30th August 2024

Lab 3 – Z-Transform

Objectives: In this lab we will compute ROC and visualize the given Z-Transforms

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3.1 ROC & stability from pole positions of Z-Transform

Consider systems which have Z-Transform of the form $X(z) = \frac{N(z)}{D(z)}$.

For simplicity assume that the numerator is constant $N(z) = 1$, i.e. system has no zeros in the finite z-plane. Also, assume all poles are in the finite z-plane.

There can be many different regions of convergence (ROC) associated with the same $H(z)$ expression. Each of which gives a different time domain signal.

(a) Write a **function** `[N,ROC,ROC_zero,S] = region_of_convergence(p)` with the input vector p containing location of poles in the Z plane (could be complex). These are the roots of polynomial $D(z)$. The outputs should be as follows:

- N – number (a positive integer) of unique ROC possible for this $H(z)$
- ROC – Nx2 matrix (of non-negative real numbers) with each row [r1, r2] indicating an ROC of the form $r_1 < |z| < r_2$
- `ROC_zero` – a scalar indicator variable taking value 1 if zero is part of one of the ROCs else it takes value of 0
- S – length N binary vector (1 if the corresponding system is stable, else 0)

>> Assume there is at least one pole in the input. The input vector has no ordering. Also, there could be multiple poles at the same location. Make sure your code can deal with the various possible inputs given in part (b).

>> You can use the inbuilt matlab functions `sort()` and `unique()`

>> In the output above, r1 could be 0 and r2 could be infinity (use Inf to denote this).

(b) Write a **matlab script** which calls the above function with the following inputs.

Example: p = -1, (**expected output: N = 2, ROC =** $\begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{Inf} \end{bmatrix}$**, ROC_zero = 1**)

p = 0	p = [0,1,2]
p = 0.3	p = [-0.5, -j]
p = [0, 0.65]	p = [0, j, -j]
p = [1, -0.85]	p = [1, -1, 2+j, 2-j]
p = [0.45, -0.45]	p = [1+j, 1+2j, 1+3j, 2-j]
p = [2,2,2]	p = [1, -1, j, -j]

3.2 Tools to visualise Z-Transform

Consider the Z-Transform given by

$$X(z) = \frac{z}{z-p} = \frac{1}{1-pz^{-1}}, \qquad p \in (-1,1)$$

(a) We will visualize the log-magnitude of Z-transform i.e. $\log|X(z)|$ in this **script**. Use `meshgrid()` command to generate a grid on the 2-D z-plane with resolution of 0.01 in a 3x3 square patch centred at origin. Compute the quantity $\log|X(z)|$ and plot using the `mesh()` command. For-loops are not needed. Use the 'rotate 3D' tool to visualize the plot and identify the locations of poles and zeros. Do this for p = 0.8 and p = 1-j.

(b) Use the matlab function `zplane()` to get pole-zero plot of this system for p = 0.8 and p = 1-j. Read documentation of this function and use it in the form `zplane(b,a)`. Note that 'b' and 'a' are coefficients of the polynomial in z^{-1}.

(c) Use the matlab function `impz()` to obtain and plot the corresponding time-domain signal. Use the form `impz(b,a,n)` with n = 51. How many different time-domain signals are possible for given Z-Transform? Which one is returned by `impz()`? Do this for p = 0.8 and p = 1-j.

3.3 System with multiple poles

Consider the Z-Transform of a system given by

$$X(z) = \frac{z^2 - (2\cos\theta)\,z + 1}{z^2 - (2r\cos\theta)z + r^2}, \qquad r \in (0,1), \theta \in [0,\pi]$$

(a) Where are the poles and zeros of this Z-Transform? How do they change as the parameters r and θ are changed?

(b) When $r = 0.95$ and $\theta = \frac{\pi}{3}$, use `zplane(b,a)` to get poles & zeros of this system and verify with your answer in part (a).

(c) Use `impz(b,a,n)` with n = 101 to plot the impulse response of this system for various combinations of (r,θ) obtained by using $r = \{0.1, 0.5, 0.9, 0.99\}$ and $\theta = \{0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi\}$. Note your observations as (i) r is changed for fixed θ and (ii) θ is changed for fixed r.