Lessons

Saturday, May 2, 2020 12:38 PM

Kinematic equations (for constant 'a')

(1) $v = u + at$

(2) $\Delta x = ut + \frac{1}{2}at^2$ || (5) $\Delta x = vt - \frac{1}{2}at^2$ || (6) $\Delta x = \langle v \rangle t$

(3) $v^2 = u^2 + 2a\Delta x$

(4) $\langle v \rangle = \frac{u+v}{2}$

Proofs:

(1) $\frac{dv}{dt} = a \Rightarrow \int_u^v dv = \int_0^t a\,dt \Rightarrow v = u + at$

(2) $\frac{dx}{dt} = v \Rightarrow \int_0^{\Delta x} dx = \int_0^t (u+at)\,dt \Rightarrow \Delta x = ut + \frac{1}{2}at^2$

(3) Plug $t = \frac{v-u}{a}$ (from (1)) into (2)

$$\Delta x = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

$$= \frac{2(uv-u^2) + (v^2+u^2-2uv)}{2a}$$

$$\Delta x = \frac{v^2-u^2}{2a}$$

$$\Rightarrow v^2 = u^2 + 2a\Delta x$$

(4) $$\langle v\rangle = \frac{\int_0^t v\,dt}{\int_0^t dt}$$

$$= \frac{\int_0^t (u+at)\,dt}{t}$$

$$= \frac{1}{t}\left[ut + \frac{1}{2}at^2\right]$$

$$= \frac{2u+at}{2}$$

$$= \frac{u+(u+at)}{2}$$

$$= \frac{u+v}{2}$$

(5) Plug u from (1) into (2)

$$\Delta x = (v-at)t + \frac{1}{2}at^2$$

$$= v - at^2 + \frac{1}{2}at^2$$

$$\Delta x = v - \frac{1}{2}at^2$$

(6) From (2),

$$\Delta x = ut + \frac{1}{2} a t^2$$

$$= \frac{t(2u + at)}{2}$$

$$= \frac{t}{2}[u + (u + at)]$$

$$= t\left[\frac{u+v}{2}\right] \quad (\text{from } 1)$$

$$= t\langle v \rangle \quad (\text{from } 4)$$

So,

$$\Delta x = \langle v \rangle t$$