Foundations

Key Concepts: Rewards, States, Actions, Policies

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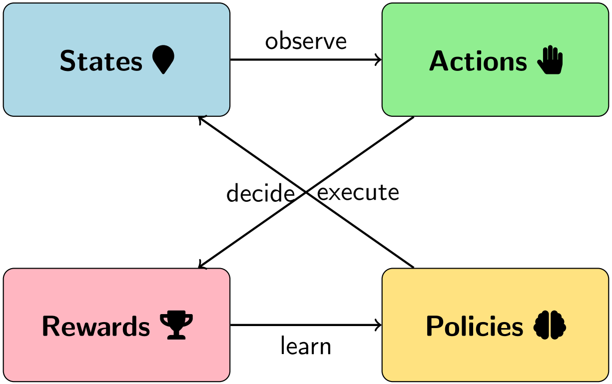
Reinforcement Learning Foundations

Today's Learning Journey

The Four Pillars of Reinforcement Learning



These four concepts form the foundation of every reinforcement learning problem

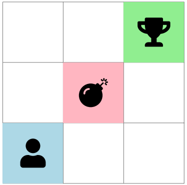
States: The World's Description

What is a State?

- A complete description of the environment at a given time
- Contains all relevant information for decision making
- Denoted as $s \in \mathcal{S}$ (state space)

Key Properties:

- **Markov Property**: Future depends only on current state
- **Observable**: Agent can perceive the state
- **Discrete or Continuous**: Finite vs infinite state spaces

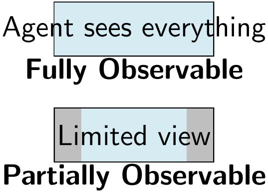


Grid World States

Types of States

Fully Observable

- Agent sees complete environment state
- s_t = environment state
- Example: Chess, Tic-tac-toe



Partially Observable

- Agent has limited view
- o_t = observation $\neq$ state
- Must maintain belief state
- Example: Poker, autonomous driving

Mathematical Representation

State space: $\mathcal{S} = \{s_1, s_2, \ldots, s_n\}$ (discrete) or $\mathcal{S} \subseteq \mathbb{R}^n$ (continuous)

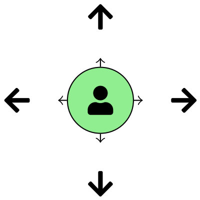
Actions: What the Agent Can Do

What is an Action?

- Choices available to the agent
- Way to influence the environment
- Denoted as $a \in \mathcal{A}$ (action space)

Types of Action Spaces:

- **Discrete**: Finite set of actions
- **Continuous**: Real-valued actions
- **State-dependent**: $\mathcal{A}(s)$ varies by state



Discrete Actions

Examples

- **Game**: {Up, Down, Left, Right, Shoot}
- **Trading**: {Buy, Sell, Hold}
- **Robot**: Joint angles $\mathbb{R}^7$ (continuous)

Action Space Characteristics

Discrete Actions: $\mathcal{A} = \{a_1, a_2, \ldots, a_k\}$. **Examples:** Atari games, Board games, Menu selection

Finite choices

Continuous Actions: $\mathcal{A} \subseteq \mathbb{R}^n$. **Examples:** Robot control, Vehicle steering, Portfolio weights

Infinite possibilities

Mathematical Formulation

Action taken at time t: $a_t \in \mathcal{A}(s_t)$ where $\mathcal{A}(s_t)$ is the set of valid actions in state s_t

Rewards: The Learning Signal

What is a Reward?

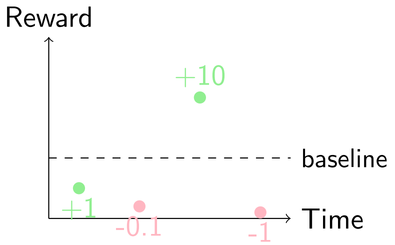
- Scalar feedback signal
- Indicates how good an action was
- Denoted as $r_t \in \mathbb{R}$
- **The only way agent learns!**

Reward Function:

$$R(s, a, s') = \mathbb{E}[r_{t+1} | s_t = s, a_t = a, s_{t+1} = s']$$

Key Principles:

- Immediate vs Delayed rewards
- Sparse vs Dense rewards
- Positive, negative, or zero



Reward Signal Over Time

Reward Hypothesis

"All goals can be described by the maximization of expected cumulative reward"

Types of Rewards

Dense Rewards: Frequent feedback, Easy to learn, May lead to myopic behavior

Shaped Rewards: Engineered guidance, Balance of both, Risk of reward hacking

Sparse Rewards: Infrequent feedback, Hard to learn, More realistic

Examples

- **Dense**: Game score every frame
- **Sparse**: Win/lose at end of game
- **Shaped**: Distance to goal + win bonus

Policies: The Agent's Strategy

What is a Policy?

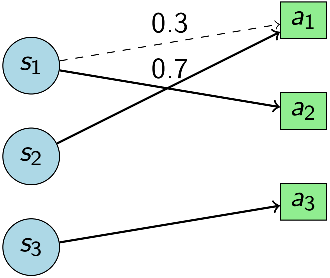
- Agent's behavior function
- Maps states to actions
- Denoted as $\pi : \mathcal{S} \to \mathcal{A}$
- **The brain of the agent**

Mathematical Definition:

$$\pi(a|s) = P(a_t = a|s_t = s)$$

Types:

- **Deterministic**: $a = \pi(s)$
- **Stochastic**: $\pi(a|s)$ (probability distribution)

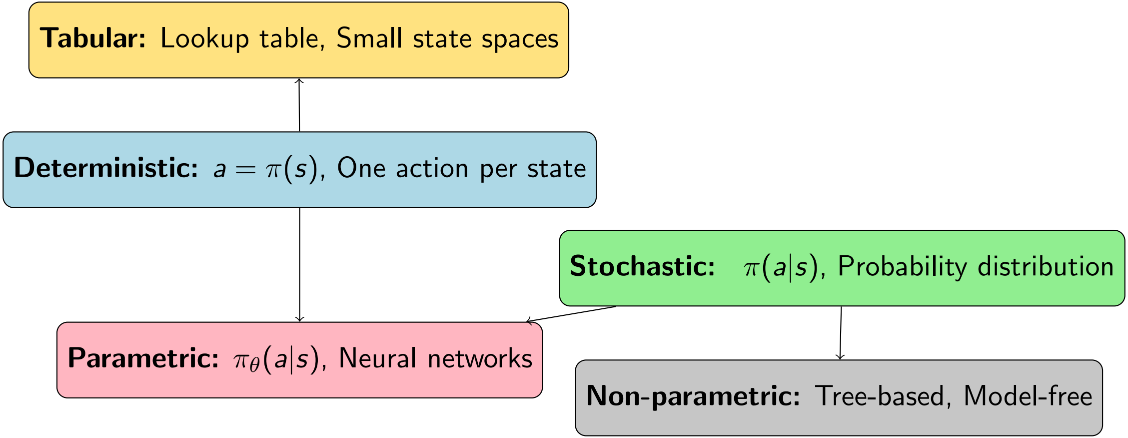


Stochastic Policy

Goal

Find optimal policy π^* that maximizes expected cumulative reward

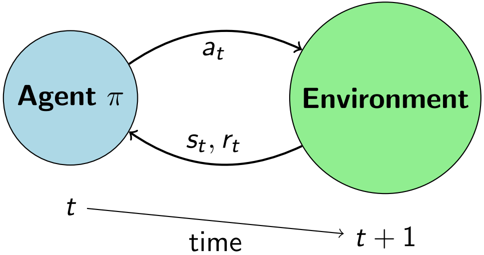
Policy Types and Representations



Policy Optimization

Learning involves finding π^* through exploration and exploitation

The RL Loop: How Components Interact



Agent observes s_t

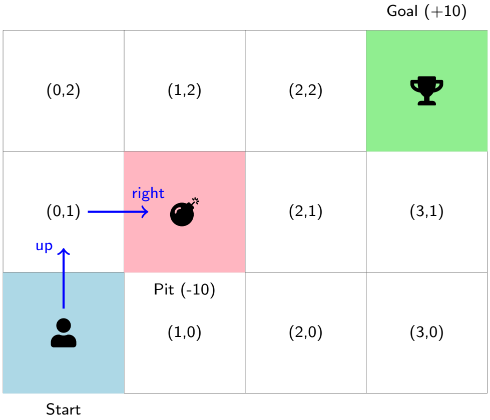
Policy selects a_t

Environment returns r_t, s_{t+1}

The Sequential Decision Process:

1. Agent observes state s_t
2. Policy π selects action a_t
3. Environment provides reward r_t and next state s_{t+1}
4. Agent updates policy and repeats

Example: Grid World Navigation



States: Grid positions
Actions: {up, down, left, right}
Rewards: Goal: +10, Pit: -10, Step: -1
Policy: Navigate to goal safely

Key Relationships

State $\rightarrow$ Action

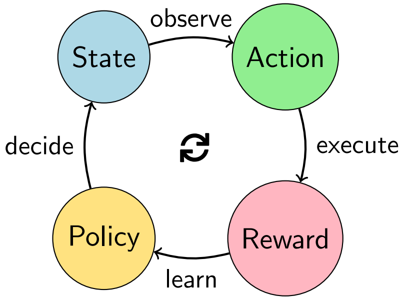
- Policy determines action selection
- $a_t \sim \pi(\cdot|s_t)$

Action $\rightarrow$ Reward

- Environment provides feedback
- $r_t = R(s_t, a_t, s_{t+1})$

Reward $\rightarrow$ Policy

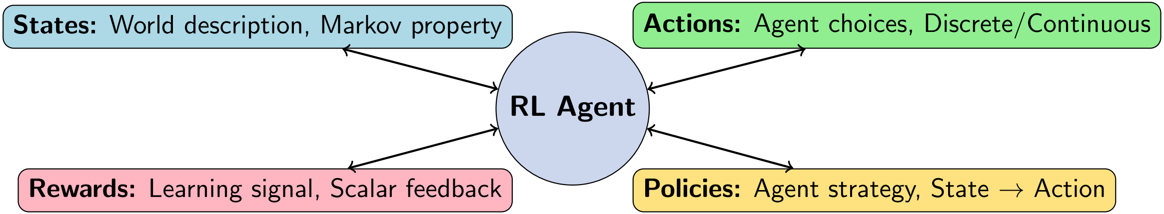
- Learning signal for improvement
- π updated to maximize rewards



Central Equation

$$\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t | \pi\right]$$

Key Takeaways



Essential Points

- These four components define every RL problem
- Agent learns optimal policy through trial and error
- Goal: Maximize expected cumulative reward
- Foundation for all RL algorithms we'll study

Next Steps

What's Coming Next?

Examples of RL applications (games, robotics, recommendation systems)

Preparation:

- Review probability theory
- Think about how to formalize the concepts learned today
- Consider: How do we measure policy quality?

These foundations will be the building blocks for everything that follows!