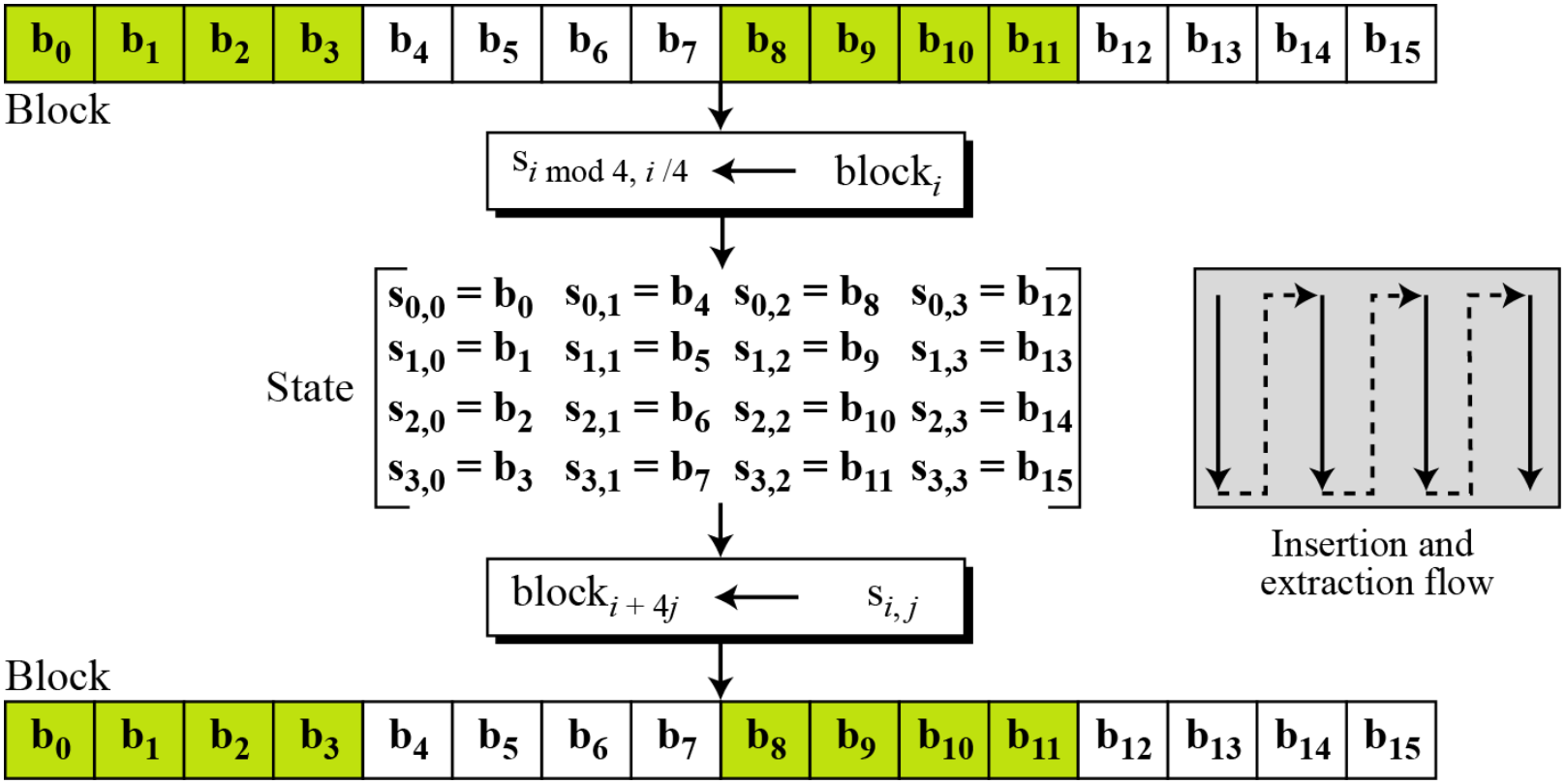
Advanced Encryption Standard (AES)

7.1.4 Continue

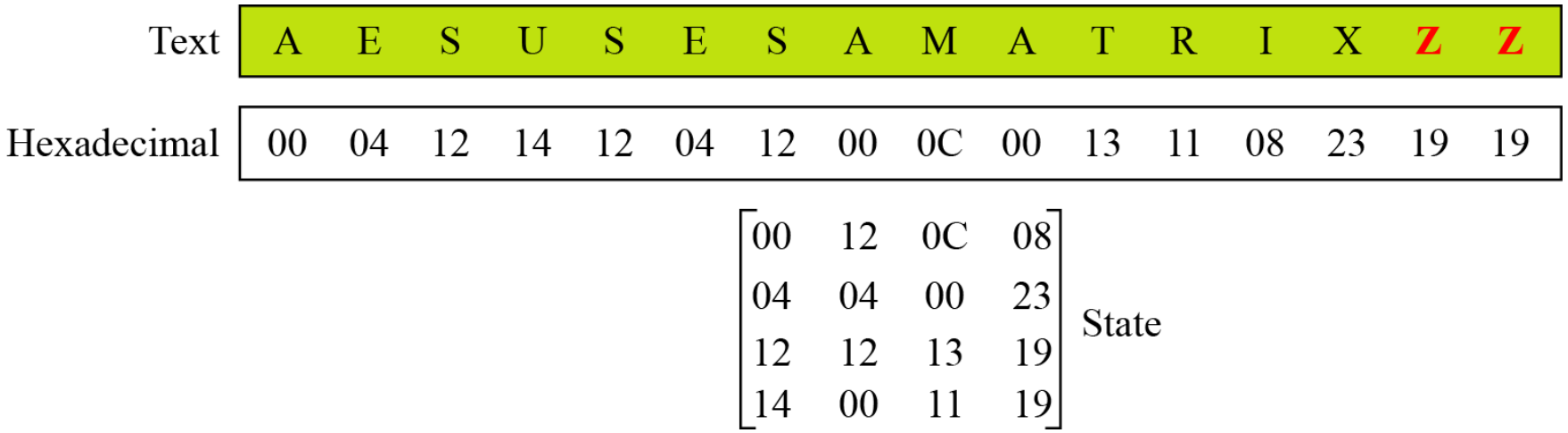
Figure 7.3 *Block-to-state and state-to-block transformation*



7.1.4 Continue

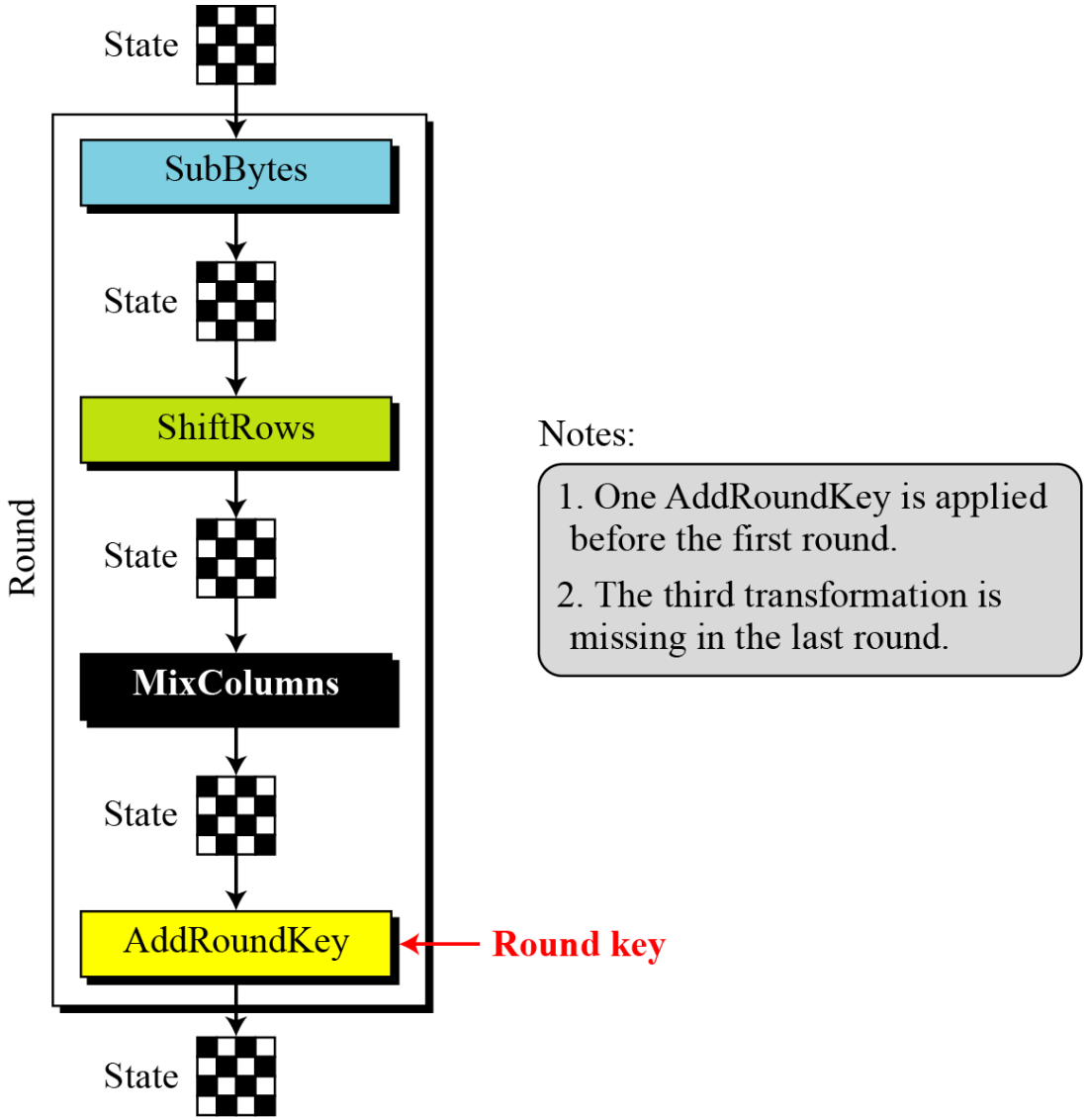
Example 7.1 Continue

Figure 7.4 *Changing plaintext to state*



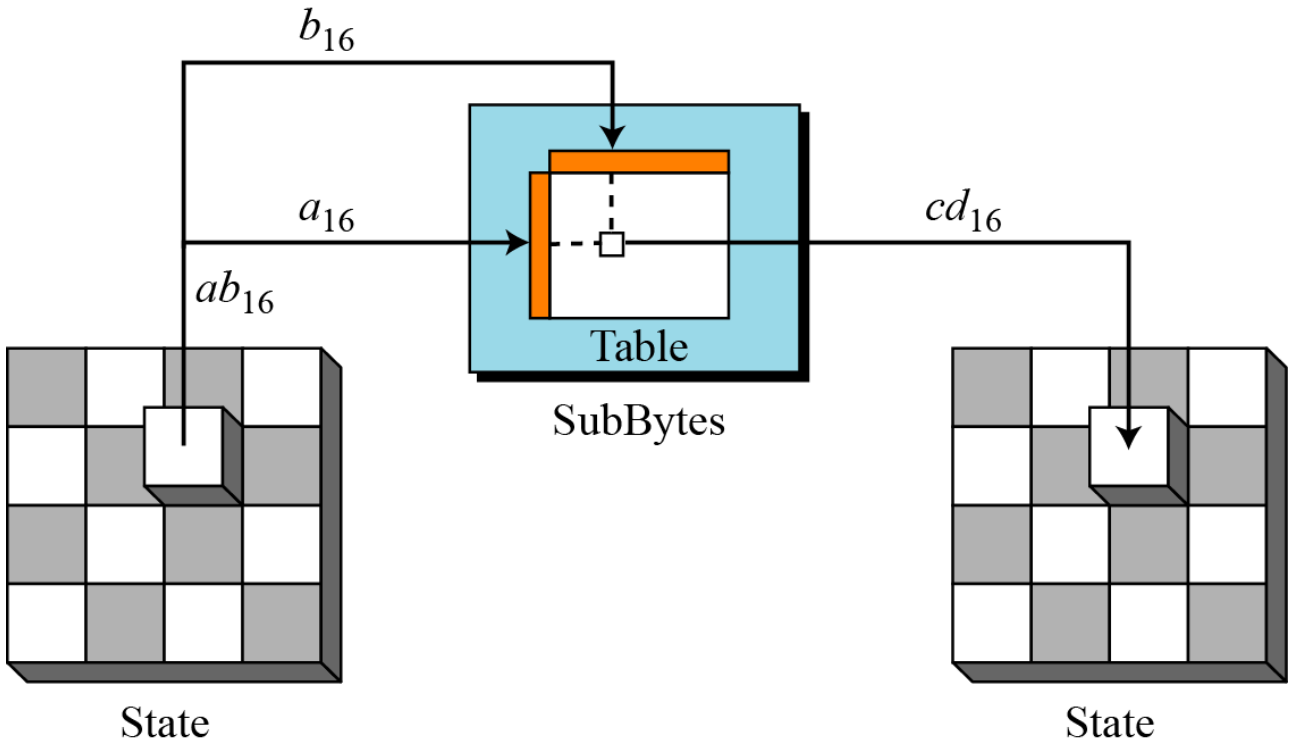
7.1.5 Structure of Each Round

Figure 7.5 *Structure of each round at the encryption site*



7.2.1 Continue

Figure 7.6 *SubBytes transformation*



7.2.1 Continue

Table 7.1 *SubBytes transformation table*

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*A*	*B*	*C*	*D*	*E*	*F*
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8

7.2.1 Continue

Table 7.1 *SubBytes transformation table (continued)*

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*A*	*B*	*C*	*D*	*E*	*F*
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	CB	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

7.2.1 Continue

InvSubBytes

Table 7.2 *InvSubBytes transformation table*

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B

7.2.1 Continue

InvSubBytes (Continued)

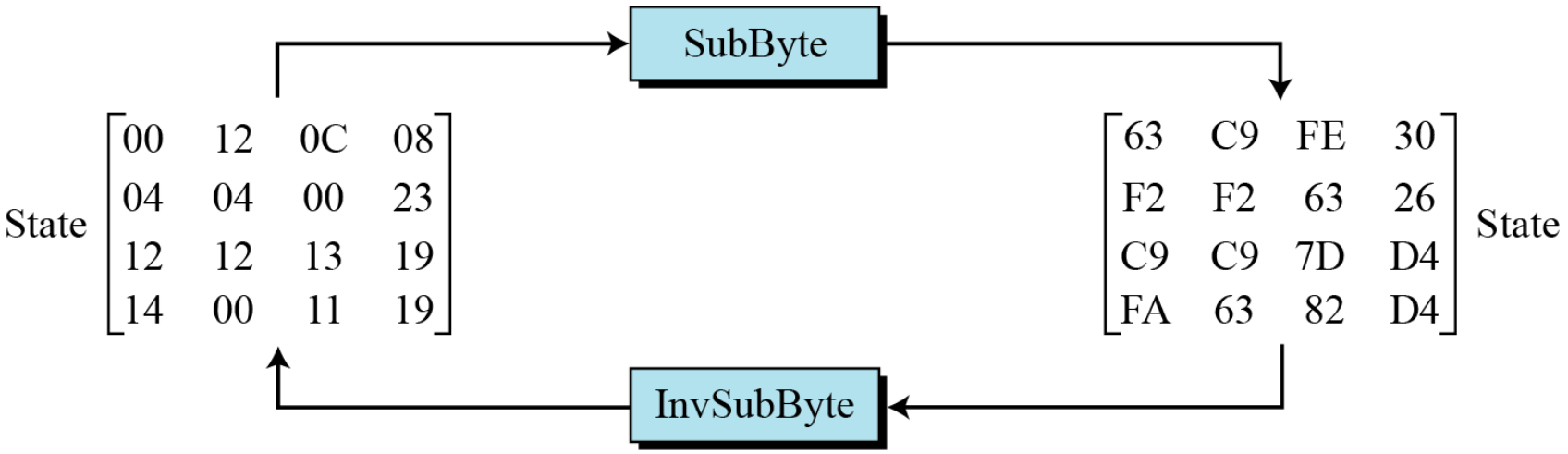
8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
E	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

7.2.1 Continue

Example 7.2

Figure 7.7 shows how a state is transformed using the SubBytes transformation. The figure also shows that the InvSubBytes transformation creates the original one. Note that if the two bytes have the same values, their transformation is also the same.

Figure 7.7 *SubBytes transformation for Example 7.2*



7.2.1 Continue

Transformation Using the GF(2^8) Field

AES also defines the transformation algebraically using the GF(2^8) field with the irreducible polynomials ($x^8 + x^4 + x^3 + x + 1$), as shown in Figure 7.8.

$$\text{subbyte:} \quad \rightarrow \quad \mathbf{d} = \mathbf{X}\,(s_{r,c})^{-1} \oplus \mathbf{y}$$

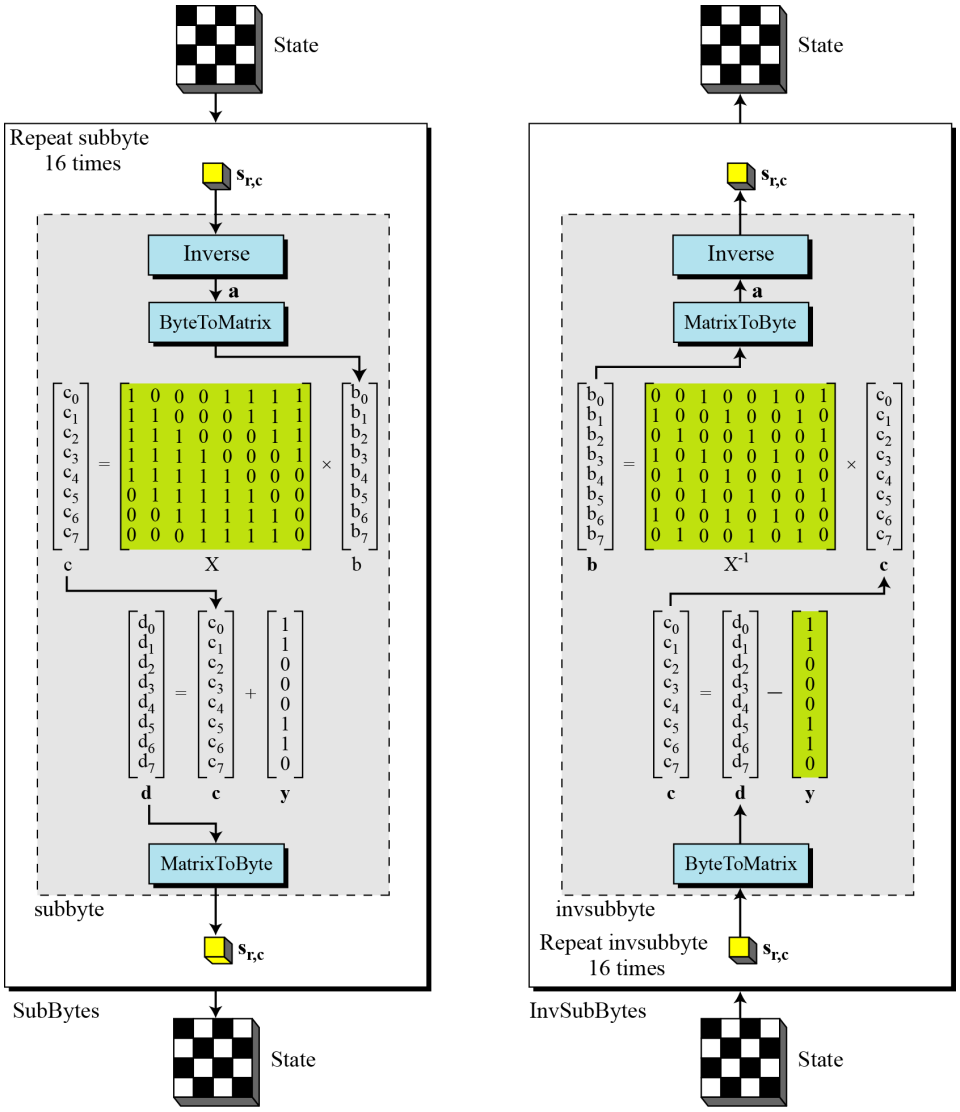
$$\text{invsubbyte:} \quad \rightarrow \quad [\mathbf{X}^{-1}(\mathbf{d} \oplus \mathbf{y})]^{-1} = [\mathbf{X}^{-1}(\mathbf{X}\,(s_{r,c})^{-1} \oplus \mathbf{y} \oplus \mathbf{y})]^{-1} = [(s_{r,c})^{-1}]^{-1} = s_{r,c}$$

Note

The SubBytes and InvSubBytes transformations are inverses of each other.

7.2.1 Continue

Figure 7.8 *SubBytes and InvSubBytes processes*



7.2.1 Continue

Example 7.3

Let us show how the byte 0C is transformed to FE by subbyte routine and transformed back to 0C by the invsubbyte routine.

1. *subbyte:*
   a. The multiplicative inverse of 0C in GF(2^8) field is B0, which means **b** is (10110000).
   b. Multiplying matrix **X** by this matrix results in **c** = (10011101)
   c. The result of XOR operation is **d** = (11111110), which is FE in hexadecimal.

2. *invsubbyte:*
   a. The result of XOR operation is **c** = (10011101)
   b. The result of multiplying by matrix $\mathbf{X}^{-1}$ is (11010000) or B0
   c. The multiplicative inverse of B0 is 0C.

7.2.1 Continue

Algorithm 7.1 *Pseudocode for SubBytes transformation*

```
SubBytes (S)
{
   for (r = 0 to 3)
     for (c = 0 to 3)
           S_r,c = subbyte (S_r,c)
}
```

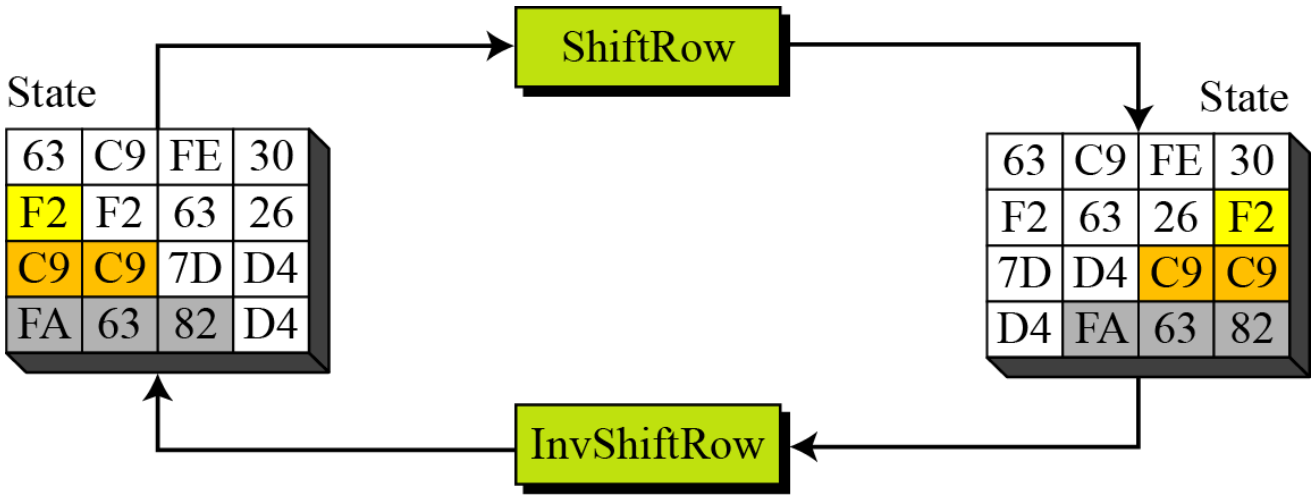
```
subbyte (byte)
{
   a ← byte^-1                 // Multiplicative inverse in GF(2^8) with inverse of 00 to be 00
    ByteToMatrix (a, b)
    for (i = 0 to 7)
    {
        c_i ← b_i ⊕ b_(i+4)mod 8 ⊕ b_(i+5)mod 8 ⊕ b_(i+6)mod 8 ⊕ b_(i+7)mod 8
        d_i ← c_i ⊕ ByteToMatrix (0x63)
   }
    MatrixToByte (d, d)
    byte ← d
}
```

7.2.2 Continue

Example 7.4

Figure 7.10 shows how a state is transformed using ShiftRows transformation. The figure also shows that InvShiftRows transformation creates the original state.

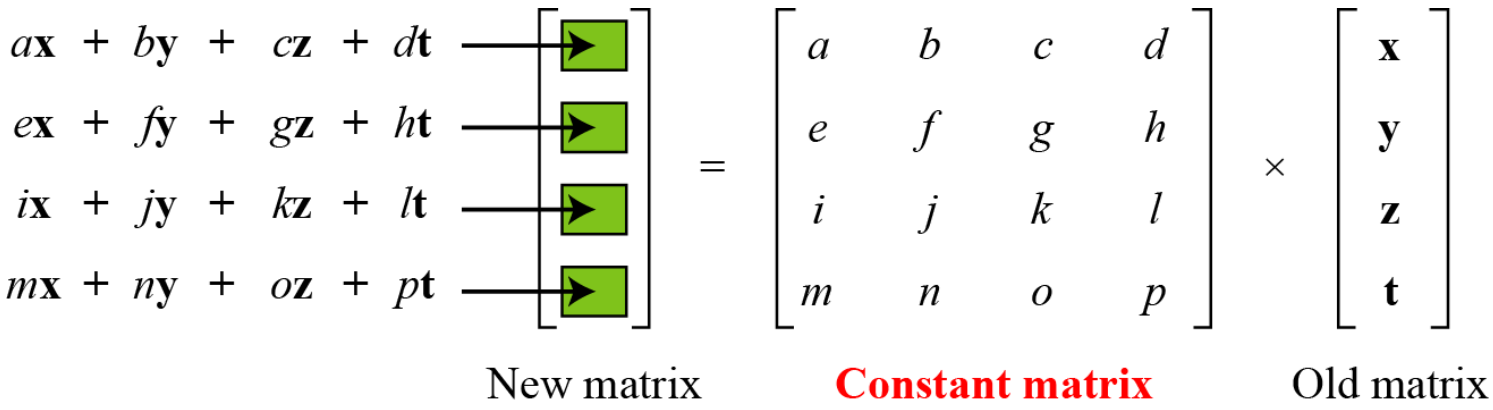
Figure 7.10 *ShiftRows transformation in Example 7.4*



7.2.3 Mixing

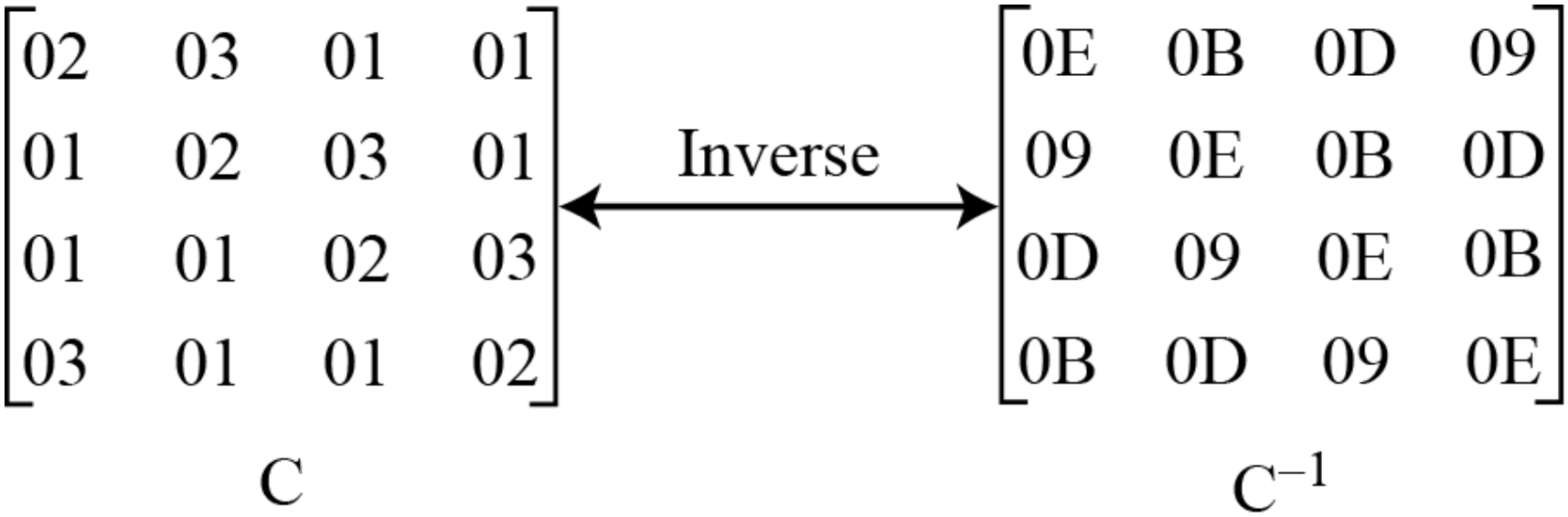
We need an interbyte transformation that changes the bits inside a byte, based on the bits inside the neighboring bytes. We need to mix bytes to provide diffusion at the bit level.

Figure 7.11 *Mixing bytes using matrix multiplication*



7.2.3 Continue

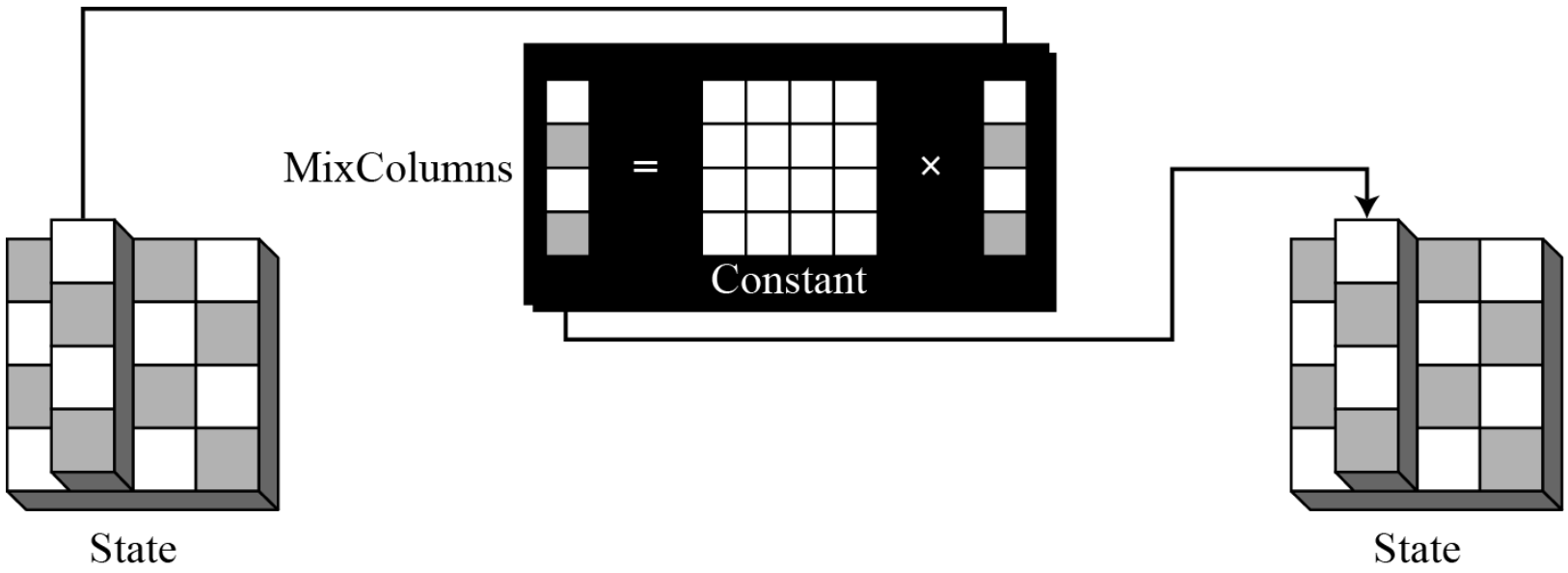
Figure 7.12 *Constant matrices used by MixColumns and InvMixColumns*



MixColumns

The MixColumns transformation operates at the column level; it transforms each column of the state to a new column.

Figure 7.13 *MixColumns transformation*



7.2.3 Continue

Algorithm 7.3 *Pseudocode for MixColumns transformation*

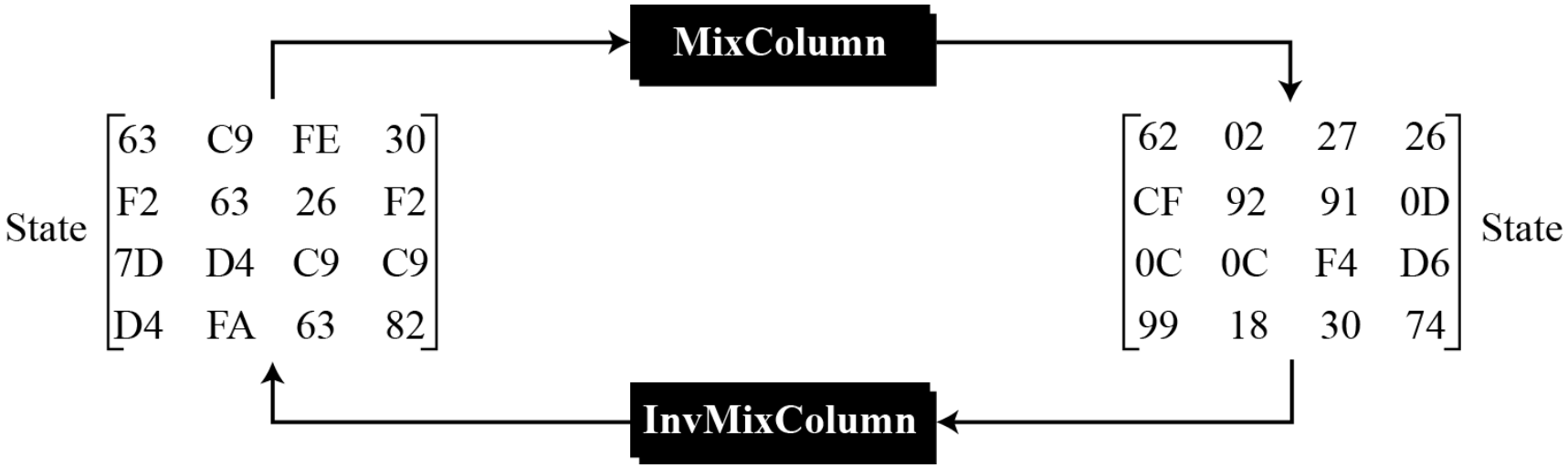
```
MixColumns (S)
{
    for (c = 0 to 3)
        mixcolumn (s_c)
}

mixcolumn (col)
{
   CopyColumn (col, t)                    // t is a temporary column

   col_0 ← (0x02) • t_0 ⊕ (0x03 • t_1) ⊕ t_2 ⊕ t_3

   col_1 ← t_0 ⊕ (0x02) • t_1 ⊕ (0x03) • t_2 ⊕ t_3

   col_2 ← t_0 ⊕ t_1 ⊕ (0x02) • t_2 ⊕ (0x03) • t_3

   col_3 ← (0x03 • t_0) ⊕ t_1 ⊕ t_2 ⊕ (0x02) • t_3
}
```

7.2.3 Continue

Example 7.5

Figure 7.14 shows how a state is transformed using the MixColumns transformation. The figure also shows that the InvMixColumns transformation creates the original one.

Figure 7.14 *The MixColumns transformation in Example 7.5*

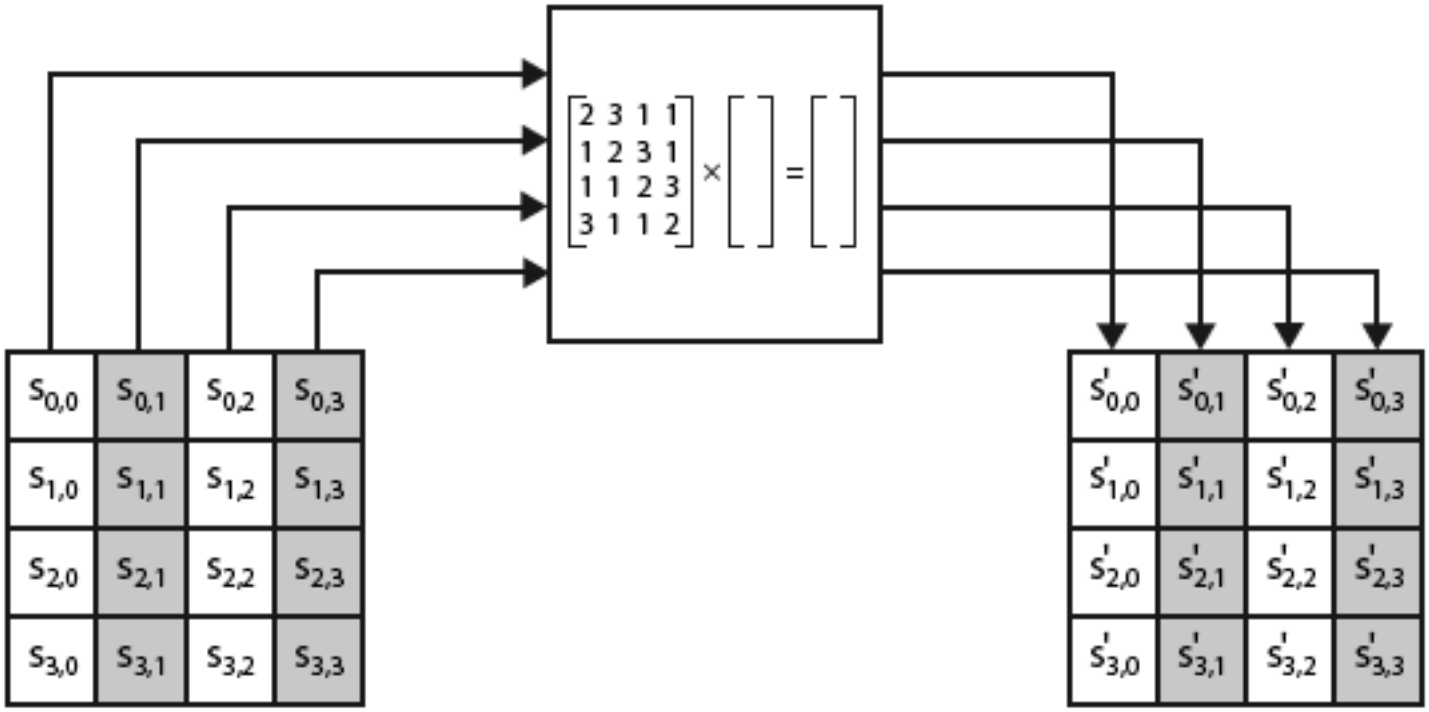


Mix Columns

- each column is processed separately
- each byte is replaced by a value dependent on all 4 bytes in the column
- effectively a matrix multiplication in GF(2^8) using prime poly $m(x) = x^8+x^4+x^3+x+1$

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

Mix Columns


2 3 1 1
1 2 3 1
1 1 2 3
3 1 1 2
×
=
$s_{0,0}$ $s_{0,1}$ $s_{0,2}$ $s_{0,3}$
$s_{1,0}$ $s_{1,1}$ $s_{1,2}$ $s_{1,3}$
$s_{2,0}$ $s_{2,1}$ $s_{2,2}$ $s_{2,3}$
$s_{3,0}$ $s_{3,1}$ $s_{3,2}$ $s_{3,3}$
$s'_{0,0}$ $s'_{0,1}$ $s'_{0,2}$ $s'_{0,3}$
$s'_{1,0}$ $s'_{1,1}$ $s'_{1,2}$ $s'_{1,3}$
$s'_{2,0}$ $s'_{2,1}$ $s'_{2,2}$ $s'_{2,3}$
$s'_{3,0}$ $s'_{3,1}$ $s'_{3,2}$ $s'_{3,3}$

Mix Columns Example

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

→

47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

$$(\{02\} \bullet \{87\}) \oplus (\{03\} \bullet \{6E\}) \oplus \{46\} \oplus \{A6\} = \{47\}$$

$$\{87\} \oplus (\{02\} \bullet \{6E\}) \oplus (\{03\} \bullet \{46\}) \oplus \{A6\} = \{37\}$$

$$\{87\} \oplus \{6E\} \oplus (\{02\} \bullet \{46\}) \oplus (\{03\} \bullet \{A6\}) = \{94\}$$

$$(\{03\} \bullet \{87\}) \oplus \{6E\} \oplus \{46\} \oplus (\{02\} \bullet \{A6\}) = \{ED\}$$

AES Arithmetic

- uses arithmetic in the finite field GF(2^8)
- with irreducible polynomial

$m(x) = x^8 + x^4 + x^3 + x + 1$

which is (100011011) or {11b}

- e.g.

{02} • {87} mod {11b} = (1 0000 1110) mod {11b}

= (1 0000 1110) xor (1 0001 1011) = (0001 0101)

Mix Columns

- can express each col as 4 equations
  - to derive each new byte in col
- decryption requires use of inverse matrix
- have an alternate characterisation
  - each column a 4-term polynomial
  - with coefficients in GF(2^8)
  - and polynomials multiplied modulo (x^4+1)

7-5 Examples

In this section, some examples of encryption/ decryption and key generation are given to emphasize some points discussed in the two previous sections.

Example 7.10

The following shows the ciphertext block created from a plaintext block using a randomly selected cipher key.

Plaintext:	00	04	12	14	12	04	12	00	0C	00	13	11	08	23	19	19
Cipher Key:	24	75	A2	B3	34	75	56	88	31	E2	12	00	13	AA	54	87
Ciphertext:	BC	02	8B	D3	E0	E3	B1	95	55	0D	6D	FB	E6	F1	82	41

7-5 Continued

Example 7.10 Continued

Table 7.7 *Example of encryption*

Round	*Input State*	*Output State*	*Round Key*
Pre-round	00 12 0C 08 04 04 00 23 12 12 13 19 14 00 11 19	24 26 3D 1B 71 71 E2 89 B0 44 01 4D A7 88 11 9E	24 34 31 13 75 75 E2 AA A2 56 12 54 B3 88 00 87
1	24 26 3D 1B 71 71 E2 89 B0 44 01 4D A7 88 11 9E	6C 44 13 BD B1 9E 46 35 C5 B5 F3 02 5D 87 FC 8C	89 BD 8C 9F 55 20 C2 68 B5 E3 F1 A5 CE 46 46 C1
2	6C 44 13 BD B1 9E 46 35 C5 B5 F3 02 5D 87 FC 8C	1A 90 15 B2 66 09 1D FC 20 55 5A B2 2B CB 8C 3C	CE 73 FF 60 53 73 B1 D9 CD 2E DF 7A 15 53 15 D4

Example 7.10 Continued

3	1A 90 15 B2 66 09 1D FC 20 55 5A B2 2B CB 8C 3C	F6 7D A2 B0 1B 61 B4 B8 67 09 C9 45 4A 5C 51 09	FF 8C 73 13 89 FA 4B 92 85 AB 74 0E C5 96 83 57
4	F6 7D A2 B0 1B 61 B4 B8 67 09 C9 45 4A 5C 51 09	CA E5 48 BB D8 42 AF 71 D1 BA 98 2D 4E 60 9E DF	B8 34 47 54 22 D8 93 01 DE 75 01 0F B8 2E AD FA
5	CA E5 48 BB D8 42 AF 71 D1 BA 98 2D 4E 60 9E DF	90 35 13 60 2C FB 82 3A 9E FC 61 ED 49 39 CB 47	D4 E0 A7 F3 54 8C 1F 1E F3 86 87 88 98 B6 1B E1
6	90 35 13 60 2C FB 82 3A 9E FC 61 ED 49 39 CB 47	18 0A B9 B5 64 68 6A FB 5A EF D7 79 8E B2 10 4D	86 66 C1 32 90 1C 03 1D 0B 8D 0A 82 95 23 38 D9

7-5 Continued

Example 7.10 Continued

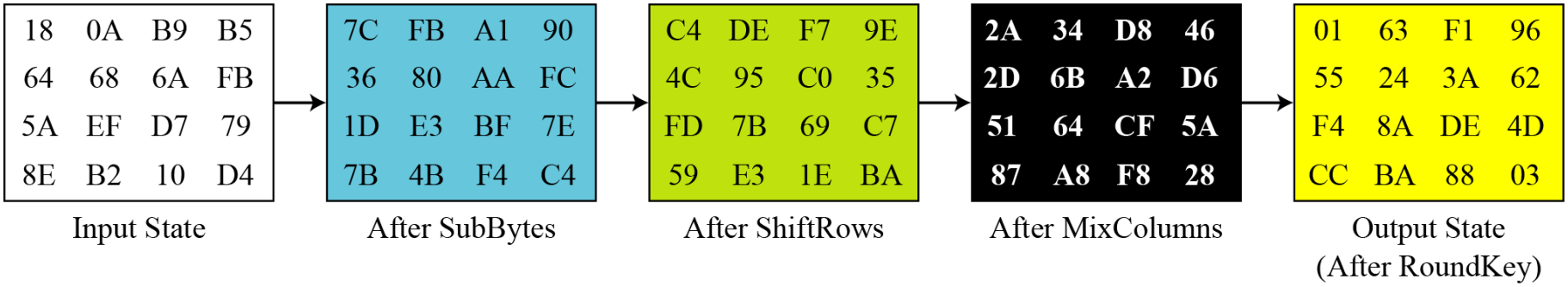
7	18 0A B9 B5 64 68 6A FB 5A EF D7 79 8E B2 10 4D	01 63 F1 96 55 24 3A 62 F4 8A DE 4D CC BA 88 03	62 04 C5 F7 83 9F 9C 81 3E B3 B9 3B B6 95 AD 74
8	01 63 F1 96 55 24 3A 62 F4 8A DE 4D CC BA 88 03	2A 34 D8 46 2D 6B A2 D6 51 64 CF 5A 87 A8 F8 28	EE EA 2F D8 61 FE 62 E3 AC 1F A6 9D DE 4B E6 92
9	2A 34 D8 46 2D 6B A2 D6 51 64 CF 5A 87 A8 F8 28	0A D9 F1 3C 95 63 9F 35 2A 80 29 00 16 76 09 77	E4 0E 21 F9 3F C1 A3 40 E3 FC 5A C7 BF F4 12 80
10	0A D9 F1 3C 95 63 9F 35 2A 80 29 00 16 76 09 77	BC E0 55 E6 02 E3 0D F1 8B B1 6D 82 D3 95 F8 41	DB D5 F4 0D F9 38 9B DB 2E D2 88 4F 26 D2 C0 40

7-5 Continued

Example 7.11

Figure 7.21 shows the state entries in one round, round 7, in Example 7.10.

Figure 7.21 *States in a single round*



7-5 Continued

Example 7.12

One may be curious to see the result of encryption when the plaintext is made of all 0s. Using the cipher key in Example 7.10 yields the ciphertext.

```
Plaintext:  00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
Cipher Key: 24 75 A2 B3 34 75 56 88 31 E2 12 00 13 AA 54 87
Ciphertext: 63 2C D4 5E 5D 56 ED B5 62 04 01 A0 AA 9C 2D 8D
```

7-5 Continued

Example 7.13

Let us check the avalanche effect that we discussed in Chapter 6. Let us change only one bit in the plaintext and compare the results. We changed only one bit in the last byte. The result clearly shows the effect of diffusion and confusion. Changing a single bit in the plaintext has affected many bits in the ciphertext.

```
Plaintext 1:  00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
Plaintext 2:  00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 01
Ciphertext 1: 63 2C D4 5E 5D 56 ED B5 62 04 01 A0 AA 9C 2D 8D
Ciphertext 2: 26 F3 9B BC A1 9C 0F B7 C7 2E 7E 30 63 92 73 13
```

7-5 Continued

Example 7.14

The following shows the effect of using a cipher key in which all bits are 0s.

Plaintext:	00	04	12	14	12	04	12	00	0c	00	13	11	08	23	19	19
Cipher Key:	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
Ciphertext:	5A	6F	4B	67	57	B7	A5	D2	C4	30	91	ED	64	9A	42	72