Chapter 4

Motion in a Plane

SCALARS AND VECTORS

Scalars : *The physical quantities which have only magnitude but no direction, are called scalar quantities.*

For example - distance, speed, work, temperature, mass, etc.

- Scalars are added, subtracted, multiplied and divided by ordinary laws of algebra.

Vectors: *For any quantity to be a vector,*

(i) it must have magnitude.

(ii) it must have direction.

(iii) it must satisfy parallelogram law of vector addition.

For example – displacement, velocity, force, etc.

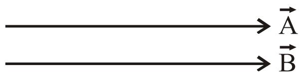
Note : Electric current has magnitude as well as direction but still it is not treated as a vector quantity because it is added by ordinary law of algebra.

Types of Vectors

(i) Like vectors : Vectors having same direction are called like vectors. The magnitude may or may not be equal.

$\vec{A}$ and $\vec{B}$ are like vectors. These are also called parallel vectors or collinear vectors.

(ii) Equal vectors : Vectors having same magnitude and same direction are called equal vectors.



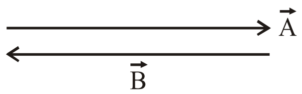
Here $\vec{A}$ and $\vec{B}$ are equal vectors $\vec{A} = \vec{B}$

Thus, equal vector is a special case of like vector.

(iii) Unlike vectors : Vectors having exactly opposite directions are called unlike vectors. The magnitude may or may not be equal.

$\vec{A}$ and $\vec{B}$ are unlike vectors.

(iv) Negative vectors : Vectors having exactly opposite direction and equal magnitudes are called negative vectors.

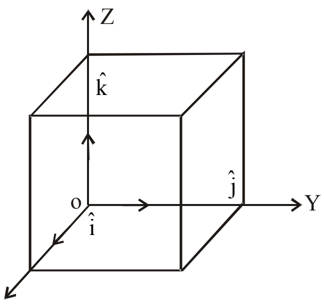


Here $\vec{A}$ and $\vec{B}$ are negative vectors, $\vec{A} = -\vec{B}$

Thus negative vectors is a special case of unlike vectors.

(v) Unit vector : Vector which has unit magnitude. It represents direction only. For example take a vector $\vec{B}$. Unit vector in the direction of $\vec{B}$ is $\frac{\vec{B}}{|\vec{B}|}$, which is denoted as $\hat{B}$. $\hat{B}$, is read as "B cap" or "B caret".

(vi) Orthogonal unit vector : A set of unit vectors, having the directions of the positive x, y and z axes of three dimensional rectangular coordinate system are denoted by $\hat{i}, \hat{j}$ and $\hat{k}$. They are called orthogonal unit vectors because angle between any of the two unit vectors is 90º.



The coordinate system which has shown in fig. is called right handed coordinate system. Such a system derives its name from the fact that right threaded screw rotated through 90º from OX to OY will advance in positive Z direction as shown in the figure.

(vii) Null vector (zero vector) : A vector of zero magnitude is called a zero or null vector. Its direction is not defined. It is denoted by **0**.

Properties of Null or Zero Vector :

(a) The sum of a finite vector $\vec{A}$ and the zero vector is equal to the finite vector

i.e., $\vec{A} + \mathbf{0} = \vec{A}$

(b) The multiplication of a zero vector by a finite number n is equal to the zero vector

i.e., $\mathbf{0}\,n = 0$

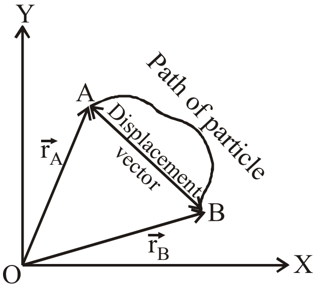
(c) The multiplication of a finite $\vec{A}$ by a zero is equal to zero vector

i.e., $\vec{A}\,\mathbf{0} = 0$

(viii) Axial vector : Vector associated with rotation about an axis i.e., produce rotation effect is called axial vector. Examples are angular velocity, angular momentum, torque etc.

(ix) Coplanar vectors : Vectors in the same plane are called coplanar vectors.

(x) Position vectors and displacement vectors : The vector drawn from the origin of the co-ordinate axes to the position of a particle is called **position vector** of the particle. If A (x_1, y_1, z_1) and B (x_2, y_2, z_2) be the positions of the particle at two different times of its motion w.r.t. the origin O, then position vector of A and B are



$$\vec r_A = \overrightarrow{OA} = x_1\hat i + y_1\hat j + z_1\hat k$$

$$\vec r_B = \overrightarrow{OB} = x_2\hat i + y_2\hat j + z_2\hat k .$$

The **displacement vector** is

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} .$$

$$= (x_2 - x_2)\hat i + (y_2 - y_1)\hat j + (z_2 - z_1)\hat k$$

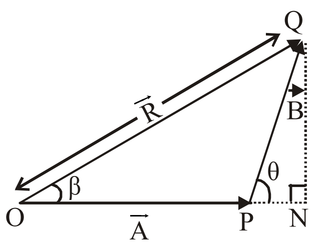
Laws of Vector Algebra

1. $\vec A + \vec B = \vec B + \vec A$ (Commutative law of addition)
2. $\vec A + (\vec B + \vec C) = (\vec A + \vec B) + \vec C$ (Associative law of addition)
3. $m\vec A = \vec A m$
4. $m(n\vec A) = (mn)\vec A$
5. $(m+n)\vec A = m\vec A + n\vec A$
6. $m(\vec A + \vec B) = m\vec A + m\vec B$

ADDITION OF VECTORS

Triangle Law of Vector Addition

It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in one order, their resultant vector is represented in magnitude and direction by the third side of the triangle taken in opposite order.



Magnitude of $\vec R$ is given by $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

where θ is the angle between $\vec A$ and $\vec B$.

Direction of $\vec R$: Let the resultant $\vec R$ makes an angle β with the direction of $\vec A$. Then from right angle triangle QNO,

$$\tan\beta = \frac{QN}{ON} = \frac{QN}{OP + PN} = \frac{B\sin\theta}{A + B\cos\theta}$$

(i) $|\vec R|$ is maximum, if $\cos\theta = 1$, $\theta = 0°$ (parallel vector)

$\xrightarrow{\vec A} \,||\, \xrightarrow{\vec B}$

$R_{max} = \sqrt{A^2 + B^2 + 2AB} = A + B$

(ii) $|\vec R|$ is minimum, if $\cos\theta = -1$, $\theta = 180°$ (opposite vector)

$\xleftarrow{\vec B}\xrightarrow{\vec A}$

$R_{min} = \sqrt{A^2 + B^2 - 2AB} = A - B$

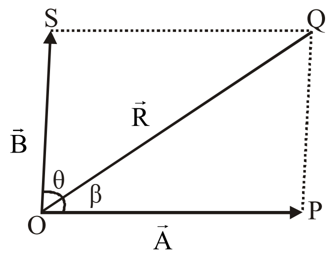
(iii) If the vectors A and B are orthogonal,

i.e., $\theta = 90^\circ, R = \sqrt{A^2 + B^2}$

Parallelogram Law of Vector Addition

It states that if two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram then their resultant is represented in magnitude and direction by the diagonal of the parallelogram.

Let the two vectors $\vec A$ and $\vec B$, inclined at angle θ are represented by sides $\overline{OP}$ and $\overline{OS}$ of parallelogram OPQS, then resultant vector $\vec R$ is represented by diagonal $\overline{OQ}$ of the parallelogram.



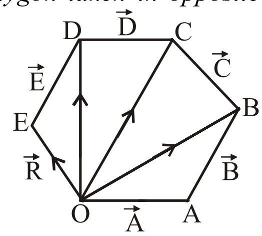
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta};\ \tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

If $\theta < 90°$, (acute angle) $\vec R = \vec A + \vec B$, $\vec R$ is called main (major) diagonal of parallelogram

If $\theta > 90°$, (obtuse angle) $\vec R = \vec A + \vec B$, $\vec R$ is called minor diagonal.

Polygon Law of Vector Addition

If a number of non zero vectors are represented by the (n–1) sides of an n-sided polygon then the resultant is given by the closing side or the n^{th} side of the polygon taken in opposite order.



So, $\vec R = \vec A + \vec B + \vec C + \vec D + \vec E$

or, $\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{OE}$

Note :

1. Resultant of two unequal vectors cannot be zero.
2. Resultant of three co-planar vectors may or may not be zero.
3. Minimum no. of coplanar vectors for zero resultant is 2 (for equal magnitude) and 3 (for unequal magnitude).
4. Resultant of three non coplanar vectors cannot be zero. Minimum number of non coplanar vectors whose sum can be zero is four.
5. Polygon law should be used only for diagram purpose for calculation of resultant vector (For addition of more than 2 vectors) we use components of vector.

Keep in Memory

1. If $\vec{A} = \vec{B}$, then $\vec{A} - \vec{B} = 0$ is a null vector.

2. Null vector or zero vector is defined as a vector whose magnitude is zero and direction indeterminate. Null vector differs from ordinary zero in the sense that ordinary zero is not associated with direction.

3. $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$ is called a unit vector. It is unitless and dimensionless vector. Its magnitude is 1. It represents direction only.

4. If $\vec{A} = \vec{B}$, then $|\vec{A}| = |\vec{B}|$ and $\hat{A} = \hat{B}$, where $\hat{A}$ and $\hat{B}$ are unit vectors of A and B respectively.

5. A vector can be divided or multiplied by a scalar.

6. Vectors of the same kind can only be added or subtracted. It is not possible to add or subtract the vectors of different kind. This rule is also valid for scalars.

7. Vectors of same as well as different kinds can be multiplied.

8. A vector can have any number of components. But it can have only three rectangular components in space and two rectangular components in a plane. Rectangular components are mutually perpendicular.

9. The minimum number of unequal non-coplanar whose vector sum is zero is 4.

10. When $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$, where $|\vec{A}|$ is modulus or magnitude of vector $\vec{A}$.

11. $\hat{i} + \hat{j}$ makes 45° with both X and Y-axes. It makes angle 90° with Z-axis.

12. $\hat{i} + \hat{j} + \hat{k}$ makes angle 54.74° with each of the X, Y and Z-axes.

13. $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$

14. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ then angle between $\vec{A}$ and $\vec{B}$ is $\dfrac{\pi}{2}$.

15. Magnitude of a vector is independent of co-ordinate axes system.

16. Component of a vector perpendicular to itself is zero.

17. (a) Resultant of two vectors is maximum when angle between the vectors is zero i.e., $\theta = 0°$

$R_{max} = A + B$

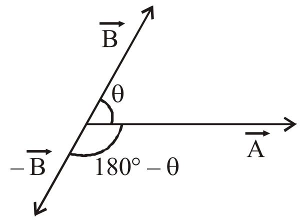
(b) Resultant of two vectors is minimum when $\theta = 180°$

$R_{min} = A - B$

(c) The magnitude of resultant of $\vec{A}$ and $\vec{B}$ can vary between $(A + B)$ and $(A - B)$

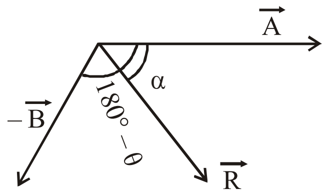
SUBTRACTION OF VECTORS

We convert vector subtraction into vector addition.



$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

If the angle between $\vec{A}$ and $\vec{B}$ is θ then the angle between



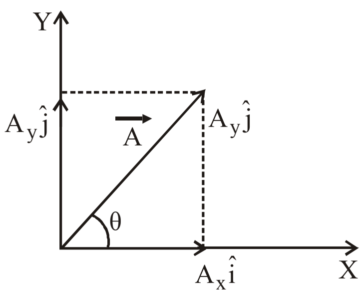
$\vec{A}$ and $-\vec{B}$ is $(180° - \theta)$.

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin(180° - \theta)}{A + B\cos(180° - \theta)} = \frac{B\sin\theta}{A - B\cos\theta}$$

RESOLUTION OF A VECTOR

Rectangular Components of a Vector in Plane



The vector $\vec{A}$ may be written as

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

where $A_x\hat{i}$ is the component of vector $\vec{A}$ in X-direction and $A_y\hat{j}$ is the component of vector $\vec{A}$ in the Y-direction.

Also $A_x = A\cos\theta$ and $A_y = A\sin\theta$

$\therefore \vec{A} = (A\cos\theta)\,\hat{i} + (A\sin\theta)\,\hat{j}$

$\Rightarrow$ A cos θ and A sin θ are the magnitudes of the components of $\vec{A}$ in X and Y-direction respectively.

Also $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

Rectangular components of a vector in 3D : Three rectangular components along X, Y and Z direction are given by $A_x\hat{i}, A_y\hat{j}, A_z\hat{k}$. Therefore,

vector $\vec{A}$ may be written as

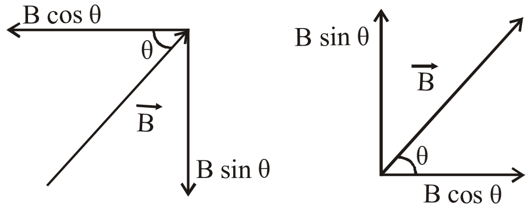
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \text{ and } A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

If α, β and γ are the angles subtended by the rectangular components of vector then

$$\cos\alpha = \frac{A_x}{A},\ \cos\beta = \frac{A_y}{A} \text{ and } \cos\gamma = \frac{A_z}{A}$$

Also, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

⚠ **CAUTION :** Do not resolve the vector at its head. The vector is always resolved at its tail.



Wrong Correct

Example 1.

X and Y component of vector A are 4 and 6 m respectively. The X and Y component of $\vec{A} + \vec{B}$ are 10 m and 9 m respectively. Calculate the length of vector B and its angle with respect to X-axis

Solution :

$$\vec{A} = 4\hat{i} + 6\hat{j} \text{ and } \vec{A} + \vec{B} = 10\hat{i} + 9\hat{j}$$

$$\therefore \vec{B} = (\vec{A} + \vec{B}) - \vec{A} = (10\hat{i} + 9\hat{j}) - (4\hat{i} + 6\hat{j}) = 6\hat{i} + 3\hat{j}$$

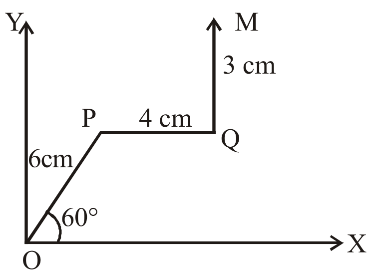
$$\therefore \text{length of } \vec{B} \text{ is } |\vec{B}| = \sqrt{6^2 + 3^2} = 3\sqrt{5}\text{m}$$

$$\text{Also } \tan\theta = \frac{B_y}{B_x} = \frac{3}{6} = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

where θ is the angle which $(\vec{A} + \vec{B})$ is making with X-axis.

Example 2.

Find the resultant of vectors given in figure



Solution :

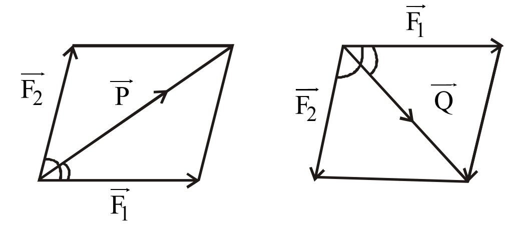
Here $\vec{OP} = 6\cos 60°\,\hat{i} + 6\sin 60°\,\hat{j} = 3\,\hat{i} + 5.2\,\hat{j}$

$\vec{PQ} = 4\,\hat{i}$ and $\vec{QM} = 3\,\hat{j}$

$\therefore \vec{OM} = \vec{OP} + \vec{PQ} + \vec{QM} = 7\,\hat{i} + 8.2\,\hat{j}$

Example 3.

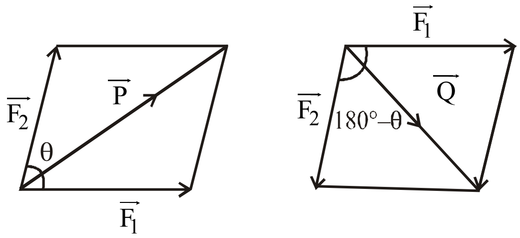
The resultant of two forces F_1 and F_2 is P. If F_2 is reversed, the resultant is Q. Show that $P^2 + Q^2 = 2(F_1^2 + F_2^2)$.



Solution :

Suppose θ be the angle between the forces $\vec{F}_1$ and $\vec{F}_2$,

then $P^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$ ......(i)



When the force F_2 is reversed, then the magnitude of their resultant is

$$Q^2 = F_1^2 + F_2^2 + 2F_1F_2\cos(180° - \theta)$$

$$= F_1^2 + F_2^2 - 2F_1F_2\cos\theta \quad \text{......(ii)}$$

Adding equations (i) and (ii),

$$P^2 + Q^2 = 2F_1^2 + 2F_2^2 = 2(F_1^2 + F_2^2)$$

Example 4.

Find the components of vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$.

Solution :

Here $\vec{A} = 2\hat{i} + 3\hat{j}$

In order to find the component of $\vec{A}$ along the direction of $\hat{i} + \hat{j}$, let us find out the unit vector along $\hat{i} + \hat{j}$. If $\hat{a}$ is the unit vector along $\hat{i} + \hat{j}$, then

$$\hat{a} = \frac{\hat{i} + \hat{j}}{|\hat{i} + \hat{j}|} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

Hence, the magnitude of the component vector of $\vec{A}$ along $\hat{i} + \hat{j}$

$$= \vec{A}.\hat{a} = (2\hat{i} + 3\hat{j}).\frac{\hat{i} + \hat{j}}{\sqrt{2}} = \frac{1}{\sqrt{2}}(2 + 3) = \frac{5}{\sqrt{2}}$$

Therefore, component vector of A along $\hat{i}+\hat{j}$

$$=(\vec{A}.\hat{a})\hat{a}=\frac{5}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)=\frac{5}{2}(\hat{i}+\hat{j})$$

Similarly, if $\hat{b}$ is the unit vector along the direction of $\hat{i}-\hat{j}$, then magnitude of the component vector of $\vec{A}$ along $\hat{i}-\hat{j}$

$$=(\vec{A}.\hat{b})=(2\hat{i}+3\hat{j}).\frac{(\hat{i}-\hat{j})}{|\hat{i}-\hat{j}|}=(2\hat{i}+3\hat{j}).\frac{(\hat{i}-\hat{j})}{\sqrt{2}}=\frac{(2-3)}{\sqrt{2}}=-\frac{1}{\sqrt{2}}$$

$\therefore$ Component vector of $\vec{A}$ along $\hat{i}-\hat{j}$

$$=(\vec{A}.\hat{b})\hat{b}=-\frac{1}{\sqrt{2}}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)=-\frac{1}{2}(\hat{i}-\hat{j})$$

Example 5.

If $0.3\hat{i}+0.4\hat{j}+c\hat{k}$ *is a unit vector, then find the value of c.*

Solution :

Unit vector is a vector of unit magnitude.

$$\therefore \quad \sqrt{0.3^2+0.4^2+c^2}=1 \quad \Rightarrow 0.09+0.16+c^2=1$$

$$\Rightarrow c^2=1-0.25=0.75 \Rightarrow c=\sqrt{0.75}$$

Example 6.

What is the vector joining the points (3, 1, 14) and (–2, –1, –6) ?

Solution :

If P and Q be the points represented by the coordinates (3, 1, 14) and (–2, –1, –6) respectively then,

$\overrightarrow{PQ}$ = p.v. of Q – p.v. of P

$$=(-2\hat{i}-\hat{j}-6\hat{k})-(3\hat{i}+\hat{j}+14\hat{k})=-5\hat{i}-2\hat{j}-20\hat{k}$$

and $\overrightarrow{QP}=-\overrightarrow{PQ}=5\hat{i}+2\hat{j}+20\hat{k}$

Example 7.

Find the angle between two vectors of magnitude 12 and 18 units, if their resultant is 24 units.

Solution :

Magnitude of first vector $(\vec{A})$ = 12; Magnitude of second vector $(\vec{B})$ = 18 and resultant of the given vectors $(\vec{R})$ = 24

$$\therefore 24=\sqrt{A^2+B^2+2AB\cos\theta}$$

$$24=\sqrt{(12)^2+(18)^2+2\times 12\times 18\cos\theta}$$

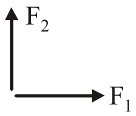
or $(24)^2=144+324+432\cos\theta$ or $432\cos\theta=108$

or $\cos\theta=\frac{108}{432}=0.25$ or $\theta=75°52'$

Example 8.

Two forces $\vec{F}_1=250N$ *due east and* $\vec{F}_2=250N$ *due north have their common initial point. Find the magnitude and direction of* $\vec{F}_2-\vec{F}_1$

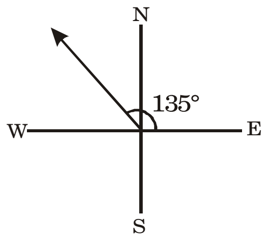
Solution :



$\vec{F}_2=250\hat{j}$, $\vec{F}_1=250\hat{i}$

$\vec{F}_2-\vec{F}_1=250\hat{j}-250\hat{i}$,

$|\vec{F}_2-\vec{F}_1|=\sqrt{250^2+250^2}=250\sqrt{2}$ N (N-W direction)



$$\tan\theta=\frac{250}{-250}=-1 \quad \Rightarrow \theta=135°$$

Example 9.

If $\vec{a},\vec{b}$ *and* $\vec{c}$ *are unit vectors such that* $\vec{a}+\vec{b}+\vec{c}=0$*, then find the angle between* $\vec{a}$ *and* $\vec{b}$*.*

Solution :

Given : $\vec{a}+\vec{b}+\vec{c}=0 \;\Rightarrow \vec{c}=-(\vec{a}+\vec{b})$

Also, $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

Let angle between $\vec{a}$ and $\vec{b}$ $=\theta$

$$\therefore \quad 1=\sqrt{1^2+1^2+2\times1\times1\times\cos\theta}$$

$$\therefore \quad \cos\theta=-1/2 \;\Rightarrow\; \theta=120°=2\pi/3$$

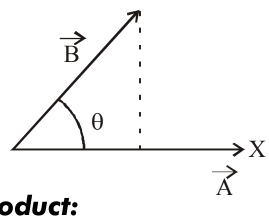
PRODUCT OF TWO VECTORS

Scalar or Dot Product

The scalar or dot product of two vectors A and B is a scalar, which is equal to the product of the magnitudes of $\vec{A}$ *and* $\vec{B}$ *and cosine of the smaller angle between them.*

i.e., $\vec{A}\,.\,\vec{B}=AB\cos\theta$

e.g. $W=\vec{F}\cdot\vec{s}; P=\vec{F}\cdot\vec{v}$



Properties of Scalar or Dot Product:

1. $\hat{i}.\hat{i}=\hat{j}.\hat{j}=\hat{k}.\hat{k}=1$

   $\hat{i}.\hat{j}=\hat{j}.\hat{k}=\hat{k}.\hat{i}=0$

2. $\vec{A}.\vec{B}=A\,(B\cos\theta)=B\,(A\cos\theta)$

   The dot product of two vectors can be interpreted as the product of the magnitude of one vector and the magnitude of the component of the other vector along the direction of the first vector.

3. $\vec{A}.\vec{B} = \vec{B}.\vec{A}$ Dot product of two vectors is **commutative**.

4. $\vec{A}.\vec{A} = A^2$

5. $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B}+\vec{A}.\vec{C}$ Dot product is **distributive**.

6. $\vec{A}.\vec{B} = (A_x\hat{i}+A_y\hat{j}+A_z\hat{k}).(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$

$= (A_x B_x + A_y B_y + A_z B_z)$

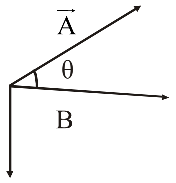
Vector or Cross Product

The vector product of two vectors is defined as a vector having magnitude equal to the product of two vectors and sine of the angle between them. Its direction is perpendicular to the plane containing the two vectors (direction of the vector is given by **right hand screw rule** or **right hand thumb rule**.

$$\vec{C} = \vec{A}\times\vec{B} = (AB\sin\theta)\,\hat{n}$$

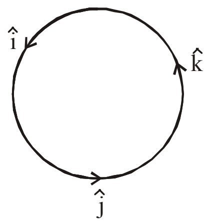
The direction of $(\vec{A}\times\vec{B})$ perpendicular to the plane containing vectors $\vec{A}$ and $\vec{B}$ in the sense of advance of a right handed screw rotated from $\vec{A}$ to $\vec{B}$ is through the smaller angle between them.

$(\vec{A}\times\vec{B}) = \vec{C}$



e.g., $\vec{v} = \vec{\omega}\times\vec{r};\ \vec{\tau} = \vec{r}\times\vec{F};\ \vec{L} = \vec{r}\times\vec{p}$

Properties of Vector or Cross Product



1. $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

2. $\hat{i}\times\hat{j} = \hat{k};\ \hat{j}\times\hat{k} = \hat{i};\ \hat{k}\times\hat{i} = \hat{j}$

3. $\vec{A}\times\vec{A} = 0$

4. $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ **(not commutative)** $[\because \vec{A}\times\vec{B} = -\vec{B}\times\vec{A}]$

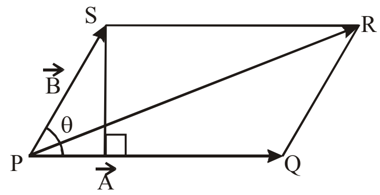
5. $\vec{A}\times(\vec{B}+\vec{C}) = (\vec{A}\times\vec{B})+(\vec{A}\times\vec{C})$ (follows **distributive** law)

6. $\vec{A}\times\vec{B} = (A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\,\hat{i} + (A_z B_x - A_x B_z)\,\hat{j} + (A_x B_y - A_y B_x)\,\hat{k}$$

7. The cross product of two vectors represents the area of the parallelogram formed by them.



$|\vec{A}\times\vec{B}| = A(B\sin\theta)$

$=$ area of parallelogram PQRS

$= 2$ (area of ΔPQR)

8. A unit vector which is perpendicular to A as well as B is

$$\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{\vec{A}\times\vec{B}}{AB\sin\theta}$$

Keep in Memory

1. $\cos\theta = \dfrac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2+A_y^2+A_z^2}\,\sqrt{B_x^2+B_y^2+B_z^2}} = \dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

2. $\tan\theta = \dfrac{|\vec{A}\times\vec{B}|}{\vec{A}\cdot\vec{B}} = \dfrac{AB\sin\theta}{AB\cos\theta}$

3. $|\vec{A}\times\vec{B}|^2 + |\vec{A}\cdot\vec{B}|^2 = A^2B^2$

4. $|\vec{A}+\vec{B}|\times|\vec{A}-\vec{B}| = 2|(\vec{B})\times(\vec{A})|$

5. If $\vec{A}+\vec{B}+\vec{C} = 0$, then $\vec{A}\times\vec{B} = \vec{B}\times\vec{C} = \vec{C}\times\vec{A}$

6. $|\vec{A}\cdot\vec{B}|^2 - |\vec{A}\times\vec{B}|^2 = A^2B^2\cos 2\theta$

7. If $|\vec{A}\times\vec{B}| = \vec{A}.\vec{B}$ then angle between $\vec{A}$ and $\vec{B}$ is $\dfrac{\pi}{4}$.

8. If $\vec{A}\parallel\vec{B}$ then $\vec{A}\times\vec{B} = \vec{0}$

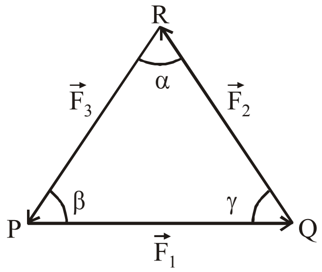
9. Division by a vector is not defined. Because, it is not possible to divide by a direction.

10. The sum and product of vectors is independent of co-ordinate axes system.

CONDITION OF ZERO RESULTANT VECTOR AND LAMI'S THEOREM

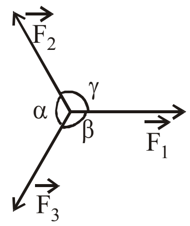
If the three vectors acting on a point object at the same time are represented in magnitude and direction by the three sides of a triangle taken in order, then their resultant is zero and the three vectors are said to be in equilibrium.

i.e. $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$



Lami's Theorem

It states that if three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces.



$$\frac{\vec{F}_1}{\sin\alpha}=\frac{\vec{F}_2}{\sin\beta}=\frac{\vec{F}_3}{\sin\gamma}$$

$$\text{or, } \frac{\vec{F}_1}{PQ}=\frac{\vec{F}_2}{QR}=\frac{\vec{F}_3}{PR}$$

Example 10.

Calculate the area of a parallelogram formed from the vectors $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$ *and* $\vec{B}=2\hat{i}-3\hat{j}+\hat{k}$, *as adjacent sides.*

Solution :

The area of a parallelogram is given by $|\vec{A}\times\vec{B}|$

Here,

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & +2 & 3\\ 2 & -3 & 1\end{vmatrix}$$

$$=\hat{i}\,[(2\times1)-(-3\times3)]+\hat{j}\,[(3\times2)-(1\times1)]$$

$$+\hat{k}\,[(1\times-3)-(2\times2)]$$

$$=11\,\hat{i}+5\,\hat{j}-7\,\hat{k}$$

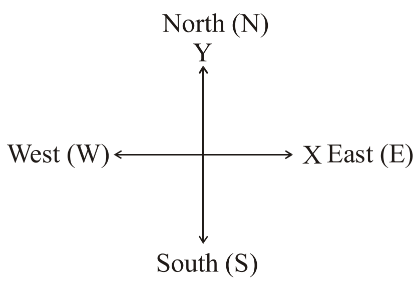
$$\therefore\ |\vec{A}\times\vec{B}|=\sqrt{(11)^2+(5)^2+(-7)^2}\cong 14$$

Example 11.

A particle suffers three displacements 4m in the northward, 2 m in the south-east and 1 m in the south-west directions. What is the displacement of the particle and the distance covered by it?

Solution :

Taking a frame of reference with the x-axis in the eastward and the y-axis in the northward direction



$$\vec{s}_1=4\hat{j},\ \vec{s}_2=2\cos45^\circ\,\hat{i}-2\sin45^\circ\,\hat{j}$$

$$\vec{s}_3=-2\cos45^\circ\,\hat{i}-2\sin45^\circ\,\hat{j}$$

$$\vec{s}=\vec{s}_1+\vec{s}_2+\vec{s}_3=4\hat{j}+\sqrt{2}\,\hat{i}-\sqrt{2}\,\hat{j}-\sqrt{2}\,\hat{i}-\sqrt{2}\,\hat{j}$$

$\therefore$ Displacement $\vec{s}=(4-2\sqrt{2})\hat{j}=(1.17)\hat{j}=(1.17)$ (northward)

And total distance covered = 4 + 2 + 1 = 7m

Example 12.

Prove that vectors $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$ *and* $\vec{B}=2\hat{i}-\hat{j}$ *are perpendicular to each other.*

Solution :

Here, $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{B}=2\hat{i}-\hat{j}$

Two vectors are perpendicular to each other if, $\vec{A}.\vec{B}=0$

Now $\vec{A}.\vec{B}=(\hat{i}+2\hat{j}+3\hat{k}).(2\hat{i}-\hat{j})=0$

$$=1\times2+2\times(-1)+3\times(0)=2-2+0=0$$

Since $\vec{A}.\vec{B}=0$, therefore vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Example 13.

Find the angle between the vectors $\vec{A}=\hat{i}+\hat{j}-2\hat{k}$ *and* $\vec{B}=-\hat{i}+2\hat{j}-\hat{k}$.

Solution :

Here $\vec{A}=\hat{i}+\hat{j}-2\hat{k}$, $\vec{B}=-\hat{i}+2\hat{j}-\hat{k}$

We know that $\vec{A}.\vec{B}=AB\cos\theta$ or $\cos\theta=\dfrac{\vec{A}.\vec{B}}{AB}$

Now $A=\sqrt{1^2+1^2+(-2)^2}=\sqrt{6}$,

$$B=\sqrt{(-1)^2+2^2+(-1)^2}=\sqrt{6}$$

$$\vec{A}.\vec{B}=(\hat{i}+\hat{j}-2\hat{k}).(-\hat{i}+2\hat{j}-\hat{k})=-1+2+3=3$$

$$\cos\theta=\frac{3}{\sqrt{6}\times\sqrt{6}}=\frac{3}{6}=\frac{1}{2}\quad\text{or}\quad\theta=60^\circ$$

Example 14.

A particle is displaced from a point (3, – 4, 5) to another point (–2, 6, – 4) under a force $2\hat{i}+3\hat{j}-\hat{k}$. Find the work done by the force.

Solution :

$$\vec{F}=2\hat{i}+3\hat{j}-\hat{k}$$

The displacement of the particle is

$\vec{s}$ = position vector of point (–2, 6, – 4) – position vector of point (3, – 4, 5)

$$\vec{s}=(-2\hat{i}+6\hat{j}-4\hat{k})-(3\hat{i}-4\hat{j}+5\hat{k})=-5\hat{i}+10\hat{j}-9\hat{k}$$

$\therefore$ work done by the force is

$$W = \vec{F}.\vec{s} = (2\hat{i} + 3\hat{j} - \hat{k}).(-5\hat{i} + 10\hat{j} - 9\hat{k})$$

$$W = (2)(-5) + (3)(10) + (-1)(-9) = 29 \text{ units.}$$

Example 15.

A force $\vec{F} = 6\hat{i} + x\hat{j}$ acting on a particle displaces it from the point A (3, 4) to the point B (1, 1). If the work done is 3 units, then find value of x.

Solution :

$$\vec{d} = -2\hat{i} - 3\hat{j}, \quad \vec{F} = 6\hat{i} + x\hat{j}$$

$$\therefore \quad W = \vec{F}.\vec{d} \ ; \ 3 = -12 - 3x \quad \Rightarrow x = -5$$

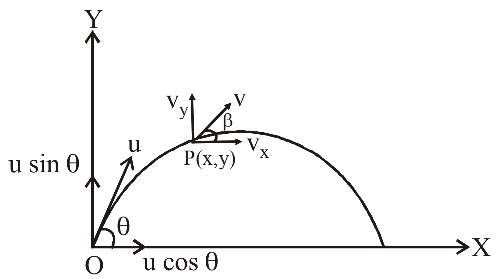
MOTION IN A PLANE OR MOTION IN TWO DIMENSIONS

The motion in which the movement of a body is restricted to a plane is called motion in a plane.

Example : A ball is thrown with some initial velocity (u) and making angle θ with harizontal.

The general approach to solve problem on this topic is to resolve the motion into two mutually perpendicular co-ordinates. One along X-axis and other along Y-axis. These two motions are independent of each other and can be treated as two separate rectilinear motions.

The velocity v and acceleration a can be resolved into its x and y components.



$$v = v_x\hat{i} + v_y\hat{j}$$

$$a = a_x i + a_y j$$

x-component of motion

$$v_x = u_x + a_x t$$

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$v_x^2 - u_x^2 = 2\, a_x x$$

$$x = \left(\frac{u_x + v_x}{2}\right) t$$

y- component of motion

$$v_y = u_y + a_y t$$

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$v_y^2 - u_y^2 = 2\, a_y y$$

$$y = \left(\frac{u_y + v_y}{2}\right) t$$

Velocity

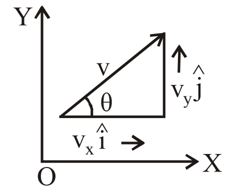
The ratio of the displacement and the corresponding time interval is called the **average velocity**.

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

Average velocity $\bar{v} = \dfrac{\Delta r}{\Delta t} = \hat{i}\dfrac{\Delta x}{\Delta t} + \hat{j}\dfrac{\Delta y}{\Delta t} = \bar{v}_x\hat{i} + \bar{v}_y\hat{j}$

Instantaneous velocity $v_{inst} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta r}{\Delta t}$

The magnitude of $v = \sqrt{v_x^{\,2} + v_y^{\,2}}$



The direction of the velocity, $\tan\theta = \dfrac{v_y}{v_x} \quad \therefore \quad \theta = \tan^{-1}\dfrac{v_y}{v_x}$

Acceleration

The **average acceleration** in a x–y plane in time interval Δt is the change in velocity divided by the time interval.

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

The magnitude of $a = \sqrt{a_x^2 + a_y^2}$

Average acceleration $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{\Delta v_x}{\Delta t}\hat{i} + \dfrac{\Delta v_y}{\Delta t}\hat{j}$

Instantaneous acceleration

$$a_{\text{inst}} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}\hat{i} + \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}\hat{j}$$

In two or three dimensions, the velocity and acceleration vectors may have any angle between 0°and 180° between them.

RELATIVE VELOCITY IN TWO DIMENSIONS

If two objects A and B moving with velocities V_A and V_B with respect to some common frame of reference, then :

(i) Relative velocity of A w.r.t B

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

(ii) Relative velocity of B w.r.t. A

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Therefore, $\vec{v}_{AB} = \vec{v}_{BA}$ and $|\vec{v}_{AB}| = |\vec{v}_{BA}|$

PROJECTILE MOTION

Projectile *is the name given to a body thrown with some initial velocity in any arbitrary direction and then allowed to move under the influence of gravity alone.*

Examples : A football kicked by the player, a stone thrown from the top of building, a bomb released from a plane.

The path followed by the projectile is called a **trajectory.**

The projectile moves under the action of two velocities:

(1) A uniform velocity in the horizontal direction, which does not change (if there is no air resistance)

(2) A uniformly changing velocity in the vertical direction due to gravity.

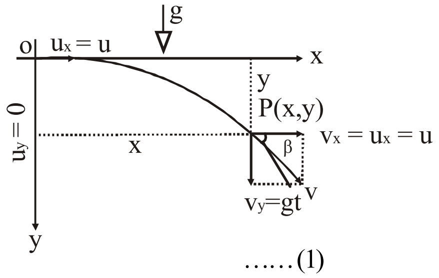
The horizontal and vertical motions are independent of each other.

Types of Projectile:

1. **Oblique projectile :** *In this, the body is given an initial velocity making an angle θ with the horizontal and it moves under the infuence of gravity along a parabolic path.*

2. Horizontal projectile : *In this, the body is given an initial velocity directed along the horizontal and then it moves under the influence of gravity along a parabolic path.*

Motion along x-axis



$u_x = u,\ a_x = 0$

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$x = ut + 0$$

$$\therefore\ t = \frac{x}{u} \qquad \ldots\ldots (1)$$

Motion along y-axis

$u_y = 0,\quad a_y = g$

$$y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow 0 + \frac{1}{2} gt^2$$

$$y = \frac{1}{2} gt^2 \qquad \ldots\ldots (2)$$

From equations (1) and (2) we get $y = \left(\frac{g}{2u^2}\right) x^2$

which is the equation of a **parabola**.

Velocity at any instant :

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$v = \sqrt{u^2 + g^2 t^2}$$

If β is the angle made by $\vec{v}$ with the horizontal, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{gt}{u}$$

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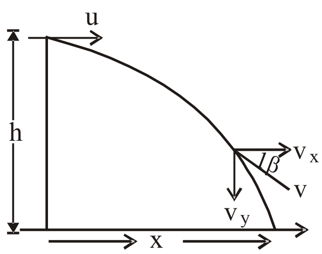
Time of flight and horizontal range:

If h is the distance of the ground from the point of projection, T is the time taken to strike the ground and R is the horizontal range of the projectile then

$$T = \sqrt{\frac{2h}{g}} \quad \text{and} \quad R = u\sqrt{\frac{2h}{g}}$$

Case 1: If the projectile is projected from the top of the tower of height 'h', in horizontal direction, then the height of tower h, range x and time of flight t are related as :

$$h = \frac{1}{2} gt^2 \quad \text{and} \quad x = vt$$



Case 2: If a particle is projected at an angle (θ) in upward direction from the top of tower of height h with velocity u, then

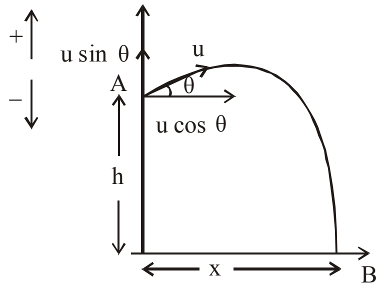
$u_y = u \sin\theta$

$a_y = -g$

$u_x = u\cos\theta$

$a_x = 0$

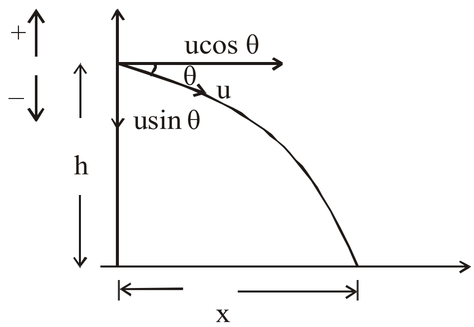
$$h = +u\sin\theta.t - \frac{1}{2} gt^2 \quad \text{and} \quad x = u\cos\theta.t$$



Case 3: If a body is projected at an angle (θ) from the top of tower in downward direction then

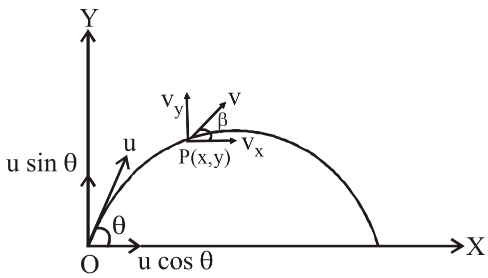
$u_y = -u\sin\theta,\ u_x = u\cos\theta,\ a_x = 0$

$$a_y = +g, -h = -u\sin\theta.t - \frac{1}{2} gt^2 \quad \text{and} \quad x = u\cos q.t$$



Equation of Trajectory

Let the point from which the projectile is thrown into space is taken as the origin, horizontal direction in the plane of motion is taken as the X-axis, the vertical direction is taken as the Y-axis, Let the projectile be thrown with a velocity u making an angle θ with the X-axis.



The components of the initial velocity in the X-direction and Y-direction are u cos θ and u sin θ respectively. Then at any instant of time t,

Motion along x – axis

$u_x = u\cos\theta,\ a_x = 0$

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$x = (u\cos\theta)\,t \quad ...(1)$$

Motion along y–axis

$u_y = u\sin\theta,\ a_y = -g$

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$y = u\sin\theta\, t + \frac{1}{2} g t^2 \quad ...(2)$$

From equations (1) and (2) we get

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta} x^2$$

which is the **equation of a parabola**. Hence the path followed by the projectile is parabolic.

Velocity at any Point

Let v_y be the vertical velocity of projectile at time t. (at P)
And v_x be the horizontal component of velocity at time t.

$$\therefore\ v_y = u\sin\theta - gt \quad ......(1)$$

$$v_x = u\cos\theta \quad ......(2)$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{u^2\cos^2\theta + u^2\sin^2\theta - 2gt\,u\sin\theta + g^2t^2}$$

$$\boxed{v = \sqrt{u^2 + g^2t^2 - 2gt\,u\sin\theta}}$$

and the instantaneous angle (β) with horizontal is given by

$$\tan\beta = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$

Time of Flight :

The time of flight of the projectile is given by

$$\boxed{T = 2t = \frac{2u\sin\theta}{g}},$$

where 't' is the **time of ascent** or **descent**.

Maximum Height :

Maximum height attained by the projectile is given by

$$\boxed{H = \frac{u^2}{2g}\sin^2\theta}.$$

In case of vertical motion, $\theta = 90^\circ$ *so maximum height attained*

$$H = \frac{u^2}{2g}$$

Horizontal Range :

The horizontal range of the projectile is given by

$$\boxed{R = \frac{u^2\sin 2\theta}{g}} \text{ and } \boxed{R_{max} = \frac{u^2}{g}} \text{ at } \theta = 45^\circ$$

($\because$ maximum value of $\sin 2\theta = 1$)

Keep in Memory

1. The horizontal range of the projectile is same at two angles of projection for θ and (90° – θ).
2. The height attained by the projectile above the ground is the largest when the angle of projection with the horizontal is 90° (vertically upward projection). In such a case time of flight is largest but the range is the smallest (zero).
3. If the velocity of projection is doubled. The maximum height attained and the range become 4 times, but the time of flight is doubled.
4. When the horizontal range of the projectile is maximum, (θ = 45°), then the maximum height attained is ¼th of the range.
5. For a projectile fired from the ground, the maximum height is attained after covering a horizontal distance equal to half of the range.

   The velocity of the projectile is minimum but not zero at the highest point, and is equal to *u* cosθ i.e. at the highest point of the trajectory, the projectile has net velocity in the horizontal direction (vertical component is zero). Horizontal component of velocity also remains same as the component of g in horizontal direction is zero i.e., no acceleration in horizontal direction.

Example 16.

A boat takes 2 hours to travel 8 km and back in still water lake. If the velocity of water is 4 km h^{-1}, the time taken for going upstream of 8 km and coming back is

(a) 2 hours
(b) 2 hours 40 minutes
(c) 1 hour and 20 minutes
(d) can't be estimated with given information

Solution : (b)

Total distance travelled by boat in 2 hours = 8 + 8 = 16 km.
Therefore speed of boat in still water, $v_b = 16/2 = 8\ \text{km h}^{-1}$.
Effective velocity when boat moves upstream = $v_b - v_\omega$
$= 8 - 4 = 4\ \text{km h}^{-1}$.
Therefore time taken to travel 8 km distance
$= 8/4 = 2\text{h}$.
Effective velocity when boat moves along the stream
$= v_b + v_\omega = 8 + 4 = 12\ \text{km h}^{-1}$.
The time taken to travel 8 km distance = 8/12 = 2/3h = 40 min.
Total time taken = 2h + 40 min = 2hours 40 min.

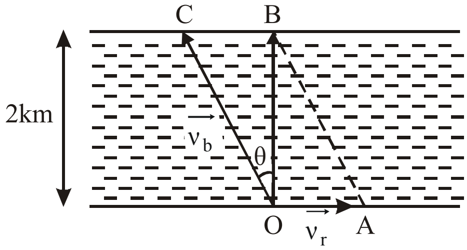
Example 17.

A boat man can row a boat with a speed of 10 km/h in still water. If the river flows at 5 km/h, the direction in which the boat man should row to reach a point on the other bank directly opposite to the point from where he started (width of the river is 2 km).

(a) is in a direction inclined at 120° to the direction of river flow
(b) is in a direction inclined at 90° to the direction of river flow.
(c) is 60° in the north-west direction
(d) is should row directly along the river flow

Solution : (a)

Refer to fig., the boat man should go along OC in order to cross the river straight (i.e. along OB).



$$\sin\theta = \frac{CB}{OC} = \frac{v_r}{v_b} = \frac{5}{10} = \frac{1}{2} = \sin 30^\circ\ ;\ \theta = 30^\circ;$$

$\therefore$ Boat man should go along in a direction inclined at $90^\circ + 30^\circ = 120^\circ$ to the direction of river flow.

Example 18.

A man swims at an angle $\theta = 120^\circ$ to the direction of water flow with a speed v_{mw} = 5km/hr relative to water. If the speed of water v_w = 3km/hr, find the speed of the man.

Solution :

$$\vec{v}_{mw} = \vec{v}_m - \vec{v}_w$$
$$\vec{v}_m = \vec{v}_{mw} + \vec{v}_w$$
$$\Rightarrow v_m = |\vec{v}_{mw} + \vec{v}_w| = \sqrt{v_{mw}^2 + v_w^2 + 2v_{mw}.v_w\cos\theta}$$
$$\Rightarrow v_m = \sqrt{5^2 + 3^2 + 2(5)(3)\cos 120^\circ}$$
$$\Rightarrow v_m = \sqrt{25+9-15} = \sqrt{19}\ \text{m/sec.}$$

Example 19.

A gun throws a shell with a muzzle speed of 98 m/sec. When the elevation is 45°, the range is found to be 900 m. How much is the range decreased by air resistance?

Solution :

Without air resistance, the expected range

$$R = \frac{u^2\sin 2\theta}{g} = \frac{(98)^2\times\sin 90}{9.8} = \frac{(98)^2}{9.8} = 980\,\text{m}$$

Decrease in range = 980 m – 900 m = 80 m

Example 20.

A particle is projected with velocity u at an angle θ with the horizontal so that its horizontal range is twice the greatest height attained. The horizontal range is

(a) u^2/g *(b) $2u^2/3g$*

(c) $4u^2/5g$ *(d) None of these*

Solution : (c)

Greatest height attained

$$H = \frac{u^2\sin^2\theta}{2g} \qquad \ldots\ldots(1)$$

Horizontal range,

$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g} \qquad \ldots\ldots(2)$$

$[\because \sin 2\theta = 2\sin\theta\cos\theta]$

Given that R = 2 H

$$\therefore \frac{2u^2\sin\theta\cos\theta}{g} = \frac{2u^2\sin^2\theta}{2g}$$

Solving we get $\tan\theta = 2$ .....(3)

Hence $\sin\theta = (2/\sqrt{5})$ and $\cos\theta = (1/\sqrt{5})$

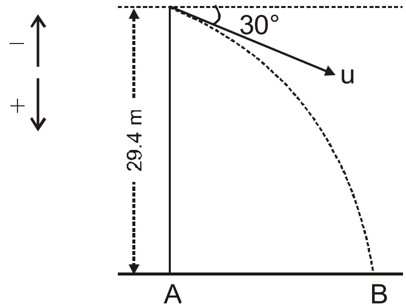
From eqn. (2) $R = \frac{2u^2}{g}\times\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{5}} = \frac{4u^2}{5g}$

Example 21.

A body is projected downwards at an angle of 30° to the horizontal with a velocity of 9.8 m/s from the top of a tower 29.4 m high. How long will it take before striking the ground?

Solution :

The situation is shown in fig.



The time taken by the body is equal to the time taken by the freely falling body from the height 29.4 m with initial velocity u sin θ. This is given by

$$u\sin\theta = \frac{9.8}{2} = 4.9\ \text{m/s}$$

Applying the formula, $s = ut + \frac{1}{2}gt^2$, we have

$$29.4 = 4.9t + \frac{1}{2}(9.8)t^2 \quad \text{or} \quad 4.9t^2 + 4.9t - 29.4 = 0$$

(because s, u and g are all in downward direction)

$t^2 + t - 6 = 0$ or t = 2 or –3

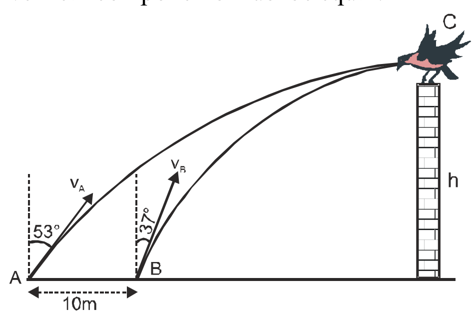
$\therefore$ Time taken to reach ground = 2 second

Example 22.

Two boys stationed at A and B fire bullets simultaneously at a bird stationed at C. The bullets are fired from A and B at angles of 53° and 37° with the vertical. Both the bullets fire the bird simultaneously. What is the value of v_A if v_B = 60 units? (Given : tan 37° = 3/4)

Solution :

The vertical components must be equal.



$\therefore\ v_A \cos 53° = v_B \cos 37°$

or $v_A = v_B \dfrac{\cos 37°}{\cos(90° - 37°)}$

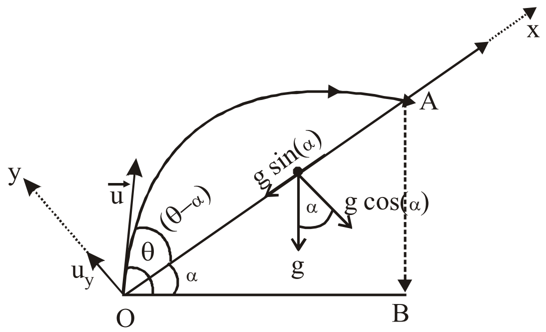
or $v_A = 60 \cot 37° = \dfrac{60}{\tan 37°}$ [$\because v_B = 60$ units]

$= \dfrac{60 \times 4}{3} = 80$ units

PROJECTILE ON AN INCLINED PLANE

Let a body is thrown from a plane OA inclined at an angle α with the horizontal, with a constant velocity u in a direction making an angle θ with the horizontal.

The body returns back on the same plane OA. Hence the net displacement of the particle in a direction normal to the plane OA is zero.



$u_x = u\cos(\theta - \alpha)$ along the incline, + x-axis)
$u_y = u\sin(\theta - \alpha)$ along the incline, + y-axis)
$a_x = g \sin\alpha$ along – x-axis, as retardation
$a_y = g\cos\alpha$ along – y-axis, as retardation

The **time of flight of the projectile** is given by

$$s = ut + \frac{1}{2}at^2$$

or $$0 = u\sin(\theta-\alpha)T - \frac{1}{2}g\cos\alpha T^2$$

$$\boxed{T = \frac{2u\sin(\theta-\alpha)}{g\cos(\alpha)}}$$

If **maximum height** above the inclined plane is H,

$$\boxed{H = \frac{u^2\sin^2(\theta-\alpha)}{2g}}$$

The **horizontal range** R of the projectile is given by

$$OB = u\cos\theta\, t = \boxed{\frac{2u^2\sin(\theta-\alpha)\cos\theta}{g\cos\alpha} = R}$$

The **range of the projectile** at the inclined plane is given by

$$OA = \boxed{\frac{OB}{\cos\alpha} = \frac{2u^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha} = R}$$

Condition for horizontal range R on the inclined plane to be maximum :

Since $R = \dfrac{u^2}{g\cos^2\alpha}[2\sin(\theta-\alpha)\cos\theta]$

$= \dfrac{u^2}{g\cos^2\alpha}[\sin(2\theta-\alpha) - \sin\alpha]$

{2 sin A cos B = sin(A+B)+sin(A–B)}

R is maximum when sin $(2\theta - \alpha)$ is maximum

i.e., $\sin(2\theta-\alpha) = 1$ or $\boxed{\theta = \dfrac{\pi}{4} + \dfrac{\alpha}{2}}$

$\Rightarrow \boxed{R_{max} = \dfrac{u^2}{g\cos^2\alpha}[1 - \sin\alpha]}$

or R_{max}(on inclined plane) $= \dfrac{R_{max}\text{(on horizontal plane)}}{1+\sin\alpha}$

where R_{max} (on horizontal plane) $= \dfrac{u^2}{2g}$.

Condition for time of flight T to be maximum :

$T = \dfrac{2u\sin(\theta-\alpha)}{g\cos\alpha}$ so **T is max when sin $(\theta - \alpha)$ is maximum**

i.e., $\sin(\theta - \alpha) = 1$ or $\boxed{\theta = \dfrac{\pi}{2} + \alpha \Rightarrow T = \dfrac{2u}{g\cos\alpha}}$

It means that if θ_1 is the angle for projectile for which T is maximum and θ_2 is the angle for which R is maximum, then $\theta_1 = 2\theta_2$.

Example 23.

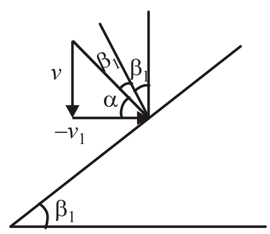
The slopes of wind screen of two cars are $\beta_1 = 30°$ and $\beta_2 = 15°$ respectively. At what ratio v_1/v_2 of the velocities of the cars will their drivers see the hailstorms bounced by windscreen of their cars in the vertical direction? Assume hailstorms falling vertically.

Solution :

From the fig $\tan\alpha = \dfrac{v}{v_1}$ and $\alpha = 90° - 2\beta_1$

where v is velocity of hail

$\tan(90° - 2\beta_1) = \cot 2\beta_1 = \dfrac{v}{v_1}$



Similarly, $\cot 2\beta_2 = \dfrac{v}{v_2}$

$$\frac{v_1}{v_2} = \frac{\cot 2\beta_2}{\cot 2\beta_1} = \frac{\cot 30}{\cot 60} = 3.$$

Example 24.

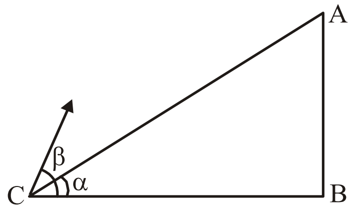
A particle is projected up an inclined plane of inclination α to the horizontal. If the particle strikes the plane horizontally then $\tan \alpha = \ldots$. Given angle of projection with the horizontal is β.

(a) 1/2 $\tan\beta$ *(b) $\tan\beta$*

(c) 2 $\tan\beta$ *(d) 3 $\tan\beta$*

Solution : (a)

If the projectile hits the plane horizontally then



$$T_{plane} = \frac{1}{2} T_{horizontal\,plane}$$

or $$\frac{2u\sin(\beta-\alpha)}{g\cos\alpha} = \frac{u\sin\beta}{g}$$

$$2\sin\beta\cos\alpha - 2\cos\beta\sin\alpha = \cos\alpha\sin\beta$$

or $$\sin\beta\cos\alpha = 2\cos\beta\sin\alpha \quad \text{or} \quad \tan\alpha = \frac{\tan\beta}{2}$$

Keep in Memory

1. Equation of trajectory of an oblique projectile in terms of range (R) is

$$y = x\tan\theta - \left(1 - \frac{x}{R}\right)$$

2. There are two unique times at which the projectile is at the same height h(< H) and the sum of these two times.

Since, $h = (u\sin\theta)t - \frac{1}{2}gt^2$ is a quadratic in time, so it has two unique roots t_1 and t_2 (say) such that sum of roots $(t_1 + t_2)$ is $\frac{2u\sin\theta}{g}$ and product (t_1t_2) is $\frac{2h}{g}$.

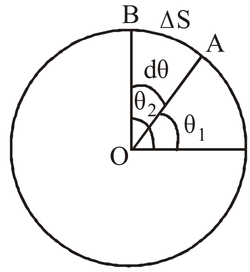
The time lapse $(t_1 - t_2)$ is $\sqrt{\frac{4u^2\sin^2\theta}{g^2} - \frac{8h}{g}}$.

UNIFORM AND NON-UNIFORM CIRCULAR MOTION

Uniform Circular Motion

An object moving in a circle with a constant speed is said to be in uniform circular motion. **Ex.** Motion of the tip of the second hand of a clock.

Angular displacement : Change in angular position is called angular displacement ($d\theta$).



Angular velocity : Rate of change of angular displacement is called angular velocity ω

$$\text{i.e., } \omega = \frac{d\theta}{dt}$$

Relation between linear velocity (v) and angular velocity (ω).

$$\vec{v} = \vec{\omega} \times \vec{r}$$

In magnitude, $v = r\omega$

Angular acceleration : Rate of change of angular velocity is called angular accelaeration.

$$\text{i.e., } \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Relation between linear acceleration and angular accelaration.

$$\vec{a} = \vec{\alpha} \times \vec{r}$$

In magnitude, $a = r\,\alpha$

Centripetal acceleration : *Acceleration acting on a body moving in uniform circular motion is called centripetal acceleration*. It arises due to the change in the direction of the velocity vector. Magnitude of certipetal acceleration is

$$a_c = \frac{v^2}{r} = r\omega^2$$

$$\because \quad \omega = \frac{2\pi}{T} = 2\pi\upsilon \qquad \left(\upsilon = \frac{1}{T} = \text{frequency}\right)$$

$$\therefore \quad a_c = 4\pi^2\upsilon^2 r$$

This acceleration is always directed radially towards the centre of the circle.

Centripetal force: *The force required to keep a body moving in uniform circular motion is called centripetal force.*

$$F_c = \frac{mv^2}{r} = mr\omega^2$$

It is always directed radially inwards.

Centrifugal force : *Centrifugal force is a fictitious force which acts on a body in rotating (non-inertial frames) frame of reference.*

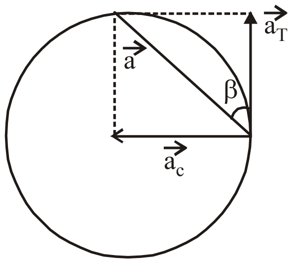
Magnitude of the centrifugal force $F = \frac{mv^2}{r}$.

This force is always directed radially outwards and is also called **corolious force**.

Non-uniform Circular Motion :

An object moving in a circle with variable speed is said to be in non-uniform circular motion.

If the angular velocity varies with time, the object has two accelerations possessed by it, centripetal acceleration (a_c) and Tangential accelaration (a_T) and both perpendicular to each other.



Net acceleration

$$a = \sqrt{a_c{}^2 + a_T{}^2}$$

$$a = \sqrt{(r^2\omega^4 + r^2\alpha^2)}$$

$$a = r\sqrt{\omega^4 + \alpha^2}$$

and, $\tan\beta = \dfrac{a_c}{a_T}$

Keep in Memory

1. Angular displacement behaves like vector, when its magnitude is very very small. It follows laws of vector addition.
2. Angular velocity and angular acceleration are axial vectors.
3. Centripetal acceleration always directed towards the centre of the circular path and is always perpendicular to the instantaneous velocity of the particle.
4. Circular motion is uniform if $a_T = r\alpha = 0$, that is angular velocity remains constant and radial acceleration

   $a_c = \dfrac{v^2}{r} = r\omega^2$ is constant.
5. When a_T or α is present, angular velocity varies with time and net acceleration is

   $$a = \sqrt{a_c{}^2 + a_T{}^2}$$
6. If $a_T = 0$ or $\alpha = 0$, no work is done in circular motion.

Example 25.

A sphere of mass 0.2 kg is attached to an inextensible string of length 0.5 m whose upper end is fixed to the ceiling. The sphere is made to describe a horizontal circle of radius 0.3 m. What will be the speed of the sphere?

Solution :

Centripetal force is provided by component T sin θ, therefore

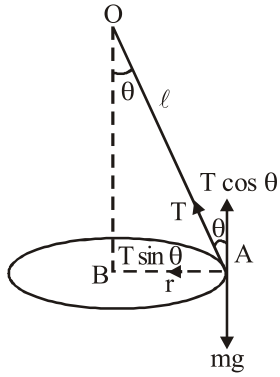
$T\sin\theta = \dfrac{mv^2}{r}$;

and, $T\cos\theta = mg$;

so,

$$\tan\theta = \frac{v^2}{rg} = \frac{r}{\sqrt{\ell^2 - r^2}};$$

$$\left[\because \tan\theta = \frac{r}{\sqrt{\ell^2 - r^2}}\right]$$



$$v = \left[\frac{r^2 g}{(\ell^2 - r^2)^{1/2}}\right]^{1/2}$$

$$= \left[\frac{0.09 \times 10}{(0.25 - 0.09)^{1/2}}\right]^{1/2}$$

$= 1.5$ m/s.

Example 26.

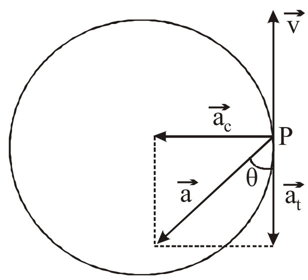
A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn ?

Solution :

Speed, $v = 27\text{ km/h} = 27 \times \dfrac{5}{18}\text{ms}^{-1} = 7.5\text{ms}^{-1}$

centripetal acceleration, $a_c = \dfrac{v^2}{r}$

or $a_c = \dfrac{(7.5)^2}{80}\text{ms}^{-2} = 0.7\text{ms}^{-2}$



P is the point at which cyclist applies brakes. At this point, tangential acceleration a_t, being negative, will act opposite to $\vec{v}$.

Total acceleration, $a = \sqrt{a_c^2 + a_t^2}$

or, $a = \sqrt{(0.7)^2 + (0.5)^2}\text{ms}^{-2} = 0.86\text{ms}^{-2}$

$\tan\theta = \dfrac{a_c}{a_t} = \dfrac{0.7}{0.5} = 1.4$

$\therefore \quad \theta = 54°28'$

CONCEPT MAP

MOTION IN A PLANE
Vector sum of two one dimensional motions along mutually perpendicular directions

Projectile motion
Motion of an object that is in flight after being projected

Total time of flight $T_f = \frac{2u\sin\theta}{g}$

Maximum height $H_{max} = \frac{u^2\sin^2\theta}{2g}$

Horizontal range $R = \frac{u^2\sin^2\theta}{g}$

Components of initial velocity

$u_x = u\cos\theta;\ u_y = u\sin\theta$

Components of acceleration at any instant $a_x = 0,\ a_y = -g$

Position after time t

$x = (u\cos\theta)t;\ y = (y\sin\theta)t - \frac{1}{2}gt^2$

Equation of trajectory

$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}.x^2$

Uniform circular motion
Motion along a circular path with uniform speed.

Angular velocity, $\omega = \frac{\theta}{t}$

Angular acceleration, $\alpha = \frac{\Delta\omega}{\Delta t}$

Centripetal acceleration (a_c)

A body moving along a circular path acted upon by an acceleration directed towards the centre along the radius

$a_c = \frac{v^2}{r} = r\omega^2 = 4\pi^2 rv^2$

Vectors
Quantities that have both magnitude and direction and obey triangle and parallelogram law of vector addition

Position and displacement vector in x-y plane

Position vector, $r = x\hat{i} + y\hat{j}$

Displacement vector, $\Delta r = \Delta x\hat{i} + \Delta y\hat{i}$

Velocity vector in x-y plane

$\vec{v} = v_x\hat{i} + v_y\hat{j}$ and

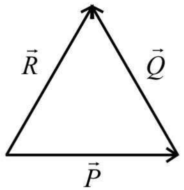
Magnitude $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

Acceleration vector

$\vec{a} = a_x\hat{i} + a_y\hat{j};\ a_x = \frac{dv_x}{dt}$ and $a_y = \frac{dv_y}{dt}$

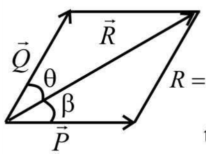
$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$

Triangle law of vector addition



Resultant $\vec{R} = \vec{P} + \vec{Q}$

Parallelogram of law of vector addition



Resultant $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$

Subtraction of two vectors

$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

Multiplication of a vector by a real number $(\lambda) = \lambda\vec{A}$

Types of vector

Zero or null vector
zero magnitude and arbitrary direction

Unit vector
A vector whose magnitude is unity $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

Equal vectors
Two vectors having same magnitude and direction

Negative vectors
$\vec{A}$ is negative of $\vec{B}$ if $\vec{A} = -\vec{B}$

Co-initial vectors
Two vectors having common starting point

EXERCISE - 1
Conceptual Questions

1. It is found that $|A+B|=|A|$. This necessarily implies,
(a) $B=0$
(b) A, B are antiparallel
(c) A, B are perpendicular
(d) $A.B \leq 0$

2. Which one of the following statements is true?
(a) A scalar quantity is the one that is conserved in a process.
(b) A scalar quantity is the one that can never take negative values.
(c) A scalar quantity is the one that does not vary from one point to another in space.
(d) A scalar quantity has the same value for observers with different orientations of the axes.

3. Two balls are projected at an angle θ and (90° – θ) to the horizontal with the same speed. The ratio of their maximum vertical heights is
(a) 1 : 1 (b) tanθ : 1
(c) 1 : tanθ (d) $\tan^2\theta : 1$

4. Which of the following is not correct ?
(a) $\vec{A}\times\vec{B}=-\vec{B}\times A$
(b) $\vec{A}\times\vec{B}\neq\vec{B}\times A$
(c) $\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$
(d) $\vec{A}\times(\vec{B}+\vec{C})=(\vec{A}\times\vec{B})+\vec{C}$

5. The greatest height to which a man can through a ball is *h*. What is the greatest horizontal distance to which he can throw the ball?
(a) 2h (b) $\frac{h}{4}$
(c) $\frac{h}{2}$ (d) None of these

6. If A and B are two vectors, then the correct statement is
(a) A + B = B + A (b) A – B = B – A
(c) A × B = B × A (d) None of these

7. Three vectors A, B and C satisfy the relation A . B = 0 and A . C = 0. The vector A is parallel to
(a) B (b) C
(c) B . C (d) B × C

8. If $\hat{n}$ is a unit vector in the direction of the vector A, then
(a) $\hat{n}=\frac{A}{|A|}$ (b) $\hat{n}=A|A|$
(c) $\hat{n}=\frac{|A|}{A}$ (d) $\hat{n}=\hat{n}\times A$

9. A projectile thrown with a speed v at an angle θ has a range R on the surface of earth. For same v and θ, its range on the surface of moon will be $\left[g_{moon}=\frac{g_{Earth}}{6}\right]$
(a) R/6 (b) R
(c) 6R (d) 36R

10. Given that A + B = R and $A^2 + B^2 = R^2$. The angle between A and B is
(a) 0 (b) π/4
(c) π/2 (d) π

11. Given that A + B = R and A = B = R. What should be the angle between A and B ?
(a) 0 (b) π/3
(c) 2π/3 (d) π

12. Let A = iA cos θ + jA sin θ be any vector. Another vector B, which is normal to A can be expressed as
(a) i B cos θ – j B sin θ (b) i B cos θ + j B sin θ
(c) i B sin θ – j B cos θ (d) i B sin θ + j B cos θ

13. Three particles A, B and C are projected from the same point with same initial speeds making angles 30°, 45° and 60° respectively with the horizontal. Which of the following statements is correct?
(a) A, B and C have unequal ranges
(b) Ranges of A and C are equal and less than that of B
(c) Ranges of A and C are equal and greater than that of B
(d) A, B and C have equal ranges

14. Two particles of equal masses are revolving in circular paths of radii r_1 and r_2 respectively with the same period. The ratio of their centripetal force is
(a) r_1/r_2 (b) $\sqrt{r_2/r_1}$
(c) $(r_1/r_2)^2$ (d) $(r_2/r_1)^2$

15. A bomb is released from a horizontal flying aeroplane. The trajectory of bomb is
(a) a parabola (b) a straight line
(c) a circle (d) a hyperbola

16. A projectile can have the same range for two angles of projection. If h_1 and h_2 are maximum heights when the range in the two cases is R, then the relation between R, h_1 and h_2 is
(a) $R=4\sqrt{h_1h_2}$ (b) $R=2\sqrt{h_1h_2}$
(c) $R=\sqrt{h_1h_2}$ (d) None of these

17. A bomb is dropped from an aeroplane moving horizontally at constant speed. If air resistance is taken into consideration, then the bomb
(a) falls on earth exactly below the aeroplane
(b) falls on the earth exactly behind the aeroplane
(c) falls on the earths ahead of the aeroplane
(d) flies with the aeroplane

18. Two vectors A and B lie in a plane, another vector C lies outside this plane, then the resultant of these three vectors i.e., A + B + C
(a) can be zero
(b) cannot be zero
(c) lies in the plane containing A + B
(d) lies in the plane containing A – B

19. A cannon on a level plane is aimed at an angle θ above the horizontal and a shell is fired with a muzzle velocity v_0 towards a vertical cliff a distance D away. Then the height from the bottom at which the shell strikes the side walls of the cliff is

(a) $D\sin\theta - \dfrac{gD^2}{2v_0^2\sin^2\theta}$

(b) $D\cos\theta - \dfrac{gD^2}{2v_0^2\cos^2\theta}$

(c) $D\tan\theta - \dfrac{gD^2}{2v_0^2\cos^2\theta}$

(d) $D\tan\theta - \dfrac{gD^2}{2v_0^2\sin^2\theta}$

20. A projectile thrown with velocity v making angle θ with vertical gains maximum height H in the time for which the projectile remains in air, the time period is

(a) $\sqrt{H\cos\theta / g}$ (b) $\sqrt{2H\cos\theta / g}$

(c) $\sqrt{4H/g}$ (d) $\sqrt{8H/g}$

21. Three particles A, B and C are thrown from the top of a tower with the same speed. A is thrown up, B is thrown down and C is horizontally. They hit the ground with speeds v_A, v_B and v_C respectively then,

(a) $v_A = v_B = v_C$ (b) $v_A = v_B > v_C$

(c) $v_B > v_C > v_A$ (d) $v_A > v_B = v_C$

22. An aeroplane flying at a constant speed releases a bomb. As the bomb moves away from the aeroplane, it will

(a) always be vertically below the aeroplane only if the aeroplane was flying horizontally

(b) always be vertically below the aeroplane only if the aeroplane was flying at an angle of 45° to the horizontal

(c) always be vertically below the aeroplane

(d) gradually fall behind the aeroplane if the aeroplane was flying horizontally.

23. In uniform circular motion, the velocity vector and acceleration vector are

(a) perpendicular to each other

(b) same direction

(c) opposite direction

(d) not related to each other

24. The time of flight of a projectile on an upward inclined plane depends upon

(a) angle of inclination of the plane

(b) angle of projection

(c) the value of acceleration due to gravity

(d) all of the above.

25. At the highest point on the trajectory of a projectile, its

(a) potential energy is minimum

(b) kinetic energy is maximum

(c) total energy is maximum

(d) kinetic energy is minimum.

EXERCISE - 2 AAJ KA TOPPER
Applied Questions

1. A projectile is projected with a kinetic energy E. Its range is R. It will have the minimum kinetic energy, after covering a horizontal distance equal to

(a) 0.25 R (b) 0.5 R

(c) 0.75 R (d) R

2. The range of a projectile when launched at an angle of 15° with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle of 45° to the horizontal

(a) 1.5 km (b) 3.0 km

(c) 6.3 km (d) 0.75 km

3. A gun fires two bullets at 60° and 30° with horizontal. The bullets strike at some horizontal distance. The ratio of maximum height for the two bullets is in the ratio

(a) 2 : 1 (b) 3 : 1

(c) 4 : 1 (d) 1 : 1

4. The angular speed of a fly-wheel making 120 revolutions/ minute is

(a) π rad/sec (b) 4π rad/sec

(c) 2π rad/sec (d) $4\pi^2$ rad/sec

5. Consider two vectors $\vec{F}_1 = 2\hat{i} + 5\hat{k}$ and $\vec{F}_2 = 3\hat{j} + 4\hat{k}$. The magnitude of the scalar product of these vectors is

(a) 20 (b) 23

(c) $5\sqrt{33}$ (d) 26

6. If range is double the maximum height of a projectile, then θ is

(a) $\tan^{-1}4$ (b) $\tan^{-1} 1/4$

(c) $\tan^{-1} 1$ (d) $\tan^{-1} 2$

7. A body is projected such that its KE at the top is 3/4th of its initial KE. What is the angle of projectile with the horizontal?

(a) 30° (b) 60° (c) 45° (d) 120°

8. Consider a vector F = 4 i – 3 j. Another vector that is perpendicular to F is

(a) $4i + 3j$ (b) $6i$

(c) $7k$ (d) $3i - 4j$

9. From a point on the ground at a distance 2 meters from the foot of a vertical wall, a ball is thrown at an angle of 45° which just clears the top of the wall and afterward strikes the ground at a distance 4 m on the other side. The height of the wall is

(a) $\dfrac{2}{3}$m (b) $\dfrac{3}{4}$m

(c) $\dfrac{1}{3}$m (d) $\dfrac{4}{3}$m

10. The velocity of projection of a body is increased by 2%. Other factors remaining unchanged, what will be the percentage change in the maximum height attained?
(a) 1% (b) 2 %
(c) 4 % (d) 8 %

11. A ball is thrown from the ground with a velocity of $20\sqrt{3}$ m/s making an angle of 60º with the horizontal. The ball will be at a height of 40 m from the ground after a time t equal to ($g = 10\ ms^{-2}$)
(a) $\sqrt{2}$ sec (b) $\sqrt{3}$ sec
(c) 2 sec (d) 3 sec

12. Forces of 4 N and 5 N are applied at origin along X-axis and Y-axis respectively. The resultant force will be
(a) $\sqrt{41}N, \tan^{-1}\left(\frac{5}{4}\right)$ (b) $\sqrt{41}N, \tan^{-1}\left(\frac{4}{5}\right)$
(c) $-\sqrt{41}N, \tan^{-1}\left(\frac{5}{4}\right)$ (d) $-\sqrt{41}N, \tan^{-1}\left(\frac{4}{5}\right)$

13. A particle moves in a circle of radius 25 cm at two revolutions per second. The acceleration of the particle in meter per second^2 is
(a) π^2 (b) $8\pi^2$
(c) $4\pi^2$ (d) $2\pi^2$

14. The vector sum of the forces of 10 N and 6 N can be
(a) 2 N (b) 8 N
(c) 18 N (d) 20 N

15. A bomb is dropped on an enemy post by an aeroplane flying horizontally with a velocity of 60 km h^{-1} and at a height of 490 m. At the time of dropping the bomb, how far the aeroplane should be from the enemy post so that the bomb may directly hit the target ?
(a) $\frac{400}{3}$ m (b) $\frac{500}{3}$ m
(c) $\frac{1700}{3}$ m (d) 498 m.

16. A projectile is thrown horizontally with a speed of 20 ms^{-1}. If g is 10 ms^{-2}, then the speed of the projectile after 5 second will be nearly
(a) 0.5 ms^{-1} (b) 5 ms^{-1}
(c) 54 ms^{-1} (d) 500 ms^{-1}

17. A ball is projected at such an angle that the horizontal range is three times the maximum height. The angle of projection of the ball is
(a) $\sin^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{4}{3}\right)$
(c) $\cos^{-1}\left(\frac{4}{3}\right)$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$

18. A body is projected horizontally from a point above the ground and motion of the body is described by the equation $x = 2t$, $y = 5t^2$ where x, and y are horizontal and vertical coordinates in metre after time t. The initial velocity of the body will be
(a) $\sqrt{29}$ m/s horizontal (b) 5 m/s horizontal
(c) 2 m/s vertical (d) 2 m/s horizontal

19. A vector $\overrightarrow{P_1}$ is along the positive x-axis. If its vector product with another vector $\overrightarrow{P_2}$ is zero then $\overrightarrow{P_2}$ could be
(a) $4\hat{j}$ (b) $-4\hat{i}$
(c) $(\hat{j}+\hat{k})$ (d) $-(\hat{i}+\hat{j})$

20. A wheel rotates with constant acceleration of 2.0 rod/s^2, if the wheel starts from rest the number of revolutions it makes in the first ten seconds will be approximately
(a) 32 (b) 24
(c) 16 (d) 8

21. A projectile of mass m is fired with velocity u making angle θ with the horizontal. Its angular momentum about the point of projection when it hits the ground is given by
(a) $\frac{2mu\sin^2\theta\cos\theta}{g}$ (b) $\frac{2mu^3\sin^2\theta\cos\theta}{g}$
(c) $\frac{mu\sin^2\theta\cos\theta}{2g}$ (d) $\frac{mu^3\sin^2\theta\cos\theta}{2g}$

22. A bucket, full of water is revolved in a vertical circle of radius 2 m. What should be the maximum time-period of revolution so that the water doesn't fall out of the bucket?
(a) 1 sec (b) 2 sec
(c) 3 sec (d) 4 sec

23. If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then angle between $\vec{a}$ & $\vec{b}$ is
(a) 45º (b) 30º
(c) 90º (d) 180º

24. If the sum of two unit vectors is a unit vector, then the magnitude of their difference is
(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) $\sqrt{5}$ (d) $\frac{1}{\sqrt{2}}$

25. Out of the following sets of forces, the resultant of which cannot be zero ?
(a) 10, 10, 10 (b) 10, 10, 20
(c) 10, 20, 20 (d) 10, 20, 40

26. The magnitude of the vector product of two vectors is $\sqrt{3}$ times the scalar product. The angle between vectors is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

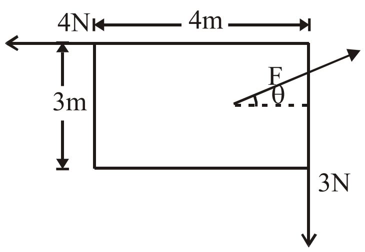
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

27. If A = B + C and the magnitudes of A, B and **C** are 5, 4 and 3 units, the angle between A and C is

(a) $\cos^{-1}(3/5)$ (b) $\cos^{-1}(4/5)$

(c) $\frac{\pi}{2}$ (d) $\sin^{-1}(3/4)$

28. A rectangular sheet of material has a width of 3 m and a length of 4 m. Forces with magnitudes of 3 N and 4N. respectively, are applied parallel to two edges of the sheet, as shown in the figure below.



A third force F, is applied to the centre of the sheet, along a line in the plane of the sheet, at an angle θ = tan 0.75 with respect to the horizontal direction. The sheet will be in translational equilibrium when F has what value?

(a) F = 3 N (b) F = 4N

(c) F = 5 N (d) F = 7N

29. The linear velocity of a rotating body is given by :

$$\vec{v} = \vec{\omega} \times \vec{r}$$

If $\vec{\omega} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{r} = 4\hat{j} - 3\hat{k}$, then the magnitude of $\vec{v}$ is

(a) $\sqrt{29}$ units (b) $\sqrt{31}$ units

(c) $\sqrt{37}$ units (d) $\sqrt{41}$ units

30. Two forces are such that the sum of their magnitudes is 18 N and their resultant is 12 N which is perpendicular to the smaller force. Then the magnitudes of the forces are

(a) 12 N, 6 N (b) 13 N, 5 N

(c) 10 N, 8 N (d) 16N, 2N.

31. For any two vectors $\vec{A}$ and $\vec{B}$, if $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$, the magnitude of $\vec{C} = \vec{A} + \vec{B}$ is

(a) A + B (b) $\sqrt{A^2 + B^2 + \sqrt{2}AB}$

(c) $\sqrt{A^2 + B^2}$ (d) $\sqrt{A^2 + B^2 + \frac{AB}{\sqrt{2}}}$

32. The angles which the vector $\vec{A} = 3\hat{i} + 6\hat{j} + 2\hat{k}$ makes with the co-ordinate axes are :

(a) $\cos^{-1}\frac{3}{7}, \cos^{-1}\frac{4}{7}, \cos^{-1}\frac{1}{7}$

(b) $\cos^{-1}\frac{3}{7}, \cos^{-1}\frac{6}{7}, \cos^{-1}\frac{2}{7}$

(c) $\cos^{-1}\frac{4}{7}, \cos^{-1}\frac{5}{7}, \cos^{-1}\frac{3}{7}$

(d) None of these

33. The resultant of two forces 3P and 2P is R. If the first force is doubled then the resultant is also doubled. The angle between the two forces is

(a) 60º (b) 120º

(c) 70º (d) 180º

34. Let $\vec{C} = \vec{A} + \vec{B}$

(a) $|\vec{C}|$ is always greater than $|\vec{A}|$

(b) It is possible to have $|\vec{C}| < |\vec{A}|$ and $|\vec{C}| < |\vec{B}|$

(c) $\vec{C}$ is always equal to $\vec{A} + \vec{B}$

(d) $\vec{C}$ is never equal to $\vec{A} + \vec{B}$

35. At what angle must the two forces (x + y) and (x – y) act so that the resultant may be $\sqrt{(x^2 + y^2)}$?

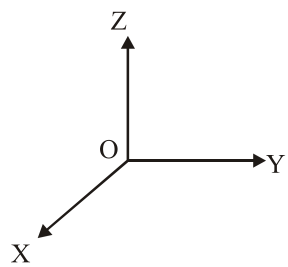
(a) $\cos^{-1}[-(x^2 + y^2)/2(x^2 - y^2)]$

(b) $\cos^{-1}[-2(x^2 - y^2)/(x^2 + y^2)]$

(c) $\cos^{-1}[-(x^2 + y^2)/(x^2 - y^2)]$

(d) $\cos^{-1}[-(x^2 - y^2)/(x^2 + y^2)]$

36. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1, –1) is



(a) $F(\hat{i} - \hat{j})$ (b) $-F(\hat{i} + \hat{j})$

(c) $F(\hat{i} + \hat{j})$ (d) $-F(\hat{i} - \hat{j})$

37. The component of vector $\vec{a} = 2\hat{i} + 3\hat{j}$ along the vector $i + j$ is

(a) $\frac{5}{\sqrt{2}}$ (b) $10\sqrt{2}$

(c) $5\sqrt{2}$ (d) 5

38. A body is projected at an angle of 30º to the horizontal with speed 30 m/s. What is the angle with the horizontal after 1.5 seconds? Take $g = 10\ m/s^2$.

(a) 0º (b) 30º
(c) 60º (d) 90º

39. A projectile is moving at 60 m/s at its highest point, where it breaks into two equal parts due to an internal explosion. One part moves vertically up at 50 m/s with respect to the ground. The other part will move at a speed of

(a) 110 m/s (b) 120 m/s
(c) 130 m/s (d) $10\sqrt{61}\ m/s$

40. A particle having a mass 0.5 kg is projected under gravity with a speed of 98 m/sec at an angle of 60º. The magnitude of the change in momentum (in N-sec) of the particle after 10 seconds is

(a) 0.5 (b) 49
(c) 98 (d) 490

41. A large number of bullets are fired in all directions with the same speed v. What is the maximum area on the ground on which these bullets will spread?

(a) $\frac{\pi v^2}{g}$ (b) $\frac{\pi v^4}{g^2}$
(c) $\pi^2 \frac{v^2}{g^2}$ (d) $\frac{\pi^2 v^4}{g^2}$

42. A force of $(3\hat{i}+4\hat{j})$ N acts on a body and displaces it by $(3\hat{i}+4\hat{j})$ m. The work done by the force is

(a) 5 J (b) 30 J
(c) 25 J (d) 10 J

43. A person aims a gun at a bird from a point at a horizontal distance of 100 m. If the gun can impact a speed of $500\ ms^{-1}$ to the bullet. At what height above the bird must he aim his gun in order to hit it? $(g = 10\ ms^{-2})$

(a) 10.4 cm (b) 20.35 cm
(c) 50 cm (d) 100 cms

44. If $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$, then the unit vector along $\vec{a} + \vec{b}$ is

(a) $\frac{3i+4k}{5}$ (b) $\frac{-3i+4k}{5}$
(c) $\frac{-3i-4k}{5}$ (d) None of these

45. A body is thrown with a velocity of $9.8\ ms^{-1}$ making an angle of 30° with the horizontal. It will hit the ground after a time

(a) 3.0 s (b) 2.0 s
(c) 1.5 s (d) 1 s

46. Two bullets are fired horizontally, simultaneously and with different velocities from the same place. Which bullet will hit the ground earlier ?

(a) It would depend upon the weights of the bullets.
(b) The slower one.
(c) The faster one.
(d) Both will reach simultaneously.

47. A cricket ball is hit with a velocity $25\ ms^{-1}$, 60° above the horizontal. How far above the ground, ball passes over a fielder 50 m from the bat (consider the ball is struck very close to the ground)?

Take $\sqrt{3} = 1.7$ and $g = 10\ ms^{-2}$

(a) 6.8 m (b) 7 m
(c) 5 m (d) 10 m

48. The equation of a projectile is

$$y = \sqrt{3}x - \frac{gx^2}{2}$$

The angle of projection is given by

(a) $\tan\theta = \frac{1}{\sqrt{3}}$ (b) $\tan\theta = \sqrt{3}$
(c) $\frac{\pi}{2}$ (d) zero.

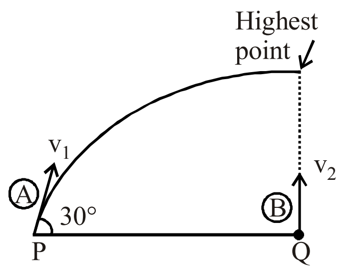
49. A body is thrown horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of the tower at a distance x from the tower. The value of x is

(a) gh (b) $\frac{gh}{2}$
(c) 2h (d) $\frac{2gh}{3}$

50. A plane flying horizontally at a height of 1500 m with a velocity of $200\ ms^{-1}$ passes directly overhead on antiaircraft gun. Then the angle with the horizontal at which the gun should be fired from the shell with a muzzle velocity of $400\ ms^{-1}$ to hit the plane, is

(a) 90° (b) 60°
(c) 30° (d) 45°

51. A projectile A is thrown at an angle of 30° to the horizontal from point P. At the same time, another projectile B is thrown with velocity v_2 upwards from the point Q vertically below the highest point. For B to collide with A, $\frac{v_2}{v_1}$ should be



(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) 4

52. The velocity of projection of oblique projectile is $(6\hat{i}+8\hat{j})\,\text{m s}^{-1}$. The horizontal range of the projectile is

(a) 4.9m (b) 9.6m
(c) 19.6m (d) 14m

53. A gun is aimed at a horizontal target. It takes $\frac{1}{2}$ s for the bullet to reach the target. The bullet hits the target x metre below the aim. Then, x is equal to

(a) $\frac{9.8}{4}$m (b) $\frac{9.8}{8}$m
(c) 9.8m (d) 19.6m.

54. The equation of trajectory of projectile is given by

$y=\frac{x}{\sqrt{3}}-\frac{gx^2}{20}$, where x and y are in metre.

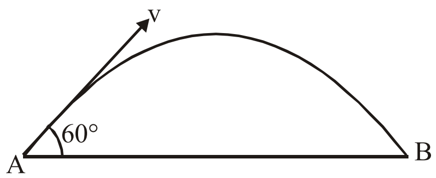
The maximum range of the projectile is

(a) $\frac{8}{3}$m (b) $\frac{4}{3}$m
(c) $\frac{3}{4}$m (d) $\frac{3}{8}$m

55. A bullet is fired with a speed of 1500 m/s in order to hit a target 100 m away. If g = 10 m/s^2. The gun should be aimed

(a) 15 cm above the target
(b) 10 cm above the target
(c) 2.2 cm above the target
(d) directly towards the target

56. A projectile of mass m is thrown with a velocity v making an angle 60° with the horizontal. Neglecting air resistance, the change in momentum from the departure A to its arrival at B, along the vertical direction is

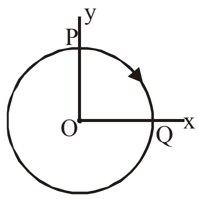


(a) 2 mv (b) $\sqrt{3}$mv
(c) mv (d) $\frac{mv}{\sqrt{3}}$

57. A person sitting in the rear end of the compartment throws a ball towards the front end. The ball follows a parabolic path. The train is moving with velocity of 20 m/s. A person standing outside on the ground also observes the ball. How will the maximum heights (y_m) attained and the ranges (R) seen by the thrower and the outside observer compare with each other?

(a) Same y_m different R (b) Same y_m and R
(c) Different y_m same R (d) Different y_m and R

58. A particle moves in a circle of radius 4 cm clockwise at constant speed 2 cm/s. If $\hat{x}$ and $\hat{y}$ are unit acceleration vectors along X and Y-axis respectively (in cm/s^2), the acceleration of the particle at the instant half way between P and Q is given by



(a) $-4(\hat{x}+\hat{y})$
(b) $4(\hat{x}+\hat{y})$
(c) $-(\hat{x}+\hat{y})/\sqrt{2}$
(d) $(\hat{x}-\hat{y})/4$

59. A projectile is thrown in the upward direction making an angle of 60° with the horizontal direction with a velocity of 147 ms^{-1}. Then the time after which its inclination with the horizontal is 45°, is

(a) 15 s (b) 10.98 s
(c) 5.49 s (d) 2.745 s

60. A cyclist moving at a speed of 20 m/s takes a turn, if he doubles his speed then chance of overturn

(a) is doubled (b) is halved
(c) becomes four times (d) becomes 1/4 times

61. A person swims in a river aiming to reach exactly on the opposite point on the bank of a river. His speed of swimming is 0.5 m/s at an angle of 120º with the direction of flow of water. The speed of water is

(a) 1.0 m/s (b) 0.5 m/s
(c) 0.25 m/s (d) 0.43 m/s

62. The position vector of a particle is

$\vec{r}=(a\cos\omega t)\hat{i}+(a\sin\omega t)\hat{j}$. The velocity of the particle is

(a) directed towards the origin
(b) directed away from the origin
(c) parallel to the position vector
(d) perpendicular to the position vector

63. A ball whose kinetic energy is E is thrown at an angle of 45º with horizontal. Its kinetic energy at highest point of flight will be

(a) E (b) E/2
(c) $\frac{E}{\sqrt{2}}$ (d) O

64. Two projectiles are fired from the same point with the same speed at angles of projection 60º and 30º respectively. Which one of the following is true?

(a) Their maximum height will be same
(b) Their range will be same
(c) Their landing velocity will be same
(d) Their time of flight will be same

65. A body of 3kg. moves in X-Y plane under the action of force given by $6t\hat{i}+4t\hat{j}$. Assuming that the body is at rest at time t = 0, the velocity of body at t = 3 sec is

(a) $9\hat{i}+6\hat{j}$ (b) $18\hat{i}+6\hat{j}$
(c) $18\hat{i}+12\hat{j}$ (d) $12\hat{i}+68\hat{j}$

66. From a 10 m high building a stone A is dropped, and simultaneously another identical stone B is thrown horizontally with an initial speed of 5 ms^{-1}. Which one of the following statements is true?
(a) It is not possible to calculate which one of the two stones will reach the ground first
(b) Both the stones (A and B) will reach the ground simultaneously
(c) A stone reaches the ground earlier than B
(d) B stone reaches the ground earlier than A

67. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces
(a) cannot be predicted
(b) are equal to each other
(c) are equal to each other in magnitude
(d) are not equal to each other in magnitude

68. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$m with constant tangential acceleration. It the velocity of particle is 80 m/sec at end of second revolution after motion has begun, the tangential acceleration is
(a) 40 π m/sec^2 (b) 40 m/sec^2
(c) 640 π m/sec^2 (d) 160 π m/sec^2

69. If $|\vec{A}\times\vec{B}|=\sqrt{3}\vec{A}.\vec{B}$ then the value of $|\vec{A}+\vec{B}|$ is
(a) $(A^2+B^2\sqrt{3}AB)^{½}$ (b) $(A^2+B^2+AB)^{½}$
(c) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{½}$ (d) A + B

70. If a vector $2\hat{i}+3\hat{j}+8\hat{k}$ is perpendicular to the vector $4\hat{j}-4\hat{i}+\alpha\hat{k}$, then the value of α is
(a) 1/2 (b) –1/2
(c) 1 (d) –1

Directions for Qs. (71 to 75) : Each question contains STATEMENT-1 and STATEMENT-2. Choose the correct answer (ONLY ONE option is correct) from the following-
(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

71. **Statement -1 :** $\vec{\tau}=\vec{r}\times\vec{F}$ and $\vec{\tau}\neq\vec{F}\times\vec{r}$
Statement -2 : Cross product of vectors is commutative.

72. **Statement -1 :** If dot product and cross product of $\vec{A}$ and $\vec{B}$ are zero, it implies that one of the vector $\vec{A}$ and $\vec{B}$ must be a null vector
Statement -2 : Null vector is a vector with zero magnitude.

73. **Statement-1** Two stones are simultaneously projected from level ground from same point with same speeds but different angles with horizontal. Both stones move in same vertical plane. Then the two stones may collide in mid air.
Statement-2 : For two stones projected simultaneously from same point with same speed at different angles with horizontal, their trajectories may intersect at some point.

74. **Statement-1 :** K.E. of a moving body given by as^2 where s is the distance travelled in a circular path refers to a variable acceleration.
Statement-2 : Acceleration varies with direction only in this case of circular motion.

75. **Statement-1 :** Centripetal and centrifugal forces cancel each other.
Statement-2 : This is because they are always equal and opposite.

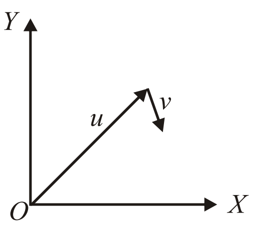
EXERCISE - 3
Exemplar & Past Years NEET/AIPMT Questions

Exemplar Questions

1. The angle between $A=\hat{i}+\hat{j}$ and $B=\hat{i}-\hat{j}$ is
(a) 45° (b) 90°
(c) –45° (d) 180°

2. Which one of the following statements is true?
(a) A scalar quantity is the one that is conserved in a process
(b) A scalar quantity is the one that can never take negative values
(c) A scalar quantity is the one that does not vary from one point to another in space
(d) A scalar quantity has the same value for observers with different orientation of the axes

3. Figure shows the orientation of two vectors *u* and *v* in the *xy*-plane.



If $u=a\hat{i}+b\hat{j}$ and $v=p\hat{i}+q\hat{j}$
Which of the following is correct?
(a) *a* and *p* are positive while *b* and *q* are negative
(b) *a, p* and *b* are positive while *q* is negative
(c) *a, q* and *b* are positive while *p* is negative
(d) *a, b, p* and *q* are all positive

4. The component of a vector r along X-axis will have maximum value if
(a) r is along positive Y-axis
(b) r is along positive X-axis
(c) r makes an angle of 45° with the X-axis
(d) r is along negative Y-axis

5. The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be
(a) 60 m (b) 71 m
(c) 100 m (d) 141 m

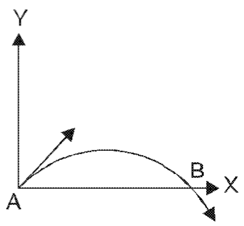
6. Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are :
(a) impulse, pressure and area
(b) impulse and area
(c) area and gravitational potential
(d) impulse and pressure

7. In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then, which of the following are necessarily true?
(a) The average velocity is not zero at any time
(b) Average acceleration must always vanish
(c) Displacements in equal time intervals are equal
(d) Equal path lengths are traversed in equal intervals

8. In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then, which of the following are necessarily true?
(a) The acceleration of the particle is zero
(b) The acceleration of the particle is bounded
(c) The acceleration of the particle is necessarily in the plane of motion
(d) The particle must be undergoing a uniform circular motion

NEET/AIPMT (2013-2017) Questions

9. The velocity of a projectile at the initial point A is $\left(2\hat{i}+3\hat{j}\right)$ m/s. It's velocity (in m/s) at point B is **[2013]**



(a) $-2\hat{i}+3\hat{j}$ (b) $2\hat{i}-3\hat{j}$
(c) $2\hat{i}+3\hat{j}$ (d) $-2\hat{i}-3\hat{j}$

10. Vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are such that $\vec{A}\cdot\vec{B}=0$ and $\vec{A}\cdot\vec{C}=0$. Then the vector parallel to $\vec{A}$ is **[NEET Kar. 2013]**
(a) $\vec{B}$ and $\vec{C}$ (b) $\vec{A}\times\vec{B}$
(c) $\vec{B}+\vec{C}$ (d) $\vec{B}\times\vec{C}$

11. A particle is moving such that its position coordinate (x, y) are
(2m, 3m) at time t = 0
(6m, 7m) at time t = 2 s and
(13m, 14m) at time t = 5s.
Average velocity vector $(\vec{V}_{av})$ from t = 0 to t = 5s is : **[2014]**
(a) $\frac{1}{5}(13\hat{i}+14\hat{j})$ (b) $\frac{7}{3}(\hat{i}+\hat{j})$
(c) $2(\hat{i}+\hat{j})$ (d) $\frac{11}{5}(\hat{i}+\hat{j})$

12. A ship A is moving Westwards with a speed of 10 km h^{-1} and a ship B 100 km South of A, is moving Northwards with a speed of 10 km h^{-1}. The time after which the distance between them becomes shortest, is : **[2015]**
(a) 5 h (b) $5\sqrt{2}$ h
(c) $10\sqrt{2}$ h (d) 0 h

13. If vectors $\vec{A}=\cos\omega t\hat{i}+\sin\omega t\hat{j}$ and $\vec{B}=\cos\frac{\omega t}{2}\hat{i}+\sin\frac{\omega t}{2}\hat{j}$ are functions of time, then the value of t at which they are orthogonal to each other is : **[2015 RS]**
(a) $t=\frac{\pi}{2\omega}$ (b) $t=\frac{\pi}{\omega}$
(c) $t=0$ (d) $t=\frac{\pi}{4\omega}$

14. The position vector of a particle $\vec{R}$ as a function of time is given by:
$\vec{R}=4\sin(2\pi t)\hat{i}+4\cos(2\pi t)\hat{j}$
Where R is in meter, t in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x-and y-directions, respectively.
Which one of the following statements is wrong for the motion of particle? **[2015 RS]**
(a) Magnitude of acceleration vector is $\frac{v^2}{R}$, where v is the velocity of particle
(b) Magnitude of the velocity of particle is 8 meter/second
(c) path of the particle is a circle of radius 4 meter.
(d) Acceleration vector is along $-\vec{R}$

15. A particle moves so that its position vector is given by $\vec{r}=\cos\omega t\hat{x}+\sin\omega t\hat{y}$. Where ω is a constant. Which of the following is true ? **[2016]**
(a) Velocity and acceleration both are perpendicular to $\vec{r}$
(b) Velocity and acceleration both are parallel to $\vec{r}$
(c) Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin
(d) Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin

16. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is : **[2016]**
(a) 0° (b) 90°
(c) 45° (d) 180°

17. The x and y coordinates of the particle at any time are x = 5t – 2t^2 and y = 10t respectively, where x and y are in meters and t in seconds. The acceleration of the particle at t = 2s is **[2017]**
(a) 5 m/s^2 (b) –4 m/s^2
(c) –8 m/s^2 (d) 0

Hints & Solutions AAJ KA TOPPER

EXERCISE - 1

1. (b) 2. (d)

3. (d) $\frac{H_1}{H_2} = \frac{u^2 \sin^2 \theta / 2g}{u^2 \sin^2 (90° - \theta)/2g} = \tan^2 \theta$

4. (d) 5. (a)

6. (a) In vector addition, the commutative law is obeyed

i.e., $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Vector subtraction does not follow commutative law.

7. (d) Since $\vec{A}.\vec{C} = 0 = \vec{A}.\vec{B}$, it means that $\vec{A}$ is perpendicular to both $\vec{C}$ & $\vec{B}$, hence $\vec{A}$ is parallel to $(\vec{B} \times \vec{C})$ or $(\vec{C} \times \vec{B})$.

8. (a) The unit vector of any vector $\vec{A}$ is defined as $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

9. (c) On earth, $R = u^2 \sin 2\theta / g$.
On moon, $g' = g/6$
$R' = u^2 \sin 2\theta / g' = 6u^2 \sin 2\theta / g = 6R$.

10. (c) $\cos\theta = \frac{R^2 - A^2 - B^2}{2AB} = \frac{R^2 - R^2}{2AB} = 0$

$\therefore \theta = \pi/2$

11. (c) $R^2 = [A^2 + B^2 + 2AB \cos\theta]$

$R^2 = R^2 + R^2 + 2R^2 \cos\theta$

$-R^2 = 2R^2 \cos\theta$ or $\cos\theta = -1/2$ or $\theta = 2\pi/3$

12. (c) The dot product should be zero.

13. (c) $R_{30°} = \frac{u^2 \sin 60°}{g} = \frac{\sqrt{3}}{2}(u^2/g)$

$R_{45°} = \frac{u^2 \sin 90}{g} = u^2/g$

$R_{60°} = \frac{u^2 \sin 120°}{g} = \frac{u^2 \cos 30°}{g} = \frac{u^2}{g}(\sqrt{3}/2)$

so $R_{30°} = R_{60°} > R_{45°}$ or $R_A = R_C > R_B$

14. (a) $F_1 = mr_1\omega^2$; $F_2 = mr_2\omega^2$

since period T is same, so ω is same, because $T = \frac{2\pi}{\omega}$.

Hence $\frac{F_1}{F_2} = \left(\frac{r_1}{r_2}\right)$

15. (a) A parabola

16. (a) $h_1 = \frac{u^2 \sin^2 \theta}{2g}$

$h_2 = \frac{u^2 \sin^2 (90 - \theta)}{2g}$, $R = \frac{u^2 \sin 2\theta}{g}$

Range R is same for angle θ and $(90° - \theta)$

$\therefore h_1 h_2 = \frac{u^2 \sin^2 \theta}{2g} \times \frac{u^2 \sin^2 (90 - \theta)}{2g}$

$= \frac{u^4 (\sin^2 \theta) \times \sin^2 (90 - \theta)}{4g^2}$ $[\because \sin(90 - \theta) = \cos\theta]$

$= \frac{u^4 (\sin^2 \theta) \times \cos^2 \theta}{4g^2}$ $[\because \sin 2\theta = 2\sin\theta\cos\theta]$

$= \frac{u^4 (\sin\theta\cos\theta)^2}{4g^2} = \frac{u^4 (\sin 2\theta)^2}{16g^2}$

$= \frac{(u^2 \sin 2\theta)^2}{16g^2} = \frac{R^2}{16}$

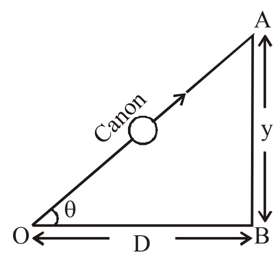
or, $R^2 = 16 h_1 h_2$ or $R = 4\sqrt{h_1 h_2}$

17. (b) If there is no resistance, bomb will drop at a place exactly below the flying aeroplane. But when we take into account air resistance, bomb will face deceleration in its velocity. So, it will fall on the earth exactly behind the aeroplane.

18. (b)

19. (c) From the resultant path of the particle, when it is projected at angle θ with its velocity u is

$y = x \tan\theta - \frac{1}{2} \frac{gx^2}{u^2 \cos^2 \theta}$

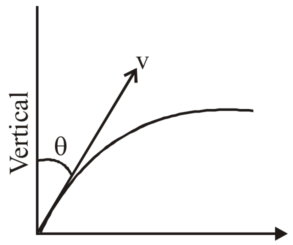


Where y denotes the instantaneous height of particle when it travels an instantaneous horizontal distance x.
here $x = D$, $u = v_o$

so $y = D\tan\theta - \frac{1}{2}\frac{gD^2}{v_0^2 \cos^2 \theta}$

20. (d) Max. height $= H = \frac{v^2 \sin^2 (90 - \theta)}{2g}$ .....(i)

Time of flight, $T = \frac{2v\sin(90 - \theta)}{g}$ ...(ii)

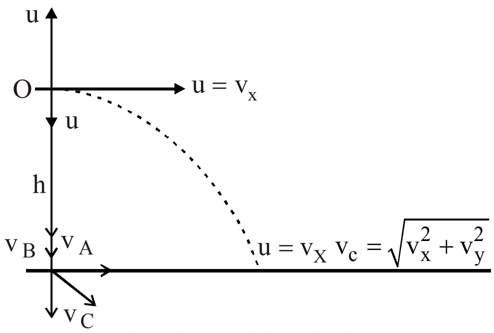


From (i), $\frac{v\cos\theta}{g} = \sqrt{\frac{2H}{g}}$,From (ii),

$T = 2\sqrt{\frac{2H}{g}} = \sqrt{\frac{8H}{g}}$

21. (a) **For A:** It goes up with velocity *u* will it reaches its maximum height (i.e. velocity becomes zero) and comes back to O and attains velocity *u*.

Using $v^2 = u^2 + 2as \Rightarrow v_A = \sqrt{u^2 + 2gh}$



For B, going down with velocity u

$\Rightarrow v_B = \sqrt{u^2 + 2gh}$

For C, horizontal velocity remains same, i.e. u. Vertical velocity= $\sqrt{0 + 2gh} = \sqrt{2gh}$

The resultant $v_C = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$.

Hence $v_A = v_B = v_C$

22. (c) Since horizontal component of the velocity of the bomb will be the same as the velocity of the aeroplane, therefore horizontal displacements remain the same at any instant of time.

23. (a) In uniform circular motion speed is constant. So, no tangential acceleration.

It has only radial acceleration $a_R = \frac{v^2}{R}$ [directed towards center]

and its velocity is always in tangential direction. So these two are perpendicular to each other.

24. (d) $T = \frac{2u\sin(\theta - \alpha)}{g\cos\alpha}$

25 (d) Velocity and kinetic energy is minimum at the highest point.

$K.E = \frac{1}{2} m v^2 \cos^2\theta$

EXERCISE - 2

1. (b) K.E. is minimum at the highest point. So, the horizontal distance is half of the range R i.e., 0.5 R.

2. (b) $R_1 = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 30}{g}$ or $1.5 = \frac{u^2}{2g}$ or $\frac{u^2}{g} = 3$

$R_2 = \frac{u^2 \sin 90}{g} = \frac{u^2}{g} = 3\,\text{km}$

3. (b) The bullets are fired at the same initial speed

$\frac{H}{H'} = \frac{u^2 \sin^2 60^\circ}{2g} \times \frac{2g}{u^2 \sin^2 30^\circ} = \frac{\sin^2 60^\circ}{\sin^2 30^\circ}$

$= \frac{(\sqrt{3}/2)^2}{(1/2)^2} = \frac{3}{1}$

4. (b)

5. (a) $\vec{F}_1 . \vec{F}_2 = (2\hat{i} + 5\hat{k}).(3\hat{j} + 4\hat{k}) = 20$

6. (d) $\frac{u^2 2\sin\theta\cos\theta}{g} = 2 \times \frac{u^2 \sin^2\theta}{2g}$ or $\tan\theta = 2$

7. (a) $\frac{1}{2} m (u\cos\theta)^2 = \frac{3}{4} \times \frac{1}{2} m u^2$

or $\cos^2\theta = \frac{3}{4}$ or $\cos\theta = \frac{\sqrt{3}}{2} = \cos 30^\circ$.

8. (c)

9. (d) $R = \frac{u^2}{g}$ or $6 = \frac{u^2}{g}$; $y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}$

or $h = 2\tan 45^\circ - \frac{4}{2\cos^2 45^\circ}\left(\frac{1}{6}\right) = 2 - \frac{2}{3} = \frac{4}{3}\,\text{m}$

10. (c) We know that, $y_m = H = \frac{(u\sin\theta)^2}{2g} = \frac{u^2\sin^2\theta}{2g}$

$\therefore \frac{\Delta H}{H} = \frac{2\Delta u}{u}$. Given $\frac{\Delta u}{u} = 2\%$

$\therefore \frac{\Delta H}{H} = 2 \times 2 = 4\%$

11. (c) As, $s = u\sin\theta t - \frac{1}{2} g t^2$

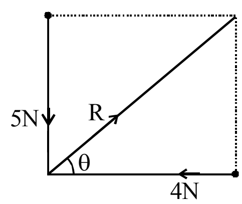
so $40 = 20\sqrt{3} \times (\sqrt{3}/2)t - \frac{1}{2} \times 10 \times t^2$

or $5t^2 - 30t + 40 = 0$ or $t^2 - 6t + 8 = 0$

or t = 2 or 4.

The minimum time t = 2s.

12. (a) $R = \sqrt{4^2 + 5^2} = \sqrt{41}\,\text{N}$



The angle θ will be given by $\tan\theta = \frac{5}{4}$ or $\theta = \tan^{-1}\left(\frac{5}{4}\right)$

13. (c) Here $T = \frac{1}{2}$ sec the required centripetal acceleration for moving in a circle is

$$a_C = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 = r\times(2\pi/T)^2$$

so $a_c = 0.25\times(2\pi/0.5)^2 = 16\pi^2\times .25 = 4.0\pi^2$

14. (b) $R_{max} = (10+6) = 16N$, $R_{min} = (10-6) = 4N$
$\Rightarrow$ Values can be from 4N to 16N

15. (b) Time taken for vertical direction motion

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 490}{9.8}} = \sqrt{100} = 10\,s$$

The same time is for horizontal direction.

$$\therefore\ x = vt = \left(60\times\frac{5}{18}\right)\times 10 = \frac{500}{3}\,m$$

16. (c) Even after 5 second, the horizontal velocity v_x will be $20\ ms^{-1}$. The vertical velocity v_y is given by

$$v_y = 0 + 10\times 5 = 50\,ms^{-1}$$

Now, $v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{20^2 + 50^2} \approx 54\ ms^{-1}$

17. (d) Given $\frac{u^2\sin 2\theta}{g} = 3\frac{u^2\sin^2\theta}{2g}$

$$\Rightarrow\quad \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

18. (d) The horizontal velocity of the projectile remains constant throughout the journey.
Since the body is projected horizontally, the initial velocity will be same as the horizontal velocity at any point.

Since, $x = 2t$, $\frac{dx}{dt} = 2$

$\therefore$ Horizontal velocity = 2 m/s
$\therefore$ Initial velocity = 2 m/s

19. (b) Vector product of parallel vectors is zero.

20. (c) For circular angular motion, the formula for angular displacement θ and angular acceleration α is

$\theta = \omega t + \frac{1}{2}\alpha t^2$ where ω = initial velocity

or $\theta = 0 + \frac{1}{2}\alpha t^2$ or $\theta = \frac{1}{2}\times(2)(10)^2$

or $\theta = 100$ radian
2π radian are covered in 1 revolution

$\therefore$ 1 radian is covered in $\frac{1}{2\pi}$ revolution

or 100 radian are covered in $\frac{100}{2\pi}$ revolution

$\therefore$ Number of revolution $= \frac{50}{3.14} = 16$

21. (b) $L = \vec{R}\times\vec{p}$

Where R = range $= \frac{u^2\sin 2\theta}{g}$

The angle between $\vec{R}$ and $\vec{p} = \theta$
Also, p = m u.

Hence, $L = \frac{u^2\sin 2\theta}{g}\times mu\sin(\theta)$

$$= \frac{2mu^3\sin^2\theta\cos\theta}{g}$$

22. (c) Let its angular velocity be ω at all points (uniform motion). At the highest point weight of the body is balanced by centrifugal force, so

$$m\omega^2 r = mg \Rightarrow \omega = \sqrt{\frac{g}{r}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{2}{10}} = \frac{2\pi}{\sqrt{5}}$$

$$= \frac{2\times 3.14}{2.1} = 3\text{ sec.}$$

23. (c) $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}| \Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$

$$\Rightarrow |a|^2 + |b|^2 + 2\vec{a}.\vec{b} = |a|^2 + |b|^2 - 2\vec{a}.\vec{b}$$

24. (a)

25. (d) $R^2 = P^2 + Q^2 - PQ\cos\theta$
$(40)^2 = (10)^2 + (20)^2 - 2\times 10\times 20\times\cos\theta$
$400\cos\theta = 500 - 1600 = -1100$

$\cos\theta = -\frac{1100}{400} = -\frac{11}{4}$ which is not possible.

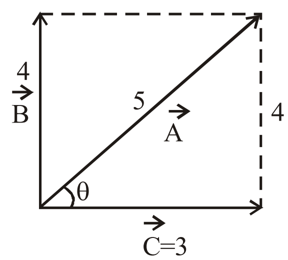
In this way, the set of forces given in option (d) can not be represented both in magnitude and direction by the sides of a triangle taken in the same order. Thus their resultant can not be zero.

26. (c) Here, $|\vec{A}\times\vec{B}| = \sqrt{3}\,|\vec{A}.\vec{B}|$

$$\Rightarrow AB\sin\theta = \sqrt{3}AB\cos\theta \ \Rightarrow \tan\theta = \sqrt{3}$$

$$\Rightarrow \theta = 60° = \frac{\pi}{3}$$

27. (a) See fig. Clearly **A** is the resultant of **B** and **C**. Further **B** is perpendicular to **C**



$\cos\theta = 3/5$ or $\theta = \cos^{-1}(3/5)$

28. (c) A body is in translational equilibrium when the components of all external forces cancel.
For the sheet : $F\cos\theta = 4$ N, $F\sin\theta = 3$ N. The magnitude of F is found by adding the squares of the components: $F^2\cos^2\theta + F^2\sin^2\theta = F^2 = 4^2 + 3^2 = 25\ N^2$. Therefore F = 5 N. The F vector points in the proper direction, since $\tan\theta = 0.75 = 3/4$.

29. (a) $\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix}$

$\vec{v} = \hat{i}[6-8] + \hat{j}[0+3] + \hat{k}[4-0]$

$\vec{v} = -2\hat{i} + 3\hat{j} + 4\hat{k} \Rightarrow |\vec{v}| = \sqrt{29}$ units.

30. (b) Use $\tan\alpha = \dfrac{P\sin\theta}{Q + P\cos\theta}$

$\Rightarrow \tan 90° = \dfrac{P\sin\theta}{Q + P\cos\theta} = \infty$

$\therefore Q + P\cos\theta = 0 \Rightarrow P\cos\theta = -Q.$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$R = \sqrt{P^2 + Q^2 - 2Q^2}$ or $R = \sqrt{P^2 - Q^2} = 12$

$144 = (P+Q)(P-Q)$ or $P - Q = 144/18 = 8.$

$\therefore$ P = 13 N and Q = 5 N.

31. (b) $|\vec{A}.\vec{B}| = |\vec{A}\times\vec{B}| \Rightarrow AB\cos\theta = AB\sin\theta$

$\Rightarrow \tan\theta = 1 \Rightarrow \cos\theta = \dfrac{1}{\sqrt{2}}$

Now, $|\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{A^2 + B^2 + 2AB.\dfrac{1}{\sqrt{2}}} = \sqrt{A^2 + B^2 + \sqrt{2}AB}$

32. (b) $\hat{A} = \dfrac{3\hat{i} + +b\hat{j} + 2\hat{k}}{\sqrt{9+36+4}} = \left(\dfrac{3}{7}\hat{i} + \dfrac{6}{7}\hat{j} + \dfrac{2}{7}\hat{k}\right)$. If α, β and γ are angles made by $\vec{A}$ with coordinate axes, then

$\cos\alpha = \dfrac{3}{7}, \cos\beta = \dfrac{6}{7}$ and $\cos\gamma = \dfrac{2}{7}$.

33. (b) Solve two equations : $R^2 = 9P^2 + 4P^2 + 12P^2\cos\theta$ and $4R^2 = 36P^2 + 4P^2 + 24P^2\cos\theta$.

34. (b)

35. (a) $\left(\sqrt{x^2+y^2}\right)^2 = (x+y)^2 + (x-y)^2 + 2(x+y)(x-y)\cos\theta$

$\Rightarrow x^2 + y^2 = x^2 + y^2 + 2xy + x^2 + y^2 - 2xy + 2(x^2 - y^2)\cos\theta$

or $2(x^2 - y^2).\cos\theta = -(x^2 + y^2)$

$\Rightarrow \theta = \cos^{-1}\left[\dfrac{-(x^2+y^2)}{2(x^2-y^2)}\right]$

36. (c) Torque $\vec{\tau} = \vec{r}\times\vec{F} = (\hat{i}-\hat{j})\times(-F\hat{k})$

$= F[-\hat{i}\times\hat{k} + \hat{j}\times\hat{k}] = F(\hat{j}+\hat{i}) = F(\hat{i}+\hat{j})$

$\left[\text{Since } \hat{k}\times\hat{i} = \hat{j} \text{ and } \hat{j}\times\hat{k} = \hat{i}\right]$

37. (a) Component of $\vec{a}$ along $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

38. (a) $u_x = 30\cos 30° = 30\sqrt{3}/2,$, $u_y = 30\sin 30°$,

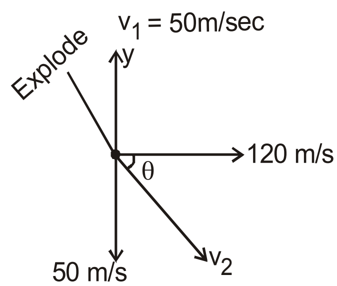
$v_y = 30\sin 30° - gt$

$v_y = 30\sin 30° - 10\times 1.5 = 0$

As vertical velocity = 0,
Angle with horizontal $\alpha = 0°$
It is a state, when a particle reach to a highest point of its path.

39. (c) From conservation of linear momentum,

$m\vec{v} = \dfrac{m}{2}\vec{v}_1 + \dfrac{m}{2}\vec{v}_2$



(here we take particle & earth as a system so in this case external force is zero & linear momentum is conserved)
Where $\vec{v}$ is velocity of particle before explosion & $\vec{v}_1, \vec{v}_2$ are velocity of its equal pieces.

here $\vec{v} = 60\hat{i}$ (in x direction),

$\vec{v}_1 = 50\hat{j}$ (in y direction)

so $\vec{v}_2 = 120\hat{i} - 50\hat{j}$ or $|\vec{v}_2| = 130$ m/sec &
[From conservation of linear momentum]

$\tan\theta = \dfrac{-50}{120}$

40. (b) There is no change in horizontal velocity, hence no change in momentum in horizontal direction. The vertical velocity at t = 10sec is

$v = 98\times\sin 60° - (9.8)\times 10 = -13.13$ m/sec

so change in momentum in vertical direction is

$= (0.5\times 98\times\sqrt{3}/2) - [-(0.5\times 13.13)]$

$= 42.434 + 6.56 = 48.997 \approx 49$

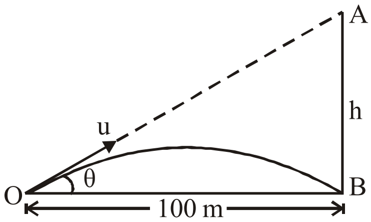
41. (b) Maximum possible horizontal range = v^2/g
Maximum possible area of the circle

$= \pi\left(\dfrac{v^2}{g}\right)^2 = \dfrac{\pi v^4}{g^2}$ $\left[\text{Here } r = \dfrac{v^2}{g}\right]$

42. (c) Here , $\vec{F} = (3\hat{i} + 4\hat{j})N$

$\vec{d} = (3\hat{i} + 4\hat{j})m \qquad \therefore\ W = \vec{F}.\vec{d} = (9 + 16)J = 25J$

43. (b) Let the gun be fired with velocity u from point O on the bird at B, making an angle θ with the horizontal direction. Therefore the height of the aims of the person is at height BA (=h) above the bird.



Here, horizontal range $= \dfrac{u^2 \sin 2\theta}{g} = 100$

or $\dfrac{500^2 \sin 2\theta}{10} = 100$ or $\sin 2\theta$

$= \dfrac{100 \times 10}{(500)^2} = \dfrac{1}{250} = \sin 14'$

or $2\theta = 14'$ or $\theta = 7' = \dfrac{7}{60} \times \dfrac{\pi}{180}$ radian

As, angle $= \dfrac{\text{arc}}{\text{radius}} \qquad \therefore\ \theta = \dfrac{AB}{OB}$

or $AB = \theta \times OB$

$= \dfrac{7}{60} \times \dfrac{\pi}{180} \times (100 \times 100)$ cm = 20.35 cm

44. (a) $\vec{a} + \vec{b} = 3\hat{i} + 4\hat{k}$

$\therefore$ Required unit vector

$= \dfrac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \dfrac{3\hat{i} + 4\hat{k}}{\sqrt{3^2 + 4^2}} = \dfrac{3\hat{i} + 4\hat{k}}{5}.$

45. (d) Time of flight $= \dfrac{2u \sin\theta}{g}$

$= \dfrac{2 \times 9.8 \times \sin 30°}{9.8} = 2 \times \dfrac{1}{2} = 1\,\text{sec}.$

46. (d) The initial velocity in the vertically downward direction is zero and same height has to be covered.

47. (c) $y = x \tan\theta - \dfrac{1}{2} \dfrac{gx^2}{u^2 \cos^2\theta}$

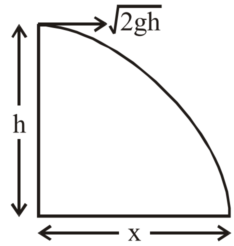
$y = 50 \tan 60° - \dfrac{10 \times 50 \times 50}{2 \times 25 \times 25 \times \cos^2 60°} = 5\,\text{m}$

48. (b) Comparing the given equation with

$y = x \tan\theta - \dfrac{gx^2}{2u^2 \cos^2\theta}$, we get

$\tan\theta = \sqrt{3}$

49. (c)



$u_y = 0,\ s_y = -h,\ a_y = -g,\ t_y = ?$

$s = ut + \dfrac{1}{2}at^2$

$\therefore -h = -\dfrac{1}{2}gt^2 \quad \Rightarrow t = \sqrt{\dfrac{2h}{g}}$

velocity $= \dfrac{x}{t}$

$\therefore x = \sqrt{2gh} \times \sqrt{\dfrac{2h}{g}} = 2h$

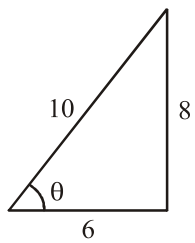
50. (b) Horizontal distance covered should be same for the time of collision.

$400 \cos\theta = 200$ or $\cos\theta = \dfrac{1}{2}$ or $\theta = 60°$

51. (c) This happen when vertical velocity of both are same.

$\therefore \quad v_2 = v_1 \sin 30°$ or $\dfrac{v_2}{v_1} = \dfrac{1}{2}$

52. (b) $\vec{v} = 6\hat{i} + 8\hat{j}$



Comparing with $\vec{v} = v_x\hat{i} + v_y\hat{j}$, we get

$v_x = 6\,\text{ms}^{-1}$ and $v_y = 8\,\text{ms}^{-1}$

Also, $v^2 = v_x{}^2 + v_y{}^2 = 36 + 64 = 100$

or $v = 10\ \text{ms}^{-1}$

$\sin\theta = \dfrac{8}{10}$ and $\cos\theta = \dfrac{6}{10}$

$R = \dfrac{v^2 \sin 2\theta}{g} = \dfrac{2v^2 \sin\theta\cos\theta}{g}$

$R = 2 \times 10 \times 10 \times \dfrac{8}{10} \times \dfrac{6}{10} \times \dfrac{1}{10} = 9.6\,\text{m}$

53. (b) $x = \dfrac{1}{2}gt^2$

$= \dfrac{1}{2} \times 9.8 \times \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{9.8}{8}$ m

54. (b) Comparing the given equation with the equation of trajectory of a projectile,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

we get, $\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$

and $2u^2\cos^2\theta = 20 \Rightarrow u^2 = \frac{20}{2\cos^2\theta}$

$$= \frac{10}{\cos^2 30°} = \frac{10}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{40}{3}$$

Now, $R_{max} = \frac{u^2}{g} = \frac{40}{3\times10} = \frac{4}{3}\text{m}$

55. (c) The bullet performs a horizontal journey of 100 cm with constant velocity of 1500 m/s. The bullet also performs a vertical journey of h with zero initial velocity and downward acceleration g.

$\therefore$ For horizontal journey, time (t) $= \frac{\text{Distance}}{\text{Velocity}}$

$$\therefore\ t = \frac{100}{1500} = \frac{1}{15}\text{sec} \quad \ldots(1)$$

The bullet performs vertical journey for this time.

For vertical journey, $h = ut + \frac{1}{2}gt^2$

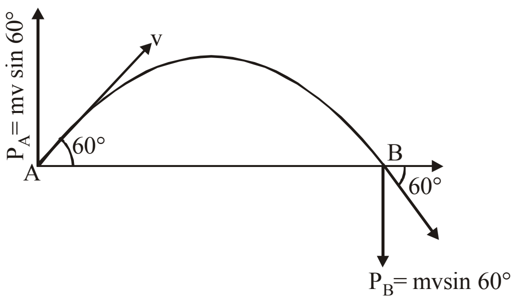
$$h = 0 + \frac{1}{2}\times10\times\left(\frac{1}{15}\right)^2$$

$$\text{or, } h = \frac{10}{2\times15\times15}\text{m} = \frac{10\times100}{2\times15\times15}\text{cm}$$

$$\text{or, } h = \frac{20}{9}\text{cm} = 2.2\text{cm}$$

The gun should be aimed $\left(\frac{20}{9}\right)$cm above the target.

56. (b)



As the figure drawn above shows that at points A and B the vertical component of velocity is v sin 60° but their directions are opposite.

Hence, change in momentum is given by :

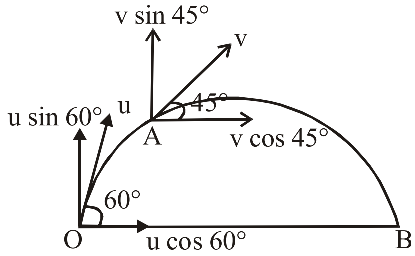
$$\Delta p = mv\sin 60° - (-mv\sin 60°) = 2mv\sin 60°$$

$$= 2mv\frac{\sqrt{3}}{2} = \sqrt{3}mv$$

57. (a) The motion of the train will affect only the horizontal component of the velocity of the ball. Since, vertical component is same for both observers, the y_m will be same, but R will be different.

58. (c) $a = \frac{v^2}{r} = 1$ cm/s. Centripetal acceleration is directed towards the centre. Its magnitude = 1. Unit vector at the mid point on the path between P and Q is $-(\hat{x}+\hat{y})/\sqrt{2}$.

59. (c)



Velocity of projectile $u = 147\ \text{ms}^{-1}$
angle of projection $\alpha = 60°$
Let, the time taken by the projectile from O to A be t where direction $\beta = 45°$. As horizontal component of velocity remains constant during the projectile motion.

$\Rightarrow\ v\cos 45° = u\cos 60°$

$\Rightarrow\ v\times\frac{1}{\sqrt{2}} = 147\times\frac{1}{2} \Rightarrow\ v = \frac{147}{\sqrt{2}}\text{ms}^{-1}$

For Vertical motion, $v_y = u_y - gt$

$\Rightarrow\ v\sin 45° = 45\sin 60° - 9.8\,t$

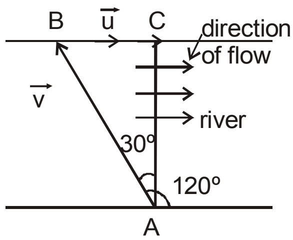
$\Rightarrow\ \frac{147}{\sqrt{2}}\times\frac{1}{\sqrt{2}} = 147\times\frac{\sqrt{3}}{2} - 9.8\,t$

$\Rightarrow\ 9.8\,t = \frac{147}{2}(\sqrt{3}-1) \Rightarrow\ t = 5.49\ \text{s}$

60. (d) When a cyclist moves on a circular path, it experiences a centrifugal force which is equal to mv^2/r. It tries to overturn the cyclist in outward direction. If speed increases twice, the value of centrifugal force too increases to 4 times its earlier value. Therefore the chance of overturning is 1/4 times.

61. (c) Here v = 0.5 m/sec. u = ?

so $\sin\theta = \frac{u}{v} \Rightarrow \frac{u}{.5} = \frac{1}{2}$ or $u = 0.25\ \text{ms}^{-1}$



62. (d) $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$

$$\vec{v} = \frac{\vec{d}(r)}{dt} = \frac{d}{dt}\{(a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}\}$$

$$= (-a\omega\sin\omega t)\hat{i} + (a\omega\cos\omega t)\hat{j}$$

$$= \omega[(-a\sin\omega t)\hat{i} + (a\cos\omega t)\hat{j}]$$

$\vec{r}.\vec{v} = 0$

$\therefore$ velocity is perpendicular to the displacement.

63. (b) Since $E = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2E}{m}}$

Now at highest point of flight, the vertical component of velocity is zero & only horizontal component is non zero. So K.E. at highest point is $E' = \frac{1}{2}m(v\cos 45^\circ)^2$

$= E/2$

64. (b) Given, $u_1 = u_2 = u, \quad \theta_1 = 60^\circ, \theta_2 = 30^\circ$

In Ist case, we know that range

$$R_1 = \frac{u^2\sin 2(60^\circ)}{g} = \frac{u^2\sin 120^\circ}{g} = \frac{u^2\sin(90^\circ + 30^\circ)}{g}$$

$$= \frac{u^2(\cos 30^\circ)}{g} = \frac{\sqrt{3}u^2}{2g}$$

In IInd case when $\theta_2 = 30^\circ$, then

$$R_2 = \frac{u^2\sin 60^\circ}{g} = \frac{u^2\sqrt{3}}{2g} \Rightarrow R_1 = R_2$$

(we get same value of ranges).

65. (a) $F = 6t\hat{i} + 4t\hat{j}$ or $a_x = \frac{6t}{3}, a_y = \frac{4t}{3}$

so $u_x = \int_0^t a_x dt = t^2 \Rightarrow (u_x)_{t=3} = 9\text{m/sec}$

and $u_y = \int_0^t a_y dt = \frac{2t^2}{3} \Rightarrow (u_y)_{t=3} = 6\text{m/sec}$

(because u_x & $u_y = 0$ at $t = 0$ sec)

66. (b) $s = \frac{1}{2}gt^2$, s and g are same for both the balls, so time of fall 't' will also be the same for both of them (s is vertical height)

67. (c) $\vec{P}$ = vector sum $= \vec{A} + \vec{B}$

$\vec{Q}$ = vector differences $= \vec{A} - \vec{B}$

Since $\vec{P}$ and $\vec{Q}$ are perpendicular

$\therefore \vec{P}.\vec{Q} = 0$

$\Rightarrow (\vec{A}+\vec{B}).(\vec{A}-\vec{B}) = 0 \Rightarrow A^2 = B^2 \Rightarrow |A| = |B|$

68. (b) Circumference of circle is $2\pi r = 40$m

Total distance travelled in two revolution is 80m. Initial velocity u = 0, final velocity v = 80 m/sec

so from

$v^2 = u^2 + 2as$

$\Rightarrow (80)^2 = 0^2 + 2 \times 80 \times a$

$\Rightarrow a = 40\text{m/sec}^2$

69. (b) $|\vec{A} \times \vec{B}| = AB\sin\theta$

$\vec{A}.\vec{B} = AB\cos\theta$

$|\vec{A} \times \vec{B}| = \sqrt{3}\ \vec{A}.\vec{B}$

$\Rightarrow AB\sin\theta = \sqrt{3}AB\cos\theta$ or $\tan\theta = \sqrt{3}$

$\therefore \theta = 60^\circ$

$$\therefore |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos 60^\circ}$$

$$= \sqrt{A^2 + B^2 + AB}$$

70. (b) For two vectors to be perpendicular to each other

$\vec{A} \cdot \vec{B} = 0$

$(2\hat{i} + 3\hat{j} + 8\hat{k}) \cdot (4\hat{j} - 4\hat{i} + \alpha\hat{k}) = 0$

$-8 + 12 + 8\alpha = 0$ or $\alpha = -\frac{4}{8} = -\frac{1}{2}$

71. (d) 72. (c) 73. (d) 74. (d) 75. (a)

EXERCISE - 3

Exemplar Questions

1. (b) Given, $A = \hat{i} + \hat{j}$

$B = \hat{i} - \hat{j}$

As we know that

$\vec{A} \cdot \vec{B} = |A||B|\cos\theta$

$(\hat{i} + \hat{j}) \cdot (\hat{i} - \hat{j}) = (\sqrt{1^2 + 1^2})(\sqrt{1^2 + 1^2})\cos\theta$

$(i + j)(i - j) = \sqrt{2} \times \sqrt{2}\cos\theta$

where θ is the angle between A and B

$$\cos\theta = \frac{1 - 0 + 0 - 1}{\sqrt{2}\sqrt{2}} = 0$$

$\therefore \quad \theta = 90^\circ$

2. (d) A scalar quantity does not depend on direction so it has the same value for observers with different orientations of the axes.

3. (b) From the diagram, $u = a\hat{i} + b\hat{j}$

As u is in the first quadrant, so both components a and b will be positive.

For $v = p\hat{i} + q\hat{j}$, as it is in positive x-direction and located downward so x-component p will be positive and y-component q will be negative.

Hence, a, b and p are positive but q is negative.

4. (b) Let r makes an angle θ with positive x-axis component of r along x-axis.

$r_x = |r|\cos\theta$

$(r_x)_{\text{maximum}} = |r|(\cos\theta)_{\text{maximum}}$

$= |r|\cos 0^\circ = |r|$

($\because \cos\theta$ is maximum of $\theta = 0^\circ$)

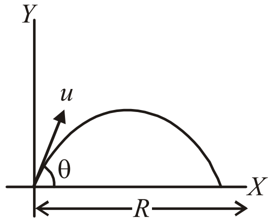
$\theta = 0^\circ$.

Hence the vector r has maximum value along positive x-axis.

5. (c) Consider, projectile is fired at an angle θ.

According to question,

$\theta = 15°$ and $R = 50$ m



Range, $R = \dfrac{u^2 \sin 2\theta}{g}$

$R = 50\,\text{m} = \dfrac{u^2 \sin(2\times 15°)}{g}$

$50\times g = u^2 \sin 30° = u^2 \times \dfrac{1}{2}$

$50 \times g \times 2 = u^2$

$u^2 = 50\times 9.8\times 2 = 100\times 9.8 = 980$

$u = \sqrt{980} = 31.304$ m/s $= 14\sqrt{5}$

$(\because g = 9.8 \text{ m/s}^2)$

Now, $\theta = 45°$;

$R = \dfrac{u^2 \sin 2\times 45°}{g} = \dfrac{u^2}{g}$

$R = \dfrac{(14\sqrt{5})^2}{g} = \dfrac{14\times 14\times 5}{9.8} = 100$ m

6. (b) As we know that,

Impulse, $I = F\Delta t = \left(\dfrac{\Delta p}{\Delta t}\right)\Delta t = \Delta p$

where F is force, Δt is time duration and Δp is change in momentum. As Δp is a vector quantity, hence impulse is also a vector quantity. Sometimes area can also be treated as vector.

7. (d) As speed is a scalar quantity, hence it will be related with path length (scalar quantity) only.

Hence, Speed $v_0 = \dfrac{\text{total distance travelled}}{\text{time taken}}$

So, total distance travelled = Path length

= (speed) × time taken

Hence, path length which is scalar and traversed in equal intervals.

8. (c) As given that in two dimensional motion the instanteous speed v_0 is positive constant and we know that acceleration is rate of change of velocity or instantaneous speed and hence it will also be in the plane of motion.

NEET/AIPMT (2013-2017) Questions

9. (b) At point B the direction of velocity component of the projectile along Y - axis reverses.

Hence, $\vec{V}_B = 2\hat{i} - 3\hat{j}$

10. (d) Vector triple product

$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B}) = 0$

$\Rightarrow \vec{A} \,||\, (\vec{B}\times\vec{C})$ $\quad [\because \vec{A}\cdot\vec{B} = 0 \text{ and } \vec{A}\cdot\vec{C} = 0]$ 1.

(a) $(\vec{A}+\vec{B})^2 = (\vec{C})^2$

$\Rightarrow A^2 + B^2 + 2\vec{A}.\vec{B} = C^2$

$\Rightarrow 3^2 + 4^2 + 2\vec{A}.\vec{B} = 5^2$

$\Rightarrow 2\vec{A}.\vec{B} = 0$

or $\Rightarrow \vec{A}.\vec{B} = 0$

$\therefore \vec{A} \perp \vec{B}$

Here $A^2 + B^2 = C^2$. Hence, $\vec{A} \perp \vec{B}$

11. (d) $\vec{v}_{av} = \dfrac{\Delta\vec{r}\text{ (displacement)}}{\Delta t\text{ (time taken)}}$

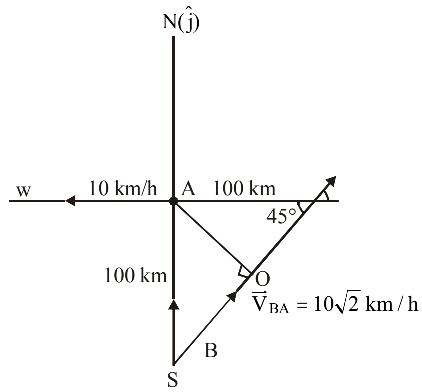
$= \dfrac{(13-2)\hat{i} + (14-3)\hat{j}}{5-0} = \dfrac{11}{5}(\hat{i}+\hat{j})$

12. (a) $\vec{V}_A = 10\left(-\hat{i}\right)$

$\vec{V}_B = 10\left(\hat{j}\right)$

$\vec{V}_{BA} = 10\,\hat{j} + 10\,\hat{i} = 10\sqrt{2}$ km / h

Distance OB = 100 cos 45° = $50\sqrt{2}$ km



Time taken to reach the shortest distance between

A and B = $\dfrac{OB}{\vec{V}_{BA}} = \dfrac{50\sqrt{2}}{10\sqrt{2}} = 5$h

13. (b) Two vectors are

$\vec{A} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$

$\vec{B} = \cos\dfrac{\omega t}{2}\hat{i} + \sin\dfrac{\omega t}{2}\hat{j}$

For two vectors $\vec{A}$ and $\vec{B}$ to be orthogonal $\vec{A}.\vec{B}=0$

$\vec{A}.\vec{B}=0=\cos\omega t.\cos\frac{\omega t}{2}+\sin\omega t.\sin\frac{\omega t}{2}$

$=\cos\left(\omega t-\frac{\omega t}{2}\right)=\cos\left(\frac{\omega t}{2}\right)$

So, $\frac{\omega t}{2}=\frac{\pi}{2}\ \therefore\ t=\frac{\pi}{\omega}$

14. (b) Here, $x=4\sin(2\pi t)$ ...(*i*)

$y=4\cos(2\pi t)$ ...(*ii*)

Squaring and adding equation (*i*) and (*ii*)

$x^2+y^2=4^2 \Rightarrow R=4$

Motion of the particle is circular motion, acceleration vector is along $-\vec{R}$ and its magnitude $=\frac{V^2}{R}$

Velocity of particle, $V=\omega R=(2\pi)(4)=8\pi$

15. (c) **Given:** Position vector

$\vec{r}=\cos\omega t\ \hat{x}+\sin\omega t\ \hat{y}$

$\therefore$ Velocity, $\vec{v}=-\omega\sin\omega t\ \hat{x}+\omega\cos\omega t\ \hat{y}$

and acceleration,

$\vec{a}=-\omega^2\cos\omega t\ \hat{x}-\omega^2\sin\omega t\ \hat{y}=-\omega^2\vec{r}$

$\vec{r}\cdot\vec{v}=0$ hence $\vec{r}\perp\vec{v}$ and

$\vec{a}$ is directed towards the origin.

16. (b) $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$

Squaring on both sides

$|\vec{A}+\vec{B}|^2=|\vec{A}-\vec{B}|^2$

$\Rightarrow \vec{A}\cdot\vec{A}+2\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{B}$

$=\vec{A}\cdot\vec{A}-2\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{B}$

$\Rightarrow 4\vec{A}\cdot\vec{B}=0 \Rightarrow 4AB\cos\theta=0$

$\Rightarrow \cos\theta=0 \Rightarrow \theta=90°$

17. (b) Given:

$x=5t-2t^2$ $\qquad y=10t$

$v_x=\frac{dx}{dt}=5-4t$ $\qquad v_y=\frac{dy}{dt}=10$

$a_x=\frac{dv_x}{dt}=-4$ $\qquad a_y=\frac{dv_y}{dt}=0$

$\vec{a}=a_x i+a_y j$ $\qquad \vec{a}=-4i\ m/s^2$

Hence, acceleration of particle at $(t=2\text{ s})=-4\text{m/s}^2$

AAJ KA TOPPER