Question 2:

Q2

a)

Ans

First computing mean

Data from normal distribution $= \vec{x}$

max likelihood estimate $= ???$

we can use gaussian distribution to solve this problem

$$\Rightarrow \arg\max_{\mu} \log P(x|\mu\sigma^2)$$

$$\Rightarrow \arg\max_{\mu} \log P(x|\mu)$$

likelihood function $\Rightarrow P(x|\mu) = \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$

Putting values in formula

$$\Rightarrow \arg\max_{\mu} \log \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$$

$$\Rightarrow \frac{d}{d\mu} \ln P(x|\mu) = 0 \quad \Rightarrow \text{Using partial derivative}$$

Putting values in formula

$$\Rightarrow \frac{d}{d\mu} \log \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x_n-\mu)^2}{2\sigma^2}\right]$$

$$\Rightarrow \frac{d}{d\mu}\left[\sum_{n=1}^{N} \log (2\pi\sigma^2)^{1/2} - \sum_{n=1}^{N}\left[\frac{(x_n-\mu)^2}{2\sigma^2}\right]\right] = 0$$

$$\Rightarrow \frac{1}{2\sigma^2} \sum_{n=1}^{N} \frac{d}{d\mu}(x_n-\mu)^2 = 0 \quad \Rightarrow \left(\text{since } \frac{d}{d\mu}\sum_{n=1}^{N} \log(2\pi\sigma^2)^{1/2} = 0\right)$$

$$\Rightarrow \sum_{n=1}^{N} 2(x_n-\mu)(-1) = 0$$

$$\Rightarrow N\mu = \sum_{n=1}^{N} x_n$$

$$\Rightarrow \frac{1}{N}\sum_{n=1}^{N} x_n \Rightarrow \text{MAP of mean}$$

Now calculating variance

$$\arg\max_{\sigma^2} \log P(x|\sigma^2)$$

$$\Rightarrow \frac{d}{d\sigma^2} \log P(x|\sigma^2) = 0$$

$$\Rightarrow \frac{d}{d\sigma^2} \log \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x_n-\mu)^2}{2\sigma^2}\right) = 0$$

$$\Rightarrow \frac{d}{d\sigma^2} \sum_{n=1}^{N} \log (2\pi\sigma^2)^{1/2} - \frac{d}{d\sigma^2} \sum_{n=1}^{N} \frac{(x_n-\mu)^2}{2\sigma^2} = 0$$

$$\Rightarrow N \frac{d}{d\sigma^2} \left[\log(2\pi\sigma^2)\right] \frac{1}{2} - \sum_{n=1}^{N} (x_n-\mu)^2 \cdot \frac{1}{2} \frac{-1}{(\sigma^2)^2} = 0$$

$$\Rightarrow \frac{N}{2} \cdot \frac{1}{\sigma^2} + \frac{1}{2} \frac{1}{(\sigma^2)^2} \sum_{n=1}^{N} (x_n-\mu)^2 = 0$$

$$\Rightarrow \frac{N}{\sigma^2} = \sum_{n=1}^{N} (x_n-\mu)^2$$

$$\Rightarrow \frac{1}{N}\sum_{n=1}^{N} (x_n-\mu)^2 \Rightarrow \text{MAP of variance}$$

Ans

Question 4:

①

Q4

Ans Courses => 1) Machine learning (ML)

2) Probability theory (PT)

3) Linear System (LS)

# of students => 1) ML (30 MS, 40 UG, 10 PhD)

~~2) P(T)~~

2) PT (10 MS, 10 UG, 0 PhD)

3) LS (30 MS, 30 UG, 4 PhD)

a) $P(ML) = 0.2$

$P(PT) = 0.2$

$P(LS) = 0.6$

P(selecting MS student) = ???

$$P(MS) = P(ML) \times \frac{\text{MS students}}{\text{Total students}} + P(PT) \times \frac{\text{MS Students}}{\text{Total students}} + P(LS)\,\frac{\text{MS}}{\text{Total}}$$

$$\Rightarrow 0.2 \times \frac{30}{80} + 0.2 \times \frac{10}{20} + 0.6 \times \frac{30}{64}$$

$$\Rightarrow 0.075 + 0.1 + 0.28$$

$$\Rightarrow 0.45$$

Q4
b)

Total undergrad. students = 80

Undergrads in "LS" = 30

Using Bayes theorm

$$P(A|B) = \frac{P(A)\,P(B|A)}{P(B)}$$

Let A = Undergrads in "LS"

Let B = Total undergrads

$$\Rightarrow \frac{P(A)\,P(B|A)}{P(B|ML)\,P(ML) + P(B|PT)\,P(PT) + P(B|A)\,P(A)}$$

$$\Rightarrow \frac{30/64 \times 0.6}{40/80 \times 0.2 + 10/20 \times 0.2 + 30/64 \times 0.6}$$

$$\Rightarrow \frac{0.2}{0.48} \Rightarrow 0.41$$

$P(A|B) \Rightarrow 0.41$ Ans

Question 5:

Q5

Ans (a)

Ans

$t = 0$

messages recieved from friends $\Rightarrow \lambda = a$

Messages relieved from anonymous users $\Rightarrow \lambda = b$

Message from friend $\Rightarrow P(\text{Friend}) = 25\% \Rightarrow 0.25$

Message from anonymous $\Rightarrow P(\text{anonymous}) = 75\% \Rightarrow 0.75$

Let $u_{friend} = x$, $u_{anonymous} = y$

$\lambda_f = \frac{1}{x}$, $\lambda_a = \frac{1}{y}$

Using Poisson distribution

$$P(a+b) = \frac{e^{-(a+b)} (a+b)^k}{k!}$$ Ans

b)

Ans

messages from anonymous users $= N$

$P(t_1, t_2) = ???$

rate $= \lambda$

$$\lambda = \frac{1}{d}$$

$$\Rightarrow \frac{1}{d} e^{-\frac{1}{d}(a)}$$

$$\Rightarrow \int_{t_1}^{t_2} \frac{1}{d} e^{-\frac{1}{d}(a)} \, da$$ Ans

c)

Ans

Messages recieved per day => 10

$T = [t_1, t_2]$

Using chain rule

$$P\left(\bigcap_{k=1}^{n} A_k\right) = \prod_{k=1}^{n} P\left(A_k \,\middle|\, \bigcap_{j=1}^{k-1} A_j\right)$$

Putting values in formula

~~P(friends, anonymous) => $\prod_{k=1}^{m}$ P friends | Anonymous A~~

~~P(a, b, c, d, t1, t2) = P(a | b, c, d, t1, t2). P(b~~

$P(t_2, t_1, d, c, b, a) = P(t_2 | t_1, d, c, b, a) \cdot P(t_1 | d, c, b, a) \cdot$

$\Rightarrow P(d | c, b, a) \cdot P(c | b, a) \cdot P(b | a) \cdot P(a)$

=> For odd roll numbers

$\Rightarrow \frac{10}{33} \cdot \frac{5}{28} \cdot \frac{10}{18} \cdot \frac{2}{8} \cdot \frac{1}{6} \cdot 0.1$

$\Rightarrow 0.3 \cdot 0.17 \cdot 0.5 \cdot 0.2 \cdot 0.16 \cdot 5$

=> 0.008 => message from friend

=> For even roll numbers

$\Rightarrow \frac{13}{54} \cdot \frac{8}{41} \cdot \frac{20}{33} \cdot \frac{4}{13} \cdot \frac{2}{9} \cdot 0.1$

$\Rightarrow 0.24 \cdot 0.1 \cdot 0.6 \cdot 0.3 \cdot 0.2 \Rightarrow 0.00099$

message from anonymous

Message was sent from friend