Chapter

5 Laws of Motion

AAJ KA TOPPER

ARISTOTLE'S FALLACY

According to Aristotelian law an external force is required to keep a body in motion. However an external force is required to overcome the frictional forces in case of solids and viscous forces in fluids which are always present in nature.

LINEAR MOMENTUM (p)

Linear momentum of a body is the quantity of motion contained in the body. Momentum $\vec{p} = m\vec{v}$

It is a vector quantity having the same direction as the direction of the velocity. Its **SI unit** is kg ms^{-1}.

NEWTON'S LAWS OF MOTION

First law : *A body continues to be in a state of rest or of uniform motion, unless it is acted upon by some external force to change its state.*

Newton's first law gives the qualitative definition of force according to which *force is that external cause which tends to change or actually changes the state of rest or motion of a body*.

Newton's first law of motion is the same as **law of inertia** given by Galileo.

Inertia *is the inherent property of all bodies because of which they cannot change their state of rest or of uniform motion unless acted upon by an external force.*

Second law : *The rate of change of momentum of a body is directly proportional to the external force applied on it and the change takes place in the direction of force applied.*

$$i.e., \quad \vec{F} = \frac{d\vec{p}}{dt} = \frac{md\vec{v}}{dt} = m\vec{a}$$

This is the equation of motion of constant mass system. For variable mass system such as rocket propulsion

$$\vec{F} = \frac{d(m\vec{v})}{dt}$$

$$\text{And,} \quad \vec{F} = \frac{m(d\vec{v})}{dt} + \vec{v}\frac{dm}{dt}$$

The **SI unit** of force is newton. (One newton force is that much force which produces an acceleration of 1ms^{-2} in a body of mass 1 kg.

The **CGS unit** of force is dyne. (1N = 10^5 dyne)

The gravitational unit of force is kg-wt (kg-f) or g-wt (g-f)

$$1\ kg\text{-}wt\ (kg\text{-}f) = 9.8\ N, \qquad 1\ g\text{-}wt\ (g\text{-}f) = 980\ dyne$$

Third law : *To every action there is an equal and opposite reaction*. For example – walking , swimming , a horse pulling a cart etc.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Action and reaction act on different bodies and hence cannot balance each other. Action and reaction occur simultaneously. Forces always occur in pairs.

EQUILIBRIUM OF A PARTICLE

A body is said to be in equilibrium when no net force acts on the body.

$$i.e., \ \Sigma\vec{F} = 0$$

Then $\Sigma F_x = 0,\ \Sigma F_y = 0$ and $\Sigma F_z = 0$

Stable equilibrium : If a body is slightly displaced from equilbrium position, it has the tendency to regain its original position, it is said to be in stable equilibrium.

In this case, *P.E. is minimum.* $\left(\frac{d^2u}{dr^2} = +ve\right)$

So, the *centre of gravity is lowest*.

Unstable equilibrium : If a body, after being displaced from the equilibrium position, moves in the direction of displacement, it is said to be in unstable equilibrium.

In this case, *P.E. is maximum.* $\left(\frac{d^2u}{dr^2} = -ve\right)$

So, the *centre of gravity is highest*.

Neutral equilibrium : If a body, after being slightly displaced from the equilibrium position has no tendency to come back or to move in the direction of displacement the equilibrium is known to be neutral.

In this case, *P.E. is constant* $\left(\frac{d^2u}{dr^2} = constant\right)$

The *centre of gravity remains at constant height.*

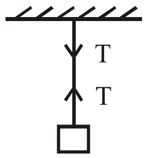
COMMON FORCES IN MECHANICS

1. **Weight :** It is the force with which the earth attracts a body and is called force of gravity, For a body of mass m, where acceleration due to gravity is g, the weight

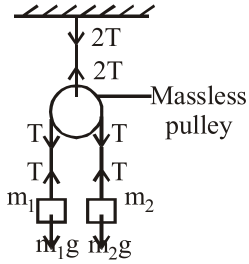
$$W = mg$$

2. **Tension :** The force exerted by the ends of a loaded/stretched string (or chain) is called tension. The tension has a sense of pull at its ends.

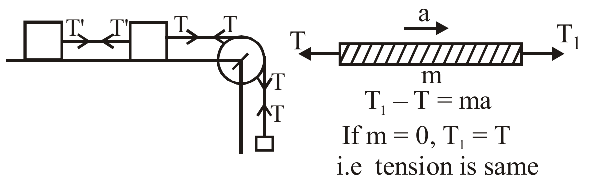
Case 1



Case 2



Case 3



$T_1 - T = ma$

If $m = 0$, $T_1 = T$

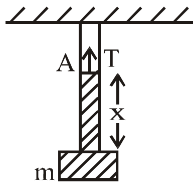
i.e tension is same

The tension in a string remains the same throughout the string if

(a) string is massless,

(b) pulley is massless or pulley is frictionless

Case 4 : String having mass



Let the total mass of the string be M and length be L. Then mass per unit length is $\frac{M}{L}$

Let x be the distance of the string from the mass m. Then the mass of the shaded portion of string is $\left(\frac{M}{L} \times x\right)$

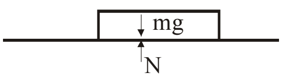
If the string is at rest then the tension T has to balance the wt of shaded portion of string and weight of mass m.

$$\therefore T = \left(m + \frac{M}{L}x\right)g$$

$\Rightarrow$ as x increases, the tension increases. Thus tension is non-uniform in a string having mass.

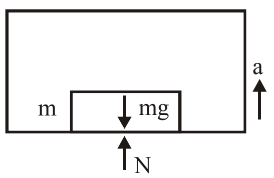
3. **Normal force :** It measures how strongly one body presses the other body in contact. It acts normal to the surface of contact.

Case 1



$N = mg$

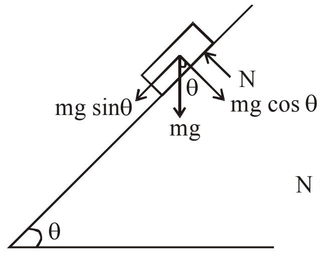
Case 2



$N - mg = ma$

$\Rightarrow N = m(g + a)$

Case 3



$N = mg\cos\theta$

4. **Spring force :** If an object is connected by spring and spring is stretched or compressed by a distance x, then restoring force on the object $F = -kx$

where k is a spring contact on force constant.

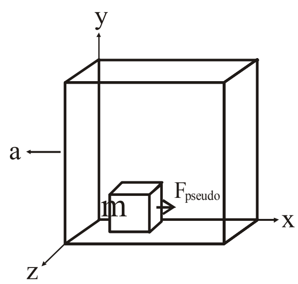
5. **Frictional force :** It is a force which opposes relative motion between the surfaces in contact. $f = \mu N$

This will be discussed in detail in later section.

6. **Pseudo force :** If a body of mass m is placed in a non-inertial frame having aceleration $\vec{a}$, then it experiences a Pseudo force acting in a direction opposite to the direction of $\vec{a}$.

$$\vec{F}_{pseudo} = -m\vec{a}$$

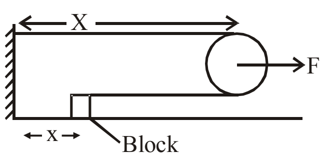
Negative sign shows that the pseudo force is always directed in a direction opposite to the direction of the acceleration of the frame.



CONSTRAINT MOTION :

When the motion of one body is dependent on the other body, the relationship of displacements, velocities and accelerations of the two bodies are called constraint relationships.

Case 1 Pulley string system :



Step 1 : Find the distance of the two bodies from fixed points.

Step 2 : The length of the string remain constant. (We use of this condition)

Therefore $X + (X - x) = \text{constant} \Rightarrow 2X - x = \text{constant}$

$$\Rightarrow \quad 2\frac{dX}{dt} - \frac{dx}{dt} = 0 \quad \Rightarrow \quad 2\frac{dX}{dt} = \frac{dx}{dt}$$

$$\Rightarrow \quad 2V_p = v_B \left[\because \frac{dX}{dt} = V_p = \text{velocity of pulley}\right.$$

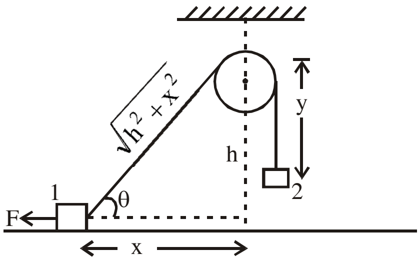
$$\left.\frac{dx}{dt} = v_B = \text{velocity of block}\right]$$

Again differentiating we get, $2a_p = a_B$

$$\left[a_p = \frac{dVp}{dt} \text{ and } a_B = \frac{dv_B}{dt}\right]$$

a_p = acceleration of pulley, a_B = acceleration of block

Case 2 Here $\sqrt{h^2 + x^2} + y = \text{constt.}$ On differentiating w.r.t 't'

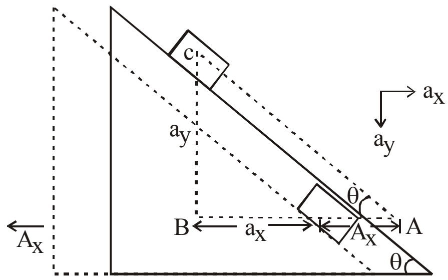


[Negative sign with dy/dt shows that with increase in time, y decreases]

$$\frac{1 \times 2x}{2\sqrt{h^2 + x^2}} \frac{dx}{dt} - \frac{dy}{dt} = 0 \Rightarrow \cos\theta\,(v_1 - v_2) = 0$$

$$\left[\because \cos\theta = \frac{x}{\sqrt{h^2 + x^2}}\right]$$

Case 3 Wedge block system : Thin lines represents the condition of wedge block at t = 0 and dotted lines at t = t



A_x = acceleration of wedge towards left

a_x, a_y = acceleration of block as shown

From ΔABC, $\tan\theta = \dfrac{a_y}{a_x + A_x}$

Frame of Reference :

Reference frames are co-ordinate systems in which an event is described.

There are two types of reference frames

(a) **Inertial frame of reference**: These are frames of reference in which Newton's laws hold good. These frames are at rest with each other or which are moving with uniform speed with respect to each other.

All reference frames present on surface of Earth are supposed to be inertial frame of reference.

(b) **Non – inertial frame of reference**: Newton's law do not hold good in non-inertial reference frame.

All accelerated and rotatory reference frames are non – inertial frame of reference. Earth is a non-intertial frame.

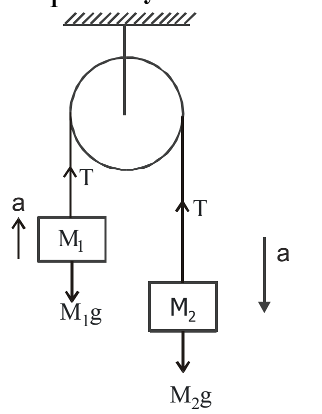
Note : When the observer is in non-inertial reference frame a pseudo force is applied on the body under observation.

Free Body Diagram (FBD) :

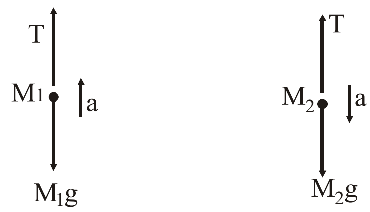
Free body diagram of a mass is a separate diagram of that mass. All forces acting on the mass are sketched. A FBD is drawn to visualise the direct forces acting on a body.

Case 1 : Masses M_1 and M_2 are tied to a string, which goes over a frictionless pulley

(a) **If $M_2 > M_1$ and they move with acceleration a**



FBD of M_1, FBD of M_2



$T - M_1g = M_1a$ $\quad\quad$ $M_2g - T = M_2a$

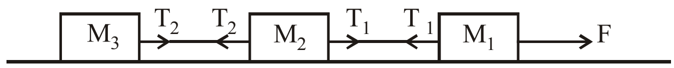
where T is the tension in the string. It gives

$$a = \frac{M_2 - M_1}{M_1 + M_2} g \text{ and } T = \frac{2M_1M_2}{M_1 + M_2} g$$

(b) **If the pulley begins to move with acceleration f, downwards**

$$\vec{a} = \frac{M_2 - M_1}{M_1 + M_2}(\vec{g} - \vec{f}) \text{ and } \vec{T} = \frac{2M_1M_2}{M_1 + M_2}(\vec{g} - \vec{f})$$

Case 2 : Three masses M_1, M_2 and M_3 are connected with strings as shown in the figure and lie on a frictionless surface. They are pulled with a force F attached to M_1.

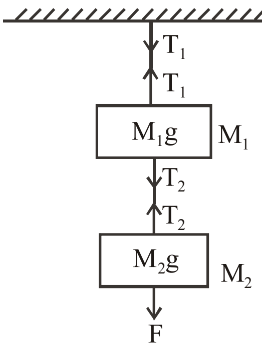


The forces on M_2 and M_3 are as follows

$$T_1 = \frac{M_2 + M_3}{M_1 + M_2 + M_3} F \text{ and } T_2 = \frac{M_3}{M_1 + M_2 + M_3} F;$$

Acceleration of the system is $a = \dfrac{F}{M_1 + M_2 + M_3}$

Case 3 : Two blocks of masses M_1 and M_2 are suspended vertically from a rigid support with the help of strings as shown in the figure. The mass M_2 is pulled down with a force F.



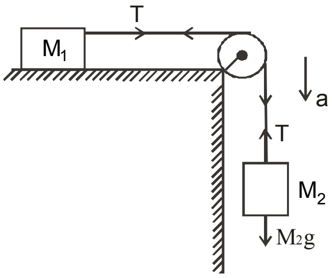
The tension between the masses M_1 and M_2 will be

$$T_2 = F + M_2g$$

Tension between the support and the mass M_1 will be

$$T_1 = F + (M_1 + M_2)g$$

Case 4 : Two masses M_1 and M_2 are attached to a string which passes over a pulley attached to the edge of a horizontal table. The mass M_1 lies on the frictionless surface of the table.



Let the tension in the string be T and the acceleration of the system be a. Then

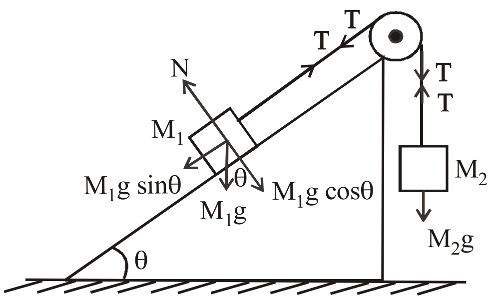
$T = M_1a$ ...(1)

$M_2g - T = M_2a$ ...(2)

Adding eqns. (1) and (2), we get

$$a = \left[\frac{M_2}{M_1 + M_2}\right]g \text{ and } T = \left[\frac{M_1M_2}{M_1 + M_2}\right]g$$

Case 5 : Two masses M_1 and M_2 are attached to the ends of a string, which passes over a frictionless pulley at the top of the inclined plane of inclination θ. Let the tension in the string be T.



(i) **When the mass M_1 moves upwards with acceleration a.**

From the FBD of M_1 and M_2,

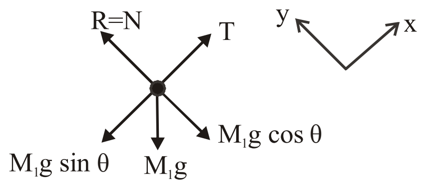
$T - M_1g\sin\theta = M_1a$ ...(1)

$M_2g - T = M_2a$ ...(2)

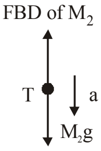
Solving eqns. (1) and (2) we get,

$$a = \left[\frac{M_2 - M_1\sin\theta}{M_1 + M_2}\right]g$$

FBD of mass M_1



FBD of M_2



$$T = \left[\frac{M_2M_1}{M_1 + M_2}\right]\frac{g}{(1+\sin\theta)}$$

(ii) **When the mass M_1 moves downwards with acceleration a.**

Equation of motion for M_1 and M_2,

$M_1g\sin\theta - T = M_1a$ ...(1)

$T - M_2g = M_2a$ ...(2)

Solving eqns. (1) and (2) we get,

$$a = \left[\frac{M_1\sin\theta - M_2}{M_1 + M_2}\right]g;\ T = \left[\frac{M_2M_1}{M_1 + M_2}\right]\frac{g}{(1+\sin\theta)}$$

(a) If $(M_2/M_1 = \sin\theta)$ then the system does not accelerate.

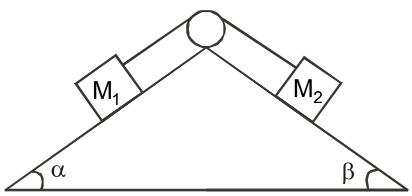
(b) Changing position of masses, does not affect the tension. Also, the acceleration of the system remains unchanged.

(c) If $M_1 = M_2 = M$ (say), then

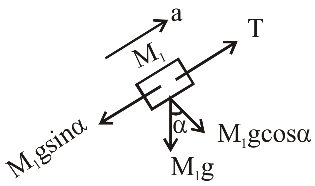
$$a = \left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)^2\left(\frac{g}{2}\right);\ T = \left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2\left(\frac{Mg}{2}\right)$$

Case 6 : Two masses M_1 and M_2 are attached to the ends of a string over a pulley attached to the top of a double inclined plane of angle of inclination α and β.

Let M_2 move downwards with acceleration a and the tension in the string be T then



<u>FBD of M_1</u>

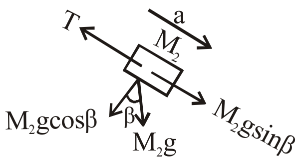


Equation of motion for M_1

$T - M_1g\sin\alpha = M_1a$

or $T = M_1g\sin\alpha + M_1a$ ...(1)

FBD of M_2



Equation of motion for M_2

$M_2g \sin\beta - T = M_2a$

or $\quad T = M_2g \sin \beta - M_2a$ ...(2)

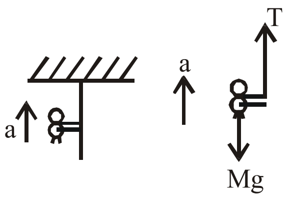
Using eqn. (1) and (2) we get,

$M_1g \sin \alpha + M_1a = M_2g \sin \beta - M_2a$

Solving we get,

$$a = \frac{(M_2 \sin\beta - M_1 \sin\alpha)g}{M_1 + M_2} \text{ and } T = \frac{M_1M_2g}{M_1 + M_2}[\sin\beta + \sin\alpha]$$

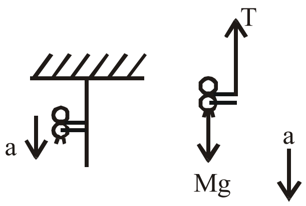
Case 7 : A person/monkey climbing a rope



(a) A person of mass M climbs up a rope with acceleration a. The tension in the rope will be M(g+a).

$T - Mg = Ma \Rightarrow T = M(g + a)$

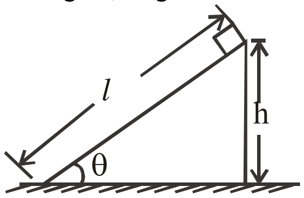
(b) If the person climbs down along the rope with acceleration a, the tension in the rope will be M(g–a).



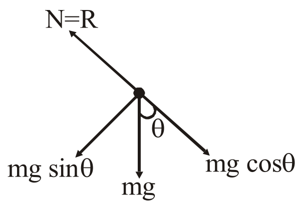
$Mg - T = Ma \Rightarrow T = M(g - a)$

(c) When the person climbs up or down with uniform speed, tension in the string will be Mg.

Case 8 : A body starting from rest moves along a smooth inclined plane of length *l*, height h and having angle of inclination θ.



FBD of body



(where N=R is normal reaction applied by plane on the body of mass m)

For downward motion, along the inclined plane,

$mg \sin\theta = ma \Rightarrow a = g \sin\theta$

By work-energy theorem loss in P.E. = gain in K.E.

$$\Rightarrow mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

Also, from the figure, $h = \ell \sin \theta$. $\quad \therefore\ v = \sqrt{2gh} = \sqrt{2g\ell\sin\theta}$

(a) Acceleration down the plane is $g \sin \theta$.

(b) Its velocity at the bottom of the inclined plane will be $\sqrt{2gh} = \sqrt{2g\ell\sin\theta}$

(c) Time taken to reach the bottom will be

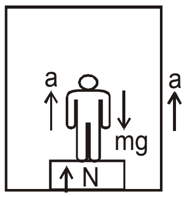
$$t = \left(\frac{2\ell}{g\sin\theta}\right)^{1/2} = \left(\frac{2h}{g\sin^2\theta}\right)^{1/2} = \frac{1}{\sin\theta\left(\frac{g}{2h}\right)^{1/2}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$$

(d) If angles of inclination are θ_1 and θ_2 for two inclined planes

Keeping the length constant then $\dfrac{t_1}{t_2} = \left(\dfrac{\sin\theta_2}{\sin\theta_1}\right)^{1/2}$

Case 9 : Weight of a man in a lift :

(i) **When lift is accelerated upward :** In this case the man also moves in upward direction with an acceleration $\vec{a}$.



Then from Newton' second law

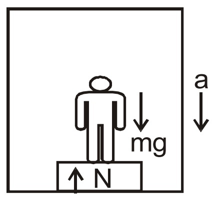
$N - mg = ma$ or $N = m(g + a)$

or $W_{app} = m(g + a) = W_o(1 + a/g)$ (as W = mg)

Where W_{app} is apparent weight of the man in the lift, W_o is the real weight, N is the reaction of lift on the man. It is clear that $N = W_{app}$

When the lift moves upward and if we measure the weight of the man by any means (such as spring balance) then we observe more weight (i.e., W_{app}) than the real weight (W_o) $W_{app} > W_o$

(ii) **When lift is accelerated downward :** In this case from Newton's second law



$mg - N = ma$

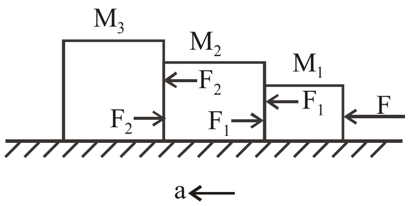
or $\quad N = m(g - a) = W_o(1 - a/g)$

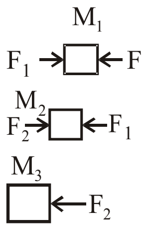
or $\quad W_{app} = W_o(1 - a/g) \qquad \{\because W_o = mg\}$

If we measure the weight of man by spring balance, we observe deficiency because $W_{app} < W_o$.

(iii) **When lift is at rest or moving with constant velocity :** From Newton's second law $N - mg = 0$ or $N = mg$
In this case spring balance gives the true weight of the man.

Case 10 : Three masses M_1, M_2 and M_3 are placed on a smooth surface in contact with each other as shown in the figure. A force F pushes them as shown in the figure and the three masses move with acceleration a,





$\Rightarrow F - F_1 = m_1 a$ ...(i)

$\Rightarrow F_1 - F_2 = m_2 a$ ...(ii)

$\Rightarrow F_2 = M_3 a$ ...(iii)

Adding eqns. (i), (ii) and (iii) we get, $a = \dfrac{F}{M_1 + M_2 + M_3}$

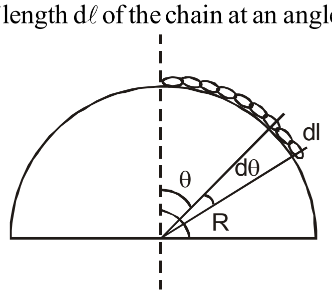
$$\Rightarrow F_2 = \frac{M_3 F}{M_1 + M_2 + M_3} \text{ and } F_1 = \frac{(M_2 + M_3)F}{M_1 + M_2 + M_3}$$

Keep in Memory

1. When a man jumps with load on his head, the apparent weight of the load and the man is zero.
2. (i) If a person sitting in a train moving with uniform velocity throws a coin vertically up, then coin will fall back in his hand.
   (ii) If the train is uniformly accelerated, the coin will fall behind him.
   (iii) If the train is retarded uniformly, then the coin will fall in front of him.

Example 1.
A chain of length ℓ is placed on a smooth spherical surface of radius R with one of its ends fixed at the top of the sphere. What will be the acceleration a of each element of the chain when its upper end is released? It is assumed that the length of chain $\ell < \left(\pi \dfrac{R}{2}\right)$.

Solution :
Let m be the mass of the chain of length ℓ. Consider an element of length $d\ell$ of the chain at an angle θ with vertical,



From figure, $d\ell = R\, d\theta$;
Mass of the element,

$$dm = \frac{m}{\ell} d\ell \text{ ; or } dm = \frac{m}{\ell}.R\, d\theta$$

Force responsible for acceleration, $dF = (dm) g \sin\theta$;

$$dF = \left(\frac{m}{\ell} R\, d\theta\right)(g \sin\theta) = \frac{mgR}{\ell} \sin\theta\, d\theta$$

Net force on the chain can be obtained by integrating the above relation between 0 to α, we have

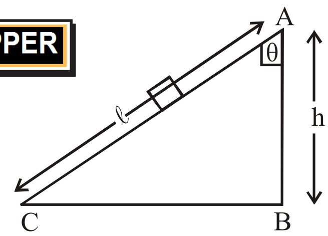
$$F = \int_0^{\alpha} \frac{mg\,R}{\ell} \sin\theta\, d\theta = \frac{mg\,R}{\ell}(-\cos\theta)_0^{\alpha} = \frac{mg\,R}{\ell}[1 - \cos\alpha]$$

$$= \frac{mg\,R}{\ell}\left[1 - \cos\frac{\ell}{R}\right];$$

$$\therefore \text{ Acceleration, } a = \frac{F}{m} = \frac{gR}{\ell}\left(1 - \cos\frac{\ell}{R}\right).$$

Example 2.
A block slides down a smooth inclined plane to the ground when released at the top, in time t second. Another block is dropped vertically from the same point, in the absence of the inclined plane and reaches the ground in t/2 second. Then find the angle of inclination of the plane with the vertical.

Solution :
If θ is the angle which the inclined plane makes with the vertical direction, then the acceleration of the block sliding down the plane of length ℓ will be g cosθ.



Using the formula, $s = ut + \frac{1}{2}at^2$, we have $s = \ell, u = 0, t = t$ and $a = g\cos\theta$.

so $\ell = 0 \times t + \dfrac{1}{2} g\cos\theta\, t^2 = \dfrac{1}{2}(g\cos\theta)t^2$ ...(i)

Taking vertical downward motion of the block, we get

$h = 0 + \dfrac{1}{2} g(t/2)^2 = \dfrac{1}{2} gt^2/4$ ...(ii)

Dividing eq^n. (ii) by (i), we get

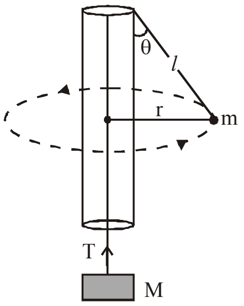
$$\frac{h}{\ell} = \frac{1}{4\cos\theta} \quad [\because \cos\theta = h/\ell]$$

or $\cos\theta = \dfrac{1}{4\cos\theta}$; or $\cos^2\theta = \dfrac{1}{4}$; or $\cos\theta = \dfrac{1}{2}$

or $\theta = 60°$

Example 3.

A large mass M and a small mass m hang at the two ends of a string that passes through a smooth tube as shown in fig. The mass m moves around a circular path in a horizontal plane. The length of the string from mass m to the top of the tube is l, and θ is the angle the string makes with the vertical. What should be the frequency (ν) of rotation of mass m so that mass M remains stationary?



Solution :

Tension in the string T = Mg.

Centripetal force on the body = $mr\omega^2 = mr(2\pi\nu)^2$. This is provided by the component of tension acting horizontally i.e. T sinθ (= Mg sinθ).

$$\therefore\ mr(2\pi\nu)^2 = Mg\sin\theta = Mgr/l.\ \text{or}\ \ \nu = \frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}$$

Example 4.

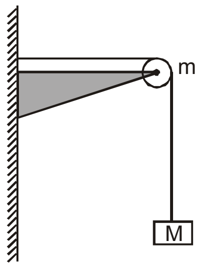
A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in fig. The force on the pulley by the clamp is given by

(a) $\sqrt{2}\,Mg$

(b) $\sqrt{2}\,mg$

(c) $[\sqrt{(M+m)^2+m^2}]\,g$

(d) $[\sqrt{(M+m)^2+M^2}]\,g$



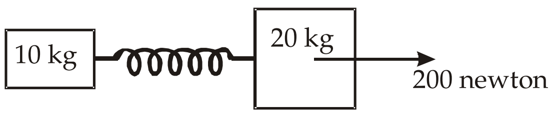
Solution : (c)

Force on the pulley by the clamp = resultant of T = (M + m)g and mg acting along horizontal and vertical respectively

$$\therefore\ F = \sqrt{[(M+m)g]^2 + (mg)^2} = [\sqrt{(M+m)^2+m^2}]g$$

Example 5.

The masses of 10 kg and 20 kg respectively are connected by a massless spring in fig. A force of 200 newton acts on the 20 kg mass. At the instant shown, the 10 kg mass has acceleration 12 m/sec^2. What is the acceleration of 20 kg mass?



Solution :

Force on 10 kg mass = 10 × 12 = 120 N

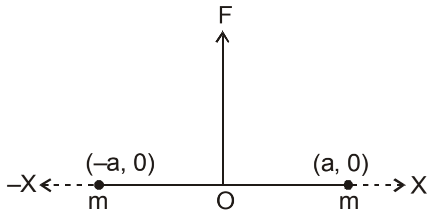
The mass of 10 kg will pull the mass of 20 kg in the backward direction with a force of 120 N.

∴ Net force on mass 20 kg = 200 – 120 = 80 N

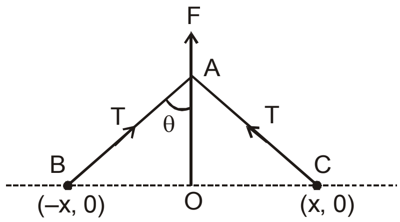
$$\text{Its acceleration}\ a = \frac{\text{force}}{\text{mass}} = \frac{80\,N}{20\,kg} = 4\,m/s^2$$

Example 6.

Two masses each equal to m are lying on X-axis at (–a, 0) and (+ a, 0) respectively as shown in fig. They are connected by a light string. A force F is applied at the origin and along the Y-axis. As a result, the masses move towards each other. What is the acceleration of each mass? Assume the instantaneous position of the masses as (– x, 0) and (x, 0) respectively



Solution :



From figure F = 2 T cos θ or T = F/(2 cos θ)

The force responsible for motion of masses on X-axis is T sin θ

$$\therefore\ m\,a = T\sin\theta = \frac{F}{2\cos\theta}\times\sin\theta$$

$$= \frac{F}{2}\tan\theta = \frac{F}{2}\times\frac{OB}{OA} = \frac{F}{2}\times\frac{x}{\sqrt{(a^2-x^2)}}$$

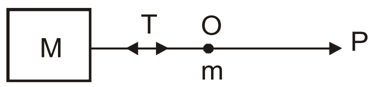
$$\text{so,}\quad a = \frac{F}{2m}\times\frac{x}{\sqrt{(a^2-x^2)}}$$

Example 7.

A block of mass M is pulled along horizontal frictionless surface by a rope of mass m. Force P is applied at one end of rope. Find the force which the rope exerts on the block.

Solution :

The situation is shown in fig



Let a be the common acceleration of the system. Here

T = M a for block

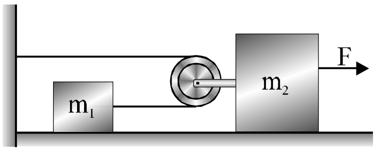
P – T = m a for rope

$$\therefore\ P - Ma = ma\ \text{or}\ P = a(M+m)\ \text{or}\ a = \frac{P}{(M+m)}$$

$$\therefore\ T = \frac{MP}{(M+m)}$$

Example 8.

In the system shown below, friction and mass of the pulley are negligible. Find the acceleration of m_2 if $m_1 = 300$ g, $m_2 = 500$ g and $F = 1.50$ N



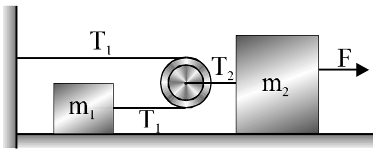
Solution :

When the pulley moves a distance d, m_1 will move a distance 2d. Hence m_1 will have twice as large an acceleration as m_2 has.

For mass m_1, $T_1 = m_1 (2a)$ ...(1)

For mass m_2, $F - T_2 = m_2(a)$ ...(2)

Putting $T_1 = \frac{T_2}{2}$ in eq^n. (1) gives $T_2 = 4m_1a$



Substituting value of T_2 in equation (2),

$F = 4m_1a + m_2a = (4m_1 + m_2)a$

Hence $a = \frac{F}{4m_1 + m_2} = \frac{1.50}{4(0.3) + 0.5} = 0.88 \text{ m/s}^2$

LAW OF CONSERVATION OF LINEAR MOMENTUM

A system is said to be isolated, when no external force acts on it. For such isolated system, the linear momentum $(\vec{P} = m\vec{v})$ is constant i.e., conserved.

The linear momentum is defined as

$$\vec{P} = m\vec{v} \quad .....(1)$$

where $\vec{v}$ is the velocity of the body, whose mass is m. The direction of $\vec{P}$ is same as the direction of the velocity of the body. It is a vector quantity. From Newton's second law,

$$\vec{F}_{ext.} = \frac{d}{dt}(m\vec{v}) = \frac{d}{dt}\vec{P} \quad .....(2)$$

i.e., time rate of change in momentum of the body is equal to total external force applied on the body.

$$\text{If } \vec{F}_{ext.} = 0 \Rightarrow \frac{d}{dt}(\vec{P}) = 0 \quad \text{or } \vec{P} = \text{constant} \quad .....(3)$$

This is called **law of conservation of momentum**.

Now let us consider a rigid body consisting of a large number of particles moving with different velocities, then total linear momentum of the rigid body is equal to the summation of individual linear momentum of all particles

i.e., $\sum_{i=1}^{n} \vec{p}_i = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...\vec{p}_n$

or $\vec{P}_{total} = \sum_{i=1}^{n} \vec{p}_i = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...+ \vec{p}_n$

where $\vec{p}_1, \vec{p}_2..............\vec{p}_n$ are individual linear momentum of first, second and nth particle respectively.

If this rigid body is isolated i.e., no external force is applied on it, then $\vec{P}_{total}$ = constant (from Newton's second law).

Further we know that internal forces (such as intermolecular forces etc.) also act inside the body, but these can only change individual linear momentum of the particles (i.e., p_1, p_2.........), but their total momentum $\vec{P}_{total}$ remains constant.

Gun Firing a Bullet

If a gun of mass M fires a bullet of mass m with velocity v. Then from law of conservation of momentum, as initially bullet & gun are at rest position i.e., initial momentum is zero, so final momentum (gun + bullet) must also be zero.

Since on firing, the bullet moves with velocity $\vec{v}_b$ in forward direction, then from Newton's third law, the gun moves in backward direction $\vec{v}_g$. So,

Initial momentum = final momentum

$$0 = \underset{\text{Momentum of bullet}}{m\vec{v}_b} + \underset{\text{Momentum of gun}}{M\vec{V}_g} \quad \therefore \overrightarrow{V_g} = \frac{-m\overrightarrow{v_b}}{M}$$

(–ve sign shows that the vel. of gun will have the opposite direction to that of bullet)

IMPULSE

According to Newton's second law the rate of change of momentum of a particle is equal to the total external force applied on it (particle) i.e.,

$$\frac{d\vec{P}}{dt} = \vec{F}_{ext} \quad ...(i)$$

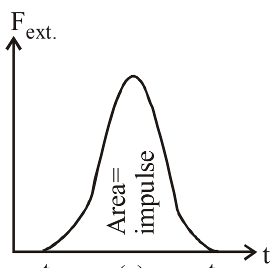
$$\text{or } d\vec{P} = \vec{F}_{ext}.dt \text{ or } \Delta\vec{P} = \vec{P}_f - \vec{P}_i = \int_{t_i}^{t_f} \vec{F}_{ext}.dt \quad ...(ii)$$

Where $\vec{P}_i$ is momentum of the particle at initial time t_i and when we apply some external force $\vec{F}_{ext}$ its final momentum is $\vec{P}_f$ at time t_f. The quantity $\vec{F}_{ext} \cdot dt$ on R.H.S in equation (ii) is called the **impulse.**

We can write equation (ii) as

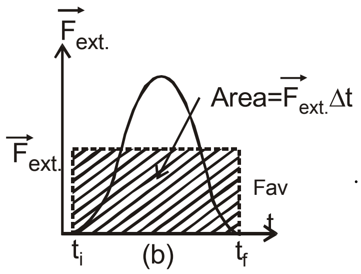
$$I = \int_{t_i}^{t_f} \vec{F}_{ext}.dt = \Delta\vec{P} \quad ...(iii)$$

So, *the impulse of the force* $\vec{F}_{ext}$ *is equal to the change in momentum of the particle. It is known as* **impulse momentum theorem.**



(a)

Force vary with time and impulse is area under force versus time curve



Force constant with time i.e., $\vec{F}_{ext.}$ constant with time (shown by horizontal line) and it would give same impulse to particle in time $\Delta t = t_f - t_i$ as time varying force described.

It is a vector quantity having a magnitude equal to the area under the force-time curve as shown in fig. (a). In this figure, it is assumed that force varies with time and is non-zero in time interval $\Delta t = t_f - t_i$. Fig.(b) shows the time averaged force $\vec{F}_{ext.}$ i.e., it is constant in time interval Δt, then equation (iii) can be written as

$$I = \vec{F}_{ext.}\int_{t_i}^{t_f} dt = \vec{F}_{ext.}(t_f - t_i) \qquad I = \vec{F}_{ext.}\Delta t \qquad ...(iv)$$

The direction of impulsive vector I is same as the direction of change in momentum. Impulse I has same dimensions as that of momentum i.e, $[MLT^{-1}]$

Rocket propulsion (A case of system of variable mass) : It is **based on principle of conservation of linear momentum**.

In rocket, the fuel burns and produces gases at high temperature. These gases are ejected out of the rocket from nozzle at the backside of rocket and the ejecting gas exerts a forward force on the rocket which accelerates it.

Let the gas ejects at a rate $r = -\frac{dM}{dt}$ and at constant velocity u w.r.t. rocket then from the conservation of linear momentum

$\frac{dv}{dt} = \frac{ru}{M} = \frac{ru}{M_0 - rt}$ where $M = M_0$ - rt and M_0 is mass of rocket with fuel and solving this equation, we get $v = u\log_e\left(\frac{M_0}{M_0 - rt}\right)$

where v = velocity of rocket w.r.t. ground.

Example 9.

Two skaters A and B approach each other at right angles. Skater A has a mass 30 kg and velocity 1 m/s and skater B has a mass 20 kg and velocity 2 m/s. They meet and cling together. Find the final velocity of the couple.

Solution :

Applying principle of conservation of linear momentum,

$$p = \sqrt{p_1^2 + p_2^2}; (m_1 + m_2)v = \sqrt{(m_1v_1)^2 + (m_2v_2)^2}$$

$$(30+20)v = \sqrt{(30\times1)^2 + (20\times2)^2} = 50$$

$$v = \frac{50}{50} = 1\ m/s$$

Example 10.

A hammer of mass M strikes a nail of mass m with velocity of u m/s and drives it 's' meters in to fixed block of wood. Find the average resistance of wood to the penetration of nail.

Solution :

Applying the law of conservation of momentum,

$$m\,u = (M+m)\,v_0 \Rightarrow v_0 = \left(\frac{M}{m+M}\right)u$$

There acceleration a can be obtained using the formula $(v^2 = u^2 + 2as)$.

Here we have $\quad 0 - v_0^2 = 2as$ or $a = v_0^2/2s$

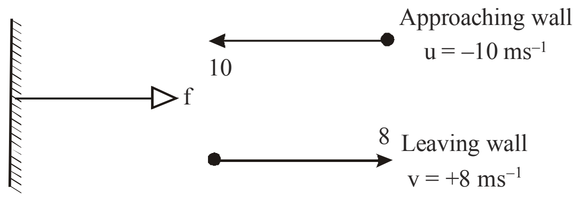
$$\therefore \quad a = \left(\frac{M}{m+M}\right)^2\frac{u^2}{2s}$$

$$\text{Resistance} = (M+m)\,a = \left(\frac{M^2}{m+M}\right)\frac{u^2}{2s}$$

Example 11.

A ball of mass 0.5 kg is thrown towards a wall so that it strikes the wall normally with a speed of 10 ms^{-1}. If the ball bounces at right angles away from the wall with a speed of $8ms^{-1}$, what impulse does the wall exert on the ball ?

Solution :



Taking the direction of the impulse J as positive and using

$$J = mv - mu$$

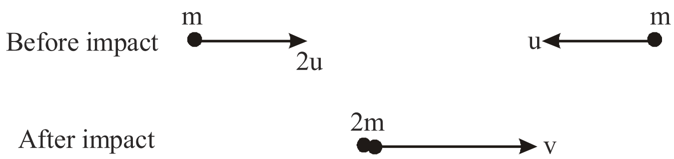
we have $\quad J = \frac{1}{2}\times 8 - \frac{1}{2}(-10) = 9$ N-s

Therefore the wall exerts an impulse of 9 N-s on the ball.

Example 12.

Two particles, each of mass m, collide head on when their speeds are 2u and u. If they stick together on impact, find their combined speed in terms of u.

Solution :



Using conservation of linear momentum (in the direction of the velocity 2u) we have

$$(m)(2u) - mu = 2m \times V \Rightarrow V = \frac{1}{2}u$$

The combined mass will travel at speed u/2.

(Note that the momentum of the second particle before impact is negative because its sense is opposite to that specified as positive.)

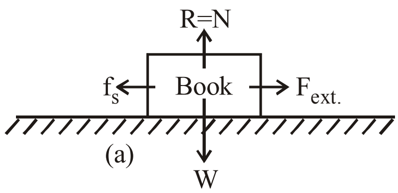
FRICTION

When a body is in motion on a rough surface, or when an object moves through water (i.e., viscous medium), then velocity of the body decreases constantly even if no external force is applied on the body. This is due to **friction.**

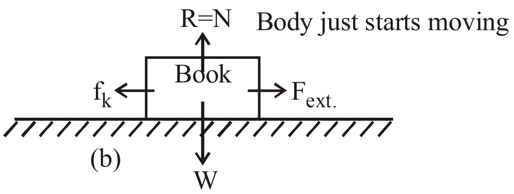
So "*an opposing force which comes into existence, when two surfaces are in contact with each other and try to move relative to one another, is called friction*".

Frictional force acts along the common surface between the two bodies in such a direction so as to oppose the relative movement of the two bodies.

(a) The force of static friction f_s *between book and rough surface is opposite to the applied external force* $F_{ext.}$ *The force of static friction* $f_s = \vec{F}_{ext}$.



(b) *When* $\vec{F}_{ext}$. *exceeds the certain maximum value of static friction, the book starts accelerating and during motion Kinetic frictional force is present.*



(c) *A graph* $\vec{F}_{ext}$. *versus* | f | *shown in figure. It is clear that* $f_{s,\ max} > f_k$

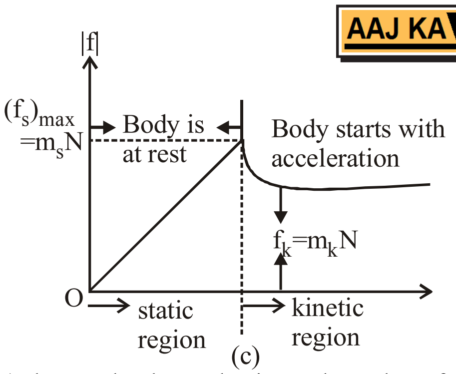


Fig.(a) shows a book on a horizontal rough surface. Now if we apply external force $\vec{F}_{ext.}$, on the book, then the book will remain stationary if $\vec{F}_{ext.}$ is not too large. If we increase $\vec{F}_{ext.}$ then frictional force f also increase up to $(f_s)_{max}$ (called maximum force of static friction or limiting friction) and $(f_s)_{max} = \mu_s N$. At any instant when $\vec{F}_{ext.}$ is slightly greater than $(f_s)_{max}$ then the book moves and accelerates to the right.

Fig.(b) when the book is in motion, the retarding frictional force become less than, $(f_s)_{max}$

Fig.(c) $(f_s)_{max}$ is equal to $\mu_k N$. When the book is in motion, we call the retarding frictional force as the force of kinetic friction f_k.

Since $f_k < (f_s)_{max}$, so it is clear that, we require more force to start motion than to maintain it against friction.

By experiment one can find that $(f_s)_{max}$ and f_k are proportional to normal force N acting on the book (by rough surface) and depends on the roughness of the two surfaces in contact.

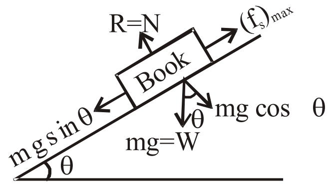
Note :

(i) The force of static friction between any two surfaces in contact is opposite to $\vec{F}_{ext.}$ and given by $f_s \le \mu_s N$ and $(f_s)_{max} = \mu_s N$ (when the body just moves in the right direction).
where N = W = weight of book and μ_s is called coefficient of static friction, f_s is called force of static friction and $(f_s)_{max}$ is called *limiting friction or maximum value of static friction.*

(ii) The force of kinetic friction is opposite to the direction of motion and is given by $f_k = \mu_k N$
where μ_k is coefficient of kinetic friction.

(iii) The value of μ_k and μ_s depends on the nature of surfaces and μ_k is always less then μ_s.

Friction on an inclined plane : Now we consider a book on an inclined plane & it just moves or slips, then by definition



$$(f_s)_{max} = \mu_s R$$

Now from figure, $f_{s,max} = mg \sin\theta$ and $R = mg \cos\theta$

$\Rightarrow \mu_s = \tan\theta$ or $\theta = \tan^{-1}(\mu_s)$

where angle θ is called the **angle of friction** or **angle of repose**

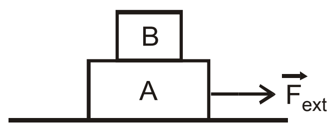
Some facts about friction :

(1) The force of kinetic friction is less than the force of static friction and the force of rolling friction is less than force of kinetic friction i.e.,

$$f_r < f_k < f_s \quad \text{or} \quad \mu_{rolling} < \mu_{kinetic} < \mu_{static}$$

hence it is easy to roll the drum in comparison to sliding it.

(2) Frictional force does not oppose the motion in all cases, infact in some cases the body moves due to it.



In the figure, book B moves to the right due to friction between A and B. If book A is totally smooth (i.e., frictionless) then book B does not move to the right. This is because of no force applies on the book B in the right direction.

Laws of limiting friction :

(i) The force of friction is independent of area of surfaces in contact and relative velocity between them (if it is not too high).

(ii) The force of friction depends on the nature of material of surfaces in contact (i.e., force of adhesion). μ depends upon nature of the surface. It is independent of the normal reaction.

(iii) The force of friction is directly proportional to normal reaction i.e., $F \propto N$ or $F = mn$.

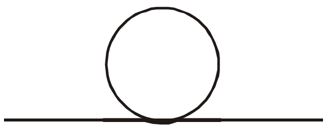
While solving a problem having friction involved, follow the given methodology

Check
(a) F_{app}
(b) Limiting friction (f_l)

- If $F_{app} < f_l$
  Body does not move and
  F_{app} = frictional force
- If $F_{app} = f_l$
  Body is on the verge of movement if the body is initially at rest
  Body moves with constant velocity
- If $F_{app} > f_l$
  Body moves with an acceleration
  $F_{app} - f_l = m.a$ in the line of action of frictional force

Rolling Friction :

The name rolling friction is a misnomer. Rolling friction has nothing to do with rolling. Rolling friction occurs during rolling as well as sliding operation.



Cause of rolling friction : When a body is kept on a surface of another body it causes a depression (an exaggerated view shown in the figure). When the body moves, it has to overcome the depression. This is the cause of rolling friction.

Note : Rolling friction will be zero only when both the bodies incontact are rigid. Rolling friction is very small as compared to sliding friction. Work done by rolling friction is zero

CONSERVATIVE AND NON-CONSERVATIVE FORCES

If work done on a particle is zero in complete round trip, the force is said to be conservative. The gravitational force, electrostatics force, elastic force etc., are conservative forces. On the other hand *if the work done on a body is not zero during a complete round trip, the force is said to be non-conservative*. The frictional force, viscous force etc. are non-conservative forces.

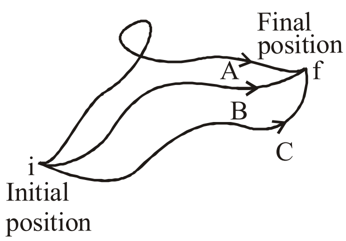


Figure shows three processes A, B and C by which we can reach from an initial position to final position. If force is conservative, then work done is same in all the three processes i.e., independent of the path followed between initial and final position.

If force is non conservative then work done from i to f is different in all three paths A,B and C.

Hence it is clear that *work done in conservative force depends only on initial & final position irrespective of the path followed between initial & final position. In case of non-conservative forces the work done depends on the path followed between initial and final position.*

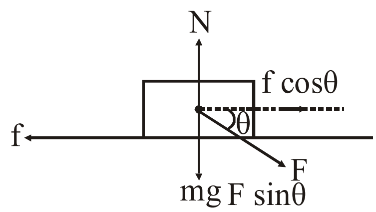
We can say also that there is no change in kinetic energy of the body in complete round trip in case of conservative force. While in case of non conservative forces, when a body return to its initial position after completing the round trip, the kinetic energy of the body may be more or less than the kinetic energy with which it starts.

Example 13.

Pushing force making an angle θ to the horizontal is applied on a block of weight W placed on a horizontal table. If the angle of friction is ϕ, then determine the magnitude of force required to move the body.

Solution :

The various forces acting on the block are shown in fig.



Here,

$\mu = \tan\phi = \dfrac{f}{N}$; or $f = N\tan\phi$ ...(i)

The condition for the block just to move is

$F\cos\theta = f = N\tan\phi$ ...(ii)

and $F\sin\theta + W = N$ ...(iii)

From (ii) and (iii),

$F\cos\theta = (W + F\sin\theta)\tan\phi = W\tan\phi + F\sin\theta\tan\phi$;

or $F\cos\theta - F\sin\theta\sin\phi/\cos\phi = W\sin\phi/\cos\phi$

or $F(\cos\theta\cos\phi - \sin\theta\sin\phi) = W\sin\phi$;

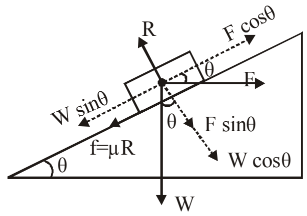
or $F\cos(\theta + \phi) = W\sin\phi$ or $F = W\sin\phi / \cos(\theta + \phi)$

Example 14.

An object of weight W is resting on an inclined plane at an angle θ to the horizontal. The coefficient of static friction is μ. Find the horizontal force needed to just push the object up the plane.

Solution :

The situation is shown in fig.



Let F be the horizontal force needed to just push the object up the plane. From figure $R = W\cos\theta + F\sin\theta$

Now $f = \mu R = \mu\,[W\cos\theta + F\sin\theta]$ ...(1)

Further, $F\cos\theta = W\sin\theta + f$ ...(2)

$F\cos\theta = W\sin\theta + \mu\,[W\cos\theta + F\sin\theta]$

$F\cos\theta - \mu F\sin\theta = W\sin\theta + \mu W\cos\theta$

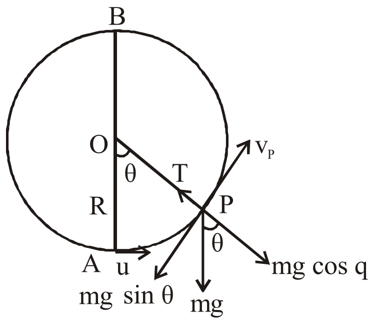
$$\therefore \quad F = \frac{W(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$$

CASES OF CIRCULAR MOTIONS

Motion in a Vertical Circle :

Let us consider a particle of mass m attached to a string of length R let the particle be rotated about its centre O.

At t = 0 the particle start with velocity u from the point A (lowest point of vertical circle) and at time t its position is P. Then the tension at point P is given by



$$T_P - mg\cos\theta = \frac{mv_P^2}{R} \quad \text{or} \quad T_P = mg\cos\theta + \frac{mv_P^2}{R} \qquad ...(1)$$

So tension at point A (lowest point of vertical circle) is

$$T_A - mg = \frac{mv_A^2}{R} \qquad (\because \theta = 0^\circ) \qquad ...(2)$$

and tension at point B (highest point of vertical circle) is

$$T_B + mg = \frac{mv_B^2}{R} \qquad (\because \theta = 180^\circ) \qquad ...(3)$$

Where $\frac{\mathbf{mv^2}}{\mathbf{r}}$ **is centripetal force** required for the particle to move in a vertical circle.

Now from law of conservation of energy

$$\frac{1}{2}mv_A^2 - \frac{1}{2}mv_B^2 = 2mgR$$

$$\text{or, } v_A^2 - v_B^2 = 4gR \qquad ...(4)$$

(change in kinetic energy of particle)

= (change in potential energy of particle)

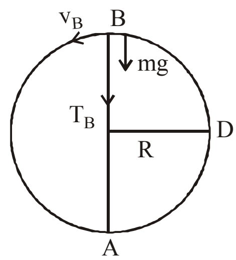
or

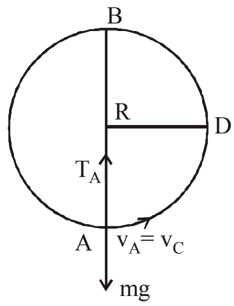
(loss in kinetic energy of the particle) = (gain in potential energy)

In conservative force system (such as gravity force) the mechanical energy (i.e., kinetic energy + potential energy) must be constant.

Total energy will be constant

Now from eq^{ns}.(2) and (3), we get





$$T_A - T_B = 2mg + \frac{m}{R}(V_A^2 - V_B^2) = 2mg + \frac{m}{R}(4gR)$$

$$\Rightarrow \quad T_A - T_B = 6mg \qquad ...(5)$$

$$\text{or} \quad T_A = T_B + 6mg \qquad ...(6)$$

So it is clear from eq^n. (6) that tension in string at lowest point of vertical circle is greater then the tension at highest point of vertical circle by 6mg.

Condition to complete a vertical circle :

If we reduce the velocity v_A in equation (2), then T_A will be reduce and at some critical velocity v_c, T_B will be zero, then put $T_B = 0$ and $v_B = v_C$ in equation (3) and we obtain

$$v_C = v_B = \sqrt{gR} \qquad ...(7)$$

In this condition the necessary centripetal force at point B is provided by the weight of the particle [see again equation (3)] then from equation (4), we get

$$v_A^2 - gR = 4gR \Rightarrow v_A = \sqrt{5gR} \qquad ...(8)$$

then the tension at the point A will be

$$T_A = mg + \frac{m(5gR)}{R} = 6mg \qquad ...(9)$$

Hence if we rotate a particle in a vertical circle and tension in string at highest point is zero, then the tension at lowest point of vertical circle is 6 times of the weight of the particle.

Some Facts of Vertical Motion :

(i) The body will complete the vertical circle if its velocity at lowest point is equal to or greater then $\sqrt{5gR}$

(ii) The body will oscillate about the lowest point if its velocity at lowest point is less then $\sqrt{2gR}$. This will happen when the velocity at the halfway mark, i.e.

$$v_D = 0 \quad \left[\because \frac{1}{2}mv_A^2 = mgR\right]$$

(iii) The string become slack and fails to describe the circle when its velocity at lowest point lies between $\sqrt{2gR}$ to $\sqrt{5gR}$

Example 15.

A mass m is revolving in a vertical circle at the end of a string of length 20 cm. By how much does the tension of the string at the lowest point exceed the tension at the topmost point?

Solution :

The tension T_1 at the topmost point is given by,

$$T_1 = \frac{m v_1^2}{20} - m g$$

Centrifugal force acting outward while weight acting downward

The tension T_2 at the lowest point, $T_2 = \frac{m v_2^2}{20} + m g$

Centrifugal force and weight (both) acting downward

$$T_2 - T_1 = \frac{m v_2^2 - m v_1^2}{20} + 2 m g \, ; \quad v_1^2 = v_2^2 - 2 g h \text{ or}$$

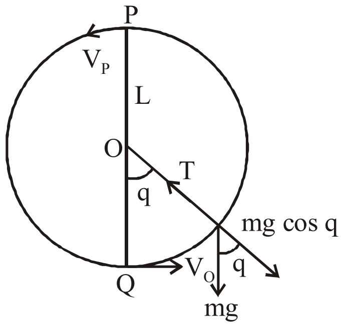
$$v_2^2 - v_1^2 = 2 g (40) = 80 g$$

$$\therefore \quad T_2 - T_1 = \frac{80 m g}{20} + 2 m g = 6 \text{ m g}$$

Example 16.

A stone of mass 1 kg tied to a light inextensible string of length L = (10/3) m is whirling in a circular path of radius L in a vertical plane. If the ratio of the maximum to the minimum tension in the string is 4 and g = 10 m/s², then find the speed of the stone at the highest point of the circle.

Solution :



The tension T in the string is given by

$$T_{max} = m\left[g + \frac{v_Q^2}{L}\right] \text{ and } T_{min} = m\left[-g + \frac{v_P^2}{L}\right]$$

According to the given problem

$$\frac{g + (v_Q^2/L)}{-g + (v_P^2/L)} = 4 \quad \text{or} \quad g + \frac{v_Q^2}{L} = -4g + 4\frac{v_P^2}{L}$$

$$\text{or} \quad g + \frac{v_P^2 + 4 g L}{L} = -4 g + 4\frac{v_P^2}{L}$$

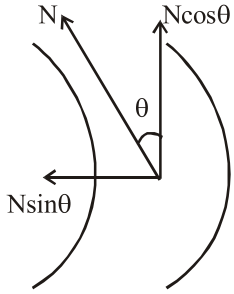
L = (10/3) m and g = 10 m/s² (given)

Solving we get $v_P = 10$ m/s.

Negotiating a Curve :

Case of cyclist

To safely negotiate a curve of radius r, a cyclist should bend at an angle θ with the vertical.



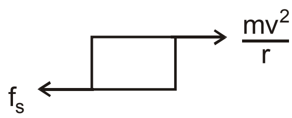
Which is given by $\tan\theta = \frac{v^2}{rg}$. Angle θ is also called as **angle of banking**.

$$N \sin\theta = \frac{mv^2}{r} \text{ and } N\cos\theta = mg$$

Case of car on a levelled road

A vehicle can safely negotiate a curve of radius r on a rough level road when coefficient of sliding friction is related to the velocity as $\mu_s \geq \frac{v^2}{rg}$.

Now consider a case when a vehicle is moving in a circle, the centrifugal force is $\frac{mv^2}{r}$ whereas m is mass of vehicle, r = radius of circle and v is its velocity.



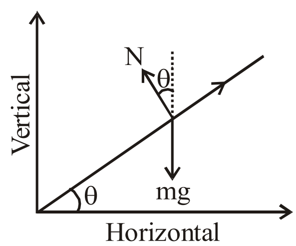
The frictional force is static since wheels are in rolling motion because point of contact with the surface is at rest

$$\therefore f_s = \frac{mv^2}{r} \qquad f_s \leq f_{max} = \mu_s mg$$

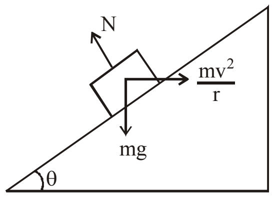
$$\frac{mv^2}{r} \leq \mu_s mg \text{ or } \mu_s \geq \frac{v^2}{rg}$$

Case of banking of road (frictionless)

A vehicle can safely negotiate a curve of radius r on a smooth (frictionless) road, when the angle θ of banking of the road is given by $\tan\theta = \frac{v^2}{rg}$.



When the banked surface is smooth, the force acting will be gravity and normal force only.



Balancing forces

$N\cos\theta = mg$ ...(1)

$N\sin\theta = \frac{mv^2}{r}$ ...(2)

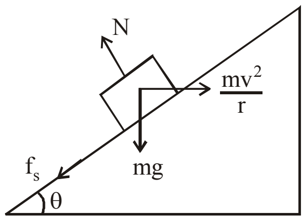
$\frac{v^2}{rg} = \tan\theta$ ...(3)

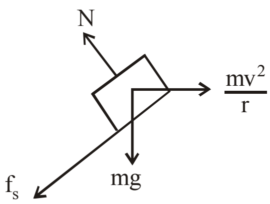
Case of banking of road (with friction)

The maximum velocity with which a vehicle can safely negotiate a curve of radius r on a rough inclined road is given by

$v^2 = \frac{rg(\mu+\tan\theta)}{1-\mu\tan\theta}$; where μ is the coefficient of friction of the rough surface on which the vehicle is moving, and θ is the angle of inclined road with the horizontal.

Suppose a vehicle is moving in a circle of radius r on a rough inclined road whose coefficient of friction is $\frac{\mu}{M}$ and angle of banking is θ.





Let velocity of object (vehicle) be V.

If we apply pseudo force on body, centrifugal force is $\frac{mv^2}{r}$ when v is max. and friction force will be acting down the slope.

Balancing the force horizontally, $\frac{mv^2}{r} = f_s\cos\theta + N\sin\theta$ ...(1)

Balancing the force vertically,

$N\cos\theta = f_s\sin\theta + mg$ ...(2)

when $v = \text{maximum}$, $f = f_{max} = f_s = \mu N$ ...(3)

From eq^n. (2),

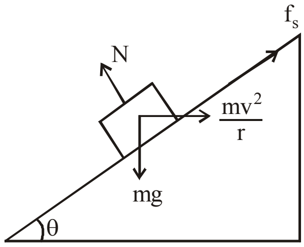
$N\cos\theta = \mu N\sin\theta + mg \Rightarrow N(\cos\theta - \mu\sin\theta) = mg$

or $N = \frac{mg}{\cos\theta - \mu\sin\theta}$

From eq^{ns}.(1) and (3), $\frac{mv^2}{r} = \frac{\mu mg\cos\theta + mg\sin\theta}{\cos\theta - \mu\sin\theta}$

$\Rightarrow \frac{mv^2}{r} = \frac{mg(\mu+\tan\theta)}{1-\mu\tan\theta} \Rightarrow v_{max}^2 = rg\frac{(\mu+\tan\theta)}{1-\mu\tan\theta}$

Now in the case of minimum velocity with which body could move in a circular motion, the direction of friction will be opposite to that one in maximum velocity case.



and $v_{min}^2 = rg\left(\frac{\mu-\tan\theta}{1+\mu\tan\theta}\right)$

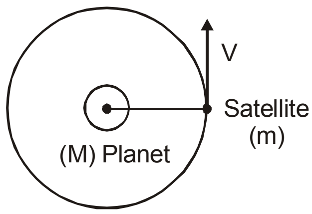
Keep in Memory

1. Whenever a particle is moving on the circular path then there must be some external force which will provide the necessary centripetal acceleration to the particle.

For examples :

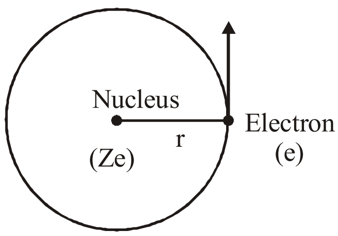
(i) Motion of satellite around a planet : Here the centripetal force is provided by the gravitational force.

i.e. $\frac{GMm}{r^2} = \frac{mv^2}{r}$



(ii) Motion of electron around the nucleus : Here the required centripetal force is provided by the Coulombian force

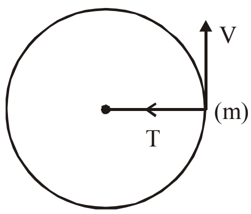
i.e. $\frac{1}{4\pi\varepsilon_0}\frac{(ze)(e)}{r^2} = \frac{mv^2}{r}$



(iii) Motion of a body in horizontal and vertical circle: Here the centripetal force is provided by the tension.

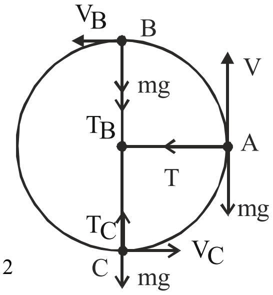
Horizontal circle

$T = \frac{mv^2}{r}$



Vertical circle

At point A, $T_A = \dfrac{mv_A^2}{r}$;

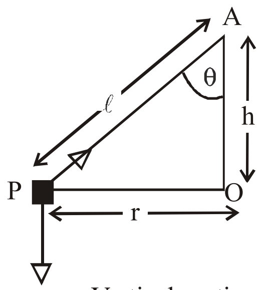


At point B, $T_B + mg = \dfrac{mv_B^2}{r}$

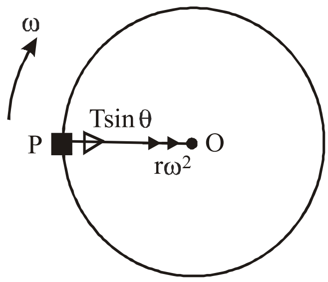
And at point C, $T_C - mg = \dfrac{mv_C^2}{r}$

CONICAL PENDULUM

Consider an inextensible string of length ℓ which is fixed at one end, A. At the other end is attached a particle P of mass m describing a circle with constant angular velocity ω in a horizontal plane.



Vertical section



Horizontal Plane

As P rotates, the string AP traces out the surface of a cone. Consequently the system is known as a conical pendulum.

Vertically, $T\cos\theta = mg$ ... (1)

Horizontally, $T\sin\theta = mr\omega^2$ ... (2)

In triangle AOP, $r = \ell\sin\theta$ ... (3)

and $h = \ell\cos\theta$ ... (4)

Several interesting facts can be deduced from these equations :

(a) It is impossible for the string to be horizontal.

This is seen from eq^n. (1) in which $\cos\theta = \dfrac{mg}{T}$ cannot be zero. Hence θ cannot be 90°.

(b) The tension is always greater than mg.

This also follows from eq^n. (1) as $\cos\theta < 1$ (θ is acute but not zero). Hence, $T > mg$

(c) The tension can be calculated without knowing the inclination of the string since, from eq^n. (2) and (3)

$T\sin\theta = m\ell\sin\theta\,\omega^2 \Rightarrow T = m\ell\omega^2$

(d) The vertical depth h of P below A is independent of the length of the string since from eq^n. (1) and (4)

$$T\frac{h}{\ell} = mg \Rightarrow T = \frac{\ell mg}{h} \quad \text{but} \quad T = m\ell\omega^2$$

Therefore $m\ell\omega^2 = \dfrac{m\ell g}{h} \Rightarrow h = \dfrac{g}{\omega^2}$

which is independent of ℓ.

Example 17.

A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. Determine the power delivered to the particle by the forces acting on it.

Solution :

Here tangential acceleration also exists which requires power. Given that centripetal acceleration

$a_c = k^2rt^2$ also, $a_c = v^2/r$;

$\therefore\ v^2/r = k^2rt^2$ or $v^2 = k^2r^2t^2$ or $v = k\,r\,t$;

Tangential acceleration, $a = \dfrac{dv}{dt} = k\,r$

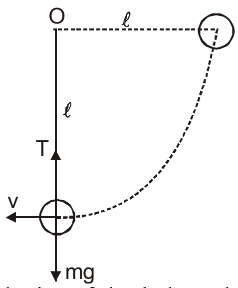
Now, force $F = ma = m\,k\,r$;

So, power, $P = F\,v = m\,k\,r \times k\,r\,t = m\,k^2\,r^2\,t.$

Example 18.

The string of a pendulum is horizontal. The mass of the bob is m. Now the string is released. What is the tension in the string in the lowest position?

Solution :



Let v be the velocity of the bob at the lowest position. In this position, The P.E. of bob is converted into K.E. Hence,

$$mg\ell = \frac{1}{2}mv^2 \quad \text{or} \quad v^2 = 2g\ell \qquad ...(1)$$

If T be the tension in the string, then

$$T - mg = \left(\frac{mv^2}{\ell}\right) \qquad ...(2)$$

From eq^{ns}. (1) and (2).

$T - mg = 2mg$ or $T = 3mg$

CONCEPT MAP

LAWS OF MOTION

Newton's laws of motion

Newton's first law of motion
Every body continues in its state of rest or of uniform motion along a straight line unless an external unbalanced force is applied to change that state. It is also called law of inertia

Newton's second law of motion
The rate of change of momentum of a body is directly proportional to applied force

$\frac{dp}{dt} \propto F \Rightarrow F = ma$

a = acceleration

Newton's third law of motion
To every action there is always an equal and opposite reaction.

$\overrightarrow{F_{AB}} = -\overrightarrow{F_{BA}}$

Action and reaction act on two different bodies

Inertia Resistance to change

Types of inertia
(i) Inertia of rest (ii) Inertia of motion

Momentum (P)
Product of mass (m) and velocity (v) of the body

$\vec{P} = m\vec{v}$

Impulse
The total change in momentum

Impulse $\vec{J}$ = force$(\vec{F})$. time duration (Δt)

= change in momentum

$= m\vec{v} - m\vec{u}$

Law of conservation of momentum
The vector sum of linear momenta of a system of particles remains constant

$\vec{P} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + ... + \vec{P}_n$

Force
An agency (push or pull) which changes or tends to change state of rest or of uniform motion of a body.

Equilibrium of a particle
The situation when net external force on the particle is zero

$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... = 0$

Centripetal force
Force required to make a body move along a circular path with a uniform speed (v) $F_c = \frac{mv^2}{R}$

centrifugal force is equal and opposite to the centripetal force.

Motion of a car on a level road

$v_{max} = \sqrt{\mu_s Rg}$

Motion of a car on a banked road
when frictional forces taken into account
Maximum safe velocity

$$v_{max} = \sqrt{Rg\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$$

when no frictional force taken into account

$v_{max} = \sqrt{Rg \tan\theta}$

Friction
Force opposing the relative motion of a body

Angle of friction
$\mu_s = \tan\theta$

Angle of repose (ϕ)
$\mu_s = \tan\phi$

Limiting friction Maximum value of static friction

Laws of limiting friction

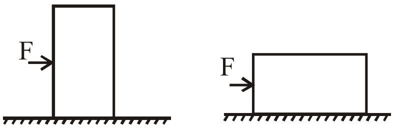
- Depends on nature of surfaces in contact
- Independent of area or shape in contact
- $F \propto R \Rightarrow F = \mu_s R$

Types of Friction

- Kinetic friction opposes actual relative motion $f_k = \mu_k R$
- Static friction opposes impending relative motion $f_s \le (f_s)_{max} = \mu_s R$
- Rolling friction $\mu_R < \mu_k < \mu_s$

EXERCISE - 1
Conceptual Questions

1. A rectangular block is placed on a rough horizontal surface in two different ways as shown, then



(a) (b)

(a) friction will be more in case (a)
(b) friction will be more in case (b)
(c) friction will be equal in both the cases
(d) friction depends on the relations among its dimensions.

2. Centripetal force :
(a) can change speed of the body.
(b) is always perpendicular to direction of motion
(c) is constant for uniform circular motion.
(d) all of these

3. When a horse pulls a cart, the horse moves down to
(a) horse on the cart.
(b) cart on the horse.
(c) horse on the earth.
(d) earth on the horse.

4. The force of action and reaction
(a) must be of same nature
(b) must be of different nature
(c) may be of different nature
(d) may not have equal magnitude

5. A body is moving with uniform velocity, then
(a) no force must be acting on the body.
(b) exactly two forces must be acting on the body
(c) body is not acted upon by a single force.
(d) the number of forces acting on the body must be even.

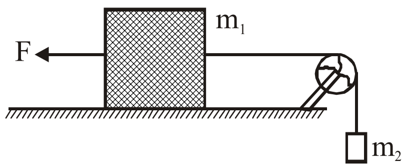
6. The direction of impulse is
(a) same as that of the net force
(b) opposite to that of the net force
(c) same as that of the final velocity
(d) same as that of the initial velocity

7. A monkey is climbing up a rope, then the tension in the rope
(a) must be equal to the force applied by the monkey on the rope
(b) must be less than the force applied by the monkey on the rope.
(c) must be greater than the force applied by the monkey on the rope.
(d) may be equal to, less than or greater the force applied by the monkey on the rope.

8. A uniform rope of length L resting on a frictionless horizontal surface is pulled at one end by a force F. What is the tension in the rope at a distance ℓ from the end where the force is applied.
(a) F
(b) $F(1+\ell/L)$
(c) F/2
(d) $F(1-\ell/L)$

9. A particle of mass m is moving with velocity v_1, it is given an impulse such that the velocity becomes v_2. Then magnitude of impulse is equal to
(a) $m(\vec{v}_2-\vec{v}_1)$
(b) $m(\vec{v}_1-\vec{v}_2)$
(c) $m\times(\vec{v}_2-\vec{v}_1)$
(d) $0.5m(\vec{v}_2-\vec{v}_1)$

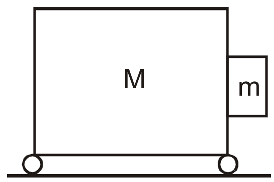
10. A constant force $F = m_2g/2$ is applied on the block of mass m_1 as shown in fig. The string and the pulley are light and the surface of the table is smooth. The acceleration of m_1 is



(a) $\dfrac{m_2 g}{2(m_1+m_2)}$ towards right
(b) $\dfrac{m_2 g}{2(m_1-m_2)}$ towards left
(c) $\dfrac{m_2 g}{2(m_2-m_1)}$ towards right
(d) $\dfrac{m_2 g}{2(m_2-m_1)}$ towards left

11. A mass is hanging on a spring balance which is kept in a lift. The lift ascends. The spring balance will show in its readings
(a) an increase
(b) a decrease
(c) no change
(d) a change depending on its velocity

12. A cart of mass M has a block of mass m attached to it as shown in fig. The coefficient of friction between the block and the cart is μ. What is the minimum acceleration of the cart so that the block m does not fall?
(a) μg
(b) g/μ
(c) μ/g
(d) $M\mu g/m$



13. A particle of mass m moving eastward with a speed v collides with another particle of the same mass moving northward with the same speed v. The two particles coalesce on collision. The new particle of mass 2m will move in the north-external direction with a velocity :

(a) v/2 (b) 2v

(c) $v/\sqrt{2}$ (d) None of these

14. A spring is compressed between two toy carts of mass m_1 and m_2. When the toy carts are released, the springs exert equal and opposite average forces for the same time on each toy cart. If v_1 and v_2 are the velocities of the toy carts and there is no friction between the toy carts and the ground, then :

(a) $v_1/v_2 = m_1/m_2$ (b) $v_1/v_2 = m_2/m_1$

(c) $v_1/v_2 = -m_2/m_1$ (d) $v_1/v_2 = -m_1/m_2$

15. Two mass m and 2m are attached with each other by a rope passing over a frictionless and massless pulley. If the pulley is accelerated upwards with an acceleration 'a', what is the value of T?

(a) $\frac{g+a}{3}$ (b) $\frac{g-a}{3}$

(c) $\frac{4m(g+a)}{3}$ (d) $\frac{m(g-a)}{3}$

16. A rider on a horse back falls forward when the horse suddenly stops. This is due to

(a) inertia of horse

(b) inertia of rider

(c) large weight of the horse

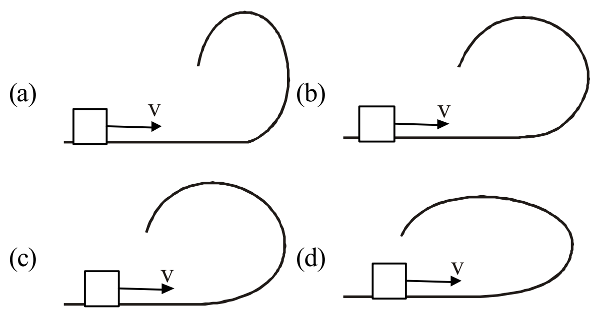
(d) losing of the balance

17. A ball of mass m is thrown vertically upwards. What is the rate at which the momentum of the ball changes?

(a) Zero (b) mg

(c) Infinity (d) Data is not sufficient.

18. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in



19. A weight W rests on a rough horizontal plane. If the angle of friction be θ, the least force that will move the body along the plane will be

(a) $W\cos\theta$ (b) $W\cot\theta$

(c) $W\tan\theta$ (d) $W\sin\theta$

20. A particle starts sliding down a frictionless inclined plane. If S_n is the distance traveled by it from time t = n – 1 sec to t = n sec, the ratio S_n/S_{n+1} is

(a) $\frac{2n-1}{2n+1}$ (b) $\frac{2n+1}{2n}$

(c) $\frac{2n}{2n+1}$ (d) $\frac{2n+1}{2n-1}$

21. A block is kept on a inclined plane of inclination θ of length ℓ. The velocity of particle at the bottom of inclined is (the coefficient of friction is μ)

(a) $[2g\ell(\mu\cos\theta - \sin\theta)]^{1/2}$ (b) $\sqrt{2g\ell(\sin\theta - \mu\cos\theta)}$

(c) $\sqrt{2g\ell(\sin\theta + \mu\cos\theta)}$ (d) $\sqrt{2g\ell(\cos\theta + \mu\sin\theta)}$

22. A bird is in a wire cage which is hanging from a spring balance . In the first case, the bird sits in the cage and in the second case, the bird flies about inside the cage. The reading in the spring balance is

(a) more in the first case

(b) less in first case

(c) unchanged

(d) zero in second case.

23. In an explosion, a body breaks up into two pieces of unequal masses. In this

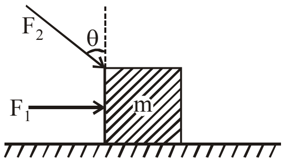
(a) both parts will have numerically equal momentum

(b) lighter part will have more momentum

(c) heavier part will have more momentum

(d) both parts will have equal kinetic energy

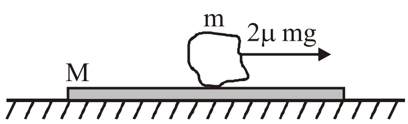
24. A block of mass m on a rough horizontal surface is acted upon by two forces as shown in figure. For equilibrium of block the coefficient of friction between block and surface is



(a) $\frac{F_1 + F_2\sin\theta}{mg + F_2\cos\theta}$ (b) $\frac{F_1\cos\theta + F_2}{mg - F_2\sin\theta}$

(c) $\frac{F_1 + F_2\cos\theta}{mg + F_2\sin\theta}$ (d) $\frac{F_1\sin\theta - F_2}{mg - F_2\cos\theta}$

25. A plate of mass M is placed on a horizontal of frictionless surface (see figure), and a body of mass m is placed on this plate. The coefficient of dynamic friction between this body and the plate is μ . If a force 2μ mg is applied to the body of mass m along the horizontal, the acceleration of the plate will be



(a) $\frac{\mu m}{M}g$ (b) $\frac{\mu m}{(M+m)}g$

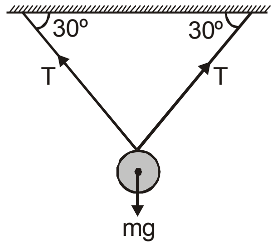
(c) $\frac{2\mu m}{M}g$ (d) $\frac{2\mu m}{(M+m)}g$.

EXERCISE - 2
Applied Questions

1. An object of mass 10 kg moves at a constant speed of 10 ms^{-1}. A constant force, that acts for 4 sec on the object, gives it a speed of 2 ms^{-1} in opposite direction. The force acting on the object is

(a) –3 N (b) –30 N
(c) 3 N (d) 30 N

2. A solid sphere of 2 kg is suspended from a horizontal beam by two supporting wires as shown in fig. Tension in each wire is approximately (g = 10 ms^{-2})



(a) 30 N
(b) 20 N
(c) 10 N
(d) 5 N

3. A toy gun consists of a spring and a rubber dart of mass 16 g. When compressed by 4 cm and released, it projects the dart to a height of 2 m. If compressed by 6 cm, the height achieved is

(a) 3m (b) 4m
(c) 4.5m (d) 6m

4. A player stops a football weighting 0.5 kg which comes flying towards him with a velocity of 10m/s. If the impact lasts for 1/50th sec. and the ball bounces back with a velocity of 15 m/s, then the average force involved is

(a) 250 N (b) 1250 N
(c) 500 N (d) 625 N

5. A car travelling at a speed of 30 km/h is brought to a halt in 4 m by applying brakes. If the same car is travelling at 60 km/h, it can be brought to halt with the same braking power in

(a) 8m (b) 16m
(c) 24m (d) 32m

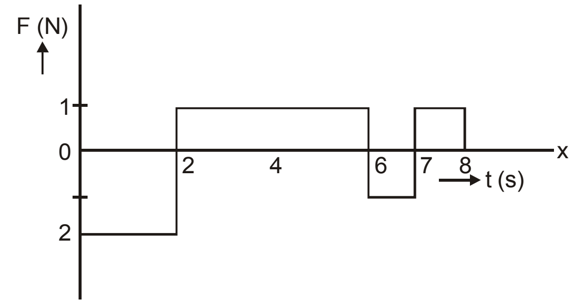
6. A body of mass 4 kg moving on a horizontal surface with an initial velocity of 6 ms^{-1} comes to rest after 3 seconds. If one wants to keep the body moving on the same surface with the velocity of 6 ms^{-1}, the force required is

(a) Zero (b) 4 N
(c) 8 N (d) 16 N

7. A machine gun has a mass 5 kg. It fires 50 gram bullets at the rate of 30 bullets per minute at a speed of 400 ms^{-1}. What force is required to keep the gun in position?

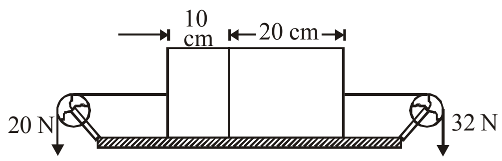
(a) 10 N (b) 5 N
(c) 15 N (d) 30 N

8. A force time graph for the motion of a body is shown in Fig. Change in linear momentum between 0 and 8s is



(a) zero (b) 4 N-s
(c) 8 Ns (d) None of these

9. Fig. shows a uniform rod of length 30 cm having a mass of 3.0 kg. The strings shown in the figure are pulled by constant forces of 20 N and 32 N. All the surfaces are smooth and the strings and pulleys are light. The force exerted by 20 cm part of the rod on the 10 cm part is



(a) 20 N (b) 24 N
(c) 32 N (d) 52 N

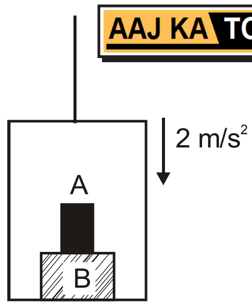
10. A force of 10 N acts on a body of mass 20 kg for 10 seconds. Change in its momentum is

(a) 5 kg m/s (b) 100 kg m/s
(c) 200 kg m/s (d) 1000 kg m/s

11. Consider the system shown in fig. The pulley and the string are light and all the surfaces are frictionless. The tension in the string is (take g = 10 m/s^2)

(a) 0 N (b) 1 N
(c) 2 N (d) 5 N

12. The elevator shown in fig. is descending with an acceleration of 2 m/s^2. The mass of the block A = 0.5 kg. The force exerted by the block A on block B is



(a) 2 N
(b) 4 N
(c) 6 N
(d) 8 N

13. Two blocks of masses 2 kg and 1 kg are placed on a smooth horizontal table in contact with each other. A horizontal force of 3 newton is applied on the first so that the block moves with a constant acceleration. The force between the blocks would be

(a) 3 newton (b) 2 newton
(c) 1 newton (d) zero

14. A 4000 kg lift is accelerating upwards. The tension in the supporting cable is 48000 N. If $g = 10 ms^{-2}$ then the acceleration of the lift is

(a) $1 \ ms^{-2}$ (b) $2 \ ms^{-2}$
(c) $4 \ ms^{-2}$ (d) $6 \ ms^{-2}$

15. A rocket has a mass of 100 kg. Ninety percent of this is fuel. It ejects fuel vapors at the rate of 1 kg/sec with a velocity of 500 m/sec relative to the rocket. It is supposed that the rocket is outside the gravitational field. The initial upthrust on the rocket when it just starts moving upwards is

(a) zero (b) 500 newton
(c) 1000 newton (d) 2000 newton

16. A 0.1 kg block suspended from a massless string is moved first vertically up with an acceleration of $5 ms^{-2}$ and then moved vertically down with an acceleration of $5 ms^{-2}$. If T_1 and T_2 are the respective tensions in the two cases, then

(a) $T_2 > T_1$
(b) $T_1 - T_2 = 1$ N, if $g = 10 ms^{-2}$
(c) $T_1 - T_2 = 1 kg f$
(d) $T_1 - T_2 = 9.8$N, if $g = 9.8 \ ms^{-2}$

17. The coefficient of friction between two surfaces is 0.2. The angle of friction is

(a) $\sin^{-1}(0.2)$ (b) $\cos^{-1}(0.2)$
(c) $\tan^{-1}(0.1)$ (d) $\cot^{-1}(5)$

18. A man weighing 80 kg is standing on a trolley weighing 320 kg. The trolley is resting on frictionless horizontal rails. If the man starts walking on the trolley along the rails at a speed of one metre per second, then after 4 seconds, his displacement relative to the ground will be :

(a) 5 metres (b) 4.8 metres
(c) 3.2 metres (d) 3.0 metres

19. Starting from rest, a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The coefficient of friction between the body and the inclined plane is:

(a) 0.33 (b) 0.25
(c) 0.75 (d) 0.80

20. A ball of mass 0.5 kg moving with a velocity of 2 m/sec strikes a wall normally and bounces back with the same speed. If the time of contact between the ball and the wall is one millisecond, the average force exerted by the wall on the ball is :

(a) 2000 newton (b) 1000 newton
(c) 5000 newton (d) 125 newton

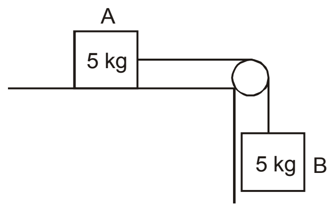
21. The mass of the lift is 100 kg which is hanging on the string. The tension in the string, when the lift is moving with constant velocity, is $(g = 9.8 \ m/sec^2)$

(a) 100 newton (b) 980 newton
(c) 1000 newton (d) None of these

22. In the question , the tension in the strings, when the lift is accelerating up with an acceleration 1 m/sec^2, is

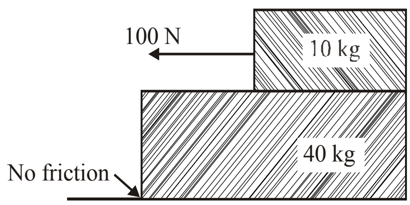
(a) 100 newton (b) 980 newton
(c) 1080 newton (d) 880 newton

23. A block of mass 5 kg resting on a horizontal surface is connected by a cord, passing over a light frictionless pulley to a hanging block of mass 5 kg. The coefficient of kinetic friction between the block and the surface is 0.5. Tension in the cord is : $(g = 9.8 \ m/sec^2)$



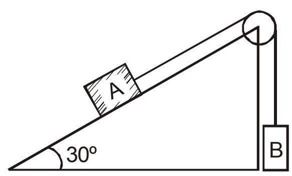
(a) 49 N (b) Zero
(c) 36.75 N (d) 2.45 N

24. A 40 kg slab rests on frictionless floor as shown in fig. A 10 kg block rests on the top of the slab. The static coefficient of friction between the block and slab is 0.60 while the kinetic friction is 0.40. The 10 kg block is acted upon by a horizontal force of 100 N. If $g = 9.8 \ m/s^2$, the resulting acceleration of the slab will be:



(a) $0.98 \ m/s^2$ (b) $1.47 \ m/s^2$
(c) $1.52 \ m/s^2$ (d) $6.1 \ m/s^2$

25. Two blocks are connected over a massless pulley as shown in fig. The mass of block A is 10 kg and the coefficient of kinetic friction is 0.2. Block A slides down the incline at constant speed. The mass of block B in kg is:



(a) 3.5 (b) 3.3
(c) 3.0 (d) 2.5

26. Two trolleys of mass m and 3m are connected by a spring. They were compressed and released at once, they move off in opposite direction and come to rest after covering distance S_1, S_2 respectively. Assuming the coefficient of friction to be uniform, ratio of distances $S_1 : S_2$ is :

(a) 1 : 9 (b) 1 : 3
(c) 3 : 1 (d) 9 : 1

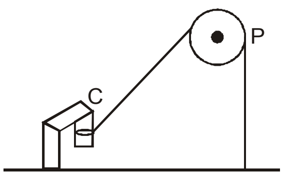
27. A particle of mass 10 kg is moving in a straight line. If its displacement, x with time t is given by $x = (t^3 - 2t - 10)$ m, then the force acting on it at the end of 4 seconds is

(a) 24 N (b) 240 N
(c) 300 N (d) 1200 N

28. When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed then the acceleration of the particle is

(a) F_1/m (b) F_2F_3/mF_1
(c) $(F_2 - F_3)/m$ (d) F_2/m

29. One end of massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 360 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 60 kg moves downwards on the rope? [Take $g = 10\ ms^{-2}$]



(a) 16 (b) 6
(c) 4 (d) 8

30. A force $\vec{F} = 8\hat{i} - 6\hat{j} - 10\hat{k}$ newton produces an acceleration of 1 ms^{-2} in a body. The mass of the body is

(a) 10 kg (b) $10\sqrt{2}$ kg
(c) $10\sqrt{3}$ kg (d) 200 kg

31. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table ?

(a) 12 J (b) 3.6 J
(c) 7.2 J (d) 1200 J

32. A body of mass 1 kg moving with a uniform velocity of $1\,ms^{-1}$. If the value of g is $5\,ms^{-2}$, then the force acting on the frictionless horizontal surface on which the body is moving is

(a) 5 N (b) 1 N
(c) 0 N (d) 10N

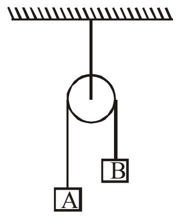
33. A trailer of mass 1000 kg is towed by means of a rope attached to a car moving at a steady speed along a level road. The tension in the rope is 400 N. The car starts to accelerate steadily. If the tension in the rope is now 1650 N, with what acceleration is the trailer moving ?

(a) 1.75 ms^{-2} (b) 0.75 ms^{-2}
(c) 2.5 ms^{-2} (d) 1.25 ms^{-2}

34. A rocket of mass 5000 kg is to be projected vertically upward. The gases are exhausted vertically downwards with velocity 1000 ms^{-2} with respect to the rocket. What is the minimum rate of burning the fuel so as to just lift the rocket upwards against gravitational attraction ?

(a) 49 kg s^{-1} (b) 147 kg s^{-1}
(c) 98 kg s^{-1} (d) 196 kg s^{-1}

35. Blocks A and B of masses 15 kg and 10 kg, respectively, are connected by a light cable passing over a frictionless pulley as shown below. Approximately what is the acceleration experienced by the system?



(a) 2.0 m/s^2
(b) 3.3 m/s^2
(c) 4.9 m/s^2
(d) 9.8 m/s^2

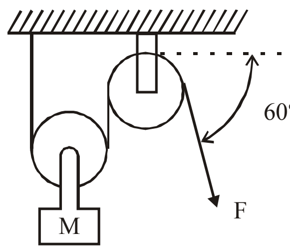
36. A 50 kg ice skater, initially at rest, throws a 0.15 kg snowball with a speed of 35 m/s. What is the approximate recoil speed of the skater?

(a) 0.10 m/s (b) 0.20 m/s
(c) 0.70 m/s (d) 1.4 m/s

37. Block A is moving with acceleration A along a frictionless horizontal surface. When a second block, B is placed on top of Block A the acceleration of the combined blocks drops to 1/5 the original value. What is the ratio of the mass of A to the mass of B?

(a) 5 : 1 (b) 1 : 4
(c) 3 : 1 (d) 2 : 1

38. A force F is used to raise a 4-kg mass M from the ground to a height of 5 m.



What is the work done by the force F? (Note : sin 60° = 0.87; cos 60° = 0.50. Ignore friction and the weights of the pulleys)

(a) 50 J (b) 100 J
(c) 174 J (d) 200 J

39. A 5000 kg rocket is set for vertical firing. The exhaust speed is 800 m/s. To give an initial upward acceleration of 20 m/s^2, the amount of gas ejected per second to supply the needed thrust will be (Take $g = 10\ m/s^2$)

(a) 127.5 kg/s (b) 137.5 kg/s
(c) 155.5 kg/s (d) 187.5 kg/s

40. A bullet is fired from a gun. The force on the bullet is given by $F = 600 - 2 \times 10^5 t$
Where, F is in newtons and t in seconds. The force on the bullet becomes zero as soon as it leaves the barrel. What is the average impulse imparted to the bullet?
(a) 1.8 N-s (b) Zero
(c) 9 N-s (d) 0.9 N-s

41. A rifle man, who together with his rifle has a mass of 100 kg, stands on a smooth surface and fires 10 shots horizontally. Each bullet has a mass 10 g and a muzzle velocity of 800 ms^{-1}. The velocity which the rifle man attains after firing 10 shots is
(a) 8 ms^{-1} (b) 0.8 ms^{-1}
(c) 0.08 ms^{-1} (d) – 0.8 ms^{-1}

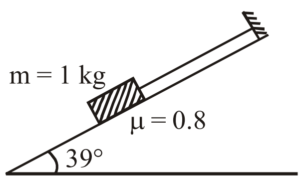
42. A block of mass 4 kg rests on an inclined plane. The inclination to the plane is gradually increased. It is found that when the inclination is 3 in 5, the block just begins to slidedown the plane. The coefficient of friction between the block and the plane is
(a) 0.4 (b) 0.6
(c) 0.8 (d) 0.75.

43. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is
(a) 60 (b) 30
(c) 15 (d) 25

44. A body of mass 2 kg is placed on a horizontal surface having kinetic friction 0.4 and static friction 0.5. If the force applied on the body is 2.5 N, then the frictional force acting on the body will be [$g = 10\ ms^{-2}$]
(a) 8 N (b) 10 N
(c) 20 N (d) 2.5 N

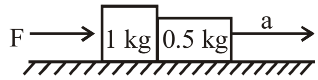
45. A bag of sand of mass m is suspended by a rope. A bullet of mass $\frac{m}{20}$ is fired at it with a velocity v and gets embedded into it. The velocity of the bag finally is
(a) $\frac{v}{20} \times 21$ (b) $\frac{20v}{21}$
(c) $\frac{v}{20}$ (d) $\frac{v}{21}$

46. For the arrangement shown in the Figure the tension in the string is [Given : $\tan^{-1}(0.8) = 39°$]



(a) 6 N (b) 6.4 N
(c) 0.4 N (d) zero.

47. A 1 kg block and a 0.5 kg block move together on a horizontal frictionless surface . Each block exerts a force of 6 N on the other. The block move with a uniform acceleration of



(a) 3 ms^{-2} (b) 6 ms^{-2}
(c) 9 ms^{-2} (d) 12 ms^{-2}

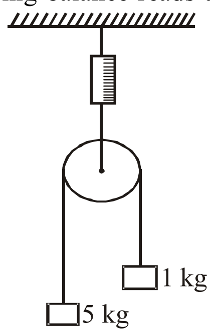
48. A body of mass 32 kg is suspended by a spring balance from the roof of a vertically operating lift and going downward from rest. At the instant the lift has covered 20 m and 50 m, the spring balance showed 30 kg and 36 kg respectively. Then the velocity of the lift is
(a) decreasing at 20 m, and increasing at 50 m
(b) increasing at 20m and decreasing at 50 m
(c) continuously decreasing at a steady rate throughout the journey
(d) constantly increasing at constant rate throughout the journey.

49. An object at rest in space suddenly explodes into three parts of same mass. The momentum of the two parts are $2p\hat{i}$ and $p\hat{j}$. The momentum of the third part
(a) will have a magnitude $p\sqrt{3}$
(b) will have a magnitude $p\sqrt{5}$
(c) will have a magnitude p
(d) will have a magnitude 2p.

50. A triangular block of mass M with angles 30°, 60°, and 90° rests with its 30°–90° side on a horizontal table. A cubical block of mass m rests on the 60°–30° side. The acceleration which M must have relative to the table to keep m stationary relative to the triangular block assuming frictionless contact is
(a) g (b) $\frac{g}{\sqrt{2}}$
(c) $\frac{g}{\sqrt{3}}$ (d) $\frac{g}{\sqrt{5}}$

51. A body of mass 1.0 kg is falling with an acceleration of 10 m/sec^2. Its apparent weight will be ($g = 10$ m/sec^2)
(a) 1.0 kg wt (b) 2.0 kg wt
(c) 0.5 kg wt (d) zero

52. In the figure a smooth pulley of negligible weight is suspended by a spring balance. Weight of 1 kg f and 5 kg f are attached to the opposite ends of a string passing over the pulley and move with acceleration because of gravity, During their motion, the spring balance reads a weight of



(a) 6 kg f
(b) less then 6 kg f
(c) more than 6 kg f
(d) may be more or less then 6 kg f

53. A particle moves so that its acceleration is always twice its velocity. If its initial velocity is 0.1 ms^{-1}, its velocity after it has gone 0.1 m is

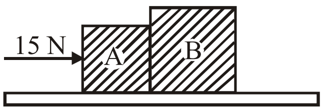
(a) 0.3 ms^{-1} (b) 0.7 ms^{-1}
(c) 1.2 ms^{-1} (d) 3.6 ms^{-1}

54. An object is resting at the bottom of two strings which are inclined at an angle of 120° with each other. Each string can withstand a tension of 20N. The maximum weight of the object that can be supported without breaking the string is

(a) 5 N (b) 10 N
(c) 20 N (d) 40 N

55. On a smooth plane surface (figure) two block A and B are accelerated up by applying a force 15 N on A. If mass of B is twice that of A, the force on B is

(a) 30 N (b) 15 N
(c) 10 N (d) 5 N



56. A 10 kg stone is suspended with a rope of breaking strength 30 kg-wt. The minimum time in which the stone can be raised through a height 10 m starting from rest is (Take $g = 10 N/kg$)

(a) 0.5 s (b) 1.0 s
(c) $\sqrt{2/3}$ s (d) 2 s

57. A ball of mass 0.4 kg thrown up in air with velocity 30 ms^{-1} reaches the highest point in 2.5 second . The air resistance encountered by the ball during upward motion is

(a) 0.88 N (b) 8800N
(c) 300 dyne (d) 300 N.

58. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1s, the force of the blow exerted by the ball on the hand of the player is equal to

(a) 150 N (b) 3 N
(c) 30 N (d) 300 N

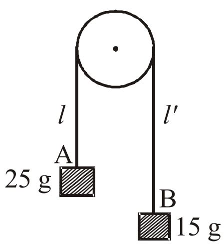
59. In the system shown in figure, the pulley is smooth and massless, the string has a total mass 5g, and the two suspended blocks have masses 25 g and 15 g. The system is released from state $\ell = 0$ and is studied upto stage $\ell' = 0$ During the process, the acceleration of block A will be

(a) constant at $\frac{g}{9}$

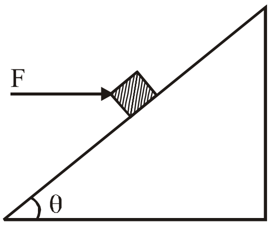
(b) constant at $\frac{g}{4}$

(c) increasing by factor of 3

(d) increasing by factor of 2



60. A horizontal force F is applied on back of mass m placed on a rough inclined plane of inclination θ. The normal reaction N is



(a) $mg\cos\theta$ (b) $mg\sin\theta$
(c) $mg\cos\theta - F\cos\theta$ (d) $mg\cos\theta + F\sin\theta$

61. The coefficient of friction between the rubber tyres and the road way is 0.25. The maximum speed with which a car can be driven round a curve of radius 20 m without skidding is ($g = 9.8\ m/s^2$)

(a) 5 m/s (b) 7 m/s
(c) 10 m/s (d) 14 m/s

62. A bucket tied at the end of a 1.6 m long string is whirled in a vertical circle with constant speed. What should be the minimum speed so that the water from the bucket does not spill when the bucket is at the highest position?

(a) 4 m/sec (b) 6.25 m/sec
(c) 16 m/sec (d) None of the above

63. A cane filled with water is revolved in a vertical circle of radius 4 meter and the water just does not fall down. The time period of revolution will be

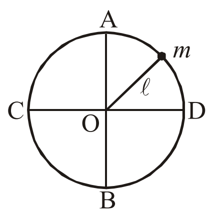
(a) 1 sec (b) 10 sec
(c) 8 sec (d) 4 sec

64. A circular road of radius r in which maximum velocity is v, has angle of banking

(a) $\tan^{-1}\left(\frac{v^2}{rg}\right)$ (b) $\tan^{-1}\left(\frac{rg}{v^2}\right)$

(c) $\tan^{-1}\left(\frac{v}{rg}\right)$ (d) $\tan^{-1}\left(\frac{rg}{v}\right)$

65. A small sphere is attached to a cord and rotates in a vertical circle about a point O. If the average speed of the sphere is increased, the cord is most likely to break at the orientation when the mass is at



(a) bottom point B (b) the point C
(c) the point D (d) top point A

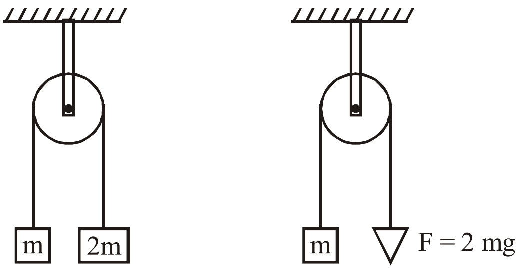
66. A person with his hand in his pocket is skating on ice at the rate of 10m/s and describes a circle of radius 50 m. What is his inclination to vertical : ($g = 10\ m/sec^2$)

(a) $\tan^{-1}(½)$ (b) $\tan^{-1}(1/5)$
(c) $\tan^{-1}(3/5)$ (d) $\tan^{-1}(1/10)$

67. When the road is dry and the coefficient of the friction is μ, the maximum speed of a car in a circular path is $10\ ms^{-1}$. If the road becomes wet and $\mu' = \frac{\mu}{2}$, what is the maximum speed permitted?

(a) $5\ ms^{-1}$ (b) $10\ ms^{-1}$
(c) $10\sqrt{2}\ ms^{-1}$ (d) $5\sqrt{2}\ ms^{-1}$

68. Two pulley arrangements of figure given are identical. The mass of the rope is negligible. In fig (a), the mass m is lifted by attaching a mass 2m to the other end of the rope. In fig (b), m is lifted up by pulling the other end of the rope with a constant downward force F = 2mg. The acceleration of m in the two cases are respectively



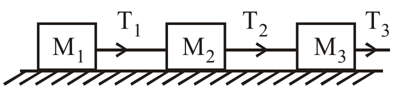
(a) (b)

(a) 3g, g (b) $g/3$, g
(c) $g/3$, 2g (d) g, $g/3$

69. The linear momentum p of a body moving in one dimension varies with time according to the equating $P = a + bt^2$ where a and b are positive constants. The net force acting on the body is

(a) proportional to t^2
(b) a constant
(c) proportional to t
(d) inversely proportional to t

70. Three blocks of masses m_1, m_2 and m_3 are connected by massless strings, as shown, on a frictionless table. They are pulled with a force $T_3 = 40$ N. If $m_1 = 10$ kg, $m_2 = 6$ kg and $m_3 = 4$kg, the tension T_2 will be

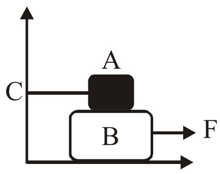


(a) 20 N (b) 40 N
(c) 10 N (d) 32 N

71. A ball of mass 400 gm is dropped from a height of 5 m. A boy on the ground hits the ball vertically upwards with a bat with an average force of 100 newton so that it attains a vertical height of 20 m. The time for which the ball remains in contact with the bat is ($g = 10\ m/s^2$)

(a) 0.12 s (b) 0.08 s
(c) 0.04 s (d) 12 s

72. Block A of weight 100 kg rests on a block B and is tied with horizontal string to the wall at C. Block B is of 200 kg. The coefficient of friction between A and B is 0.25 and that between B and surface is $\frac{1}{3}$. The horizontal force F necessary to move the block B should be ($g = 10\ m/s^2$)



(a) 1050 N (b) 1450 N
(c) 1050 N (d) 1250 N

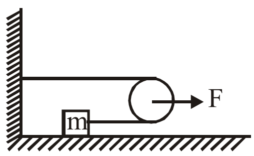
73. An open topped rail road car of mass M has an initial velocity v_0 along a straight horizontal frictionless track. It suddenly starts raising at time t = 0. The rain drops fall vertically with velocity u and add a mass m kg/sec of water. The velocity of car after t second will be (assuming that it is not completely filled with water)

(a) $v_0 + m\frac{u}{M}$ (b) $\frac{mv_0}{M + mt}$
(c) $\frac{Mv_0 + ut}{M + ut}$ (d) $v_0 + \frac{mut}{M + ut}$

74. A ball mass m falls vertically to the ground from a height h_1 and rebounds to a height h_2. The change in momentum of the ball of striking the ground is

(a) $m\sqrt{2g(h_1 + h_2)}$ (b) $n\sqrt{2g(m_1 + m_2)}$
(c) $mg(h_1 - h_2)$ (d) $m(\sqrt{2gh_1} - \sqrt{2gh_2})$

75. In the given figure, the pulley is assumed massless and frictionless. If the friction force on the object of mass m is f, then its acceleration in terms of the force F will be equal to



(a) $(F - f)/m$ (b) $\left(\frac{F}{2} - f\right)/m$
(c) F/m (d) None of these

76. A smooth block is released at rest on a 45° incline and then slides a distance 'd'. The time taken to slide is 'n' times as much to slide on rough incline than on a smooth incline. The coefficient of friction is

(a) $\mu_k = \sqrt{1-\frac{1}{n^2}}$ (b) $\mu_k = 1-\frac{1}{n^2}$

(c) $\mu_s = \sqrt{1-\frac{1}{n^2}}$ (d) $\mu_s = 1-\frac{1}{n^2}$

77. The upper half of an inclined plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by

(a) $2\cos\phi$ (b) $2\sin\phi$

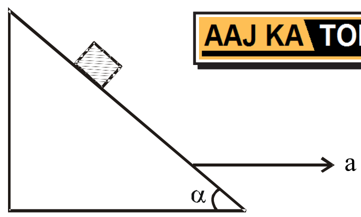
(c) $\tan\phi$ (d) $2\tan\phi$

78. A particle of mass 0.3 kg subject to a force $F = -kx$ with $k = 15$ N/m . What will be its initial acceleration if it is released from a point 20 cm away from the origin ?

(a) 15 m/s^2 (b) 3 m/s^2

(c) 10 m/s^2 (d) 5 m/s^2

79. A block is kept on a frictionless inclined surface with angle of inclination 'α'. The incline is given an acceleration 'a' to keep the block stationary. Then a is equal to



(a) $g \operatorname{cosec}\alpha$ (b) $g/\tan\alpha$

(c) $g\tan\alpha$ (d) g

80. Consider a car moving on a straight road with a speed of 100 m/s . The distance at which car can be stopped is $[\mu_k = 0.5]$

(a) 1000 m (b) 800 m

(c) 400 m (d) 100 m

81. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is

(a) $\frac{g\sin\theta}{1-MR^2/I}$ (b) $\frac{g\sin\theta}{1+I/MR^2}$

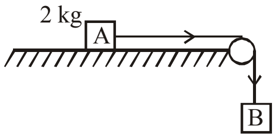
(c) $\frac{g\sin\theta}{1+MR^2/I}$ (d) $\frac{g\sin\theta}{1-I/MR^2}$

82. A block of mass *m* is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block (g is acceleration due to gravity) will be

(a) $mg/\cos\theta$ (b) $mg\cos\theta$

(c) $mg\sin\theta$ (d) mg

83. The coefficient of static friction, μ_s, between block A of mass 2 kg and the table as shown in the figure is 0.2. What would be the maximum mass value of block B so that the two blocks do not move? The string and the pulley are assumed to be smooth and massless. $(g = 10\ m/s^2)$



(a) 0.4 kg (b) 2.0 kg

(c) 4.0 kg (d) 0.2 kg

84. A body under the action of a force $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$, acquires an acceleration of 1 m/s^2. The mass of this body must be

(a) 10 kg (b) 20 kg

(c) $10\sqrt{2}$ kg (d) $2\sqrt{10}$ kg

85. A conveyor belt is moving at a constant speed of 2m/s. A box is gently dropped on it. The coefficient of friction between them is $\mu = 0.5$. The distance that the box will move relative to belt before coming to rest on it taking $g = 10\ ms^{-2}$, is

(a) 1.2 m (b) 0.6 m

(c) zero (d) 0.4 m

86. A person of mass 60 kg is inside a lift of mass 940 kg and presses the button on control panel. The lift starts moving upwards with an acceleration 1.0 m/s^2. If $g = 10\ ms^{-2}$, the tension in the supporting cable is

(a) 8600 N (b) 9680 N

(c) 11000 N (d) 1200 N

87. The upper half of an inclined plane of inclination θ is perfectly smooth while lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom, if the coefficient of friction between the block and lower half of the plane is given by

(a) $\mu = \frac{2}{\tan\theta}$ (b) $\mu = 2\tan\theta$

(c) $\mu = \tan\theta$ (d) $\mu = \frac{1}{\tan\theta}$

88. A bridge is in the from of a semi-circle of radius 40m. The greatest speed with which a motor cycle can cross the bridge without leaving the ground at the highest point is $(g = 10\ m\ s^{-2})$ (frictional force is negligibly small)

(a) 40 $m\ s^{-1}$ (b) 20 $m\ s^{-1}$

(c) 30 $m\ s^{-1}$ (d) 15 $m\ s^{-1}$

89. An explosion breaks a rock into three parts in a horizontal plane. Two of them go off at right angles to each other. The first part of mass 1 kg moves with a speed of 12 ms^{-1} and the second part of mass 2 kg moves with speed 8 ms^{-1}. If the third part flies off with speed 4 ms^{-1} then its mass is

(a) 5 kg (b) 7 kg
(c) 17 kg (d) 3 kg

90. Two particles of masses m and M (M > m) are connected by a cord that passes over a massless, frictionless pulley. The tension T in the string and the acceleration a of the particles is

(a) $T = \frac{2mM}{(M-m)}g; a = \frac{Mm}{(M+m)}g$

(b) $T = \frac{2mM}{(M+m)}g; a = \left(\frac{M-m}{(M+m)}\right)g$

(c) $T = \left(\frac{m-M}{(M+m)}\right)g; a = \left(\frac{Mm}{(M+m)}\right)g$

(d) $T = \left(\frac{mM}{(M+m)}\right)g; a = \left(\frac{2Mm}{(M+m)}\right)g$

91. A bullet of mass m is fired from a gun of mass M. The recoiling gun compresses a spring of force constant k by a distance d. Then the velocity of the bullet is

(a) $kd\sqrt{M/m}$ (b) $\frac{d}{M}\sqrt{km}$

(c) $\frac{d}{m}\sqrt{kM}$ (d) $\frac{kM}{m}\sqrt{d}$

92. A spring of force constant k is cut into two pieces whose lengths are in the ratio 1 : 2. What is the force constant of the longer piece ?

(a) $\frac{k}{2}$ (b) $\frac{3k}{2}$

(c) $2k$ (d) $3k$

93. A motor cycle is going on an overbridge of radius R. The driver maintains a constant speed. As the motor cycle is ascending on the overbridge, the normal force on it

(a) increases (b) decreases
(c) remains the same (d) fluctuates erratically

94. A body of mass M hits normally a rigid wall with velocity V and bounces back with the same velocity. The impulse experienced by the body is

(a) MV (b) 1.5 MV
(c) 2 MV (d) zero

95. A heavy uniform chain lies on horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over one edge of the table is

(a) 20% (b) 25%
(c) 35% (d) 15%

96. A body of mass 5 kg explodes at rest into three fragments with masses in the ratio 1 : 1 : 3. The fragments with equal masses fly in mutually perpendicular directions with speeds of 21 m/s. The velocity of heaviest fragment in m/s will be

(a) $7\sqrt{2}$ (b) $5\sqrt{2}$

(c) $3\sqrt{2}$ (d) $\sqrt{2}$

97. Two bodies of masses m and 4m are moving with equal kinetic energies. The ratio of their linear momenta will be

(a) 1 : 4 (b) 4 : 1
(c) 1 : 2 (d) 2 : 1

Directions for Qs. (98 to 100) : Each question contains STATEMENT-1 and STATEMENT-2. Choose the correct answer (ONLY ONE option is correct) from the following-

(a) Statement -1 is false, Statement-2 is true

(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(d) Statement -1 is true, Statement-2 is false

98. **Statement -1 :** The work done in bringing a body down from the top to the base along a frictionless incline plane is the same as the work done in bringing it down the vartical side.

Statement -2 : The gravitational force on the body along the inclined plane is the same as that along the vertical side.

99. **Statement -1 :** On a rainy day, it is difficult to drive a car or bus at high speed.

Statement -2 : The value of coefficient of friction is lowered due to wetting of the surface.

100. **Statement -1 :** The two bodies of masses M and m ($M > m$) are allowed to fall from the same height if the air resistance for each be the same then both the bodies will reach the earth simultaneously.

Statement -2 : For same air resistance, acceleration of both the bodies will be same.

EXERCISE - 3
Exemplar & Past Years NEET/AIPMT Questions

Exemplar Questions

1. A ball is travelling with uniform translatory motion. This means that
(a) it is at rest
(b) the path can be a straight line or circular and the ball travels with uniform speed
(c) all parts of the ball have the same velocity (magnitude and direction) and the velocity is constant
(d) the centre of the ball moves with constant velocity and the ball spins about its centre uniformly

2. A metre scale is moving with uniform velocity. This implies
(a) the force acting on the scale is zero, but a torque about the centre of mass can act on the scale
(b) the force acting on the scale is zero and the torque acting about centre of mass of the scale is also zero
(c) the total force acting on it need not be zero but the torque on it is zero
(d) neither the force nor the torque need to be zero

3. A cricket ball of mass 150 g has an initial velocity $\vec{u} = (3\hat{i} + 4\hat{j})\text{ms}^{-1}$ and a final velocity $\vec{v} = -(3\hat{i} + 4\hat{j})\text{ms}^{-1}$, after being hit. The change in momentum (final momentum-initial momentum) is (in kgms^{-1})
(a) zero (b) $-(0.45\hat{i} + 0.6\hat{j})$
(c) $-(0.9\hat{j} + 1.2\hat{j})$ (d) $-5(\hat{i} + \hat{j})\hat{i}$

4. In the previous problem (3), the magnitude of the momentum transferred during the hit is
(a) zero (b) 0.75 kg-m s^{-1}
(c) 1.5 kg-m s^{-1} (d) 1.4 kg-m s^{-1}

5. Conservation of momentum in a collision between particles can be understood from
(a) Conservation of energy
(b) Newton's first law only
(c) Newton's second law only
(d) both Newton's second and third law

6. A hockey player is moving northward and suddenly turns westward with the same speed to avoid an opponent. The force that acts on the player is
(a) frictional force along westward
(b) muscle force along southward
(c) frictional force along sotuh-west
(d) muscle force along south-west

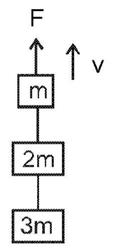
7. A body of mass 2 kg travels according to the law $x(t) = pt + qt^2 + rt^3$ where, $q = 4$ ms^{-2}, $p = 3$ ms^{-1} and $r = 5$ ms^{-3}. The force acting on the body at $t = 2$s is
(a) 136 N (b) 134 N
(c) 158 N (d) 68 N

8. A body with mass 5 kg is acted upon by a force $\vec{F} = (-3\hat{i} + 4\hat{j})$ N. If its initial velocity at $t = 0$ is $v = (6\hat{i} - 12\hat{j})$ ms^{-1}, the time at which it will just have a velocity along the y-axis is
(a) never (b) 10 s
(c) 2 s (d) 15 s

9. A car of mass m starts from rest and acquires a velocity along east, $\vec{v} = v\hat{i}$ $(v > 0)$ in two seconds. Assuming the car moves with uniform acceleration, the force exerted on the car is
(a) $\frac{mv}{2}$ eastward and is exerted by the car engine
(b) $\frac{mv}{2}$ eastward and is due to the friction on the tyres exerted by the road
(c) more than $\frac{mv}{2}$ eastward exerted due to the engine and overcomes the friction of the road
(d) $\frac{mv}{2}$ exerted by the engine

NEET/AIPMT (2013-2017) Questions

10. Three blocks with masses m, 2 m and 3 m are connected by strings as shown in the figure. After an upward force F is applied on block m, the masses move upward at constant speed v. What is the net force on the block of mass 2m? (g is the acceleration due to gravity) ***[2013]***

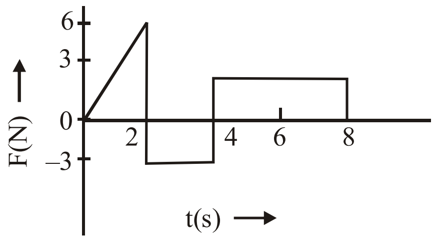


(a) 2 mg
(b) 3 mg
(c) 6 mg
(d) zero

11. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A bob is suspended from the roof of the car by a light wire of length 1.0 m. The angle made by the wire with the vertical is ***[NEET Kar. 2013]***
(a) 0° (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

12. A balloon with mass 'm' is descending down with an acceleration 'a' (where a < g). How much mass should be removed from it so that it starts moving up with an acceleration 'a'? ***[2014]***
(a) $\frac{2ma}{g + a}$ (b) $\frac{2ma}{g - a}$
(c) $\frac{ma}{g + a}$ (d) $\frac{ma}{g - a}$

13. The force 'F' acting on a particle of mass 'm' is indicated by the force-time graph shown below. The change in momentum of the particle over the time interval from zero to 8 s is : ***[2014]***



(a) 24 Ns (b) 20 Ns
(c) 12 Ns (d) 6 Ns

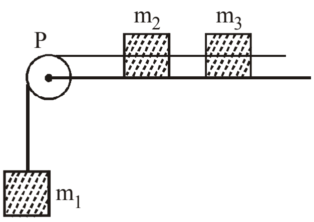
14. A system consists of three masses m_1, m_2 and m_3 connected by a string passing over a pulley P. The mass m_1 hangs freely and m_2 and m_3 are on a rough horizontal table (the coefficient of friction = μ). The pulley is frictionless and of negligible mass. The downward acceleration of mass m_1 is : (Assume $m_1 = m_2 = m_3 = m$) ***[2014]***

(a) $\frac{g(1-g\mu)}{g}$

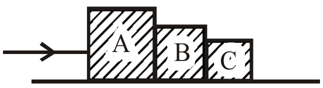
(b) $\frac{2g\mu}{3}$

(c) $\frac{g(1-2\mu)}{3}$

(d) $\frac{g(1-2\mu)}{2}$



15. Three blocks A, B and C of masses 4 kg, 2 kg and 1 kg respectively, are in contact on a frictionless surface, as shown. If a force of 14 N is applied on the 4 kg block then the contact force between A and B is ***[2015]***



(a) 6 N (b) 8 N
(c) 18 N (d) 2 N

16. A block A of mass m_1 rests on a horizontal table. A light string connected to it passes over a frictionless pulley at the edge of table and from its other end another block B of mass m_2 is suspended. The coefficient of kinetic friction between the block and the table is μ_k. When the block A is sliding on the table, the tension in the string is ***[2015]***

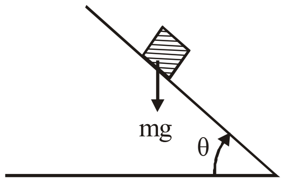
(a) $\frac{(m_2 - \mu k\, m_1)g}{(m_1 + m_2)}$ (b) $\frac{m_1 m_2 (1+\mu_k)g}{(m_1 + m_2)}$

(c) $\frac{m_1 m_2 (1-\mu_k)g}{(m_1 + m_2)}$ (d) $\frac{(m_2 + \mu_k\, m_1)g}{(m_1 + m_2)}$

17. Two stones of masses m and 2 m are whirled in horizontal circles, the heavier one in radius $\frac{r}{2}$ and the lighter one in radius r. The tangential speed of lighter stone is n times that of the value of heavier stone when they experience same centripetal forces. The value of n is : ***[2015 RS]***

(a) 3 (b) 4
(c) 1 (d) 2

18. A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30° the box starts to slip and slides 4.0 m down the plank in 4.0s. The coefficients of static and kinetic friction between the box and the plank will be, respectively : ***[2015 RS]***



(a) 0.6 and 0.5 (b) 0.5 and 0.6
(c) 0.4 and 0.3 (d) 0.6 and 0.6

19. What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop ? ***[2016]***

(a) $\sqrt{gR}$ (b) $\sqrt{2gR}$

(c) $\sqrt{3gR}$ (d) $\sqrt{5gR}$

20. One end of string of length l is connected to a particle of mass 'm' and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in circle with speed 'v' the net force on the particle (directed towards centre) will be (T represents the tension in the string) : ***[2017]***

(a) $T + \frac{mv^2}{l}$ (b) $T - \frac{mv^2}{l}$

(c) Zero (d) T

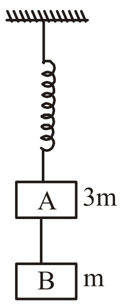
21. Two blocks A and B of masses 3 m and m respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in figure. The magnitudes of acceleration of A and B immediately after the string is cut, are respectively : ***[2017]***

(a) $\frac{g}{3}, g$

(b) g, g

(c) $\frac{g}{3}, \frac{g}{3}$

(d) $g, \frac{g}{3}$



Hints & Solutions

EXERCISE - 1

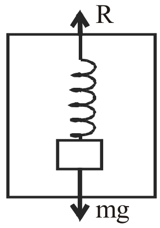
1. (c) 2. (b) 3. (d) 4. (a) 5. (c)
6. (a) 7. (a)

8. (d) Let n be the mass per unit length of rope. Therefore, mass of rope = nL.
Acceleration in the rope due to force F will be a = F/nL.
Mass of rope of length $(L - \ell)$ will be $n(L - \ell)$.
Therefore, tension in the rope of length $(L - \ell)$, is equal to pulling force on it
$= n(L - \ell)\, a = n(L - \ell) \times F/nL = F(1 - \ell/L)$

9. (a) Impulse = change in momentum = $m\vec{v}_2 - m\vec{v}_1$

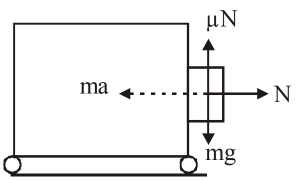
10. (a) Let a be the acceleration of mass m_2 in the downward direction. Then
$T - m_2(g/2) = m_1 a$ ....(i)
and $m_2 g - T = m_2 a$ ....(ii)
Adding eqs. (1) and (2), we get
$(m_1 + m_2)a = m_2 g - m_2(g/2) = m_2 g/2$
$\therefore\ a = \dfrac{m_2 g}{2(m_1 + m_2)}$

11. (a) Let acceleration of lift = a and
let reaction at spring balance = R



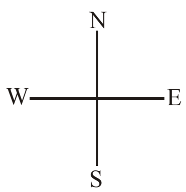
Applying Newton's law
$R - mg = ma \Rightarrow R = m(g + a)$
thus net weight increases,
So reading of spring balance increases.

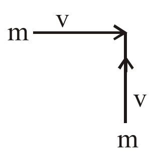
12. (b) See fig.



If a = acceleration of the cart, then N = ma
$\therefore\ \mu N = mg$ or $\mu\, ma = mg$ or $a = g/\mu$

13. (c) $p_1 = mv$ northwards, $p_2 = mv$ eastwards





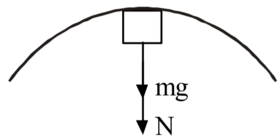
Let p = momentum after collision. Then,
$\vec{p} = \vec{p}_1 + \vec{p}_2$ or $|\vec{p}| = \sqrt{(mv)^2 + (mv)^2}$
$2mv' = mv\sqrt{2}$ or $v' = \dfrac{v}{\sqrt{2}}$ m/sec

14. (c) Applying law of conservation of linear momentum
$m_1 v_1 + m_2 v_2 = 0,\ \dfrac{m_1}{m_2} = -\dfrac{v_2}{v_1}$ or $\dfrac{v_1}{v_2} = -\dfrac{m_2}{m_1}$

15. (c) The equations of motion are
$2\,mg - T = 2ma$
$T - mg = ma \Rightarrow T = 4ma$ & $a = g/3$ so $T = 4mg/3$
If pulley is accelerated upwards with an accleration a, then tension in string is
$T = \dfrac{4m}{3}(g + a)$

16. (b) Inertia is resistance to change.
17. (b) The time rate of change of momentum is force.

18. (a) At the highest point of the track, $N + mg = \dfrac{mv'^2}{r}$



where r is the radius of curvature at that point and v′ is the speed of the block at that point.

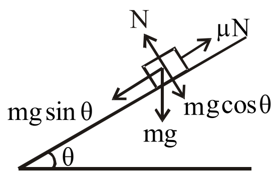
Now $N = \dfrac{mv'^2}{r} - mg$

N will be maximum when r is minimum (v′ is the same for all cases). Of the given tracks, (a) has the smallest radius of curvature at the highest point.

19. (c) $f = \mu W$
$f = W\tan\theta \qquad [\therefore\ \mu = \tan\theta]$

20. (a) $S_n = \dfrac{a}{2}(2n - 1)$
$S_{n+1} = \dfrac{a}{2}(2n + 1)$
$\dfrac{S_n}{S_{n+1}} = \dfrac{2n - 1}{2n + 1}$

21. (b) From the F.B.D.
$N = mg\cos\theta$
$F = ma = mg\sin\theta - \mu N$
$\Rightarrow\ a = g(\sin\theta - \mu\cos\theta)$



Now using, $v^2 - u^2 = 2as$

or, $v^2 = 2 \times g(\sin\theta - \mu\cos\theta)\ell$

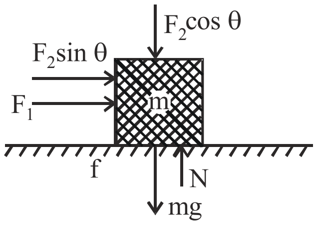
(ℓ = length of incline)

or, $v = \sqrt{2g\ell(\sin\theta - \mu\cos\theta)}$

22. (a) Based on Newton's third law of motion.

23. (a) If m_1, m_2 are masses and u_1, u_2 are velocity then by conservation of momentum $m_1u_1 + m_2u_2 = 0$ or $|m_1u_1| = |m_2u_2|$

24. (a) Here, on resolving force F_2 and applying the concept of equilibrium



$N = mg + F_2\cos\theta$, and $f = \mu N$

$\therefore\ f = \mu[mg + F_2\cos\theta]$ ...(i)

Also $f = F_1 + F_2\sin\theta$ ...(ii)

From (i) and (ii)

$\mu[mg + F_2\cos\theta] = F_1 + F_2\sin\theta$

$\Rightarrow \mu = \dfrac{F_1 + F_2\sin\theta}{mg + F_2\cos\theta}$

25. (a) The frictional force acting on M is μmg

$\therefore$ Acceleration $= \dfrac{\mu mg}{M}$

EXERCISE - 2

1. (b) Here $u = 10\ ms^{-1}$, $v = -2\ ms^{-1}$, $t = 4$ s, a = ?

Using $a = \dfrac{v-u}{t} = \dfrac{-2-10}{4} = -3\ m/s^2$

$\therefore$ Force, $F = ma = 10 \times (-3) = -30$ N

2. (b) $2T\cos 60^\circ = mg$

or $T = mg = 2 \times 10 = 20$ N.

3. (c) If k is the spring factor, then P.E. of the spring compressed by distance x $\left(= \frac{1}{2}kx^2\right)$ will equal to gain in P.E. of the dart (= mgh) i.e. $\frac{1}{2}kx^2 = mgh$

$\therefore\ \frac{1}{2}k(4)^2 = 16 \times g \times 200$ ....(i)

and $\frac{1}{2}k(6)^2 = 16 \times g \times h$ ...(ii)

On solving, (i) and (ii), we get h = 450 cm = 4.5 m.

4. (d) Here m = 0.5 kg ; u = – 10 m/s ; t = 1/50 s ; v = + 15 ms^{-1}

Force = m (v– u)/t = 0.5 (10 + 15) × 50 = 625 N

5. (b) As, $(1/2)mv^2 = Fs$

So $\frac{1}{2}m(30)^2 = F \times 4$ and $\frac{1}{2}m(60)^2 = F \times s$

$\therefore\ s/4 = (60)^2/(30)^2 = 4$ or $s = 4 \times 4 = 16$ m.

6. (c) Acceleration, $a = \dfrac{v-u}{t} = \dfrac{0-6}{3} = -2\ ms^{-2}$

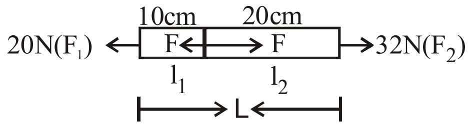
Force = m×a = 4×2 = 8 N

7. (a) Force required $= \dfrac{\text{change in momentum}}{\text{time taken}}$

$= \dfrac{(50 \times 10^{-3} \times 30) \times 400 - (5 \times 0)}{60} = 10$ N

8. (a) Change in momentum = Force × time = Area which the force-time curve encloses with time axis.

9. (b)



It is clear $F_2 > F_1$, so rod moves in right direction with an acceleration a, whereas a is given by

$(F_2 – F_1) = mL \times a$...............(i)

where m is mass of rod per unit length.

Now consider the motion of length l_1 from first end, then

$F – F_1 = ml_1a$.................(ii)

Dividing eq (ii) by (i), we get

$\dfrac{F - F_1}{F_2 - F_1} = \dfrac{l_1}{L}$ or $F = (F_2 - F_1) \times \dfrac{l_1}{L} + F_1$

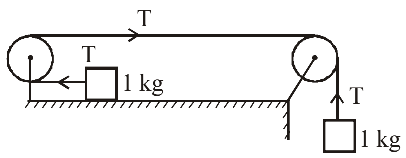
here $l_1 = 10$ cm., L = 30 cm., $F_1 = 20$ N, $F_2 = 32$N

so F = 24 N

10. (b) Change in momentum = F × t

= 10 × 10 = 100 Ns or 100 kg. m/s

11. (d) See fig.



From figure, 1 g – T = 1 a ...(i)

and T = 1 a ....(ii)

From eqs. (i) and (ii), we get

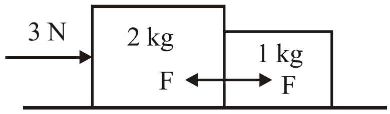
1g – 1a = 1a or 2a = g

$\therefore$ a = (g/2) = (10/2) = 5 m/s^2

So, T = ma = 1 × 5 = 5 N

12. (b) R = mg – ma = 0.5 × 10 – 0.5 × 2 = 5 – 1 = 4 N

13. (c) See fig. Let F be the force between the blocks and a their common acceleration. Then for 2 kg block,



$3 - F = 2a$ ...(1)

for 1 kg block, $F = 1 \times a = a$ ....(2)

$\therefore 3 - F = 2F$ or $3F = 3$ or $F = 1$ newton

14. (b) $T = m(g + a)$

$48000 = 4000(10 + a)$

$\Rightarrow a = 2\ ms^{-2}$

15. (b) Initial thrust on the rocket $= \dfrac{\Delta m}{\Delta t} v_{rel}$

$= 500 \times 1 = 500$ N

where $\dfrac{\Delta m}{\Delta t}$ = rate of ejection of fuel.

16. (b) $T_1 = m(g + a) = 0.1(10 + 5) = 1.5N$

$T_2 = m(g - a) = 0.1(10 - 5) = 0.5N$

$\Rightarrow T_1 - T_2 = (1.5 - 0.5)N = 1N$

17. (d) Angle of friction $= \tan^{-1}\mu$
18. (c) Displacement of the man on the trolley $= 1 \times 4 = 4$m

Now applying conservation of linear momentum

$80 \times 1 + 400\,v = 0$ or $v = -\dfrac{1}{5}$ m/sec.

The distance travelled by the trolley

$= -0.2 \times 4 = -0.8$ m.

(In opposite direction to the man.)

Thus, the relative displacement of the man with the ground $= (4 - 0.8) = 3.2$ m.

19. (c) In presence of friction $a = (g\sin\theta - \mu g\cos\theta)$

$\therefore$ Time taken to slide down the plane

$$t_1 = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g(\sin\theta - \mu\cos\theta)}}$$

In absence of friction $t_2 = \sqrt{\dfrac{2s}{g\sin\theta}}$

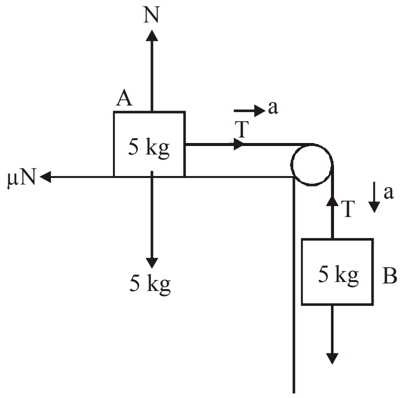
Given : $t_1 = 2t_2$

$$\therefore t_1^2 = 4t_2^2 \text{ or } \frac{2s}{g(\sin\theta - \mu\cos\theta)} = \frac{2s \times 4}{g\sin\theta}$$

$\sin\theta = 4\sin\theta - 4\mu\cos\theta$

$$\mu = \frac{3}{4}\tan\theta = \frac{3}{4} = 0.75 \text{ (since } \theta = 45°)$$

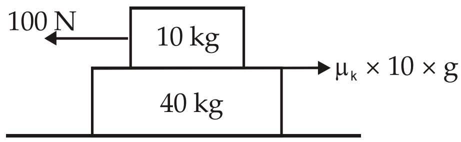
20. (a) $F = \dfrac{mv - (-mv)}{t} = \dfrac{2mv}{t} = \dfrac{2 \times 0.5 \times 2}{10^{-3}} = 2 \times 10^3$ N
21. (b) $T = m(g + a) = 100(9.8 + 0) = 980$ N
22. (c) $T = m(g+a) = 100(9.8+1) = 1080$N
23. (c) For block A, $T - \mu N = 5a$ and $N = 5g$



for block B, $5g - T = 5a$

$\Rightarrow T = 36.75$N, $a = 2.45$ m/sec^2

24. (a) Force on the slab (m = 40 kg) = reaction of frictional force on the upper block



$\therefore 40a = \mu_k \times 10 \times g$ or $a = 0.98$ m/sec^2

25. (b) Considering the equilibrium of B

$-m_Bg + T = m_Ba$

Since the block A slides down with constant speed.

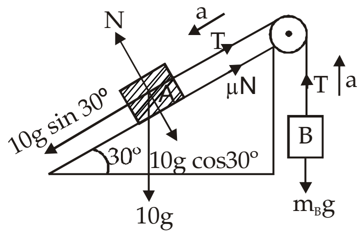
$a = 0$.

Therefore $T = m_Bg$

Considering the equilibrium of A, we get

$10a = 10g\sin 30° - T - \mu N$

where $N = 10g\cos 30°$



$$\therefore 10a = \frac{10}{2}g - T - \mu \times 10g\cos 30°$$

but $a = 0$, $T = m_Bg$

$$0 = 5g - m_Bg - \frac{0.2\sqrt{3}}{2} \times 10 \times g$$

$\Rightarrow m_B = 3.268 \approx 3.3$ kg

26. (d) $mv_1 + 3mv_2 = 0$ or $\dfrac{v_1}{v_2} = -3$

Now $\dfrac{1}{2}mv_1^2 = F.S_1 = \mu.mg.S_1$

$\dfrac{1}{2}(3m)v_2^2 = F.S_2 = \mu.3mg.S_2$

or $\dfrac{S_1}{S_2} = \dfrac{v_1^2}{v_2^2} = \dfrac{9}{1}$

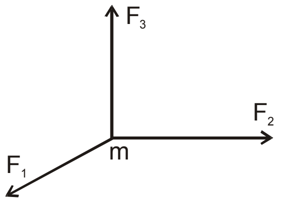
27. (b) $m = 10\,kg,\ x = (t^3 - 2t - 10)\,m$

$$\frac{dx}{dt} = v = 3t^2 - 2 \qquad \frac{d^2x}{dt^2} = a = 6t$$

At the end of 4 seconds, $a = 6 \times 4 = 24\ m/s^2$

$F = ma = 10 \times 24 = 240\,N$

28. (a)

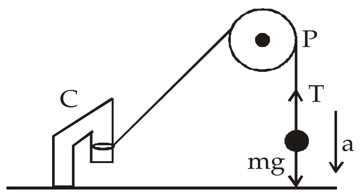


The formula for force is given by $F_1 = ma$

Acceleration of the particle $a = \frac{F_1}{m}$,

because F_1 is equal to the vector sum of F_2 & F_3.

29. (c)



$mg - T = ma$

$$\frac{60 \times 10 - 360}{60} = a$$

$a = 4\ ms^{-2}$

30. (b) $m = \dfrac{\sqrt{8^2 + (-6)^2 + (-10)^2}}{1} = 10\sqrt{2}kg$

31. (b) Mass of over hanging chain $m' = \frac{4}{2} \times (0.6)kg$

Let at the surface PE = 0

C.M. of hanging part = 0.3 m below the table

$$U_i = -m'gx = -\frac{4}{2} \times 0.6 \times 10 \times 0.30$$

$\Delta U = m'gx = 3.6J$ = Work done in putting the entire chain on the table

32. (a) Weight of body = m g = 5 N

33. (d) Here, the force of friction is 400N.

$F_{net} = (1650 - 400) = 1250N$

$$\therefore\ a = \frac{1250}{1000} = 1.25ms^{-2}$$

34. (a) $\dfrac{dm}{dt} = \dfrac{mg}{v_r} = \dfrac{5000 \times 9.8}{1000} = 49kg\,s^{-1}$

35. (a) Two external forces, F_A and F_B, act on the system and move in opposite direction. Let's arbitrarily assume that the downward direction is positive and that F_A provides downward motion while F_B provides upward motion.

$F_A = (+15\ kg)\ (9.8\ m/s^2) = 147\ N$

and $F_B = (-10\ kg)(9.8\ m/s^2) = -98\ N$

$F_{total} = F_A + F_B = 147\ N + (-98\ N) = 49\ N$

The total mass that must be set in motion is $15\ kg + 10\ kg = 25kg$

Since $F_{total} = m_{total}a,\ a = F_{total} / m_{total}$

$= 49\ N / 25\ kg \cong 2\ m/s^2$

36. (a) Momentum is always conserved. Since the skater and snowball are initially at rest, the initial momentum is zero. Therefore, the final momentum after the toss must also be zero.

$P_{skater} + P_{snowball} = 0$

or $m_{skater}v_{skater} + m_{snowball}v_{snowball} = 0$

$v_{skater} = -m_{snowball}v_{snowball} / m_{skater}$

$$= \frac{-(0.15kg)(35m/s)}{(50kg)} = -0.10m/s$$

The negative sign indicates that the momenta of the skater and the snowball are in opposite directions.

37. (b) Apply Newton's second law

$F_A = F_{AB}$, therefore :

$m_A\,a_A = (m_A + m_B)a_{AB}$ and $a_{AB} = a_A / 5$

Therefore : $m_A\,a_A = (m_A + m_B)a_A/5$ which reduces to $4\,m_A = m_B$ or 1 : 4

38. (d) Work is the product of force and distance. The easiest way to calculate the work in this pulley problem is to multiply the net force or the weight *mg* by the distance it is raised: 4 kg x 10 m/s^2 x 5 m = 200 J.

39. (d) **Given :** Mass of rocket (m) = 5000 Kg

Exhaust speed (v) = 800 m/s

Acceleration of rocket (a) = 20 m/s^2

Gravitational acceleration (g) = 10 m/s^2

We know that upward force

$F = m\,(g + a) = 5000\,(10 + 20)$

$= 5000 \times 30 = 150000\ N.$

We also know that amount of gas ejected

$$\left(\frac{dm}{dt}\right) = \frac{F}{v} = \frac{150000}{800} = 187.5\ kg/s$$

40. (d) Given $F = 600 - 2 \times 10^5\,t$

The force is zero at time t, given by

$$0 = 600 - 2 \times 10^5\,t$$

$$\Rightarrow \quad t = \frac{600}{2 \times 10^5} = 3 \times 10^{-3}\ \text{seconds}$$

$$\therefore \text{Impulse} = \int_0^t F dt = \int_0^{3\times10^{-3}} (600 - 2 \times 10^5\,t)\,dt$$

$$= \left[600t - \frac{2 \times 10^5\,t^2}{2}\right]_0^{3\times10^{-3}}$$

$$= 600 \times 3 \times 10^{-3} - 10^5(3 \times 10^{-3})^2$$

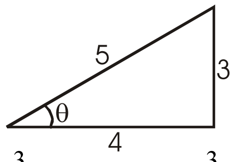
$$= 1.8 - 0.9 = 0.9Ns$$

41. (b) According to law of conservation of momentum,

$$100v = -\frac{10}{1000} \times 10 \times 800$$

ie, $v = 0.8\ ms^{-1}$.

42. (d) $\sin\theta = \frac{3}{5}$



$\therefore \tan\theta = \frac{3}{4} \Rightarrow \mu = \tan\theta = \frac{3}{4} = 0.75$

43. (b) The condition to avoid skidding, $v = \sqrt{\mu r g}$

$= \sqrt{0.6 \times 150 \times 10} = 30$ m/s.

44. (d) Limiting friction $= 0.5 \times 2 \times 10 = 10$N
The applied force is less than force of friction, therefore the force of friction is equal to the applied force.

45. (d) Applying law of conservation of momentum
Momentum of bullet = Momentum of sand-bullet system

$\frac{m}{20}v = \left(m + \frac{m}{20}\right)V = \frac{21}{20}mV$

46. (d) Here $\tan\theta = 0.8$
where θ is angle of repose

$\theta = \tan^{-1}(0.8) = 39°$

The given angle of inclination is equal to the angle of repose. So the 1 kg block has no tendency to move.

$\therefore$ $mg\sin\theta$ = force of friction

$\Rightarrow T = 0$

47. (d) For 0.5 kg block, 6 = 0.5 a

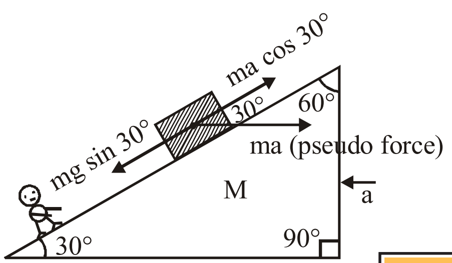
48. (b) While moving down, when the lift is accelerating the weight will be less and when the lift is decelerating the weight will be more.

49. (b) Total momentum $= 2p\hat{i} + p\hat{j}$
Magnitude of total momentum

$= \sqrt{(2p)^2 + p^2} = \sqrt{5p^2} = \sqrt{5}p$

This must be equal to the momentum of the third part.

50. (c)



$ma\cos 30° = mg\sin 30°$

$\therefore a = \frac{g}{\sqrt{3}}$

51. (d) Apparent weight when mass is falling down is given by $W' = m(g - a)$

$\therefore \quad W' = 1 \times (10 - 10) = 0$

52. (b) Reading of spring balance

$2T = \frac{4m_1m_2}{m_1 + m_2} = \frac{4 \times 5 \times 1}{6} = \frac{10}{3}$ kgf

53. (a) $a = 2v$ (given)

$\Rightarrow v\frac{dv}{ds} = 2v$

or $dv = 2\,ds$

$\int_{0.1}^{v} dv = 2[s]_0^{0.1} = 0.2$

$v - 0.1 = 0.2$

$\Rightarrow v = 0.3\text{ms}^{-1}$

54. (c) If W is the maximum weight, then
$W = 2T\cos 60°$
or $W = T = 20$N

55. (c) The acceleration of both the blocks $= \frac{15}{3x} = \frac{5}{x}$

$\therefore$ Force on $B = \frac{5}{x} \times 2x = 10$ N

56. (b) The maximum acceleration that can be given is a
$\therefore 30g = 10g + 10a$

$\Rightarrow a = 2g = 20\text{ms}^{-2}$

We know that $s = ut + \frac{1}{2}at^2$

$\therefore t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 10}{20}} = 1$ s

57. (a) Let the air resistance be F. Then
$mg + F = ma \Rightarrow F = m[a - g]$

Here $a = \frac{30}{2.5} = 12\text{ms}^{-2}$

58. (c) $F = \frac{m(v-u)}{t} = \frac{0.15(0-20)}{0.1} = 30$ N

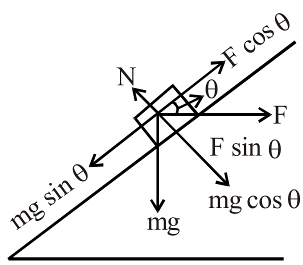
59. (c) Considering the two masses and the rope a system, then

Initial net force $= [25 - (15+5)]g = 5g$

Final net force $= [(25+5) - 15]g = 15$ g

$\Rightarrow$ (acceleration)$_{final}$ = 3 (acceleration)$_{initial}$

60. (d)



From figure $N = mg\cos\theta + F\sin\theta$

61. (b) $\mu mg = mv^2/r$ or $v = \sqrt{\mu g r}$

or $v = \sqrt{(0.25 \times 9.8 \times 20)} = 7 \text{ m/s}$

62. (a) Since water does not fall down, therefore the velocity of revolution should be just sufficient to provide centripetal acceleration at the top of vertical circle. So,

$v = \sqrt{(gr)} = \sqrt{\{10 \times (1.6)\}} = \sqrt{(16)} = 4$ m/sec.

63. (d) The speed at the highest point must be $v \geq \sqrt{rg}$

Now $v = r\omega = r(2\pi/T)$

$\therefore\ r(2\pi/T) > \sqrt{rg}$ or $T < \dfrac{2\pi r}{\sqrt{rg}} < 2\pi\sqrt{\left(\dfrac{r}{g}\right)}$

$\therefore\ T = 2\pi\sqrt{\left(\dfrac{4}{9.8}\right)} = 4\,\text{sec}$

64. (a) From figure,

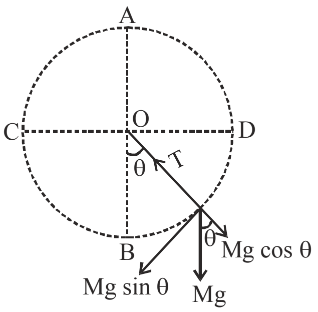
$N\sin\theta = \dfrac{mv^2}{r}$ .... (i)

$N\cos\theta = mg$ ...... (ii)

Dividing, we get

$\tan\theta = \dfrac{v^2}{rg}$ or $\theta = \tan^{-1}\dfrac{v^2}{rg}$

65. (a) In the case of a body describing a vertical circle,



$T - mg\cos\theta = \dfrac{mv^2}{l}$ $\quad T = mg\cos\theta + \dfrac{mv^2}{l}$

Tension is maximum when cos θ = +1 and velocity is maximum

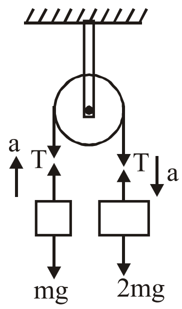
Both conditions are satisfied at θ = 0° (i.e. at lowest point B)

66. (b) Since surface (ice) is frictionless, so the centripetal force required for skating will be provided by inclination of boy with the vertical and that angle is given as

$\tan\theta = \dfrac{v^2}{rg}$ where v is speed of skating & r is radius of circle in which he moves.

67. (d) $v_{max} = \sqrt{\mu gr}$

68. (b) Let a and a' be the accelerations in both cases respectively. Then for fig (a),



(a)

$T - mg = ma$ ...(1)

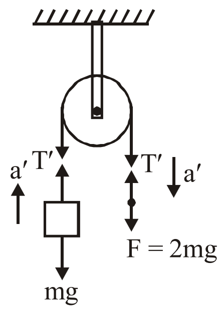
and $2mg - T = 2ma$ ...(2)

Adding (1) and (2), we get

$mg = 3ma$

$\therefore\ a = \dfrac{g}{3}$

For fig (b),



(b)

$T' - mg = ma'$ ...(3)

and $2mg - T' = 0$ ...(4)

Solving (3) and (4)

$a' = g$

$\therefore\ a = \dfrac{g}{3}$ and $a' = g$

69. (c) Linear momentum, $P = a + bt^2$

$\dfrac{dP}{dt} = 2bt$ (on differentiation)

$\therefore$ Rate of change of momentum, $\dfrac{dP}{dt} \propto t$

By 2nd law of motion, $\dfrac{dP}{dt} \propto F$

$\therefore\ F \propto t$

70. (d) For equilibrium of all 3 masses,

$T_3 = (m_1 + m_2 + m_3)a$ or

$a = \dfrac{T_3}{m_1 + m_2 + m_3}$

For equilibrium of m_1 & m_2

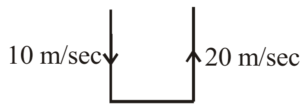
$T_2 = (m_1 + m_2).a$

or, $T_2 = \dfrac{(m_1 + m_2)T_3}{m_1 + m_2 + m_3}$

Given $m_1 = 10$ kg, $m_2 = 6$ kg, $m_3 = 4$ kg, $T_3 = 40$ N

$\therefore\ T_2 = \dfrac{(10+6).40}{10+6+4} = 32\text{N}$

71. (a) Velocity of ball after dropping it from a height of 5m



(using $v^2 = u^2 + 2gh$)

$v^2 = 0 + 2 \times 10 \times 5 \Rightarrow v = 10$ m/s

Velocity gained by ball by force exerted by bat

$0 = u^2 - 2gh$

$u^2 = 2 \times 10 \times 20$ or $u = 20$ m/s

Change in momentum = m(u + v)

= 0.4 (20 + 10) = 12 kg m/s

$F = \dfrac{\Delta P}{\Delta t}$ or $\Delta t = \dfrac{\Delta P}{F}$

$\Delta t = \dfrac{12}{100} = 0.12\,\text{sec}$

72. (d) F_1 = Force of friction between B and A

$= \mu_1 m_1 g$

$= 0.25 \times 100 \times g = 25$ g newton

F_2 = Force of friction between (A + B) and surface

$= \mu_2 m_2 g = \mu_2$ (mass of A and B) g

$= \dfrac{1}{3}(100+200)g = \dfrac{300}{3}g = 100g$ newton

$\therefore\ F = F_1 + F_2$

$= 25\,g + 100\,g = 25g = 125 \times 10$ N

$\therefore F = 1250$ N

73. (b) The rain drops falling vertically with velocity u do not affect the momentum along the horizontal track. A vector has no component in a perpendicular direction

Rain drops add to the mass of the car

Mass added in t sec = (mt) kg

Momentum is conserved along horizontal track.

Initial mass of car = M

Initial velocity of car $= v_0$

Final velocity of (car + water) = v

Mass of (car + water) after time t = (M + mt)

$\therefore$ final momentum = initial momentum

$(M + mt)v = Mv_0$

$\therefore\ v = \dfrac{Mv_0}{(M + mt)}$

74. (d) Let v_1 = velocity when height of free fall is h_1

v_2 = velocity when height of free rise is h_2

$\therefore\ v_1^2 = u^2 + 2gh_1$ for free fall

or

For free rise after impact on ground

$0 = v_2^2 - 2gh_2$ or $v_2^2 = 2gh_2$

Initial momentum = mv_1

Final momentum = mv_2

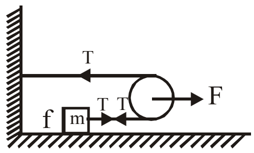
$\therefore$ Change in momentum = $m(v_1 - v_2)$

$= m(\sqrt{2gh_2} - \sqrt{gh_2})$

75. (b) T = tension is the string

$\therefore$ Applied force F = 2T

T = F/2 ...(i)



For block of mass m, force of friction due to surface f.

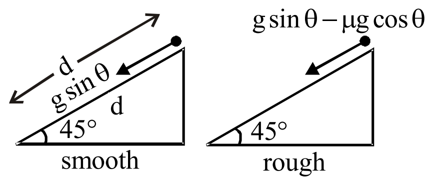
For sliding the block

T – f = force on the block = mass × acceleration

or acceleration of block $= \dfrac{T-f}{m}$. Put T from (i)

$\therefore$ Acceleration $= \dfrac{\frac{F}{2} - f}{m}$

76. (b)



When surface is smooth

When surface is rough

$d = \dfrac{1}{2}(g\sin\theta)t_1^2,\quad d = \dfrac{1}{2}(g\sin\theta - \mu g\cos\theta)t_2^2$

$t_1 = \sqrt{\dfrac{2d}{g\sin\theta}},\quad t_2 = \sqrt{\dfrac{2d}{g\sin\theta - \mu g\cos\theta}}$

According to question, $t_2 = nt_1$

$n\sqrt{\dfrac{2d}{g\sin\theta}} = \sqrt{\dfrac{2d}{g\sin\theta - \mu g\cos\theta}}$

μ, applicable here, is coefficient of kinetic friction as the block moves over the inclined plane.

$n = \dfrac{1}{\sqrt{1-\mu_k}}$ $\left(\because\ \cos 45° = \sin 45° = \dfrac{1}{\sqrt{2}}\right)$

$n^2 = \dfrac{1}{1-\mu_k}$ or $1 - \mu_k = \dfrac{1}{n^2}$

or $\mu_k = 1 - \dfrac{1}{n^2}$

77. (d) Acceleration of block while sliding down upper half $= g\sin\phi$;
retardation of block while sliding down lower half $= -(g\sin\phi - \mu g\cos\phi)$
For the block to come to rest at the bottom, acceleration in I half = retardation in II half.
$g\sin\phi = -(g\sin\phi - \mu g\cos\phi)$
$\Rightarrow \mu = 2\tan\phi$
Alternative method : According to work-energy theorem, $W = \Delta K = 0$
(Since initial and final speeds are zero)
$\therefore$ Work done by friction + Work done by gravity $= 0$

i.e., $-(\mu\, mg\cos\phi)\dfrac{\ell}{2} + mg\ell\sin\phi = 0$

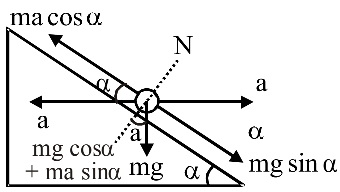
or $\dfrac{\mu}{2}\cos\phi = \sin\phi$ or $\mu = 2\tan\phi$

78. (c) Mass (m) = 0.3 kg $\Rightarrow F = m.a = -15x$

$a = -\dfrac{15}{0.3}x = \dfrac{-150}{3}x = -50x$

$a = -50 \times 0.2 = 10\,m/s^2$

79. (c) From free body diagram,



For block to remain stationary,
$mg\sin\alpha = ma\cos\alpha \Rightarrow a = g\tan\alpha$

80. (a) $v^2 - u^2 = 2as$ or $0^2 - u^2 = 2(-\mu_k g)s$

$-100^2 = 2 \times -\dfrac{1}{2} \times 10 \times s$

s = 1000 m

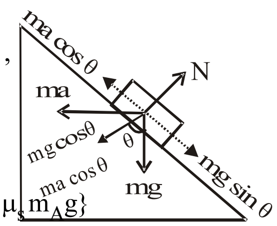
81. (b) This is a standard formula and should be memorized.

$$a = \frac{g\sin\theta}{1 + \dfrac{I}{MR^2}}$$

82. (a) $N = m\,a\sin\theta + mg\cos\theta$ ......(1)
also $m\,g\sin\theta = m\,a\cos\theta$ ......(2)
from (2) $a = g\tan\theta$

$\therefore N = mg\dfrac{\sin^2\theta}{\cos\theta} + mg\cos\theta$,

or $N = \dfrac{mg}{\cos\theta}$

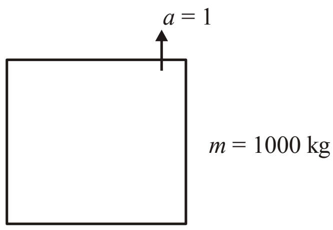


83. (a) $m_B g = \mu_s m_A g$ $\{\because m_A g = \mu_s m_A g\}$
$\Rightarrow m_B = \mu_s m_A$
or $m_B = 0.2 \times 2 = 0.4$ kg

84. (c) $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$,
$|F| = \sqrt{36 + 64 + 100} = 10\sqrt{2}$ N $\left(\because F = \sqrt{F_x^2 + F_y^2 + F_z^2}\right)$
$a = 1\ ms^{-2}$
$\because F = ma$
$\therefore m = \dfrac{10\sqrt{2}}{1} = 10\sqrt{2}$ kg

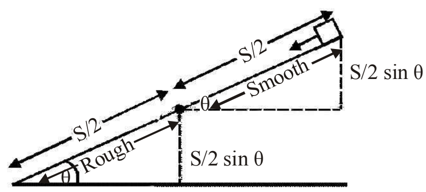
85. (d) Frictional force on the box $f = \mu mg$
$\therefore$ Acceleration in the box
$a = \mu g = 5\ ms^{-2}$
$v^2 = u^2 + 2as$
$\Rightarrow 0 = 2^2 + 2 \times (5)\,s$
$\Rightarrow s = -\dfrac{2}{5}$ w.r.t. belt
$\Rightarrow$ distance = 0.4 m

86. (c)



Total mass = (60 + 940) kg = 1000 kg
Let T be the tension in the supporting cable, then
$T - 1000g = 1000 \times 1$
$\Rightarrow T = 1000 \times 11 = 11000$ N

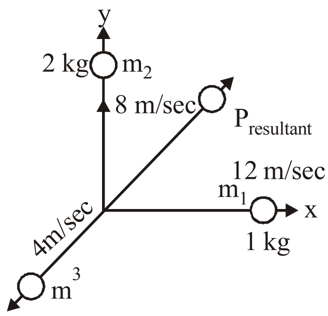
87. (b)



For upper half of inclined plane
$v^2 = u^2 + 2a\,S/2 = 2\,(g\sin\theta)\,S/2 = gS\sin\theta$
For lower half of inclined plane
$0 = u^2 + 2\,g\,(\sin\theta - \mu\cos\theta)\,S/2$
$\Rightarrow -gS\sin\theta = gS\,(\sin\theta - \mu\cos\theta)$
$\Rightarrow 2\sin\theta = \mu\cos\theta$
$\Rightarrow \mu = \dfrac{2\sin\theta}{\cos\theta} = 2\tan\theta$

88. (b) $v = \sqrt{gr} = \sqrt{10 \times 40} = 20\ m\,s^{-1}$

89. (a)



$$P_{resultant} = \sqrt{12^2 + 16^2}$$

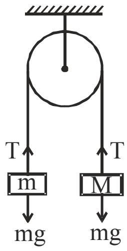
$$= \sqrt{144 + 256} = 20$$

$m_3 v_3 = 20$ (momentum of third part)

or, $m_3 = \dfrac{20}{4} = 5$ kg

90. (b) Mg – T = Ma

T – mg = ma



On solving, we get

$$a = \frac{(M - m)g}{M + m}$$

and $T = \dfrac{2Mmg}{M + m}$

91. (c) Let velocity of bullet be v. If velocity of gun is V, then mv + MV = 0

$$\Rightarrow V = -\frac{mv}{M}$$

As spring compresses by d, so

$$\frac{1}{2}kd^2 = \frac{1}{2}MV^2$$

or $\dfrac{1}{2}kd^2 = \dfrac{1}{2}M\left(\dfrac{mv}{M}\right)^2$

$$\Rightarrow v = \frac{d}{m}\sqrt{kM}$$

92. (b) Here, $l_2 = 2l_1$

As for a spring, force constant $k \propto \dfrac{1}{l}$

$$\therefore k_1 \propto \frac{1}{l_1}, k_2 \propto \frac{1}{l_2}, k \propto \frac{1}{l_1 + l_2}$$

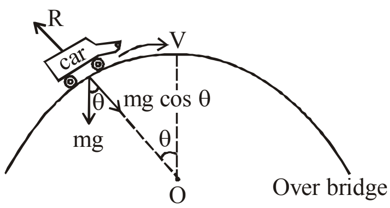
$\dfrac{k_1}{k} = \dfrac{l_1 + l_2}{l_1}$ and $\dfrac{k_2}{k} = \dfrac{l_1 + l_2}{l_2}$

or $k_1 = k\left[1 + \dfrac{l_2}{l_1}\right]$ and $k_2 = k\left[1 + \dfrac{l_1}{l_2}\right]$

$\therefore k_1 = k[1 + 2] = 3k$ [using (i)]

$k_2 = k\left[1 + \dfrac{1}{2}\right] = \dfrac{3}{2}k$ [using (i)]

93. (a) $R = mg\cos\theta - \dfrac{mV^2}{r}$



When θ decreases, cosθ increases i.e. R increases

94. (c) Impulse experienced by the body

= change in momentum = MV – (–MV) = 2MV.

95. (a) The force of friction on the chain lying on the table should be equal to the weight of the hanging chain. Let

ρ = mass per unit length of the chain

μ = coefficient of friction

l = length of the total chain

x = length of hanging chain

Now, $\mu(l - x)\rho g = x\rho g$ or $\mu(l - x) = x$

or $\mu l = (\mu + 1)x$ or $x = \mu l/(\mu + 1)$

$$\therefore x = \frac{0.25l}{(0.25 + 1)} = \frac{0.25l}{1.25} = 0.2l$$

$$\frac{x}{l} = 0.2 = 20\%$$

96. (a) Masses of the pieces are 1, 1, 3 kg. Hence

$$(1 \times 21)^2 + (1 \times 21)^2 = (3 \times V)^2$$

That is, $V = 7\sqrt{2}$ m/s

97. (c) $\dfrac{K_1}{K_2} = \dfrac{p_1^2}{m_1} \times \dfrac{m_2}{p_2^2}$ $[\because p = mv \Rightarrow K = \dfrac{p^2}{2m}]$

Hence, $\dfrac{p_1}{p_2} = \sqrt{\dfrac{M_1}{M_2}} = \sqrt{\dfrac{1}{4}} = \dfrac{1}{2}$

98. (d) Work done in moving an object against gravitational force depends only on the initial and final position of the object, not upon the path taken. But gravitational force on the body along the inclined plane is not same as that along the vertical and it varies with angle of inclination.

99. (b) On a rainy day, the roads are wet. Wetting of roads lowers the coefficient of friction between the types and the road. Therefore, grip on a road of car reduces and thus chances of skidding increases.

100. (a) The force acting on the body of mass M are its weight Mg acting vertically downward and air resistance F acting vertically upward.

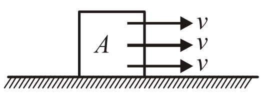
$\therefore$ Acceration of the body, $a = g - \dfrac{F}{M}$

Now M > m, therefore, the body with larger mass will have great acceleration and it will reach the ground first.

EXERCISE - 3

Exemplar Questions

1. (c) In a uniform translatory motion if all the parts of the body moves with (same velocity in same straight line, so the velocity is constant.



The situation is shown in (figure) where a body A is in unfirom translatory motion.

2. (b) According to question we have to apply Newton's second law of motion, in terms of force and change in momentum.

We know that $F = \dfrac{dp}{dt}$

As given that the meter scale is moving with uniform velocity, hence

Force $(F) = m \times 0 = 0$

No change in its velocity i.e., acceleration of it zero by Newton's second law.

Hence, net or resultant force must act on body is zero.

$\vec{\tau} = \vec{r} \times \vec{F}$, so,

As all part of the scale is moving with uniform velocity and total force is zero, hence, torque will also be zero.

3. (c) As given that,

Mass of the ball = 150 g = 0.15 kg

$\vec{u} = (3\hat{i} + 4\hat{j})$ m/s

$\vec{v} = -(3\hat{i} + 4\hat{j})$ m/s

(Δp) Change in momentum

= Final momentum – Initial momentum

$= m\vec{v} - m\vec{u}$

$= m(\vec{v} - \vec{u}) = (0.15)[-(3\hat{i} + 4\hat{j}) - (3\hat{i} + 4\hat{j})]$

$= (0.15)[-6\hat{i} - 8\hat{j}]$

$= -[0.15 \times 6\hat{i} + 0.15 \times 8\hat{j}]$

$= -[0.9\hat{i} + 1.20\hat{j}]$

$\Delta p = -[0.9\hat{i} + 1.2\hat{j}]$

Hence verifies option (c).

4. (c) From previous solution

$\Delta p = -(0.9\hat{i} + 1.2\hat{j}) = -0.9\hat{i} - 1.2\hat{j}$

Magnitude of $|\Delta p| = \sqrt{(-0.9)^2 + (-1.2)^2}$

$= \sqrt{0.81 + 1.44}$

$= \sqrt{2.25} = 1.5$ kg-m s^{-1}

Verifies the option (c).

5. (d) By Newton's second law :

$\vec{F}_{ext} = \dfrac{dp}{dt}$

As $\vec{F}_{ext}$ in law of conservation of momentum is zero.

If $F_{ext} = 0$, $dp = 0 \Rightarrow p =$ constant

Hence, momentum of a system will remain conserve if external force on the system is zero.

In case of collision between particle equal and opposite forces will act on individual particle by Newton's third law.

So, $\vec{F}_{12} = -\vec{F}_{21}$ $\quad (\because \vec{F}_{ext} = 0)$

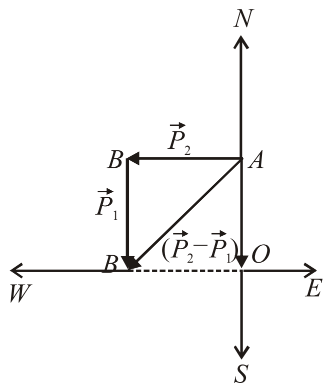
Total force on the system will be zero.

$\dfrac{d\vec{p}_{12}}{dt} = -\dfrac{d\vec{p}_{21}}{dt}$ or $d\vec{p}_{12} = -d\vec{p}_{21}$

$\Rightarrow (d\vec{p}_{12} + d\vec{p}_{21} = 0)$

So prove the law of conservation of momentum and verifies the option (d).

6. (c) Consider the adjacent diagram.



The force on player is due to rate of change of momentum. The direction of force acting on player will be the same as the direction of change in momentum.

Let $OA = P_1$ i.e., Initial momentum of player northward

$AB = P_2$ i.e., Final momentum of player towards west.

Clearly, $OB = OA + OB$

Change in momentum

$= P_2 - P_1$

$= AB - OA = AB + (-OA)$

= Clearly resultant AR will be along south-west.

So, it will be also the direction of force on player.

7. (a) As given that, mass = 2 kg

$p = 3$ m/s, $q = 4$ m/s^2, $r = 5$ m/s^3

As given equation is

$x(t) = pt + qt^2 + rt^3$

$v = \dfrac{ds(t)}{dt} = p + 2qt + 3rt^2$

$a = \dfrac{dv}{dt} = \dfrac{d^2x(t)}{dt^2} = 0 + 2q + 6rt$

$\left[\dfrac{d^2x(t)}{dt^2}\right]_{(t=2)} = 2q + 12r$

$= 2q + 12r$

$= 2 \times 4 + 12 \times 5$

$= 8 + 60 = 68$ m/s^2

Force acting on body $(\vec{F}) = ma$

$= 2 \times 68 = 136$ N.

8. (b) As given that mass $= m = 5$ kg

Acting force $= \vec{F} = (-3\hat{i} + 4\hat{j})$ N

Initial velocity at $t = 0$, $\vec{u} = (6\hat{i} - 12\hat{j})$ m/s

Retardation, $\hat{a} = \frac{\vec{F}}{m} = \left(-\frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}\right)$ m/s^2

As the final velocity along Y-component only. So its x-component must be zero.

From $v_x = u_x + a_x t$, for X-component only,

$$0 = 6\hat{i} + \frac{-3\hat{i}}{5}t$$

$$\frac{3\hat{i}}{5}(t) = 6\hat{i}$$

$$t = \frac{5 \times 6}{3} = 10 \text{ s}$$

$t = 10$ sec. Hence verifies the option (b).

9. (b) As given that mass of the car $= m$

As car starts from rest, $u = 0$

Velocity acquired along east $(\vec{v}) = v\hat{i}$

Time $(t) = 2$ s.

We know that, $v = u + at$

$\Rightarrow v\hat{i} = 0 + a \times 2$

$\Rightarrow \vec{a} = \frac{v}{2}\hat{i}$

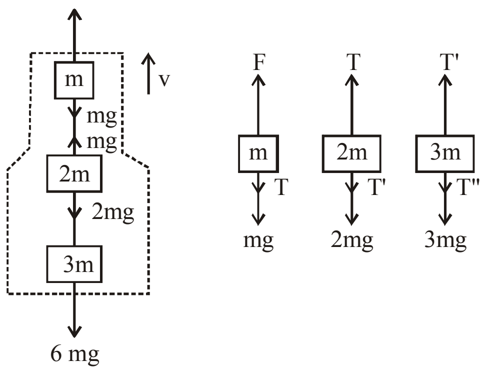
(Force, by engine is internal force)

$\vec{F} = m\vec{a} = \frac{mv}{2}\hat{i}$

Hence, force acting on the car is $\frac{mv}{2}$ towards east due to force of friction is $\frac{mv}{2}\hat{i}$ which moves the car in eastward direction. Hence, force by engine is internal force.

NEET/AIPMT (2013-2017) Questions

10. (d)



From figure

$F = 6$ mg,

As speed is constant, acceleration a = 0

$\therefore$ 6 mg = 6ma = 0, F = 6 mg

$\therefore$ T = 5 mg , T$'$ = 3 mg

T$''$ = 0

F_{net} on block of mass 2 m

$= T - T' - 2\text{ mg} = 0$

ALTERNATE:

$\because$ v = constant

so, a = 0, Hence, $F_{net} = ma = 0$

11. (d) Given; speed = 10 m/s; radius $r = 10$ m

Angle made by the wire with the vertical

$$\tan\theta = \frac{v^2}{rg} = \frac{10^2}{10 \times 10} = 1$$

$$\Rightarrow \theta = 45° = \frac{\pi}{4}$$

12. (a) Let upthrust of air be F_a then

For downward motion of balloon

$F_a = mg - ma$

$mg - F_a = ma$

For upward motion

$F_a - (m - \Delta m)g = (m - \Delta m)a$

Therefore $\Delta m = \frac{2ma}{g + a}$

13. (c) Change in momentum,

$\Delta p = \int F dt$

= Area of F-t graph

= ar of Δ – ar of $\square$ + ar of $\square$

$= \frac{1}{2} \times 2 \times 6 - 3 \times 2 + 4 \times 3 = 12$ N-s

14. (c) Acceleration

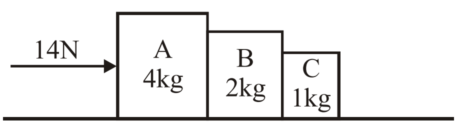
$$= \frac{\text{Net force in the direction of motion}}{\text{Total mass of system}}$$

$$= \frac{m_1 g - \mu(m_2 + m_3)g}{m_1 + m_2 + m_3} = \frac{g}{3}(1 - 2\mu)$$

$(\because m_1 = m_2 = m_3 = m \text{ given})$

15. (a) Acceleration of system $a = \frac{F_{net}}{M_{total}}$

$$= \frac{14}{4 + 2 + 1} = \frac{14}{7} = 2 \text{ m/s}^2$$



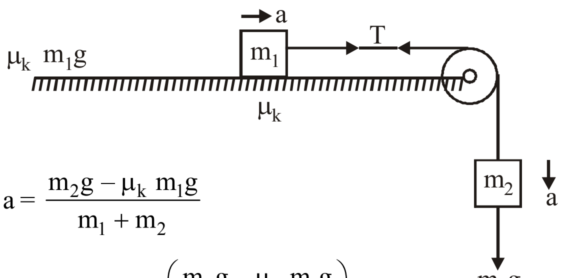
The contact force between A and B

$= (m_B + m_C) \times a = (2 + 1) \times 2 = 6N$

16. (b) For the motion of both the blocks

$m_1 a = T - \mu_k m_1 g$

$m_2 g - T = m_2 a$



$$a = \frac{m_2g - \mu_k\ m_1g}{m_1 + m_2}$$

$$m_2g - T = (m_2)\left(\frac{m_2g - \mu_k\ m_1g}{m_1 + m_2}\right)$$

solving we get tension in the string

$$T = \frac{m_1m_2g\,(1+\mu_k)\,g}{m_1 + m_2}$$

17. (d) According to question, two stones experience same centripetal force

i.e. $F_{C_1} = F_{C_2}$

or, $\frac{mv_1^2}{r} = \frac{2mv_2^2}{(r/2)}$ or, $V_1^2 = 4V_2^2$

So, $V_1 = 2V_2$ i.e., $n = 2$

18. (a) Coefficient of static friction,

$$\mu_s = \tan 30° = \frac{1}{\sqrt{3}} = 0.577 \cong 0.6$$

$$S = ut + \frac{1}{2}at^2$$

$$4 = \frac{1}{2}a(4)^2 \Rightarrow a = \frac{1}{2} = 0.5$$

[$\because$ s = 4m and t = 4s given]

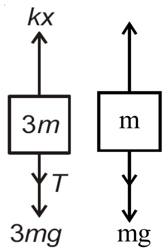
$a = g\sin\theta - \mu_k(g)\cos\theta$

$$\Rightarrow \mu_k = \frac{0.9}{\sqrt{3}} = 0.5$$

19. (d) To complete the loop a body must enter a vertical loop of radius R with the minimum velocity $v = \sqrt{5gR}$.

20. (d) Net force on particle in uniform circular motion is centripetal force $\left(\frac{mv^2}{l}\right)$ which is provided by tension in string so the net force will be equal to tension i.e., T.

21. (a)



Before cutting the string

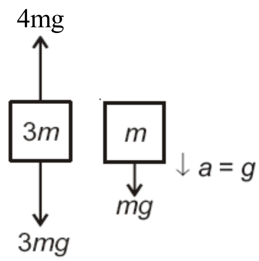
$kx = T + 3\,mg$ ...(i)

$T = mg$ ...(ii)

$\Rightarrow \quad kx = 4mg$

After cutting the string $T = 0$

$$a_A = \frac{4mg - 3mg}{3m}$$



$$a_A = \frac{g}{3} \uparrow$$

and $a_B = \frac{mg}{m} = g \downarrow$