δ ABC, with side AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that $\triangle ABD \sim \triangle ECF$.

3.13. In two concentric circles, prove that all chords of the outer circle which touch the inner circle, are of equal length.

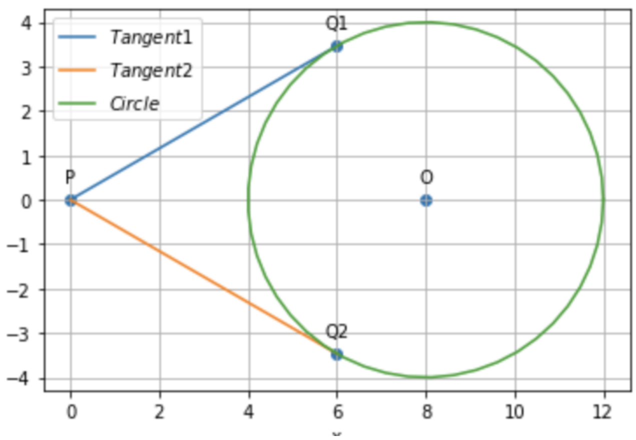

Fig. 3.10: Pair of Tangents to a Circle

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(-0.15000000000000002, 3.15, -0.2, 4.2)
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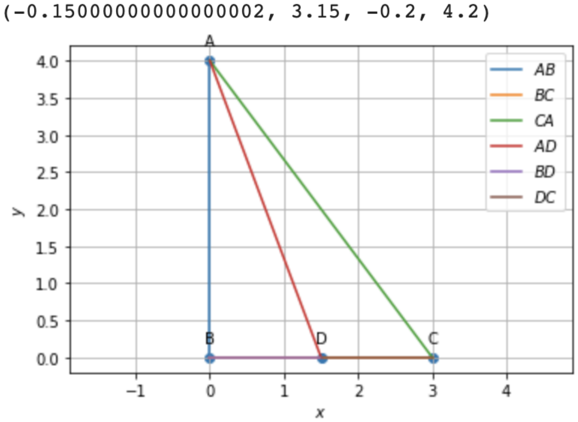

Fig. 3.11: Triangle 1

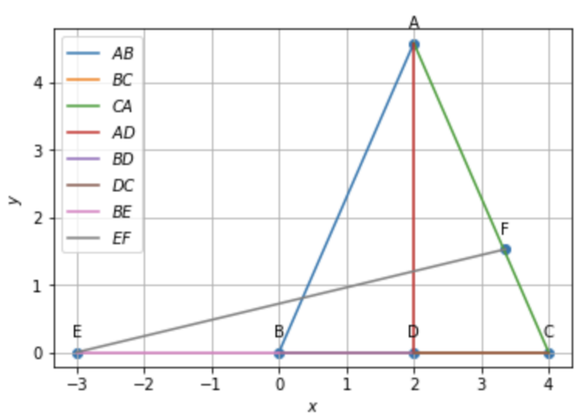

Fig. 3.12: Triangle 2

Input Parameters	Output Parameters
a=3	$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}$
c=4	$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix}$
$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\mathbf{D} = \frac{\mathbf{C}}{\mathbf{2}}$

TABLE 3.11

Input Parameters	Output Parameters
a=4	$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}$
$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\mathbf{D} = \frac{\mathbf{C}}{\mathbf{2}}, \theta_1 = \cos^{-1}(\frac{a}{2c})$
c=5	$\mathbf{A} = \begin{pmatrix} c\cos\theta_1 \\ c\sin\theta_1 \end{pmatrix}$
s = 3	$\mathbf{E} = \begin{pmatrix} -s \\ 0 \end{pmatrix}, \mathbf{F} = \begin{pmatrix} 0.84a, \sin(\frac{b}{3}) \end{pmatrix}$

TABLE 3.12

Solution: Norm of $\|x\| = c$, where $\mathbf{x}$ is the contact point on the circle.
Considering $\|px\|^2 = (px)^T(px)$

$$p = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$p^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\|px\|^2 = \|x\|^2.I$$

Which states that norm is unchanged with respect to the rotation, hence we can say that all chords of the outer circle which touch the inner circle, are of equal length.

4 Trigonometry

4.1. A boy standing on a horizontal plane finds a bird, flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of a 20 m high building finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on the opposite side of the bird. Find the distance of the bird from the girl.

Solution: Let A be the position of the bird and E and C be the positions of the girl and

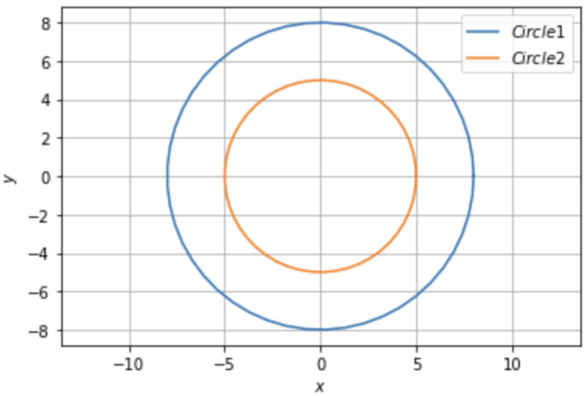

Fig. 3.13: Circles with radius 8 & 5

Input Parameters	Output Parameters
a = 4	$\mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix}$
s = 2	$\mathbf{F} = \begin{pmatrix} a+s \\ 0 \end{pmatrix}$, $\mathbf{A} = \begin{pmatrix} a \\ a\tan\theta \end{pmatrix}$
$\theta = 30°$	$\mathbf{D} = \begin{pmatrix} a \\ 0.4a\tan\theta \end{pmatrix}$
$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\mathbf{E} = \begin{pmatrix} a+s \\ 0.4a\tan\theta \end{pmatrix}$

TABLE 4.1

the boy, respectively.

$\angle ACB$=30°,$\angle AED$=45°AC=100 m,EF=20 m

In the $\triangle ACB$ we have

$$\sin 30 = \frac{AB}{AC}$$

AB = 50m
AD = AB -BD
AD = 50-EF = 50-20 = 30m
In the $\triangle ADE$ we have

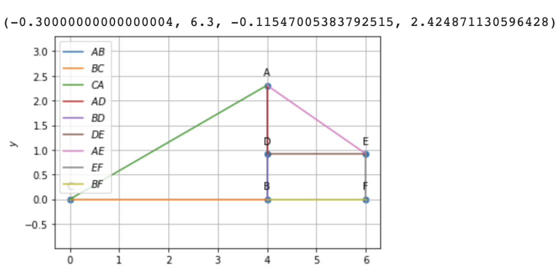

Fig. 4.1

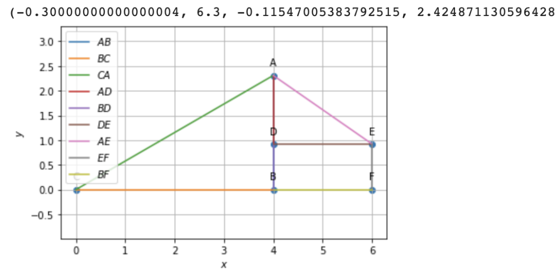

Fig. 4.1

$$\sin 45 = \frac{AD}{AE}$$

AE = 43.2m
Therefore, the distance of bird from the girl is 42.3 m.

4.2. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a height of 3600 $\sqrt{3}$ m, then find the speed of the plane.

Input Parameters	Output Parameters
a = 4	$\mathbf{B} = \begin{pmatrix} a/\tan\theta_1 \\ 0 \end{pmatrix}$
$\theta_1 = 45°$	$\mathbf{C} = \begin{pmatrix} a/\tan\theta_1 \\ a \end{pmatrix}$
$\theta_2 = 30°$	$\mathbf{E} = \begin{pmatrix} a/\tan\theta_2 \\ a \end{pmatrix}$
$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\mathbf{D} = \begin{pmatrix} a/\tan\theta_2 \\ 0 \end{pmatrix}$

TABLE 4.2

Solution: Let A be the point on the ground and the distance CE be the distance travelled by plane in 30 seconds. So,

In the $\triangle ACB$ we have

$$\tan 60° = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{3600\sqrt{3}}{AB}$$

(-0.3464101615137755, 7.274613391789285, -0.2, 4.2)

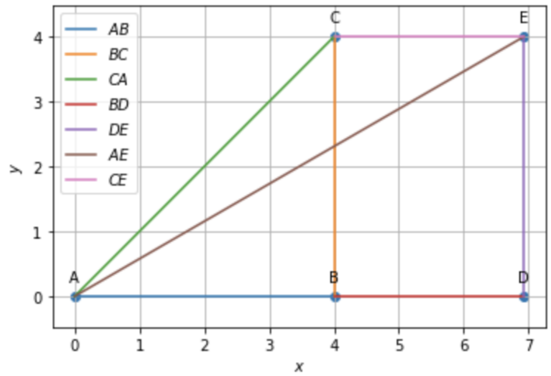

Fig. 4.2

AB = 3600m

In the $\triangle ADE$ we have

$$\tan 30° = \frac{DE}{AD}$$

$$\frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{AD}$$

AD = 10800m
Distance travelled = AD - AB = 10800-3600 = 7200
Speed is:

$$\frac{\text{Distance travelled}}{\text{Time taken}} = \frac{7200}{30} = 240\text{m/s}.$$

4.3. Prove that

$$\frac{\tan\theta}{1-\tan\theta} - \frac{\cot\theta}{1-\cot\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$$

Solution: Replace $\cot\theta$ with $\frac{1}{\tan\theta}$

$$\frac{\tan\theta}{1-\tan\theta} - \frac{\frac{1}{\tan\theta}}{1-\frac{1}{\tan\theta}}$$

$$\frac{1+\tan\theta}{1-\tan\theta}$$

Converting them into $\sin\theta$ and $\cos\theta$ terms we get $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$
Hence proved.

4.4. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

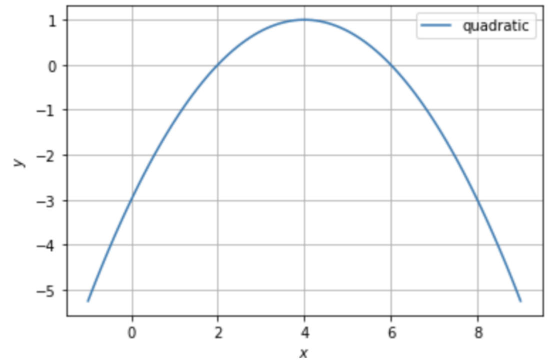

Fig. 5.1

Solution: .

$$(\cos\theta+\sin\theta)^2 = (\sqrt{2}\cos\theta)^2$$
$$\cos^2\theta + \sin^2\theta + 2\cos\theta sin\theta = 2\cos^2\theta$$
$$cos^2\theta - \sin^2\theta = 2\cos\theta sin\theta$$
$$(cos\theta + \sin\theta)(cos\theta - \sin\theta) + 2\cos\theta sin\theta$$
$$(\sqrt{2}\cos\theta)(cos\theta - \sin\theta) = 2\cos\theta sin\theta$$

From the above we can say that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

4.5. Prove that:

$$\frac{(1+\tan\theta+\cot\theta)(\sin\theta-\cos\theta)}{\sec^3\theta - \operatorname{cosec}^3\theta} = \sin^2\theta\cos^2\theta$$

Converting everything into $\sin\theta$ and $\cos\theta$ terms, we get

$$\frac{(\sin^2\theta\cos^2\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)(\sin\theta-\cos\theta)}{\sin^3\theta-\cos^3\theta}$$

From the basic algebra formula $a^3 - b^3 = (a^2+b^2+ab)(a-b)$, hence

$$\frac{(1+\tan\theta+\cot\theta)(\sin\theta-\cos\theta)}{\sec^3\theta - \operatorname{cosec}^3\theta} = \sin^2\theta\cos^2\theta$$

5 Algebra

5.1. Find the value of k for which x=2 is the solution of $kx^2 + 2x - 3 = 0$.
Solution: Substituting x=2 in given quadratic equation $kx^2 + 2x - 3 = 0$

$$4k + 4 - 3 = 0$$
$$k = -\frac{1}{4}$$

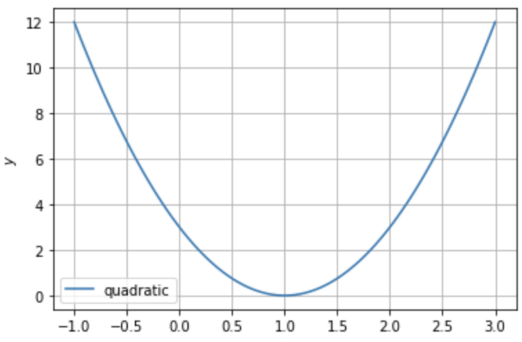

Fig. 5.2

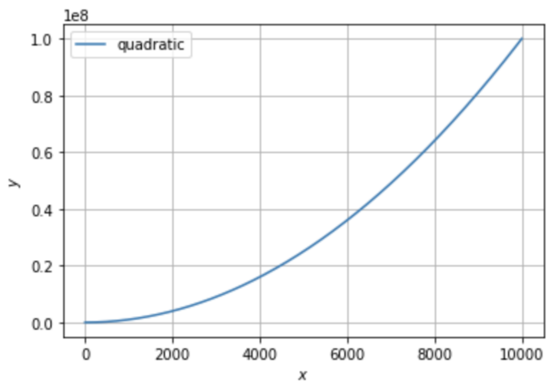

Fig. 5.3

5.2. Find the values of k for which the quadratic equation has $3x^2 + kx + 3 = 0$ has real and equal roots.
Solution: For real and equal roots if an quadratic equation $ax^2+bx+c = 0$, $b^2-4ac = 0$

$$k^2 - 4 * 3 * 3 = 0$$

$$k = \pm 6$$

5.3. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution: Let the speed of the train be s km/hr and the time taken be t hours.
Distance = Speed × Time
360 = s × t

$$t = \frac{360}{s}$$

Increased speed of the train can be written as s + 5
New time to cover the same distance = t - 1

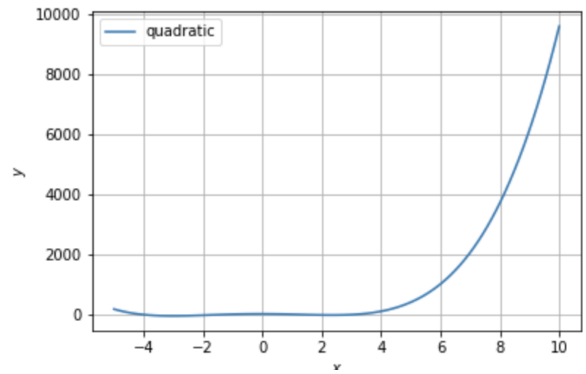

Fig. 5.4

$(s+5)(t-1) = 360$
$st - s + 5t - 5 = 360$

$$360 - s + 5\left(\frac{360}{s}\right) - 5 = 360$$

$s^2 + 5s - 1800 = 0$
Simplifying the above equation we get,
s = 40 and s = - 45
Speed of the train cannot be a negative value.
Therefore, speed of the train is 40 km /hr.

5.4. Find all the zeroes of the polynomial $x^4 + x^3 - 14x^2 - 2x + 24$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.
Solution: let $x^4 + x^3 - 14x^2 - 2x + 24$ be $f(x)$, Given $\sqrt{2}$ and $-\sqrt{2}$. are two zeroes of $f(x)$.

$x - \sqrt{2}, x + \sqrt{2}$ roots of $f(x)$
$x^2 - 2$ is the factor of $f(x)$

$$\begin{array}{r|l} & x^2 + x - 12 \\ \hline x^2 - 2) & x^4 + x^3 - 14x^2 - 2x + 24 \\ & -x^4 \quad + 2x^2 \\ \hline & x^3 - 12x^2 - 2x \\ & -x^3 \quad + 2x \\ \hline & -12x^2 + 0x + 24 \\ & 12x^2 \quad - 24 \\ \hline & 0x \quad + 0 \end{array}$$

Therefore the roots of the polynomial equation are $(x^2-2)(x^2+x-12) = 0$
$(x^2-2)(x+4)(x-3) = 0$
Other zeroes are -4 and 3

5.5. Solve for x : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}; a \neq b \neq$, $x \neq 0, x \neq -(a+b)$

$$\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$
$$\frac{-(a+b)}{xa+bx+x^2} = \frac{(a+b)}{ab}$$
$$-ab = xa + bx + x^2$$
$$xa + bx + x^2 + ab = 0$$
$$x(x+b) + a(x+b) = 0$$
$$x = -a; x = -b$$