Assignment No 5: Laplace Equation

EE18B120: Sarvani Cheruvu

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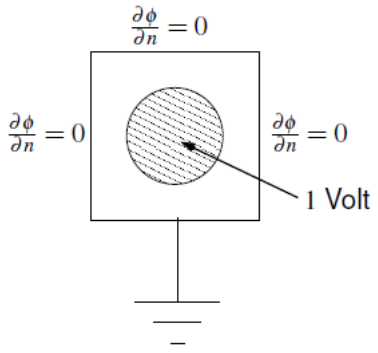
1 Introduction

We solve for the currents in a resistor. The current is dependent on the shape of the resistor.

2 The Problem

We are given a resistor as shown.



In order to solve the given problem, we solve $\nabla^2\phi = 0$ in 2 dimensions.

3 Solution

Laplace equation in the Cartesian co-ordinates is written as: $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$
This can equivalently be written as: $\phi_{i,j} = (\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1})/4$

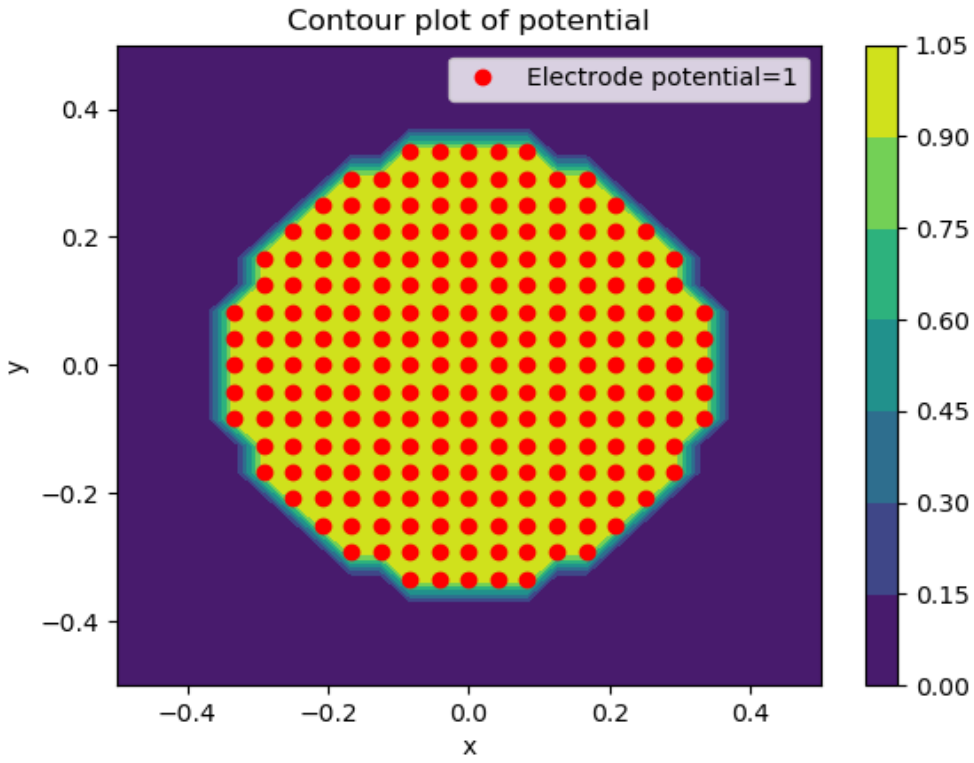
3.1 Allocate and initialize the potential array

Once the libraries are imported and parameters are given some default values (other than the values entered by the user), we initialize the potential array.

```
phi=np.zeros((Ny,Nx))
y=np.linspace(-0.5,0.5,num=Ny)
```

```
x=np.linspace(-0.5,0.5,num=Nx)
Y,X=meshgrid(y,x)
ii=np.where(X*X+Y*Y<=radius*radius)
phi[ii]=1.0
```

A contour plot of potential is obtained, as shown below.



3.2 Perform iteration

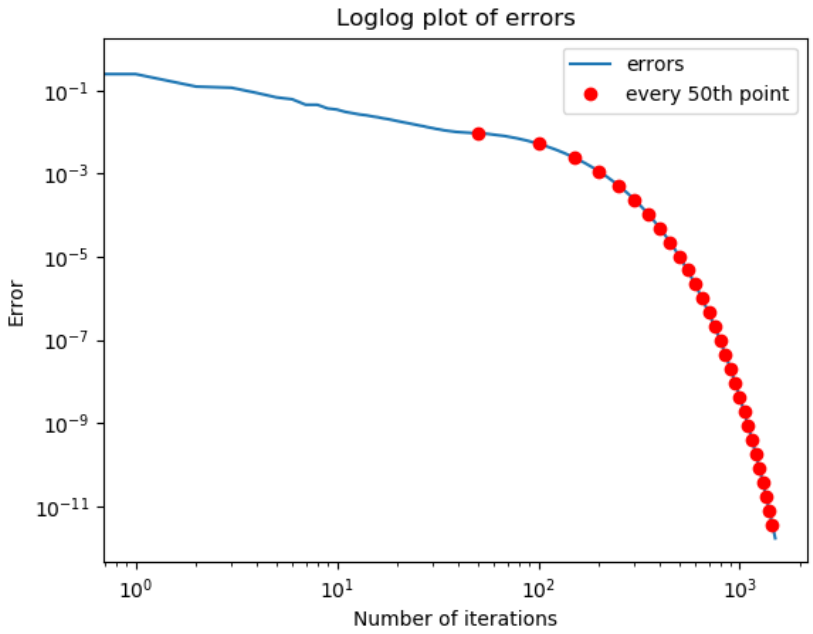
For every iteration, a copy of ϕ is saved, the array is updated, boundaries are asserted and the errors are calculated using the following code.

```
for k in range(Niter):
        oldphi=phi.copy()
        #update phi array
        phi[1:-1,1:-1]=0.25*(phi[1:-1,0:-2]+phi[1:-1,2:]+phi[0:-2,1:-1]+phi[2:,1:
        #boundaries
        phi[1:-1,0]=phi[1:-1,1]
        phi[1:-1,Nx-1]=phi[1:-1,Ny-2]
        phi[0,1:-1]=phi[1,1:-1]
        phi[ii]=1
```

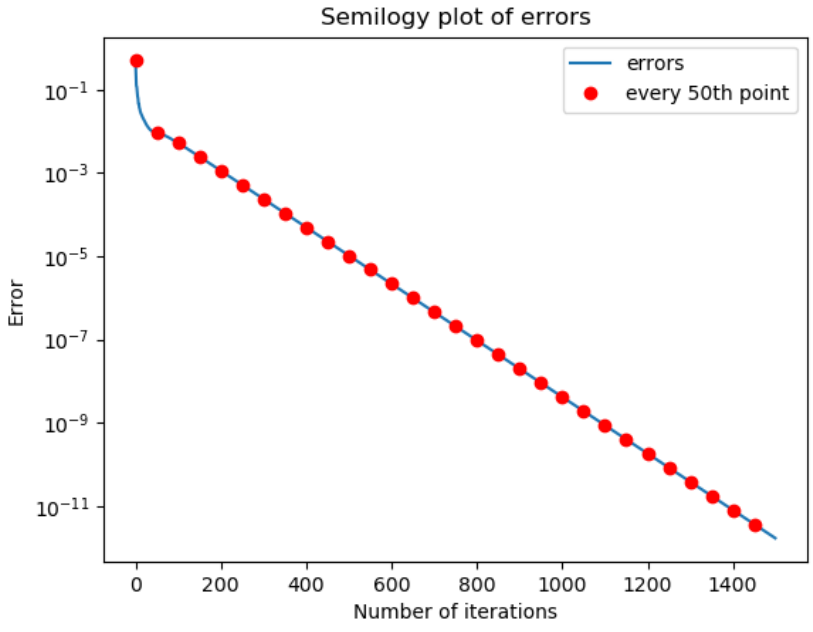
```
#errors
errors[k]=(abs(phi-oldphi)).max()
```

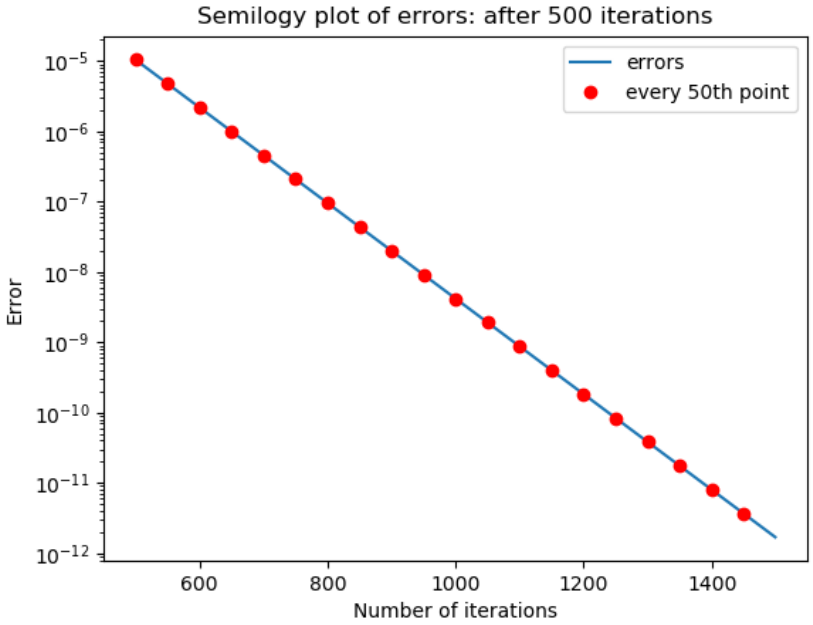
3.3 Errors

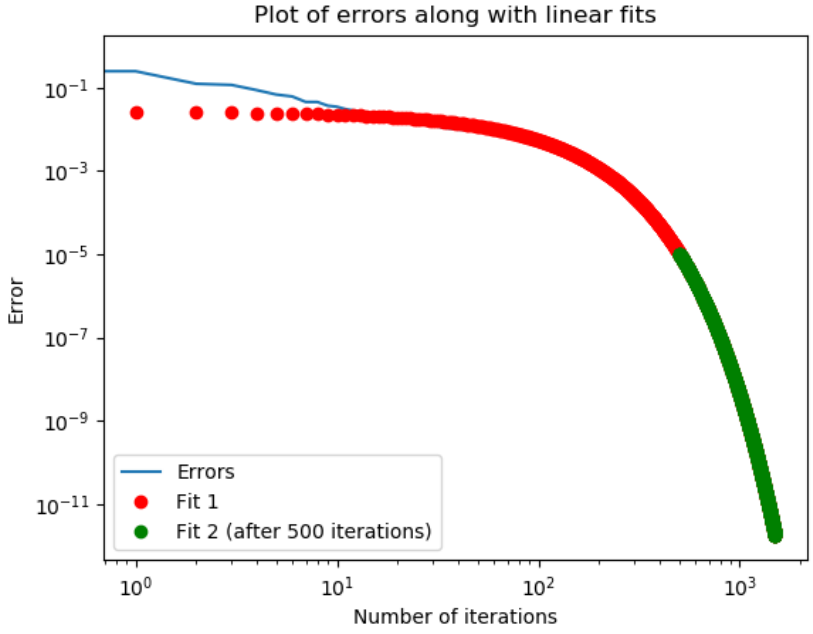
After the computation of errors, a linear fit is obtained for all iterations, as well as for iterations, 500 onwards. The following plots are obtained.




Loglog plot of errors: after 500 iterations
errors
every 50th point
Error
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
6×10^2
10^3
Number of iterations

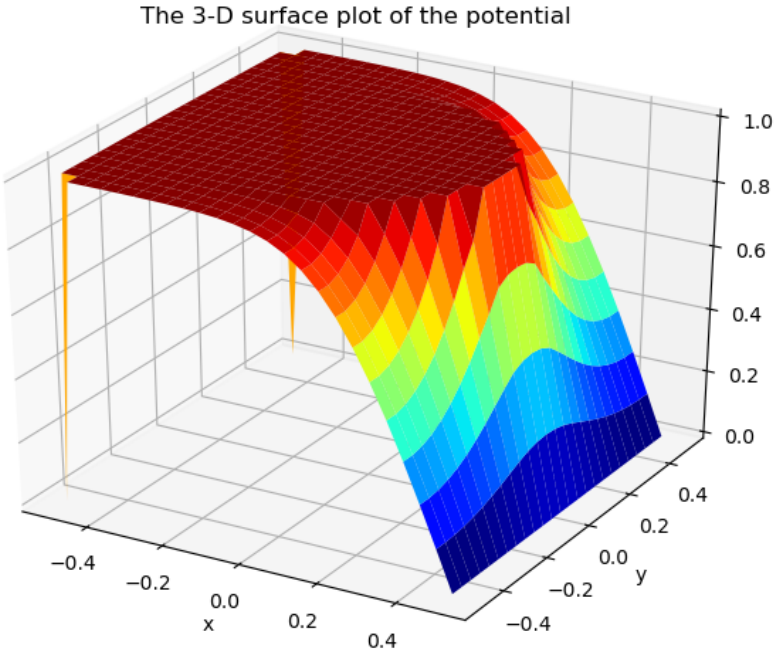

Semilogy plot of errors
errors
every 50th point
Error
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
200
400
600
800
1000
1200
1400
Number of iterations

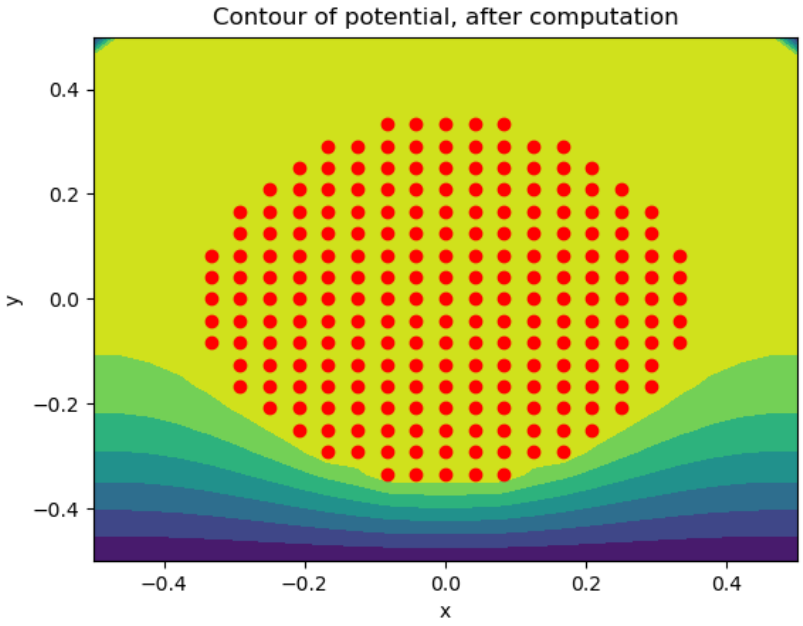

Semilogy plot of errors: after 500 iterations
errors
every 50th point
Error
Number of iterations
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
600
800
1000
1200
1400

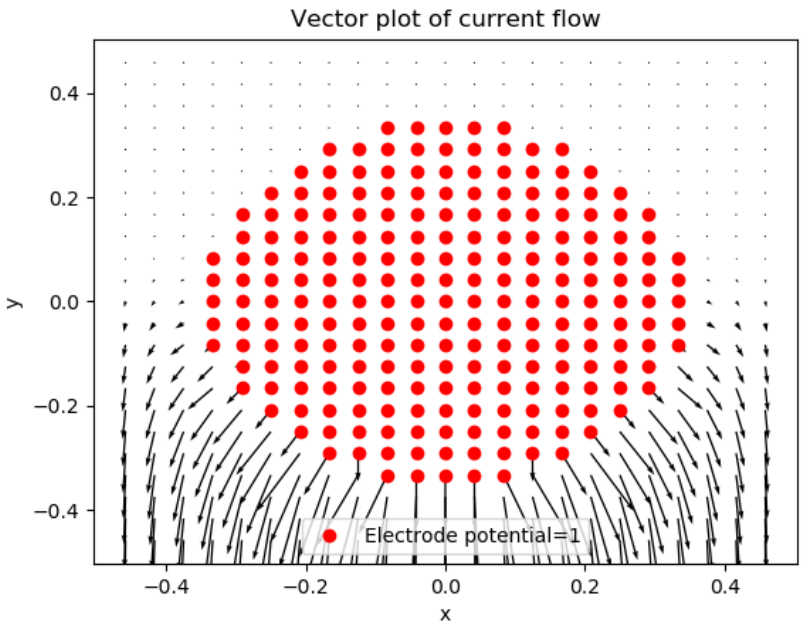

Plot of errors along with linear fits
Errors
Fit 1
Fit 2 (after 500 iterations)
Error
Number of iterations
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{0}
10^{1}
10^{2}
10^{3}

3.4 Plots

The following plots are obtained.







4 Conclusion

The error varies exponentially. To find the stopping condition for the iterative process, simply set the error to zero. The current fills the entire cross-section and then flows in the direction of the grounded electrode. The above results have been coded and plotted appropriately.