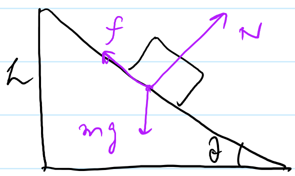
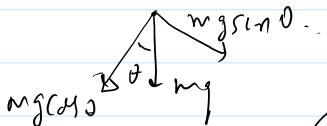
Solutions

Saturday, May 2, 2020   11:55 AM

1.1



Let us consider only vertical motion



$N = mg\cos\theta$

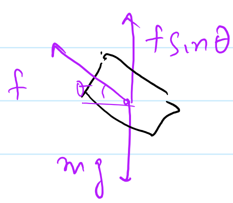
$f = \mu mg\cos\theta$

$\cancel{ma = mg\sin\theta - f}$

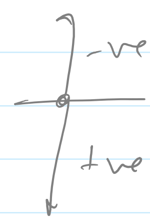
$\cancel{\Rightarrow a_d = g(\sin\theta - \mu\cos\theta)}$ ~~down~~

~~When going up,~~ $\cancel{a_u = g(\sin\theta + \mu\cos\theta)}$

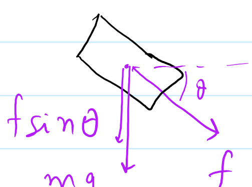
Downward motion



$$ma_d = mg - f\sin\theta$$
$$= mg - \mu mg\cos\theta\sin\theta$$
$$\Rightarrow a_d = g(1 - \mu\sin\theta\cos\theta)$$



Upward motion



$$ma_u = mg + f\sin\theta$$
$$\Rightarrow a_u = g(1 + \mu\sin\theta\cos\theta)$$

$v = u + at$

$x = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2ax$

So, using $x = ut + \frac{1}{2}at^2$,

$h = 0 + \frac{1}{2}a_d t_d^2 \Rightarrow \boxed{t_d = \sqrt{\frac{2h}{a_d}}}$

For upward motion,

$v = u_u + a_u t_u \Rightarrow 0 = u_u + a_u t_u \Rightarrow u_u = -a_u t_u$

Then,

$v = u_u + a_u t_u \Rightarrow 0 = u_u + a_u t_u \Rightarrow u_u = -a_u t_u$

Then,

$$v^2 = u^2 + 2ax \Rightarrow 0 = (-a_u t_u)^2 + 2a_u(-h)$$

$$\Rightarrow t_u^2 = \frac{2a_u h}{a_u^2}$$

$$\Rightarrow \boxed{t_u = \sqrt{\frac{2h}{a_u}}}$$

$a_u > a_d$

$$\Rightarrow \sqrt{\frac{1}{a_d}} > \sqrt{\frac{1}{a_u}} \Rightarrow t_d > t_u$$

Method 2 (Krotov-inspired)

While going down,

$v = u + at$

$\Rightarrow v_d = 0 + a_d t$

$= 0 + (a_g - a_f)t$

$\Rightarrow v_d = a_g t - a_f t$

Now,

$s = \langle v \rangle t_d$

$= (a_g t_d - a_f t_d)/2$

where s is dist. travelled on the incline.

$\Rightarrow u_u = a_g t + a_f t$

Now,

$s = \langle v \rangle t_u$

$= (a_g t_u + a_f t_u)/2$

[see "Lessons" for derivation of $\langle v \rangle t$]

Thus, $$t_d \frac{(a_g - a_f)}{2} = t_u(a_g + a_f)$$

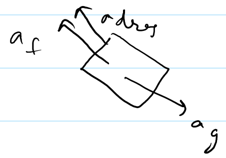
$$\Rightarrow \frac{t_d}{t_u} = \frac{(a_g + a_f)}{(a_g - a_f)} > 1 \Rightarrow t_d > t_u$$

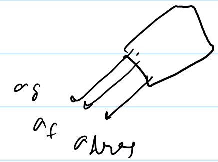
Air drag:

Drag has effects ~ to friction. It will oppose motion

$a_d = a_g - a_f - a_{drag}$ $\qquad$ $a_u = a_g + a_f + a_{drag}$

$$a_d = a_g - a_f - a_{drag} \qquad a_u = a_g + a_f + a_{drag}$$





Thus,

$$\frac{t_d}{t_u} = \frac{(a_g + a_f + a_{drag})}{(a_g - a_f - a_{drag})} > 1$$. So, the result is unchanged

<u>Krotov's solution</u>

In absence of friction,

$$t_u = t_d \quad \left(\text{as } \frac{1}{2} m v^{*2} = mgh, \quad s = \langle v \rangle t = \frac{v^* t}{2}\right)$$

In presence of friction,

$$v_d < v^*$$
$$u_u > v^* \quad \Rightarrow \quad v_d < v^* < u_u$$

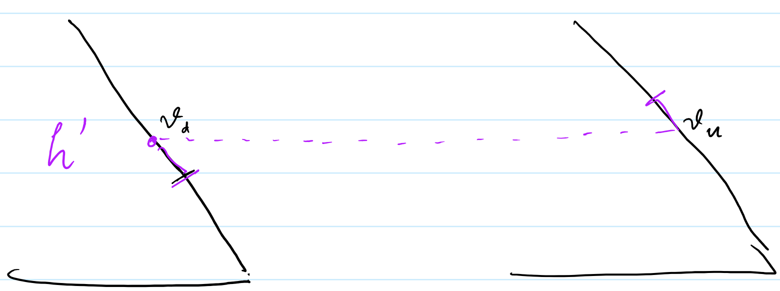
So,

$$s = \frac{\langle v \rangle t}{2} \Rightarrow \langle v_u \rangle t_u = \langle v_d \rangle t_d \quad (\text{as } s \text{ is same})$$

$$\Rightarrow \frac{u_u}{2} t_u = \frac{v_d}{2} t_d$$

$$\Rightarrow \frac{t_d}{t_u} = \frac{u_u}{v_d}$$. As $u_u > v_d$, $\frac{t_d}{t_u} > 1$

<u>Air drag</u>



Let $\bar{\ }$ denote drag

Let $\bar{\ }$ denote drag

$\bar{v}_d < v_d$

$\bar{v}_u > v_u$ (we need to push initially at a higher velocity b/c of drag. So, every point in the journey will have a higher velocity in case of drag vs case of no drag)

Thus, for dt, assuming $v_x(t)$ constant,

$$\delta s = \bar{v}_x \, \delta t_x$$

So,

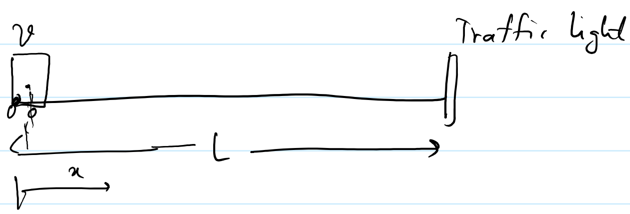
$$\delta s = \bar{v}_d \, \delta t_d = \bar{v}_u \, \delta t_u$$

Now,

$$\bar{v}_d < \bar{v}_u \quad \text{[why?]}$$

$$\text{So, } \delta t_d > \delta t_u \Rightarrow \int \delta t_d = \int \delta t_u \Rightarrow t_d > t_u$$

1-2



$L = 400$

$v = 54 \times \frac{5}{18} = 15\, m/s$

$t = 60\, s$

$a = -0.3\, m/s^2$

We can compute the distance using $x = ut + \frac{1}{2}at^2$, but that would give us the final displa-

-cement, including negative motion, if any. So, let us check when the object comes to rest.

$v = u + at$

$\Rightarrow \quad 0 = 15 - 0.3t \Rightarrow t = 50\, s$

Any motion given by the kinematic equations b/w $t = 50$ and $t = 60\, s$ will be in the negative direction, which is not possible with a braking direction, which is not possible with a braking system. This could happen on a system which is using negative currents on a motor to stop a robot though.

Now, the distance at the point of halt is our final distance. So,

$$x_{halt} = \frac{v^2-u^2}{2a} = \frac{0^2-(15)^2}{2(-0.3)} = \frac{225}{6} = 375\,m$$

So, distance from traffic light $= L - x_{halt} = 400-375 = 25\,m$

Incorrect solution

Dist from traffic light $= L - x$

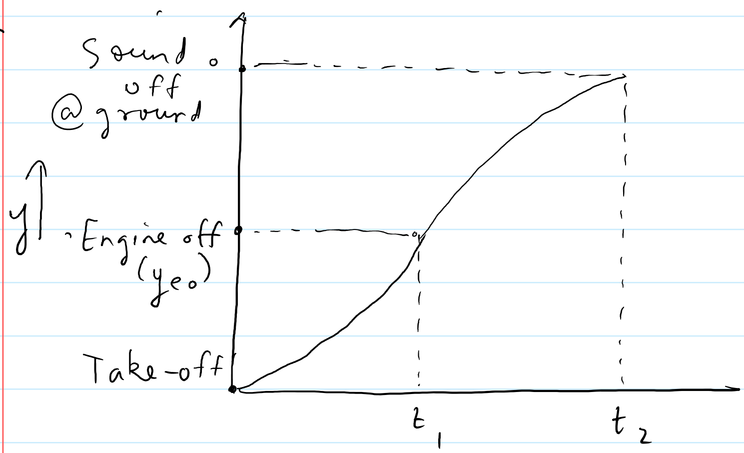
$$= L - ut - \frac{1}{2}at^2$$

$$= 400 - 15(60) - \frac{1}{2}(-0.3)(60)^2$$

$$= 400 - 900 + 540$$

$$= 40\,m$$

1.3



Given: $v(0) = 0$

$c = 320\,m/s$

$a = 3\,m/s^2$

$t_2 = 30\,s$

To find: $v(t_1)$

$\Delta t = t_2 - t_1$ = time taken by sound of last instance of engine vibration to travel from copter to ground

$$y_{eo} = c\,\Delta t$$

$$\Rightarrow y_{eo} = c(t_2 - t_1) \quad -(1)$$

Also, $y_{e0} = 0 + \frac{1}{2} a t_1^2$ — (2)

From (1) & (2)

$c(t_2 - t_1) = \frac{1}{2} a t_1^2$

$\Rightarrow \quad a t_1^2 + 2c\, t_1 - 2c\, t_2 = 0$

So,

$$t_1 = \frac{(-2c) \pm \sqrt{4c^2 + 8act_2}}{2a}$$

$$= \frac{-c \pm \sqrt{c^2 + 2act_2}}{a}$$

Using this, we get

$v(t_1) = a t_1$

$= -c \pm \sqrt{c^2 + 2act_2}$

As $v(t_1) > 0$,

$$v_s = \sqrt{c^2 + 2act_2} - c$$

$$= \sqrt{c(c + 2at_2)} - c$$

$$= \sqrt{320(320 + 2.3.30)} - 320$$

$$= \sqrt{320(500)} - 320$$

$$= \sqrt{16.1000} - 320$$

$$= 400 - 320$$

$$= 80 \text{ m/s}$$