DPP - Daily Practice Problems

Chapter-wise Sheets

Date : ________ **Start Time :** ________ **End Time :** ________

PHYSICS

CP07

SYLLABUS : Gravitation

Max. Marks : 180 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 45 MCQs. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

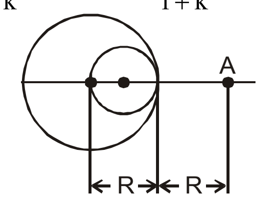
1. The radius of a planet is $1/4^{th}$ of R_e and its acc. due to gravity is 2g. What would be the value of escape velocity on the planet, if escape velocity on earth is v_e.

(a) $\frac{v_e}{\sqrt{2}}$ (b) $v_e\sqrt{2}$ (c) $2\ v_e$ (d) $\frac{v_e}{2}$

2. A projectile is fired vertically from the Earth with a velocity kv_e where v_e is the escape velocity and k is a constant less than unity. The maximum height to which projectile rises, as measured from the centre of Earth, is

(a) $\frac{R}{k}$ (b) $\frac{R}{k-1}$ (c) $\frac{R}{1-k^2}$ (d) $\frac{R}{1+k^2}$

3. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at A, distance 2R from the centre of the sphere.



A spherical cavity of radius R/2 is now made in the sphere as shown in the figure. The sphere with cavity now applies a gravitational force F_2 on the same particle placed at A. The ratio F_2/F_1 will be

(a) 1/2 (b) 3 (c) 7 (d) 1/9

4. A geostationary satellite is orbiting the earth at a height of 5R above that surface of the earth, R being the radius of the earth. The time period of another satellite in hours at a height of 2R from the surface of the earth is :

(a) 5 (b) 10 (c) $6\sqrt{2}$ (d) $\frac{6}{\sqrt{2}}$

5. A satellite of mass m is orbiting around the earth in a circular orbit with a velocity v. What will be its total energy?

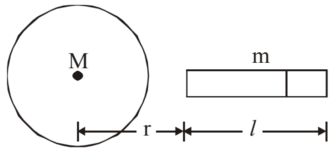
(a) $(3/4)\ mv^2$ (b) $(1/2)\ mv^2$

(c) mv^2 (d) $-(1/2)m\ v^2$

RESPONSE GRID					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ

Space for Rough Work

6. The gravitational force of attraction between a uniform sphere of mass M and a uniform rod of length l and mass m oriented as shown is



(a) $\frac{GMm}{r(r+l)}$ (b) $\frac{GM}{r^2}$ (c) Mmr^2+l (d) $(r^2+l)mM$

7. If the gravitational force between two objects were proportional to 1/R (and not as $1/R^2$) where R is separation between them, then a particle in circular orbit under such a force would have its orbital speed v proportional to

(a) $1/R^2$ (b) R^0 (c) R^1 (d) 1/R

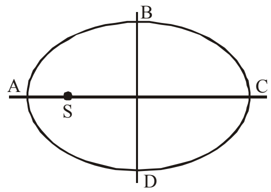
8. A satellite of mass m revolves around the earth of radius R at a height 'x' from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

(a) $\frac{gR^2}{R+x}$ (b) $\frac{gR}{R-x}$ (c) gx (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

9. A body is projected up with a velocity equal to 3/4th of the escape velocity from the surface of the earth. The height it reaches from the centre of the earth is (Radius of the earth = R)

(a) $\frac{10R}{9}$ (b) $\frac{16R}{7}$ (c) $\frac{9R}{8}$ (d) $\frac{10R}{3}$

10. A Planet is revolving around the sun.



Which of the following is correct option?

(a) The time taken in travelling DAB is less than that for BCD

(b) The time taken in travelling DAB is greater than that for BCD

(c) The time taken in travelling CDA is less than that for ABC

(d) The time taken in travelling CDA is greater than that for ABC

11. The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2m on the surface of A. What is the height of jump by the same person on the planet B?

(a) $\frac{2}{3}$m (b) $\frac{2}{9}$m (c) 18m (d) 6m

12. If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero, then the satellite will

(a) continue to move in its orbit with same speed

(b) move tangentially to the original orbit with same speed

(c) become stationary in its orbit

(d) move towards the earth

13. Mass M is divided into two parts xM and $(1-x)$M. For a given separation, the value of x for which the gravitational attraction between the two pieces becomes maximum is

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) 1 (d) 2

14. The potential energy of a satellite, having mass m and rotating at a height of 6.4×10^6 m from the earth surface, is

(a) $-mgR_e$ (b) $-0.67\,mgR_e$

(c) $-0.5\,mgR_e$ (d) $-0.33\,mgR_e$

15. If the radius of the earth were to shrink by 1%, with its mass remaining the same, the acceleration due to gravity on the earth's surface would

(a) decrease by 1% (b) decrease by 2%

(c) increase by 1% (d) increase by 2%

16. Suppose the law of gravitational attraction suddenly changes and becomes an inverse cube law i.e. $F \propto \frac{1}{r^3}$, but still remaining a central force. Then

(a) Kepler's law of area still holds

(b) Kepler's law of period still holds

(c) Kepler's law of area and period still holds

(d) neither the law of area nor the law of period still holds

17. Four equal masses (each of mass M) are placed at the corners of a square of side a. The escape velocity of a body from the centre O of the square is

(a) $4\sqrt{\frac{2GM}{a}}$ (b) $\sqrt{\frac{8\sqrt{2}GM}{a}}$ (c) $\frac{4GM}{a}$ (d) $\sqrt{\frac{4\sqrt{2}GM}{a}}$

18. If the gravitational force had varied as $r^{-5/2}$ instead of r^{-2}; the potential energy of a particle at a distance 'r' from the centre of the earth would be directly proportional to

(a) r^{-1} (b) r^{-2} (c) $r^{-3/2}$ (d) $r^{-5/2}$

19. A particle of mass 'm' is kept at rest at a height 3R from the surface of earth, where 'R' is radius of earth and 'M' is mass of earth. The minimum speed with which it should be projected, so that it does not return back, is (g is acceleration due to gravity on the surface of earth)

(a) $\left(\frac{GM}{R}\right)^{\frac{1}{2}}$ (b) $\left(\frac{GM}{2R}\right)^{\frac{1}{2}}$ (c) $\left(\frac{gR}{4}\right)^{\frac{1}{2}}$ (d) $\left(\frac{2g}{4}\right)^{\frac{1}{2}}$

RESPONSE GRID					
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ
	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ
	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	

Space for Rough Work

20. The ratio between the values of acceleration due to gravity at a height 1 km above and at a depth of 1 km below the Earth's surface is (radius of Earth is R)

(a) $\frac{R-2}{R-1}$ (b) $\frac{R}{R-1}$ (c) $\frac{R-2}{R}$ (d) 1

21. The weight of an object in the coal mine, sea level and at the top of the mountain, are respectively W_1, W_2 and W_3 then

(a) $W_1 < W_2 > W_3$ (b) $W_1 = W_2 = W_3$
(c) $W_1 < W_2 < W_3$ (d) $W_1 > W_2 > W_3$

22. The period of moon's rotation around the earth is nearly 29 days. If moon's mass were 2 fold its present value and all other things remain unchanged, the period of moon's rotation would be nearly

(a) $29\sqrt{2}$ days (b) $29/\sqrt{2}$ days
(c) 29×2 days (d) 29 days

23. The mean radius of earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g. What will be the radius of the orbit of a geostationary satellite ?

(a) $(R^2g/\omega^2)^{1/3}$ (b) $(Rg/\omega^2)^{1/3}$
(c) $(R^2\omega^2/g)^{1/3}$ (d) $(R^2g/\omega)^{1/3}$

24. In order to make the effective acceleration due to gravity equal to zero at the equator, the angular velocity of rotation of the earth about its axis should be ($g = 10\ ms^{-2}$ and radius of earth is 64000 km)

(a) Zero (b) $\frac{1}{800}$ rad sec $^{-1}$
(c) $\frac{1}{80}$ rad sec^{-1} (d) $\frac{1}{8}$ rad sec $^{-1}$

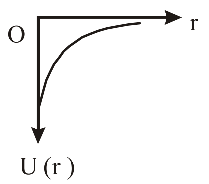
25. A body weighs 72 N on the surface of the earth. What is the gravitational force on it due to earth at a height equal to half the radius of the earth from the surface?

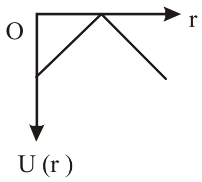
(a) 32 N (b) 28 N (c) 16 N (d) 72 N

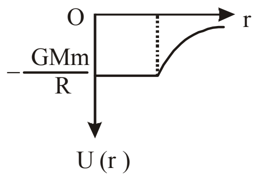
26. A body weighs W newton at the surface of the earth. Its weight at a height equal to half the radius of the earth, will be

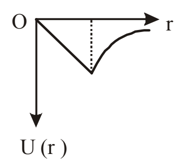
(a) $\frac{W}{2}$ (b) $\frac{2W}{3}$ (c) $\frac{4W}{9}$ (d) $\frac{8W}{27}$

27. A shell of mass M and radius R has a point mass m placed at a distance r from its centre. The graph of gravitational potential energy U(r) vs distance r will be

(a)


(b)


(c)


(d)


28. The largest and the shortest distance of the earth from the sun are r_1 and r_2. Its distance from the sun when it is at perpendicular to the major-axis of the orbit drawn from the sun

(a) $(r_1 + r_2)/4$ (b) $(r_1 + r_2)/(r_1 - r_2)$
(c) $2r_1 r_2/(r_1 + r_2)$ (d) $(r_1 + r_2)/3$

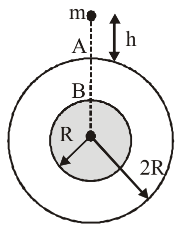
29. A planet is moving in an elliptical orbit around the sun. If T, V, E and L stand respectively for its kinetic energy, gravitational potential energy, total energy and magnitude of angular momentum about the centre of force, then which of the following is correct ?

(a) T is conserved
(b) V is always positive
(c) E is always negative
(d) L is conserved but direction of vector L changes continuously

30. The earth is assumed to be sphere of radius R. A platform is arranged at a height R from the surface of Earth. The escape velocity of a body from this platform is kv, where v is its escape velocity from the surface of the earth. The value of k is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\sqrt{2}$

31. A solid sphere of mass M and radius R is surrounded by a spherical shell of same mass M and radius 2R as shown. A small particle of mass m is released from rest from a height h [<< R] above the shell. There is a hole in the shell.



What time will it enter the hole at A ?

(a) $2\sqrt{\frac{hR^2}{GM}}$ (b) $\sqrt{\frac{2hR^2}{GM}}$
(c) $\sqrt{\frac{hR^2}{GM}}$ (d) $\sqrt{\frac{3hR^2}{GM}}$

32. A body starts from rest from a point distance R_0 from the centre of the earth. The velocity acquired by the body when it reaches the surface of the earth will be (R represents radius of the earth).

(a) $2GM\left(\frac{1}{R}-\frac{1}{R_0}\right)$ (b) $\sqrt{2GM\left(\frac{1}{R_0}-\frac{1}{R}\right)}$
(c) $GM\left(\frac{1}{R}-\frac{1}{R_0}\right)$ (d) $2GM\sqrt{\left(\frac{1}{R}-\frac{1}{R_0}\right)}$

RESPONSE GRID					
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ
	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ
	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ		

Space for Rough Work

33. A satellite of mass M is moving in a circle of radius R under a centripetal force given by $(-k/R^2)$, where k is a constant. Then

(a) The kinetic energy of the particle is $\frac{k}{12}R$

(b) The total energy of the particle is $\left(-\frac{k}{2R}\right)$

(c) The kinetic energy of the particle is $\left(-\frac{k}{R}\right)$

(d) The potential energy of the particle is $\left(\frac{k}{2R}\right)$

34. The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct?

(a) $d=\frac{3h}{2}$ (b) $d=\frac{h}{2}$ (c) $d=h$ (d) $d=2h$

35. Two identical geostationary satellites are moving with equal speeds in the same orbit but their sense of rotation brings them on a collision course. The debris will

(a) fall down
(b) move up
(c) begin to move from east to west in the same orbit
(d) begin to move from west to east in the same orbit

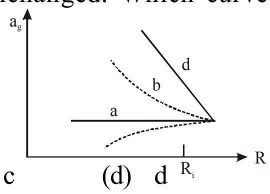
36. A diametrical tunnel is dug across the Earth. A ball is dropped into the tunnel from one side. The velocity of the ball when it reaches the centre of the Earth is .... (Given : gravitational potential at the centre of Earth $= -\frac{3}{2}\frac{GM}{R}$)

(a) $\sqrt{R}$ (b) $\sqrt{gR}$ (c) $\sqrt{2.5gR}$ (d) $\sqrt{7.1gR}$

37. A satellite revolves around the earth of radius R in a circular orbit of radius 3R. The percentage increase in energy required to lift it to an orbit of radius 5R is

(a) 10% (b) 20% (c) 30% (d) 40%

38. A (nonrotating) star collapses onto itself from an initial radius R_i with its mass remaining unchanged. Which curve in figure best gives the gravitational acceleration a_g on the surface of the star as a function of the radius of the star during the collapse



(a) a (b) b (c) c (d) d

39. If the earth is treated as a sphere of radius R and mass M; its angular momentum about the axis of its rotation with period T, is

(a) $\frac{\pi MR^3}{T}$ (b) $\frac{MR^2\pi}{T}$ (c) $\frac{2\pi MR^2}{5T}$ (d) $\frac{4\pi MR^2}{5T}$

40. A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius 1.01 R. The period of second satellite is larger than the first one by approximately

(a) 0.5% (b) 1.0% (c) 1.5% (d) 3.0%

41. A uniform spherical shell gradually shrinks maintaining its shape. The gravitational potential at the centre

(a) increases (b) decreases
(c) remains constant (d) cannot say

42. The depth d at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value at the surface of the earth, is [R = radius of the earth]

(a) $\frac{R}{n}$ (b) $R\left(\frac{n-1}{n}\right)$ (c) $\frac{R}{n^2}$ (d) $R\left(\frac{n}{n+1}\right)$

43. Radius of moon is 1/4 times that of earth and mass is 1/81 times that of earth. The point at which gravitational field due to earth becomes equal and opposite to that of moon, is (Distance between centres of earth and moon is 60R, where R is radius of earth)

(a) 5.75 R from centre of moon
(b) 16 R from surface of moon
(c) 53 R from centre of earth
(d) 54 R from centre of earth

44. If earth is supposed to be a sphere of radius R, if g_{30} is value of acceleration due to gravity at lattitude of 30° and g at the equator, the value of $g - g_{30}$ is

(a) $\frac{1}{4}\omega^2 R$ (b) $\frac{3}{4}\omega^2 R$ (c) $\omega^2 R$ (d) $\frac{1}{2}\omega^2 R$

45. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R?

(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$ (c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{2R}$

RESPONSE GRID					
	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ
	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ	42. ⓐⓑⓒⓓ
	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ	45. ⓐⓑⓒⓓ		

DAILY PRACTICE PROBLEM DPP CHAPTERWISE CP07 - PHYSICS

Total Questions	45	Total Marks	180
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	45	Qualifying Score	60
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

Space for Rough Work

1. (a) The escape velocity on the earth is defined as

$$v_e = \sqrt{2g_eR_e}$$

Where R_e & g_e are the radius & acceleration due to gravity of earth.
Now for planet $g_P=2g_e$, $R_P=R_e/4$

So $v_P = \sqrt{2g_PR_P} = \sqrt{2\times 2g_e \times R_e/4} = \frac{v_e}{\sqrt{2}}$

2. (c) Applying conservation of energy principle, we get

$$\frac{1}{2}mk^2v_e^2 - \frac{GMm}{R} = -\frac{GMm}{r}$$

$$\Rightarrow \frac{1}{2}mk^2\frac{2GM}{R} - \frac{GMm}{R} = -\frac{GMm}{r}$$

$$\Rightarrow \frac{k^2}{R} - \frac{1}{R} = -\frac{1}{r} \Rightarrow \frac{1}{r} = \frac{1}{R} - \frac{k^2}{R}$$

$$\Rightarrow \frac{1}{r} = \frac{1}{R}(1-k^2) \Rightarrow r = \frac{R}{1-k^2}$$

3. (d) The gravitational force due to the whole sphere at A point is

$F_1 = \frac{GM_em_o}{(2R)^2}$, where m_0 is the assumed rest mass at point A.
In the second case, when we made a cavity of radius (R/2), then gravitational force at point A is

$F_2 = \frac{GM_em_o}{(R+R/2)^2}$ $\quad\therefore F_2/F_1 = 1/9$

4. (c) According to Kepler's law of period $T^2 \propto R^3$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} = \frac{(6R)^3}{(3R)^3} = 8$$

$$\frac{24\times 24}{T_2^2} = 8$$

$$T_2^2 = \frac{24\times 24}{8} = 72 = 36\times 2$$

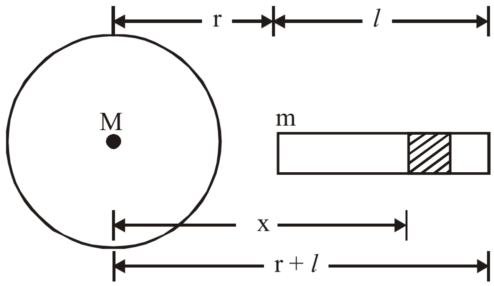
$$T_2 = 6\sqrt{2}$$

5. (d) Total energy $= -\text{K E} = \frac{\text{PE}}{2}$

$\text{K.E} = \frac{1}{2}mv^2$

$\therefore$ Total energy $= -\frac{1}{2}mv^2$

6. (a) The force of attraction between sphere and shaded position $dF = GM\frac{\left(\frac{m}{l}dx\right)}{x^2}$



$$F = \int_r^{r+l}\frac{GMm}{lx^2}dx = \frac{GMm}{l}\int_r^{r+l}\frac{1}{x^2}dx$$

$$= \frac{GMm}{l}\int_r^{r+l}x^{-2}dx = \frac{GMm}{l}\left[\frac{x^{-2+1}}{-2+1}\right]_r^{r+l}$$

$$= -\frac{GMm}{l}\left[x^{-1}\right]_r^{r+l} = -\frac{GMm}{l}\left[\frac{1}{x}\right]_r^{r+l} = \frac{GMm}{r(r+l)}$$

7. (b) $F = \frac{k}{R} = \frac{Mv^2}{R}$. Hence $v \propto R^0$

8. (d) $\frac{mv^2}{(R+x)} = \frac{GmM}{(R+x)^2}$ also $g = \frac{GM}{R^2}$

$$\therefore \frac{mv^2}{(R+x)} = m\left(\frac{GM}{R^2}\right)\frac{R^2}{(R+x)^2}$$

$$\therefore \frac{mv^2}{(R+x)} = mg\frac{R^2}{(R+x)^2}$$

$$\therefore v^2 = \frac{gR^2}{R+x} \Rightarrow v = \left(\frac{gR^2}{R+x}\right)^{1/2}$$

9. (b) $v = \frac{3}{4}v_e$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{3}{4}v_e\right)^2 = \frac{9}{32}mv_e^2$$

$$= \frac{9}{32}m\left(\frac{2GM}{R}\right)$$

$$\text{K.E.} = \frac{9}{16}\frac{GMm}{R}\ ;\ \text{P.E.} = -\frac{GMm}{R}$$

Total energy $= \text{K.E.} + \text{P.E.} = -\frac{7}{16}\frac{GMm}{R}$

Let the height above the surface of earth be h, then

$$\text{P.E.} = -\frac{GMm}{h}$$

$$-\frac{7}{16}\frac{GMm}{R} = -\frac{GMm}{h} \quad \therefore\ h = \frac{16R}{7}$$

10. (a) When closer to the sun, velocity of planet will be greater. So time taken in covering a given area will be less.

11. (c) Applying conservation of total mechanical energy principle

$$\frac{1}{2}mv^2 = mg_A h_A = mg_B h_B$$

$$\Rightarrow g_A h_A = g_B h_B$$

$$\Rightarrow h_B = \left(\frac{g_A}{g_B}\right)h_A = 9 \times 2 = 18\text{ m}$$

12. (b) Due to inertia of motion it will move tangentially to the original orbit with same velocity.

13. (a) $F \propto x\text{M} \times (1-x)\text{M} = x\text{M}^2(1-x)$

For maximum force, $\frac{dF}{dx} = 0$

$$\Rightarrow \frac{dF}{dx} = \text{M}^2 - 2x\text{M}^2 = 0 \Rightarrow x = 1/2$$

14. (c) Mass of the satellite = m and height of satellite from earth (h) = 6.4×10^6 m.
We know that gravitational potential energy of the satellite at height

$$\text{h} = -\frac{\text{GM}_\text{e}\text{m}}{\text{R}_\text{e}+\text{h}} = -\frac{\text{gR}_\text{e}^2\text{m}}{2\text{R}_\text{e}} = -\frac{\text{gR}_\text{e}\text{m}}{2} = -0.5\,\text{mgR}_\text{e}$$

(where, $\text{GM}_\text{e} = \text{gR}_\text{e}^2$ and $\text{h} = \text{R}_\text{e}$)

15. (d) Acceleration due to gravity on earth's surface

$$g = G\frac{M}{R^2}$$

This implies that as radius decreases, the acceleration due to gravity increases.

$$\frac{\Delta g}{g} = -2\frac{\Delta R}{R} \quad \text{But } \frac{\Delta R}{R} = -1\%$$

('–' sign is due to shrinking of earth)

$$\therefore \quad \frac{\Delta g}{g} = -2 \times (-1\%) = 2\%$$

16. (a) According to kepler's law of area

$$\frac{dA}{dt} = \frac{L}{2m}$$

For central forces, torque = 0

$\therefore$ L = constant

$$\therefore \quad \frac{dA}{dt} = \text{constant}$$

17. (b) Potential energy of particle at the centre of square

$$= -4\left(\frac{\text{GMm}}{\frac{a}{\sqrt{2}}}\right)$$

$$\therefore -4\left(\frac{\text{GMm}}{a/\sqrt{2}}\right) + \frac{1}{2}\text{mv}^2 = 0 \Rightarrow \text{v}^2 = \frac{8\sqrt{2}\,\text{GM}}{a}$$

18. (c) The potential energy for a conservative force is defined as

$$\text{F} = \frac{-\text{dU}}{\text{dr}} \text{ or } \text{U} = -\int_\infty^r \vec{\text{F}}.\vec{\text{dr}} \quad \ldots\ldots\text{(i)}$$

$$\text{or } \text{U}_\text{r} = \int_\infty^r \frac{\text{GM}_1\text{M}_2}{\text{r}^2}\text{dr} = \frac{-\text{GM}_1\text{M}_2}{\text{r}} \quad \ldots\ldots\text{(ii)}$$

$(\because \text{U}_\infty = 0)$

If we bring the mass from the infinity to the centre of earth, then we obtain work, 'so it has negative (gravitational force do work on the object) sign & potential energy decreases. But if we bring the mass from the surface of earth to infinite, then we must do work against gravitational force & potential energy of the mass increases.

Now in equation (i) if $\text{F} = \frac{\text{GM}_1\text{M}_2}{\text{r}^{5/2}}$ instead of

$\text{F} = \frac{\text{GM}_1\text{M}_2}{\text{r}^2}$ then

$$\text{U}_\text{r} = \int_\infty^r \frac{\text{GM}_1\text{M}_2}{\text{r}^{5/2}}\text{dr} = \frac{-2}{3}\frac{\text{GM}_1\text{M}_2}{\text{r}^{3/2}}$$

$$\Rightarrow \text{U}_\text{r} \propto \frac{1}{\text{r}^{+3/2}}$$

19. (b) As we know, the minimum speed with which a body is projected so that it does not return back is called escape speed.

$$V_e = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{4R}}$$

$$= \left(\frac{GM}{2R}\right)^{\frac{1}{2}} \quad (\because h = 3R)$$

20. (a) Acceleration due to gravity at a height h above the earth's surface is

$$\text{g}_\text{h} = \text{g}\left(1 - \frac{2\text{h}}{\text{R}}\right)$$

Acceleration due to gravity at a depth d below the earth's surface is

$$\text{g}_\text{d} = \text{g}\left(1 - \frac{\text{d}}{\text{R}}\right)$$

$$\text{Now, } \frac{\text{g}_\text{h}}{\text{g}_\text{d}} = \frac{\left(1 - \frac{2\text{h}}{\text{R}}\right)}{\left(1 - \frac{\text{d}}{\text{R}}\right)} = \frac{(\text{R} - 2\text{h})}{(\text{R} - \text{d})}$$

As h = 1 km, d = 1 km

$$\therefore \quad \frac{\text{g}_\text{h}}{\text{g}_\text{d}} = \frac{\text{R} - 2}{\text{R} - 1}$$

21. (a) At the surface of earth, the value of $g = 9.8 m/sec^2$. If we go towards the centre of earth or we go above the surface of earth, then in both the cases the value of g decreases.

Hence $W_1 = mg_{mine}$, $W_2 = mg_{sea\ level}$, $W_3 = mg_{moun}$

So $W_1 < W_2 > W_3$ (g at the sea level = g at the suface of earth)

22. (d) Time period does not depend upon the mass of satellite

23. (a) $T = \dfrac{2\pi r}{v_0} = \dfrac{2\pi r}{(gR^2/r)^{1/2}} = \dfrac{2\pi r^{3/2}}{\sqrt{gR^2}} = \dfrac{2\pi}{\omega}$

Hence, $r^{3/2} = \dfrac{\sqrt{gR^2}}{\omega}$ or $r^3 = \dfrac{gR^2}{\omega^2}$

or, $r = (gR^2/\omega^2)^{1/3}$

24. (b) $g' = g - \omega^2 R\cos^2\lambda$

To make effective acceleration due to gravity zero at equator $\lambda = 0$ and $g' = 0$

$\therefore 0 = g - \omega^2 R \Rightarrow \omega = \sqrt{\dfrac{g}{R}} = \dfrac{1}{800}\dfrac{rad}{s}$

25. (a) $mg = 72$ N (body weight on the surface)

$g = \dfrac{GM}{R^2}$

At a height $H = \dfrac{R}{2}$,

$g' = \dfrac{GM}{\left(R + \dfrac{R}{2}\right)^2} = \dfrac{4}{9}\dfrac{GM}{R^2}$

Body weight at height $H = \dfrac{R}{2}$,

$mg' = m \times \dfrac{4}{9}\dfrac{GM}{R^2}$

$= m \times \dfrac{4}{9} \times g = \dfrac{4}{9}mg$

$= \dfrac{4}{9} \times 72 = 32$ N

26. (c) At a height h,

$g' = g\dfrac{R^2}{(R+h)^2} \Rightarrow mg' = mg\left(\dfrac{R}{R+h}\right)^2$

$\Rightarrow W' = W\left(\dfrac{R}{R+h}\right)^2$

Here, $h = R/2$

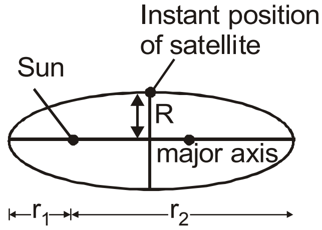
$\therefore W' = \dfrac{4}{9}W$

27. (c) Gravitational P.E. = m × gravitational potential

U = mV, so the graph of U will be same as that of V for a spherical shell.

28. (c) Applying the properties of ellipse, we have

$\dfrac{2}{R} = \dfrac{1}{r_1} + \dfrac{1}{r_2} = \dfrac{r_1 + r_2}{r_1 r_2}$



$R = \dfrac{2 r_1 r_2}{r_1 + r_2}$

29. (c) In a circular or elliptical orbital motion, torque is always acting parallel to displacement or velocity. So, angular momentum is conserved. In attractive field, potential energy is negative. Kinetic energy changes as velocity increase when distance is less. So, option (c) is correct.

30. (a) Here, $v = \sqrt{\dfrac{2GM}{R}}$ and $kv = \sqrt{\dfrac{2GM}{R+R}}$.

Solving $k = \dfrac{1}{\sqrt{2}}$

31. (a) $g = \dfrac{G(2M)}{(2R)^2} = \dfrac{GM}{2R^2}$

From $h = \dfrac{1}{2}gt^2$ $[\because U = 0]$

$t = \sqrt{\dfrac{2h}{g}} = 2\sqrt{\dfrac{hR^2}{GM}}$

32. (b) $P.E. = \int_{R_0}^{R} \dfrac{GMm}{r^2}dr = -GMm\left[\dfrac{1}{R} - \dfrac{1}{R_0}\right]$

The K.E. acquired by the body at the surface $= \dfrac{1}{2}mv^2$

$\therefore \dfrac{1}{2}mv^2 = -GMm\left[\dfrac{1}{R} - \dfrac{1}{R_0}\right]$

$v = \sqrt{2GM\left(\dfrac{1}{R_0} - \dfrac{1}{R}\right)}$

33. (b) $\dfrac{mv^2}{R} = \dfrac{k}{R^2}$ or $mv^2 = \dfrac{k}{R}$

Kinetic energy $= \dfrac{1}{2}mv^2 = \dfrac{k}{2R}$

In case of satellites P.E = – 2 K.E

and T.E = P. E + K. E

Total energy $= \dfrac{k}{2R} - \dfrac{k}{R} = -\dfrac{k}{2R}$

34. (d) Variation of g with altitude is,

$g_h = g\left[1 - \dfrac{2h}{R}\right]$;

variation of g with depth is,

$$g_d = g\left[1-\frac{d}{R}\right]$$

Equating g_h and g_d, we get $d = 2h$

35. (a) The total momentum will be zero and hence velocity will be zero just after collisiion. The pull of earth will make it fall down.

36. (b) Loss in potential energy = Gain in kinetic energy

$$-\frac{GMm}{R}-\left(-\frac{3}{2}\frac{GMm}{R}\right)=\frac{1}{2}mv^2$$

$$\Rightarrow \frac{GMm}{2R}=\frac{1}{2}mv^2 \Rightarrow v=\sqrt{\frac{GM}{R}}=\sqrt{gR}$$

37. (d)

38. (b) $g \propto \frac{1}{R^2}$

R decreasing g increase hence, curve b represents correct variation.

39. (d) Angular momentum, $L = I\omega$; moment of inertia of sphere along the axis passing through centre of mass, $I=\frac{2}{5}MR^2$ and $\omega=\frac{2\pi}{T}$.

Putting these values, $L=\frac{4\pi\ MR^2}{5T}$

40. (c) $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

$$T_1 = 2\pi\sqrt{\frac{R^3}{GM}},\quad T_2 = 2\pi\sqrt{\frac{(1.01R)^3}{GM}}$$

$$\frac{T_2-T_1}{T_1}\times 100 = 1.5\%$$

41. (a) The gravitational potential at the centre of uniform spherical shell is equal to the gravitational potential at the surface of shell i.e.,

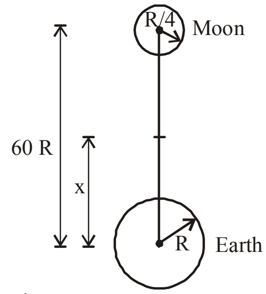
$V=\frac{-GM}{a}$, where a is radius of spherical shell

Now, if the shell shrinks then its radius decrease then density increases, but mass is constant. so from above expression if a decreases, then V increases.

42. (b) $g' = g\left(1-\frac{d}{R}\right) \Rightarrow \frac{g}{n} = g\left(1-\frac{d}{R}\right)$

$$\Rightarrow d = \left(\frac{n-1}{n}\right)R$$

43. (d) $E_{earth} = E_{moon}$



$$\Rightarrow \frac{GM}{x^2}=\frac{GM/81}{(60R-x)^2}$$

$$\Rightarrow \frac{1}{x}=\frac{1}{9\,(60R-x)}$$

$\Rightarrow x = 54$ R from centre of earth.

44. (b) Acceleration due to gravity at lattitude' λ ' is given by

$$g_\lambda = g_e - R_e\omega^2\cos^2\lambda$$

At equator, $\lambda = 90° \Rightarrow \cos\lambda = \cos 90° = 0$

or $\quad g_\lambda = g_e = g$ (as given in question)

At 30°, $g_{30} = g - R\omega^2\cos^2 30 = g - \frac{3}{4}R\omega^2$

or, $g - g_{30} = \frac{3}{4}R\omega^2$

45. (a) As we know,

Gravitational potential energy $=\frac{-GMm}{r}$

and orbital velocity, $v_0 = \sqrt{GM/R+h}$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m\frac{GM}{3R}-\frac{GMm}{3R}$$

$$= \frac{GMm}{3R}\left(\frac{1}{2}-1\right)=\frac{-GMm}{6R}$$

$$E_i = \frac{-GMm}{R}+K$$

$$E_i = E_f$$

Therefore minimum required energy, $K=\frac{5GMm}{6R}$