Assignment No 6: The Laplace Transform

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March 18, 2020

1 Introduction

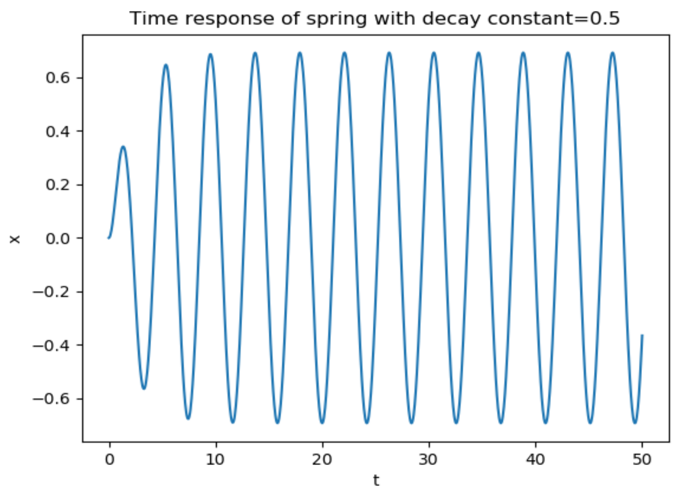
This assignment deals with analysis of Linear Time-Invariant Systems with numerical tools in python. We use the signals toolbox for this, which is part of scipy.

2 The Assignment

2.1 Time response of a spring

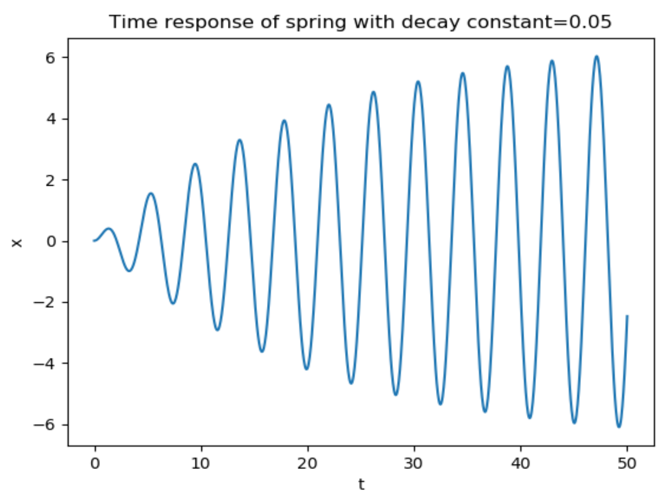
The Laplace transform of the time domain function $f(t) = \cos(1.5t)e^{-0.5t}u(t)$ is $\frac{s+0.5}{(s+0.5)^2+2.25}$.
We solve for the time response of a spring satisfying $x'' + 2.25x = f(t)$.
Use the following command for decay=0.5:

```
t,x=sp.impulse(f(0.5),None,np.linspace(0,50,1000))
```



Use the following command for decay=0.05:

```
t,x=sp.impulse(f(0.5),None,np.linspace(0,50,1000))
```



2.2 Varying frequency

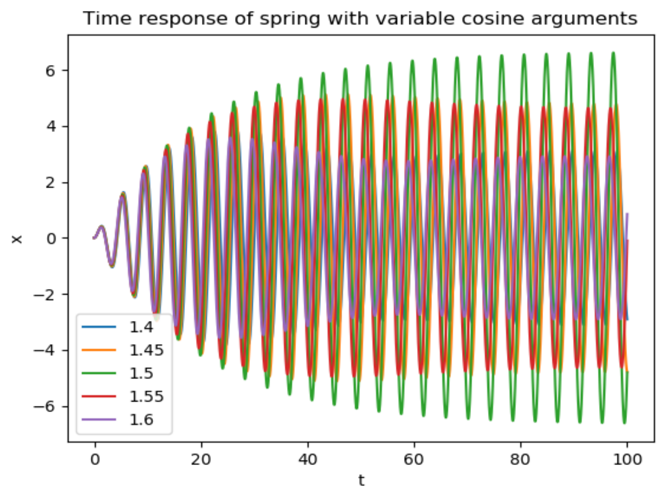
The problem is considered to be an LTI system. The system transfer function can be obtained as follows:

```
transfunction=sp.lti([1],[1,0,2.25])
```

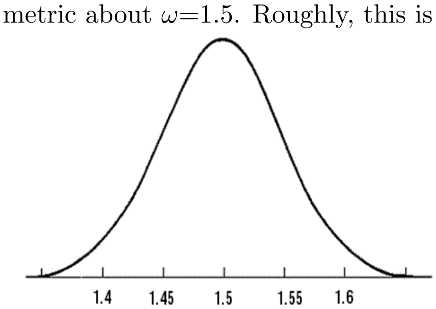
The frequency of the cosine in $f(t)$ is varied from 1.4 to 1.6. Use the following piece of code:

```
for i in range(len(k)):
        t,y,svec=sp.lsim(transfunction,np.exp(-0.05*t)*np.cos(k[i]*t),t)
```

The responses are plotted below.



The amplitude of the output $x(t)$ varies in a bell curve graph with frequency, reaching its maximum when the frequency of the natural system = frequency of the driving function. Frequency of the natural system is 1.5, which is why the amplitude is maximum at 1.5. Moreover, the plots are similar at points symmetric about ω=1.5. Roughly, this is the pictorial depiction of the situation.



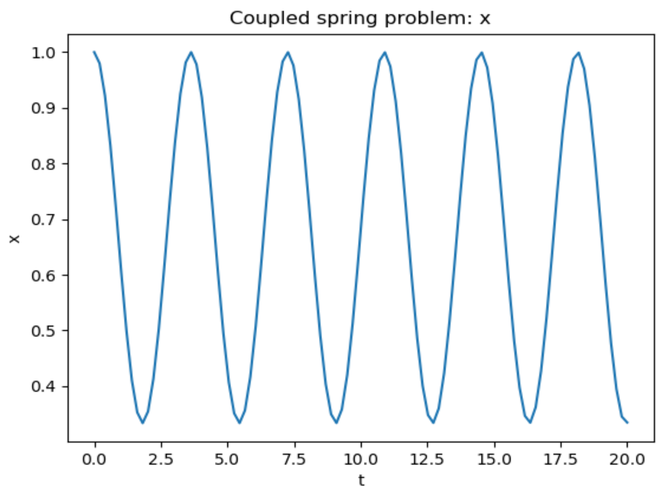
2.3 Coupled spring problem

The following problem is being solved for:

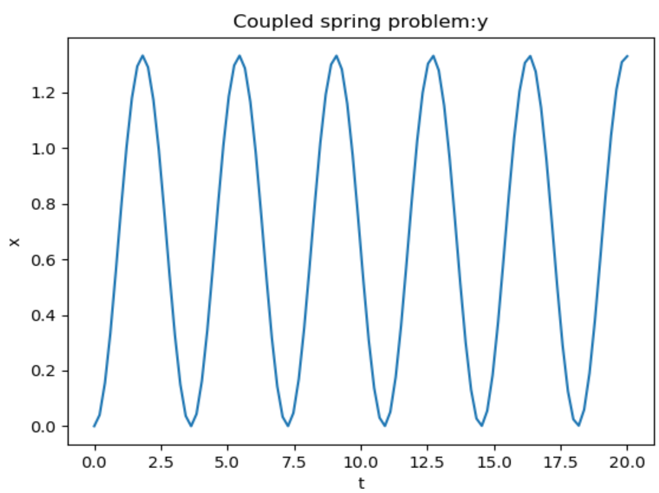
$x'' + (x - y) = 0$

$y'' + 2(y - x) = 0$, given the respective initial conditions.

The time evolution of x is plotted below.

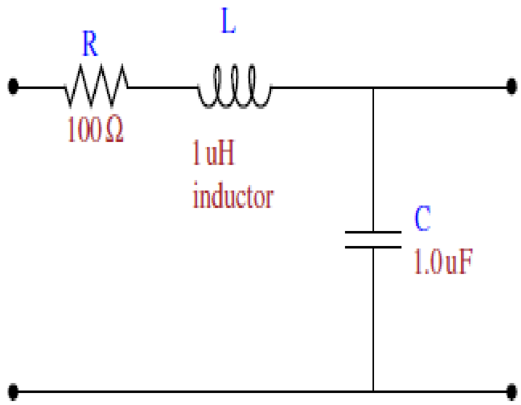


The time evolution of y is plotted below.

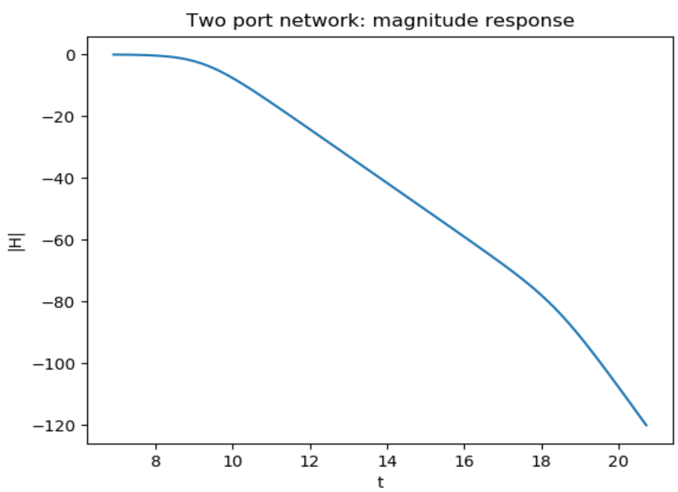


2.4 Two-port network

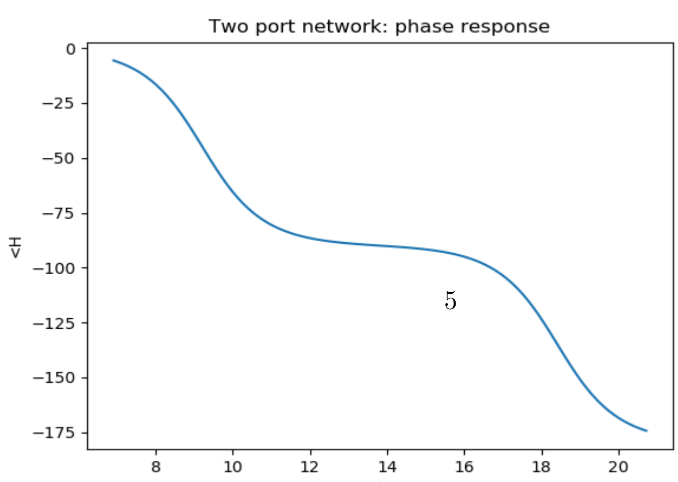
The magnitude and phase response of the following two-port network is to be obtained.



The magnitude response is plotted below.

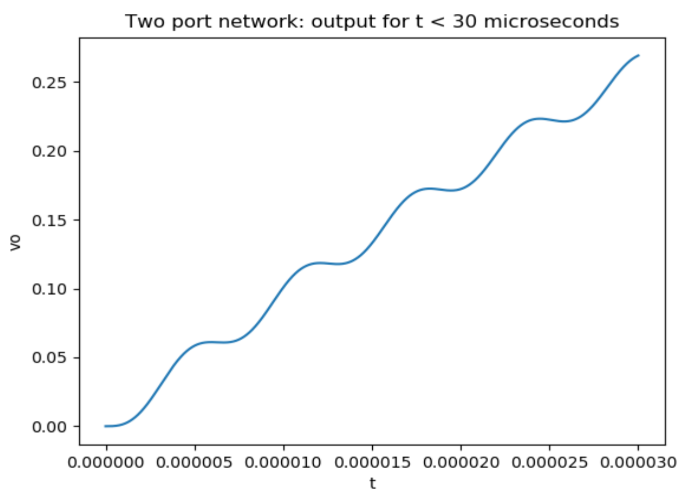


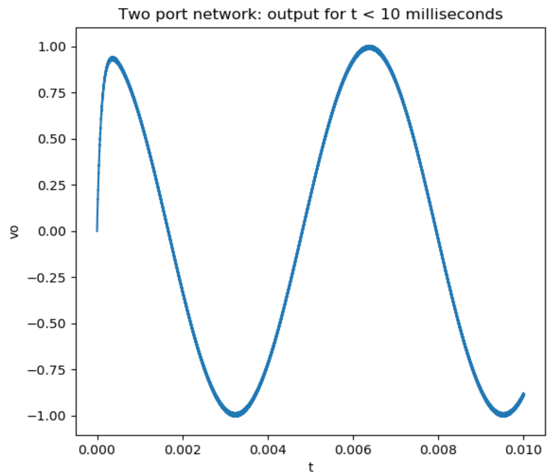
The phase response is plotted below.



Suppose the input is $v_i(t) = \cos(10^3 t)u(t) - \cos(10^6)u(t)$. The output voltage $v_o(t)$ is obtained using the following piece of code.

```
def f(n,test,title):
        t=np.linspace(0,n,30000)
        vi=np.cos((1e3)*t)-np.cos((1e6)*t)
        t,vo,svec=sp.lsim(twoport,vi,t)
        plott(title,'t','vo',t,vo)
        plt.show()
f(30e-6,1,'Two port network: output for t < 30 microseconds')
f(10e-3,1,'Two port network: output for t < 10 milliseconds')
```





The subtraction of the two cosines can be written as: $\cos(10^3 t) - \cos(10^6) = 2\sin\frac{10^3+10^6}{2}\sin\frac{10^6-10^3}{2}$ As $10^6 >> 10^3$, this expression approximately ends up being $1 - \cos(10^6 t)$, with some variation. The response in the long run reflects this.

3 Conclusion

The solution for a driven damped oscillator, represented as two different physical systems (spring and RLC network) were obtained in the Laplace domain.