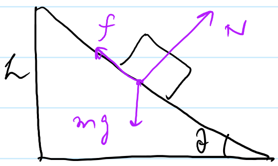
Solutions

Saturday, May 2, 2020 11:55 AM

1.1



Let us consider only vertical motion

$N = mg\cos\theta$

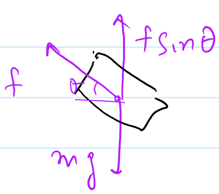
$f = \mu mg\cos\theta$

$ma = mg\sin\theta - f$

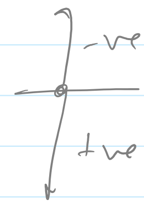
$\Rightarrow a_d = g(\sin\theta - \mu\cos\theta)$ down

When going up, $a_u = g(\sin\theta + \mu\cos\theta)$

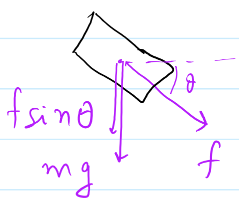
Downward motion



$$ma_d = mg - f\sin\theta$$
$$= mg - \mu mg\cos\theta\sin\theta$$
$$\Rightarrow a_d = g(1 - \mu\sin\theta\cos\theta)$$



Upward motion



$$ma_u = mg + f\sin\theta$$
$$\Rightarrow a_u = g(1 + \mu\sin\theta\cos\theta)$$

$v = u + at$

$x = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2ax$

So, using $x = ut + \frac{1}{2}at^2$,

$$h = 0 + \frac{1}{2}a_d t_d^2 \Rightarrow \boxed{t_d = \sqrt{\frac{2h}{a_d}}}$$

For upward motion,

$$v = u_u + a_u t_u \Rightarrow 0 = u_u + a_u t_u \Rightarrow u_u = -a_u t_u$$

Then,

$$v^2 = u^2 + 2ax \Rightarrow 0 = (-a_u t_u)^2 + 2a_u(-h)$$

$v^2 = u^2 + 2ax \Rightarrow 0 = (-a_u t_u)^2 + 2a_u(-h)$

$$\Rightarrow t_u^2 = \frac{2a_u h}{a_u^2}$$

$$\Rightarrow \boxed{t_u = \sqrt{\frac{2h}{a_u}}}$$

$a_u > a_d$

$\Rightarrow \sqrt{\frac{1}{a_d}} > \sqrt{\frac{1}{a_u}} \Rightarrow t_d > t_u$

Method 2 (Krotov-inspired)

While going down,

$v = u + at$

$\Rightarrow v_d = 0 + a_d t$

$= 0 + (a_g - a_f)t$

$\Rightarrow v_d = a_g t - a_f t$

Now,

$s = \langle v \rangle t_d$

$= (a_g t_d - a_f t_d)/2$

where

s is dist. travelled on the incline.

$\Rightarrow u_u = a_g t + a_f t$

Now,

$s = \langle v \rangle t_u$

$= (a_g t_u + a_f t_u)/2$

[see "Lessons" for derivation of $\langle v \rangle t$]

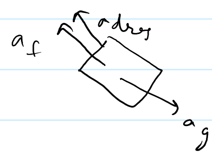
Thus, $t_d \frac{(a_g - a_f)}{2} = t_u (a_g + a_f)$

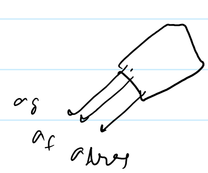
$$\Rightarrow \frac{t_d}{t_u} = \frac{(a_g + a_f)}{(a_g - a_f)} > 1 \Rightarrow t_d > t_u$$

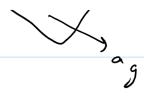
Air drag:

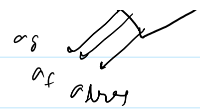
Drag has effects ~ to friction. It will oppose motion

$a_d = a_g - a_f - a_{drag}$ $\qquad$ $a_u = a_g + a_f + a_{drag}$









Thus,

$$\frac{t_d}{t_u} = \frac{(a_g + a_f + a_{drag})}{(a_g - a_f - a_{drag})} > 1$$. So, the result is unchanged

<u>Krotov's solution</u>

In absence of friction,

$$t_u = t_d \quad \left(\text{as } \frac{1}{2} m v^{*2} = mgh, \quad s = \langle v \rangle t = \frac{v^* t}{2}\right)$$

In presence of friction,

$$v_d < v^*$$
$$u_u > v^* \quad \Rightarrow \quad v_d < v^* < u_u$$

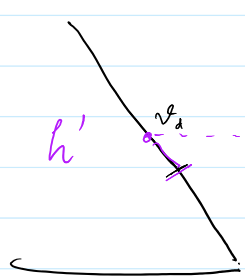
So,

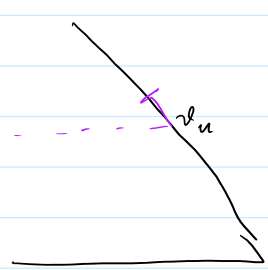
$$s = \frac{\langle v \rangle t}{2} \quad \Rightarrow \quad \langle v_u \rangle t_u = \langle v_d \rangle t_d \quad (\text{as } s \text{ is same})$$

$$\Rightarrow \quad \frac{u_u}{2} t_u = \frac{v_d}{2} t_d$$

$$\Rightarrow \quad \frac{t_d}{t_u} = \frac{u_u}{v_d}. \quad \text{As } u_u > v_d, \ \frac{t_d}{t_u} > 1$$

<u>Air drag</u>





Let ‾ denote drag.
$\bar{v}_d < v_d$

$\bar{v}_u > v_u$ (we need to push initially at a higher velocity b/c of drag. So, every point in the journey will have a higher velocity in case of decy vs case of nodrag

Thus, for dt, assuming $\bar{v}_x(t)$ constant,

$$\delta s = \bar{v}_x \, \delta t_x$$

So,

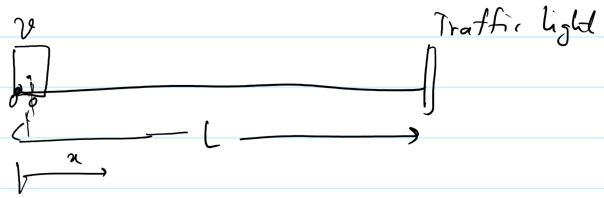
$$\delta s = \bar{v}_d \, \delta t_d = \bar{v}_u \, \delta t_u$$

Now,

$$\bar{v}_d < \bar{v}_u \quad \text{[why?]}$$

So, $\delta t_d > \delta t_u \Rightarrow \int \delta t_d = \int \delta t_u \Rightarrow t_d > t_u$

1.2



$L = 400$

$v = 54 \times \frac{5}{18} = 15 \, m/s$

$t = 60 \, s$

$a = -0.3 \, m/s^2$

We can compute the distance using $x = ut + \frac{1}{2}at^2$, but that would give us the final displa--cement, including negative motion, if any. So, let us check when the object comes to rest.

$v = u + at$

$\Rightarrow \quad 0 = 15 - 0.3t \Rightarrow t = 50 \, s$

Any motion given by the kinematic equations b/w $t = 50$ and $t = 60 \, s$ will be in the negative direction, which is not possible with a braking system. This could happen on a system which is using negative currents on a motor to stop a robot though.

Now, the distance at the point of halt is our final distance. So,

$$x_{halt} = \frac{v^2 - u^2}{2a} = \frac{0^2 - (15)^2}{2(-0.3)} = \frac{225}{.6} = 375 \, m$$

So, distance from traffic light $= L - x_{halt} = 400 - 375 = 25 \, m$

Incorrect solution

Dist from traffic light $= L - x$

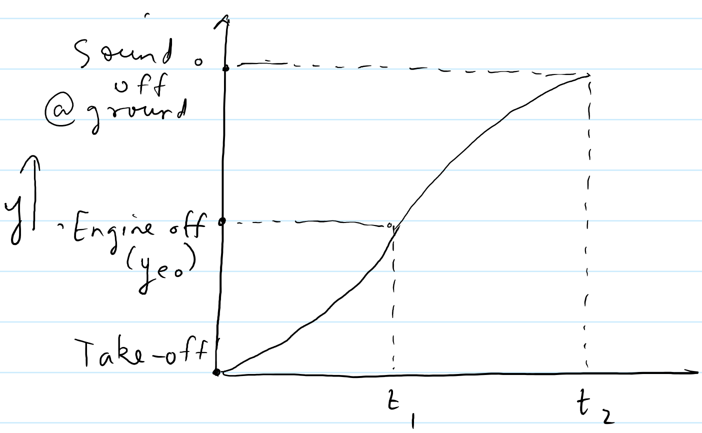
$= L - ut - \frac{1}{2} at^2$

$= 400 - 15(60) - \frac{1}{2}(-0.3)(60)^2$

$= 400 - 900 + 540$

$= 40\,m$

1.3



Given: $v(0) = 0$

$c = 320\,m/s$

$a = 3\,m/s^2$

$t_2 = 30\,s$

To find: $v(t_1)$

$\Delta t = t_2 - t_1$ = time taken by sound of last instance of engine vibration to travel from copter to ground

$y_{eo} = c\,\Delta t$

$\Rightarrow y_{eo} = c(t_2 - t_1)$ — (1)

Also, $y_{eo} = 0 + \frac{1}{2} a t_1^2$ — (2)

From (1) & (2)

$c(t_2 - t_1) = \frac{1}{2} a t_1^2$

$\Rightarrow a t_1^2 + 2c\,t_1 - 2c\,t_2 = 0$

So, $t_1 = \dfrac{(-2c) \pm \sqrt{4c^2 + 8act_2}}{2a}$

$= \dfrac{-c \pm \sqrt{c^2 + 2act_2}}{}$

a

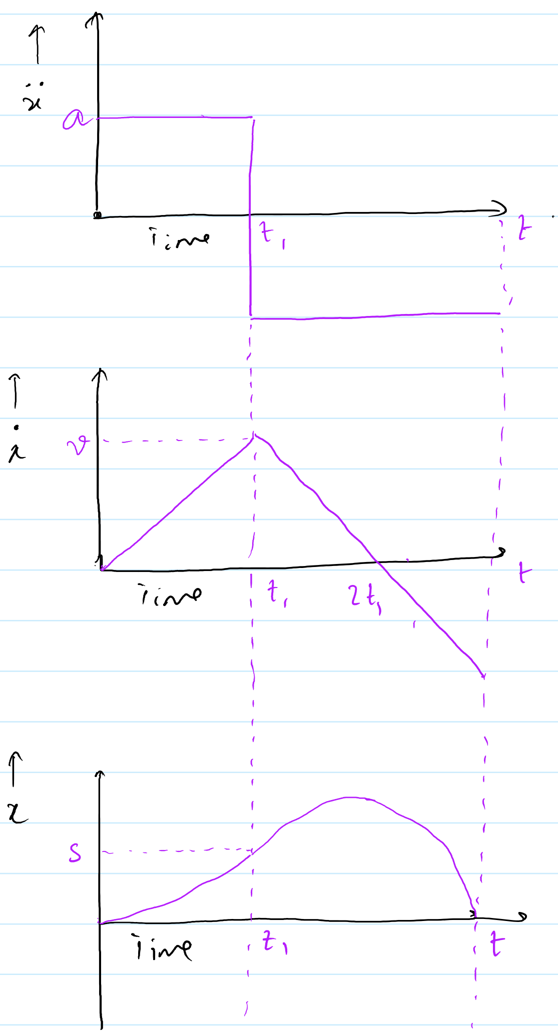
Using this, we get

$$v(t_1) = a t_1$$
$$= -c \pm \sqrt{c^2 + 2act_2}$$

As $v(t_1) > 0$,

$$v_s = \sqrt{c^2 + 2act_2} - c$$
$$= \sqrt{c(c + 2at_2)} - c$$
$$= \sqrt{320(320 + 2.3.30)} - 320$$
$$= \sqrt{320(500)} - 320$$
$$= \sqrt{16.1000} - 320$$
$$= 400 - 320$$
$$= 80\,m/s$$

1.4



First leg of motion

$$s = 0 + \frac{1}{2} a t_1^2 \quad -(1)$$
$$v = 0 + a t_1 \quad -(2)$$

Second leg of motion

$$-s = v(t - t_1) - \frac{1}{2} a (t - t_1)^2$$

Plugging in (2), we get

$$s = -(at_1)(t - t_1) + \frac{1}{2} a (t - t_1)^2 \quad -(3)$$

Using (1), we get

$$\cancel{\frac{1}{2} a t_1^2} = -at_1 t + a t_1^2 + \frac{1}{2} a t^2 + \cancel{\frac{1}{2} a t_1^2} - a t_1 t$$
$$\Rightarrow 0 = a t_1^2 + \frac{1}{2} a t^2 - 2 a t_1 t$$
$$\Rightarrow t^2 - (4t_1) t + 2t_1^2 = 0$$

So,

$$t = \frac{4t_1 \pm \sqrt{(4t_1)^2 - 4(2t_1^2)}}{2}$$

$$= \frac{4t_1 \pm \sqrt{16t_1^2 - 8t_1^2}}{2}$$

$$= (2 \pm \sqrt{2})t_1$$

$$= (2+\sqrt{2})t_1, \ (2-\sqrt{2})t_1$$

From the figures, we know that time elapsed would be at least $2t_1$. This is the time required for the body to come back to rest, before it starts moving backward.

So,

$$t \neq 2t_1 - \sqrt{2}t_1.$$

Thus,

$$t = (2+\sqrt{2})t_1$$

Method-2 (Krotov)

Replace $(t-t_1)$ in eq^n (3) above by t_2. Then, we have

$$s = -at_1t_2 + \frac{1}{2}at_2^2$$

Use (1).

$$\frac{1}{2}at_1^2 = -at_1t_2 + \frac{1}{2}at_2^2$$

Thus, $t_2^2 - 2t_1t_2 - t_1^2 = 0$

$$t_2 = \frac{(2t_1) \pm \sqrt{4t_1^2 + 4t_1^2}}{2}$$

$$= \frac{2t_1 \pm 2\sqrt{2}t_1}{2}$$

$$= t_1(1 \pm \sqrt{2})$$

As time cannot be < 0,

$$t_2 = t_1(1+\sqrt{2})$$

So,

$$t = t_1 + t_2 = (2+\sqrt{2})t_1$$