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Topics:

1. One-sample T-test ✓
2. Paired T-Test ✓
3. KS-Test ✓
4. Z-proportions test ✓
5. χ^2-test ✓

Same framework as earlier

1 / 5

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Ops:

Questions → tab ONLY ← ....

→ "END ....

→ Questions about earlier topics
↳ "END ...."

Chat → Interactivity & Yes/NO

2 / 5

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TA's → video chat — Tricky, follow-up

Problem-solving-sessions → least answered Questions

Next week

MWF + 1/2 sessions

3 / 5

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Q

2-sample T-test

Med 1 μ_1

$S_1^{n_1}: \; x_1, x_2 \ldots \; x_{n_1}$

$S_2^{n_2}: \; x_1', x_2' \ldots \; x_{n_2}'$

Med 2 μ_2

non-gaussian

Pop mean & Std-dev are finite

Compare means $\mu_1 = \mu_2$

Y / No

no Transform

Transform

4 / 5

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Y N
6 34

T-test

→ are obs are indep

compare means

→ n_1 & n_2 are small; do NOT know σ_1 & σ_2

→ $\boxed{\text{CLT}}$: pop mean & std-dev are finite

5 / 5

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$\boxed{\text{DOF}}$

ML | stats — $(n_1 + n_2) - (2)$

→ # indep obs − # params to estimate

{ 2-Sample T-test

Ques: $\mu_1 = \mu_2$ or not

$\boxed{n_1}$ obs

$\boxed{n_2}$ obs

m_1 ✓

m_2 ✓

6 / 10

2-Sample T-test

$$\text{Test-statistic} = \frac{m_1 - m_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t\,\text{dist}(\nu)$$

$\nu \rightarrow n_1 + n_2 - 2$

(e.g)

$\overbrace{x_1 \; x_2 \; \cdots \; x_n}^{\to\, m = \text{sample-mean}} \leftarrow n \text{ obs}$

$$\text{sample-var} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - m)^2$$

DOF $\rightarrow$ $n - 1$

n ← obs

1 ↪ sample mean

Chrome File Edit View History Bookmarks Profiles Tab Window Help

X

baby weights in India

1-Sample T-test

Given :- $x_1 \; x_2 \; \dots \; x_n$

(WHO)

Task: check if Indian babies have a mean weight of $\mu_0 = 2.8$ kgs

↳ world's

$\bar{X}$: sample mean

μ = mean Indian baby weight

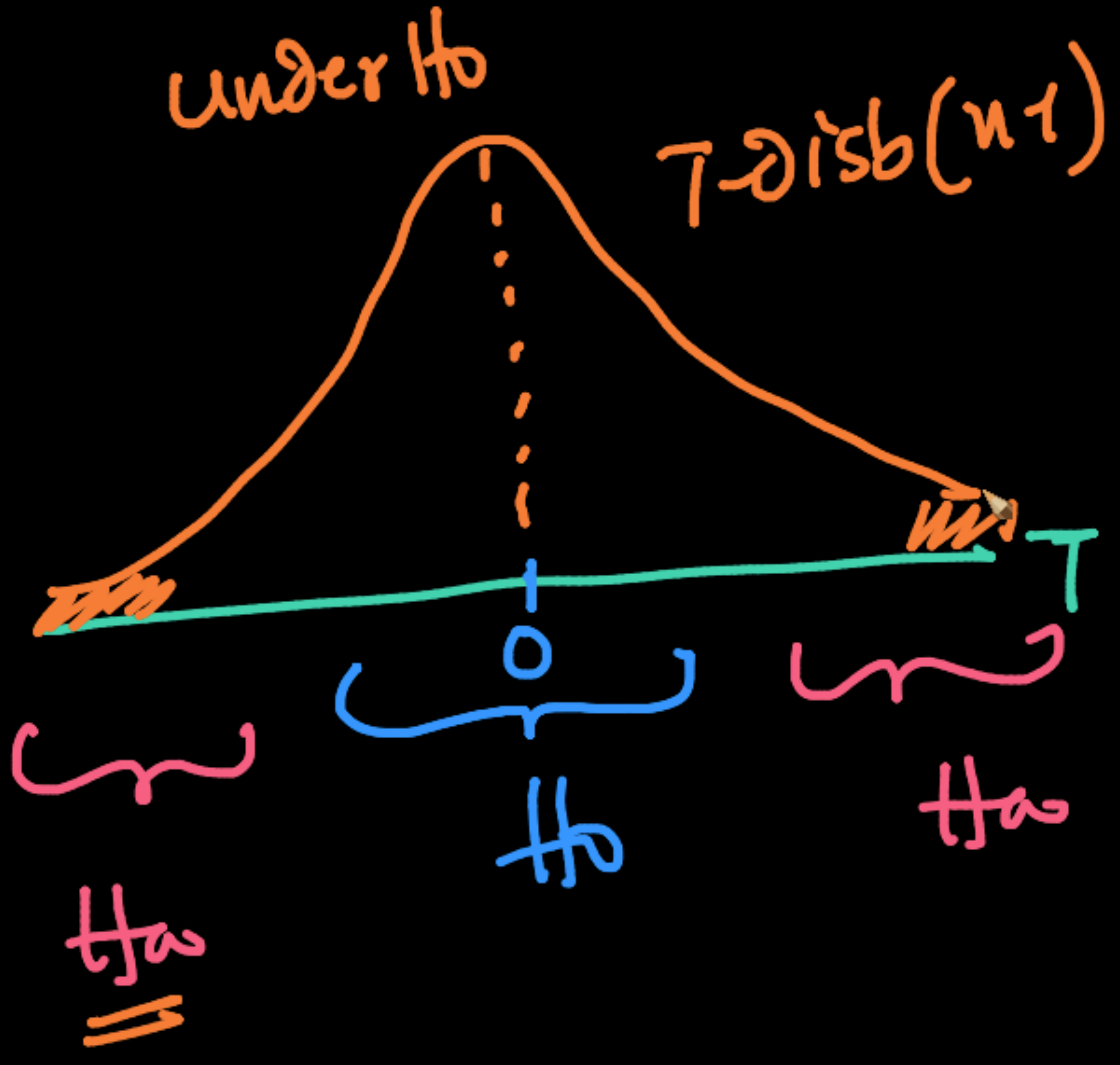
9 / 10

Framework:

✓ ① $\begin{cases} H_0: & \mu = \mu_0 \\ H_a: & \mu \neq \mu_0 \end{cases}$

$\mu_0 \rightarrow$ 2.8 kgs

$\mu_0 \rightarrow$ 2.8 kgs



② $\left\{ T = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}} \right.$

$\bar{X} \rightarrow$ sample mean

$\mu_0 \rightarrow$ 2.8

$s \rightarrow$ sample Std.dev of n obs

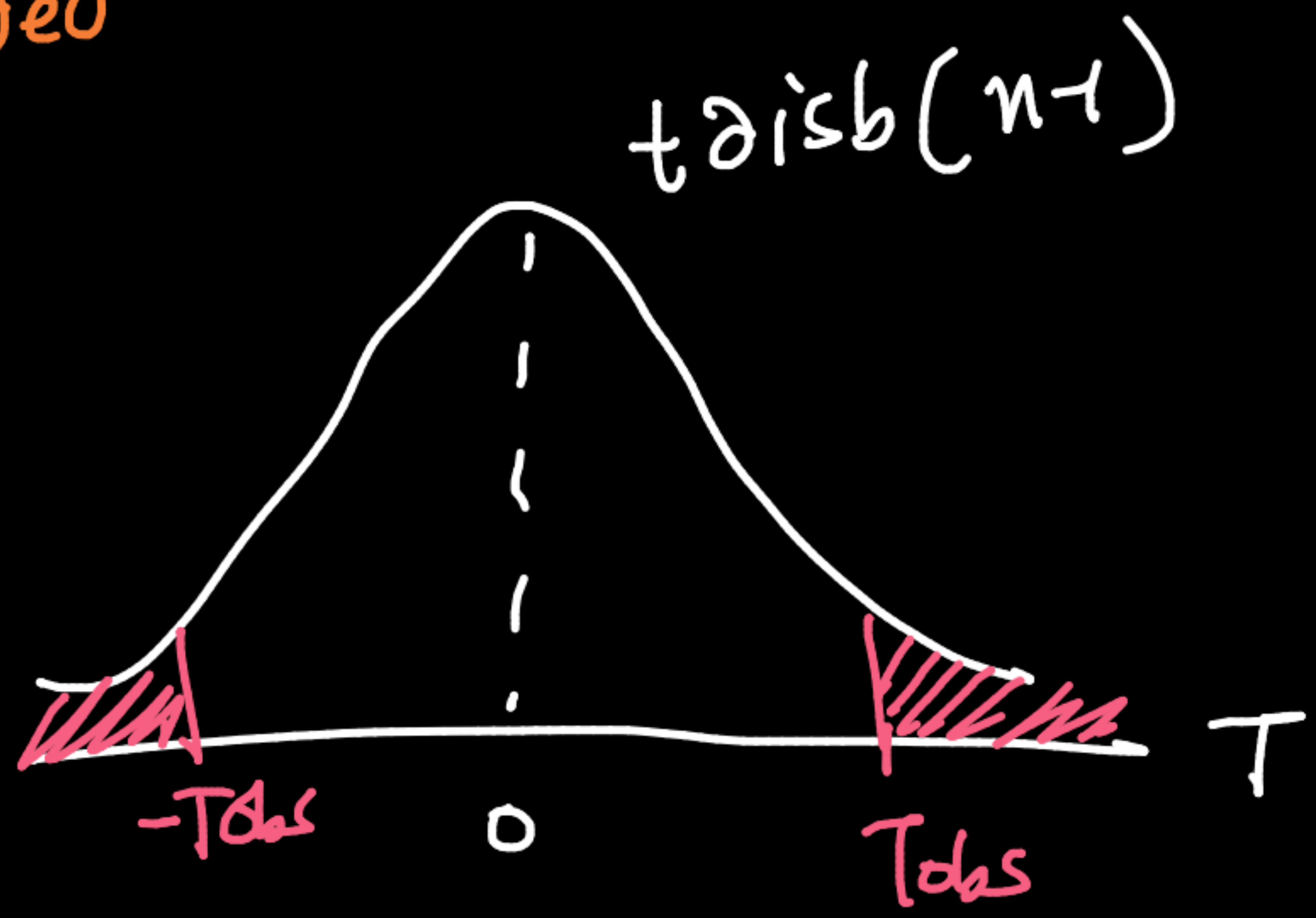
$T \sim t(\nu = n-1)$ ✓

3. 2-sided or 1-sided

4. p-val

5. $\alpha = 5\%$

p-val vs α



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Dependent / "Paired" T-Test

$\bar{d} = \frac{d_1 + d_2 + \cdots + d_n}{n}$

Task!

pop. mean of diff. . .

0

(1) Ho : mean diff is $\boxed{\mu_d}$

{Ha: mean diff is not μ_d

(2) $T = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} \sim t(\nu = n-1)$

St ID	Test 1	Test 2	diff = d
1	80	82	$-2 \to d_1$
2	86	81	$5 \to d_2$
⋮	⋮	⋮	⋮
n-1	50	89	-39
n	56	58	$-2 \to d_n$

Remedial / PS

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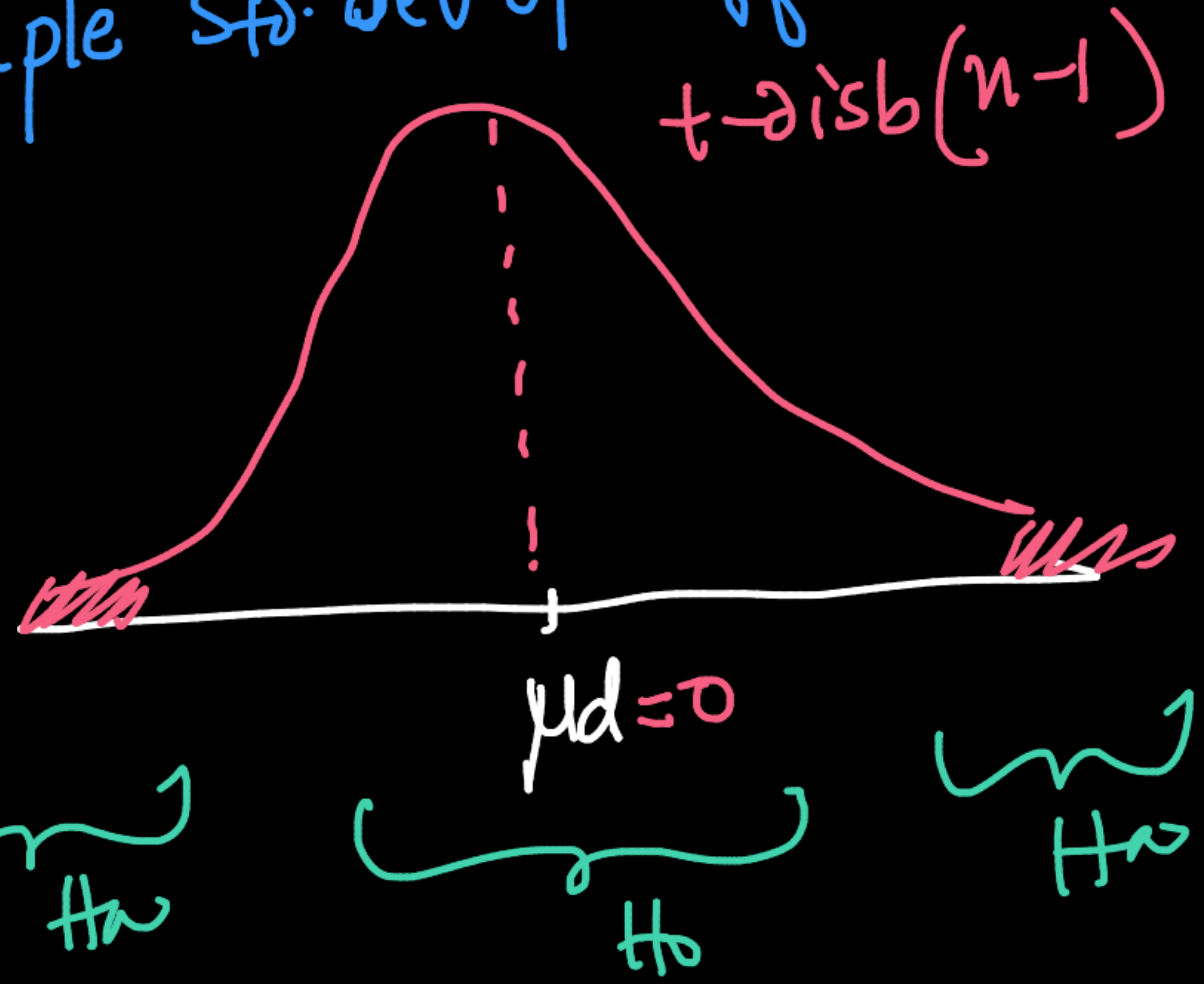
$\bar{d}$: sample mean of diff

s_d : sample std. dev of diff



(3) 2-sided

(4) p-val

(5) $\alpha = 5\%$

p-val vs α

$$\begin{cases} H_0: \mu_1 = \mu_2 \Rightarrow \mu_d = 0 \\ H_a: \mu_d \neq 0 \Rightarrow \mu_1 \neq \mu_2 \rightarrow \text{2-sided} \end{cases}$$

diff = 1st - 2nd

$H_0: \mu_d = 0 \Rightarrow \mu_1 = \mu_2$

$H_a: \mu_2 > \mu_1 \Rightarrow \mu_d < 0$

Q:-

$X = [8.5, 8.5, \ldots\ldots 8.5]$ → $n_1 = 20$

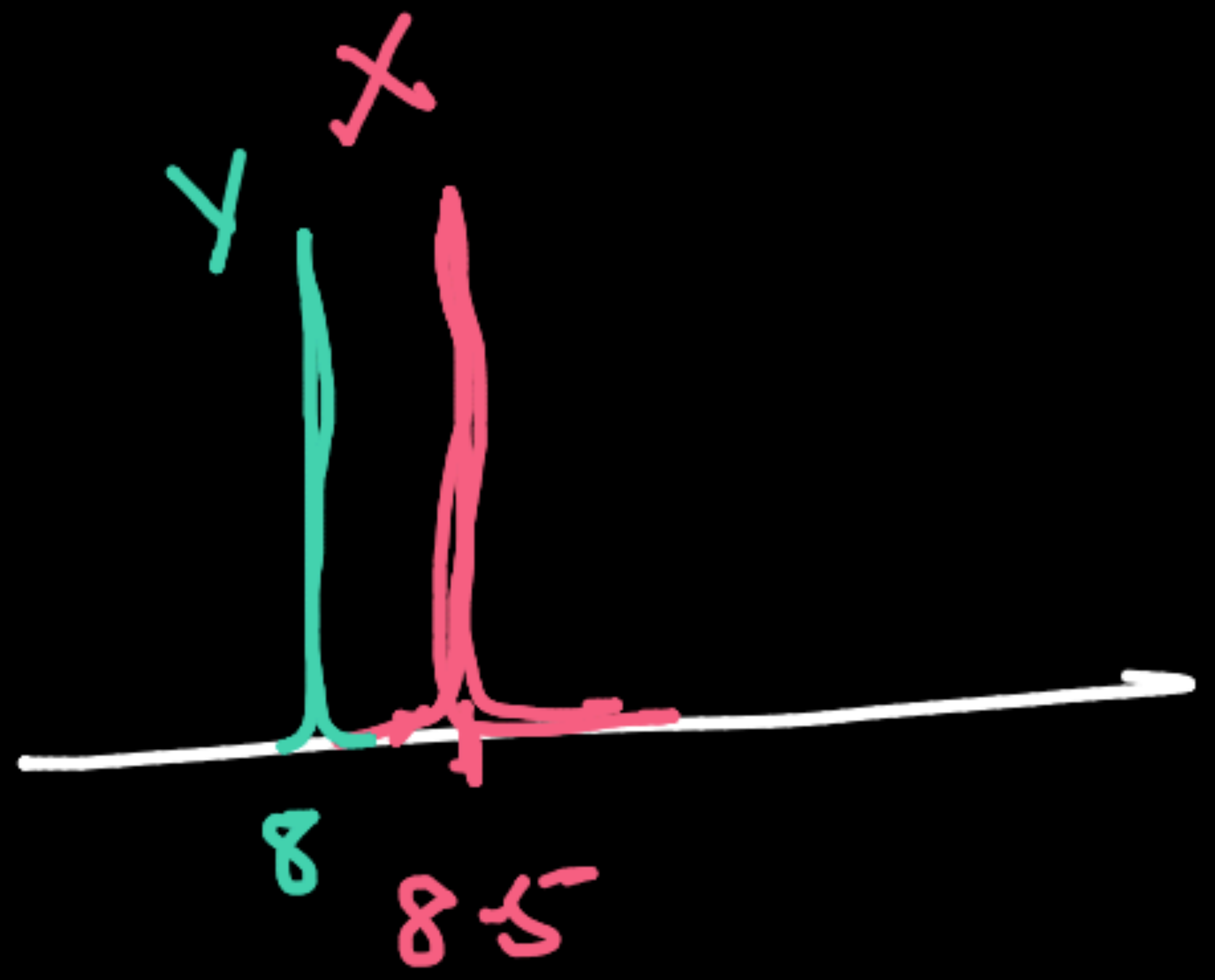
$Y = [8, 8 \ldots\ldots 8]$ → $n_2 = 25$

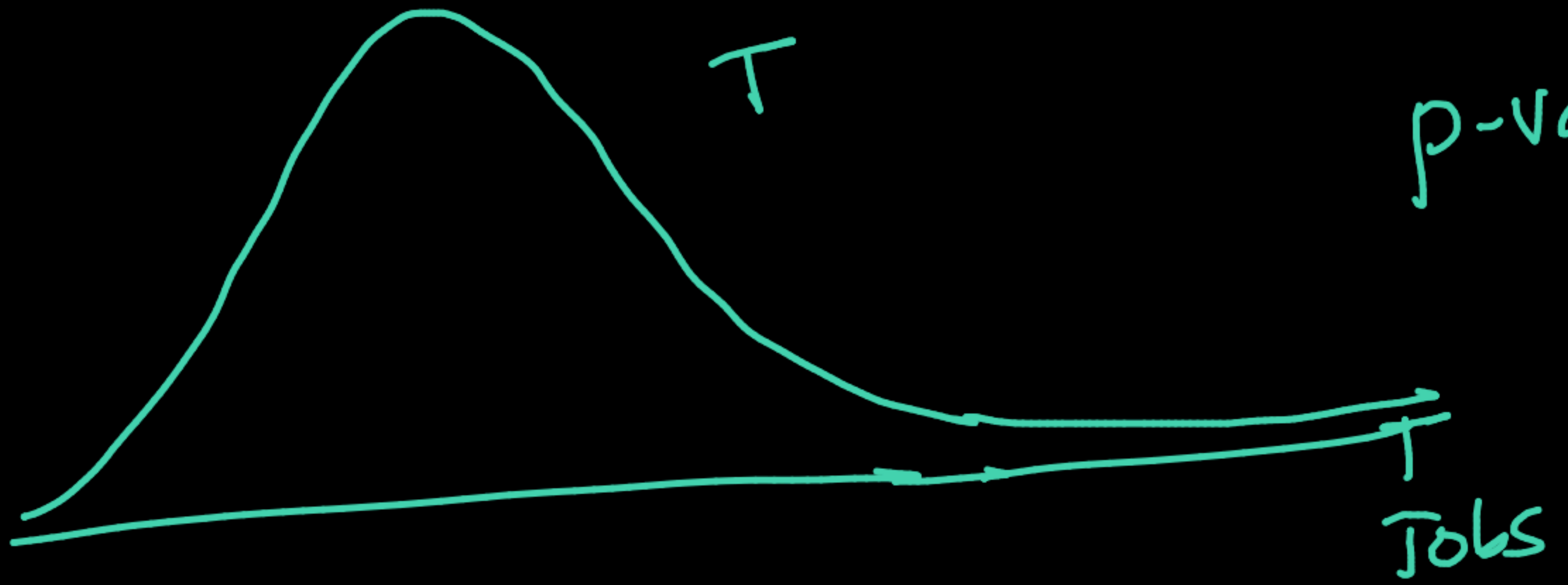
$Z = [1, 1, \ldots\ldots 1]$

t-test

$\mu_x = \mu_y$?

$T = \boxed{\infty} = \dfrac{M_x - M_y}{\sqrt{\frac{S_x^2}{n_1} + \frac{S_y^2}{n_2}}}$ → undefined





p-val =

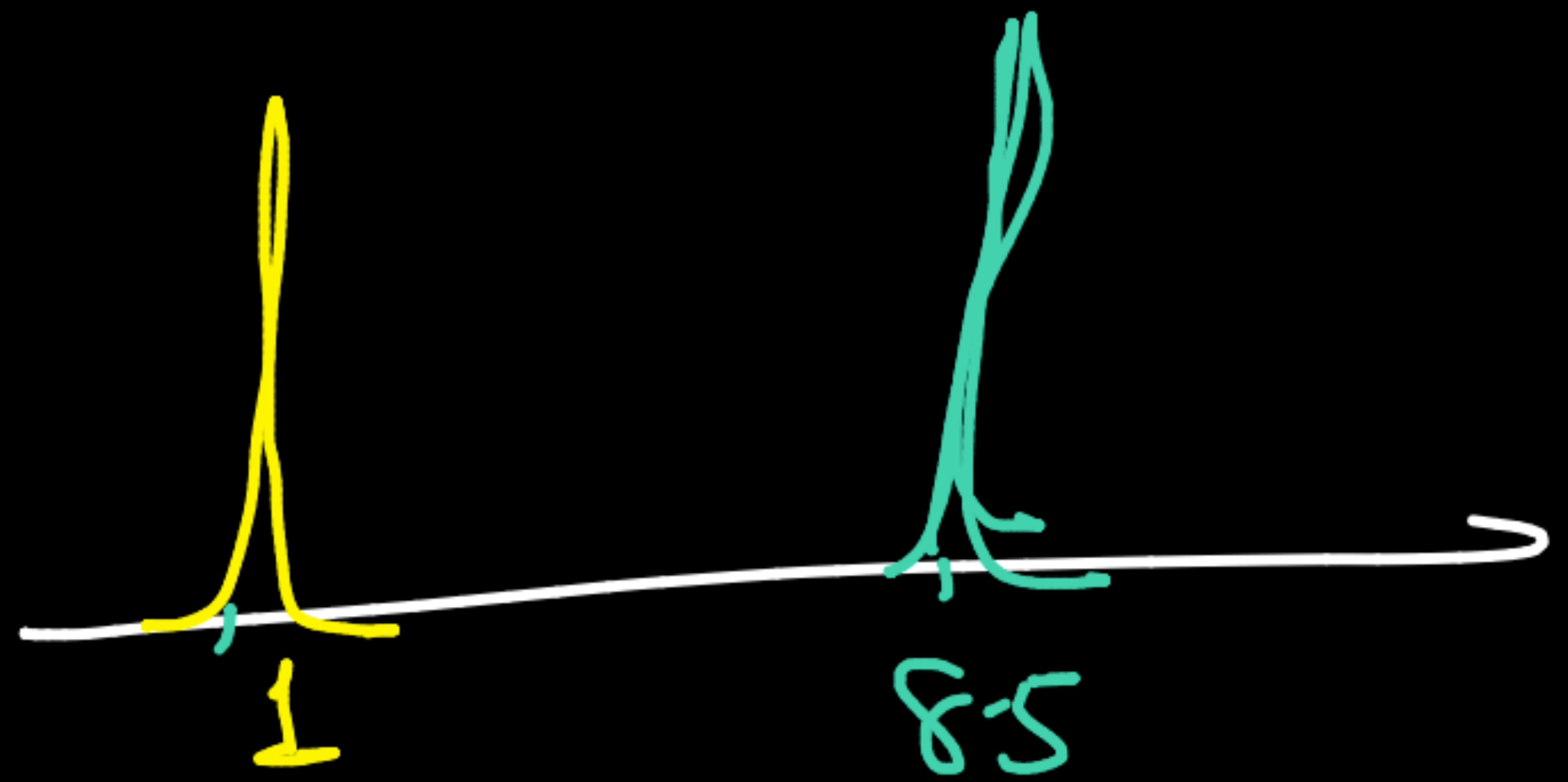
$T_{obs} \to \infty$; p-val $\to \underline{0} \Rightarrow$

reject H_0

$\mu_x \neq \mu_z$

$X = [\,8.5 \quad 8.5 \quad \ldots \quad 8.5\,]$ → 25 times

$Z = [\,1 \quad 1 \quad 1 \quad 1 \quad \ldots \quad 1\,]$ → 30 times

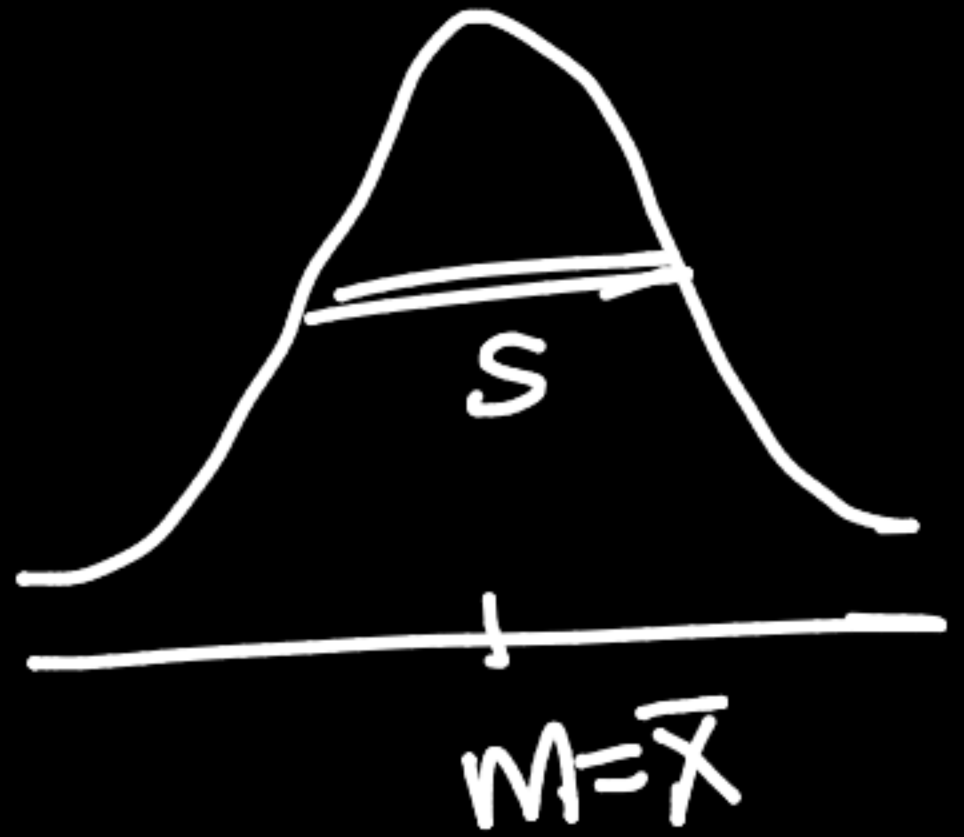


Q

$$X \sim N(\mu, \sigma)$$

$$\downarrow$$

$$x_1 \; x_2 \; \ldots \; x_n$$



$$\left\{ \begin{array}{l} \text{as } n \to \infty \\ \quad s \to \sigma \\ \quad M = \bar{X} \to \mu \end{array} \right.$$

Q

2-Sample indep T-test

obj

Med 1 Med 2

$\mu_1 = \mu_2$ or not

paired t-test

obj

diff 1st & 2nd

is the diff $= \mu_d$ or not

'0'

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St-Id Test1 Test2 diff?

paired

dependent

i $\boxed{80}$ $\longleftrightarrow$ $\boxed{85}$

j 55 $\longleftrightarrow$ 65

20 / 20

Numerical Measures
Probability
Sampling
Principles of Testing
Univariate Inferential Tests
Bivariate Relationships
Common Mistakes and Tables
Cummulative Reviews
Statistics Quizzes

Test for population mean

Hypothesis test

Formula: $z = \dfrac{\bar{x} - \Delta}{\dfrac{\sigma}{\sqrt{n}}}$

where $\bar{x}$ is the sample mean, Δ is a specified value to be tested, σ is the population standard deviation, and *n* is the size of the sample. Look up the significance level of the *z*-value in the standard normal table (Table in Appendix B).

A herd of 1,500 steer was fed a special high-protein grain for a month. A random sample of 29 were weighed and had gained an average of 6.7 pounds. If the standard deviation of weight gain for the entire herd is 7.1, test the hypothesis that the average weight gain per steer for the month was more than 5 pounds.

null hypothesis: H_0: μ = 5

alternative hypothesis: H_a: μ > 5

$$z = \frac{6.7 - 5}{\frac{7.1}{\sqrt{29}}} = \frac{1.7}{1.318} = 1.289$$

Tabled value for $z \le 1.28$ is 0.8997

1 – 0.8997 = 0.1003

$\mu \quad X \quad x_1 \; x_2 \; . \; . \; . \; x_n$

$H_0: \mu = 5$

$H_a: \mu \neq 5$

$$T = \frac{\bar{X} - 5}{\sigma/\sqrt{n}}$$

$Z(0,1)$

Back to Top

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Q

T-test

$\boxed{H_0:}\ \mu_1 = \mu_2$

$H_0: \mu_1 \neq \mu_2$

$$T = \frac{m_1 - m_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

disb of T under H_0?

22 / 22

Q

$\boxed{\mu}_d$ = pop. mean of differences Test 1 & Test 2

95% CI of this

CLT + BS

$d_1, d_2 \ldots d_n \rightarrow$ n obs of diff

$\underbrace{d_1, d_2 \ldots d_n}$

$\downarrow$

$\bar{d}$: sample mean of diff

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Q

stdId	Test 1	Test2
10	5	85 (possible)
i	55	90 / 65 ✓

Q

Z-test

$\mu_1 \to \boxed{X} \to m_1, s_1$

$\mu_2 \to \boxed{Y} \to m_2, s_2$

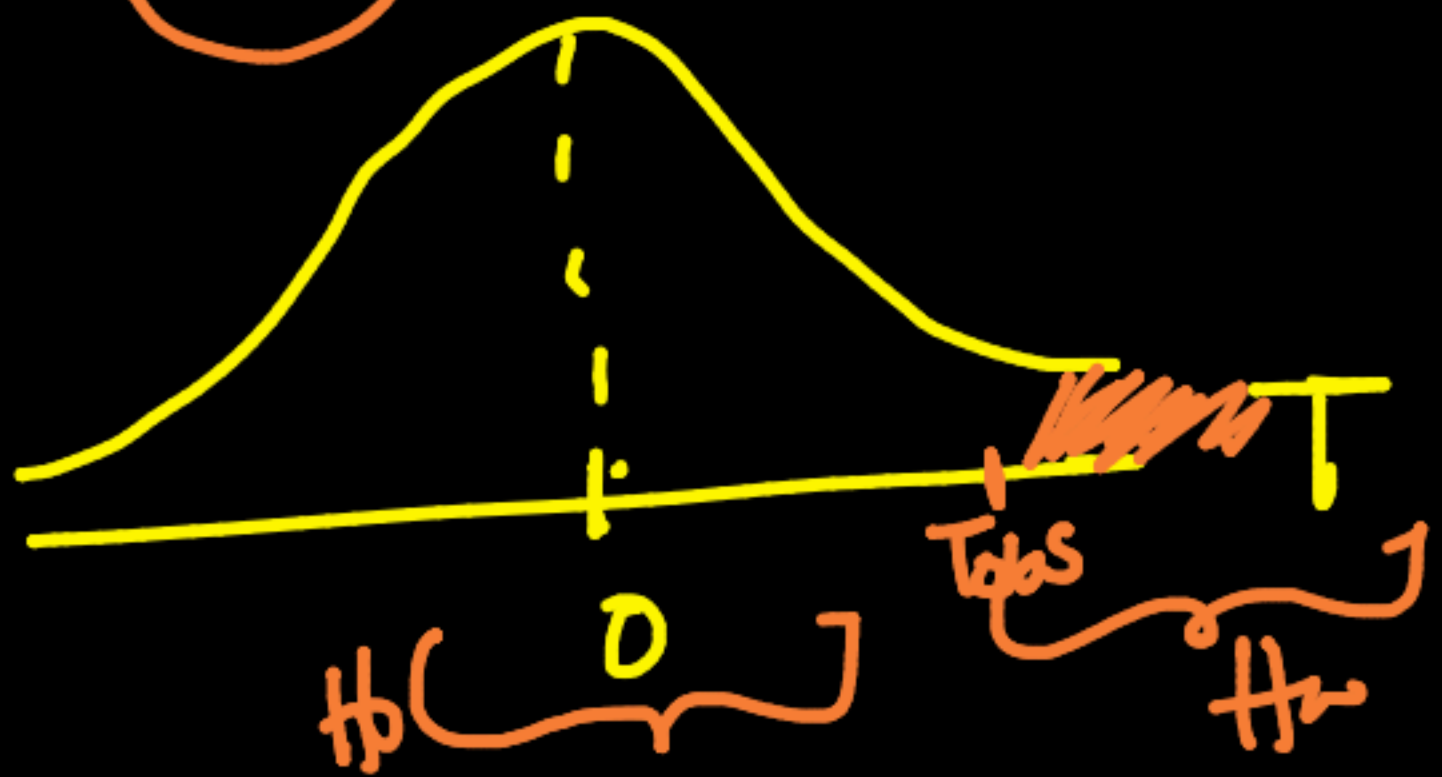
$$T = \frac{m_1 - m_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$

Under H_0:

$T \sim Z(0,1)$



Z-Test

T-Test

$\Big\} \rightarrow$ Comparision of Means rec times Med1

X: $x_1 \quad x_2 \; \ldots \ldots \; x_n$

Y: $x_1 \quad x_2 \; \ldots \ldots \; x_m$

rec times Med2

– 2 sample

KS-test ✓

Task: dist of X = dist of Y

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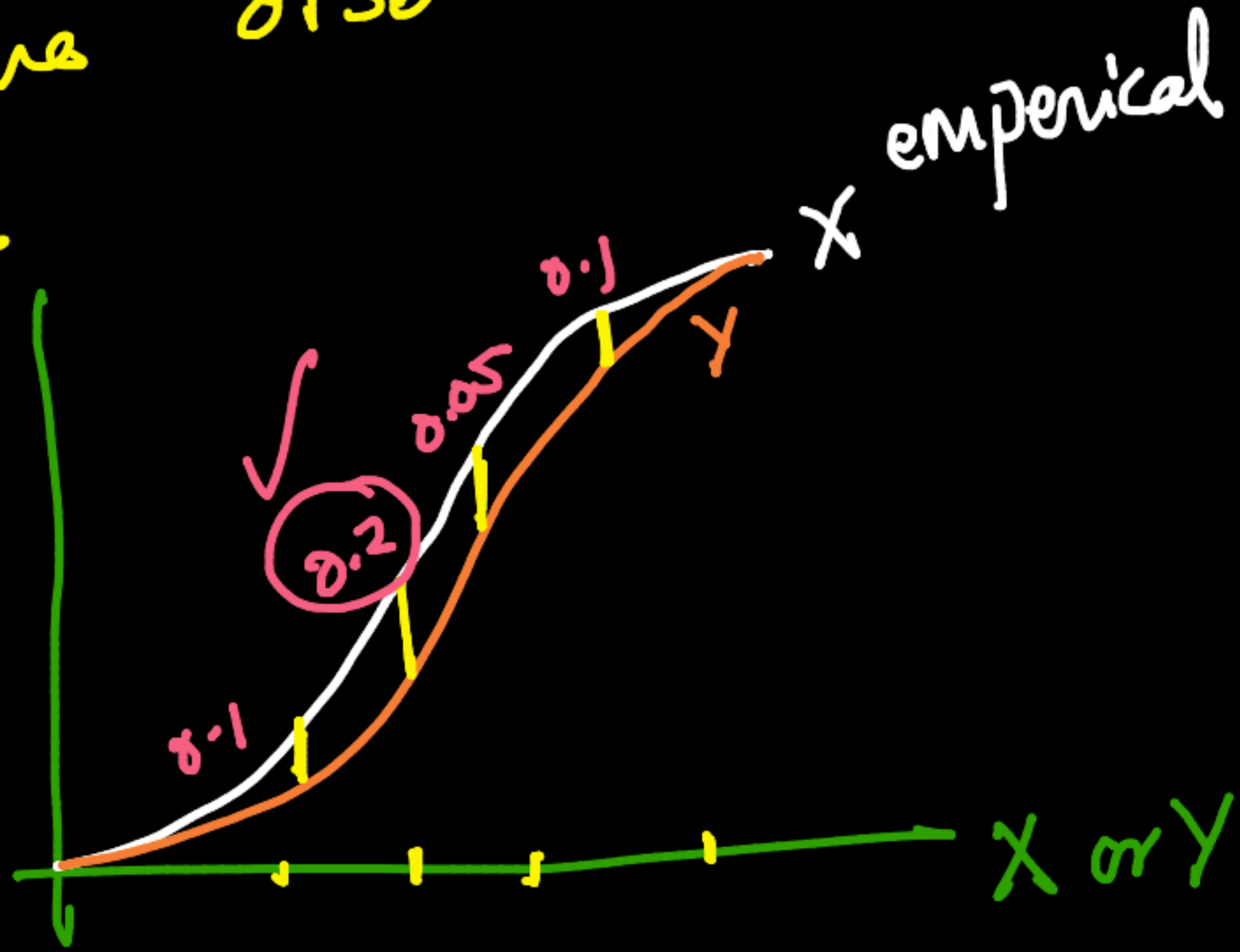
① H_0: $\overline{X \sim Y}$ have same distb

H_a: $X \not\sim Y$ not same distb

② T = max |gap b/w CDF of X & Y|

= Supremum |gap b/w CDFs|



28 / 29

J_{KS}

Under H_0: close to zero

Under H_a: larger +ve value

Right tailed test

3

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$\sqrt{n} \cdot T_{obs}$

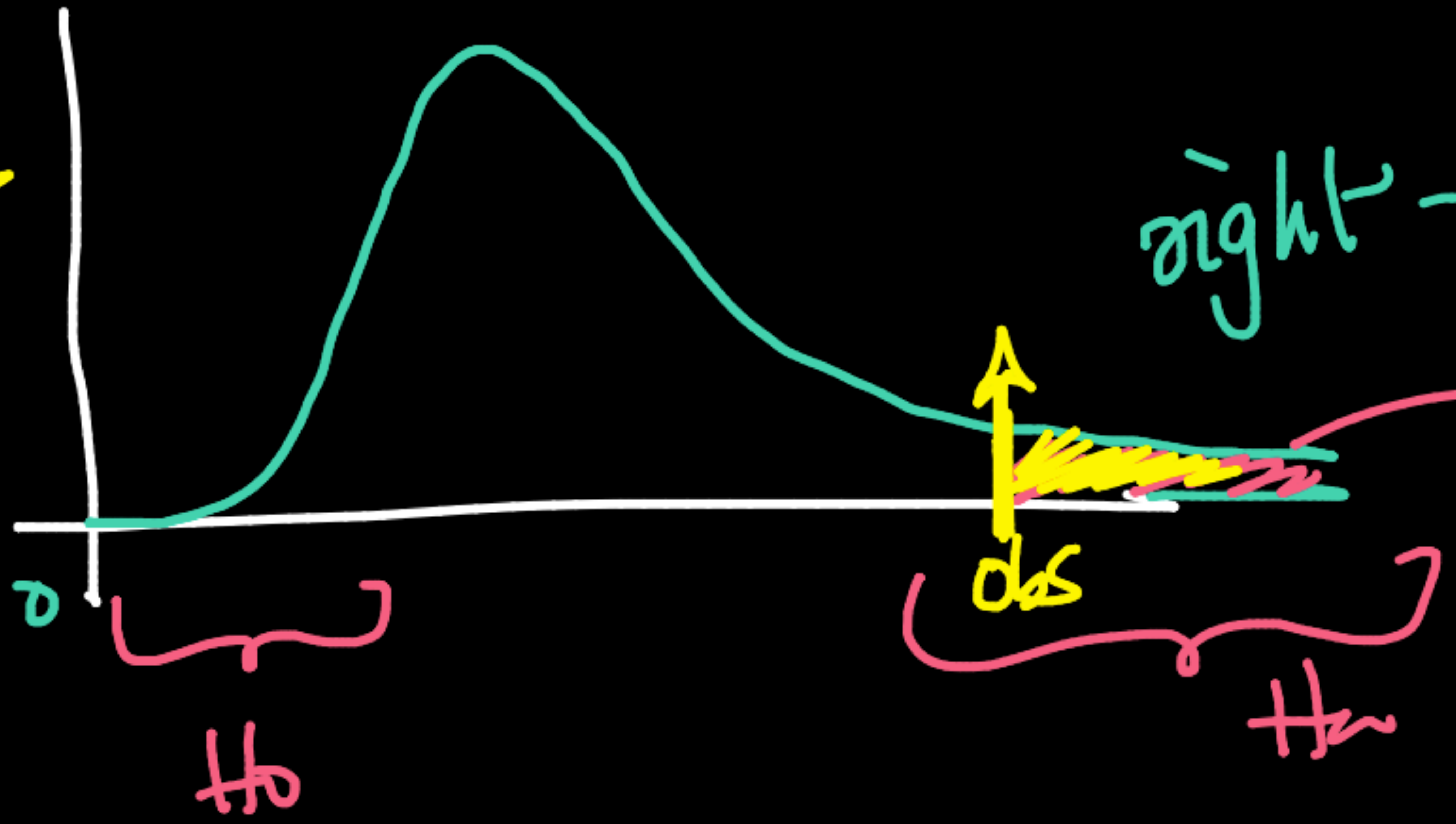
$\sqrt{n} \cdot T \sim$ Kolmogorov-dist as $n \to \infty$

T = max|gap|

Brownian motion in Physics

KS-Test

PDF of K-dist



right tailed dist

p-val vs α

Major statistical packages among which SAS `PROC NPAR1WAY`,[14] Stata `ksmirnov`[15] implement the KS test under the assumption that $F(x)$ is continuous, which is more conservative if the null distribution is actually not continuous (see [16] [17] [18]).

Two-sample Kolmogorov–Smirnov test [edit]

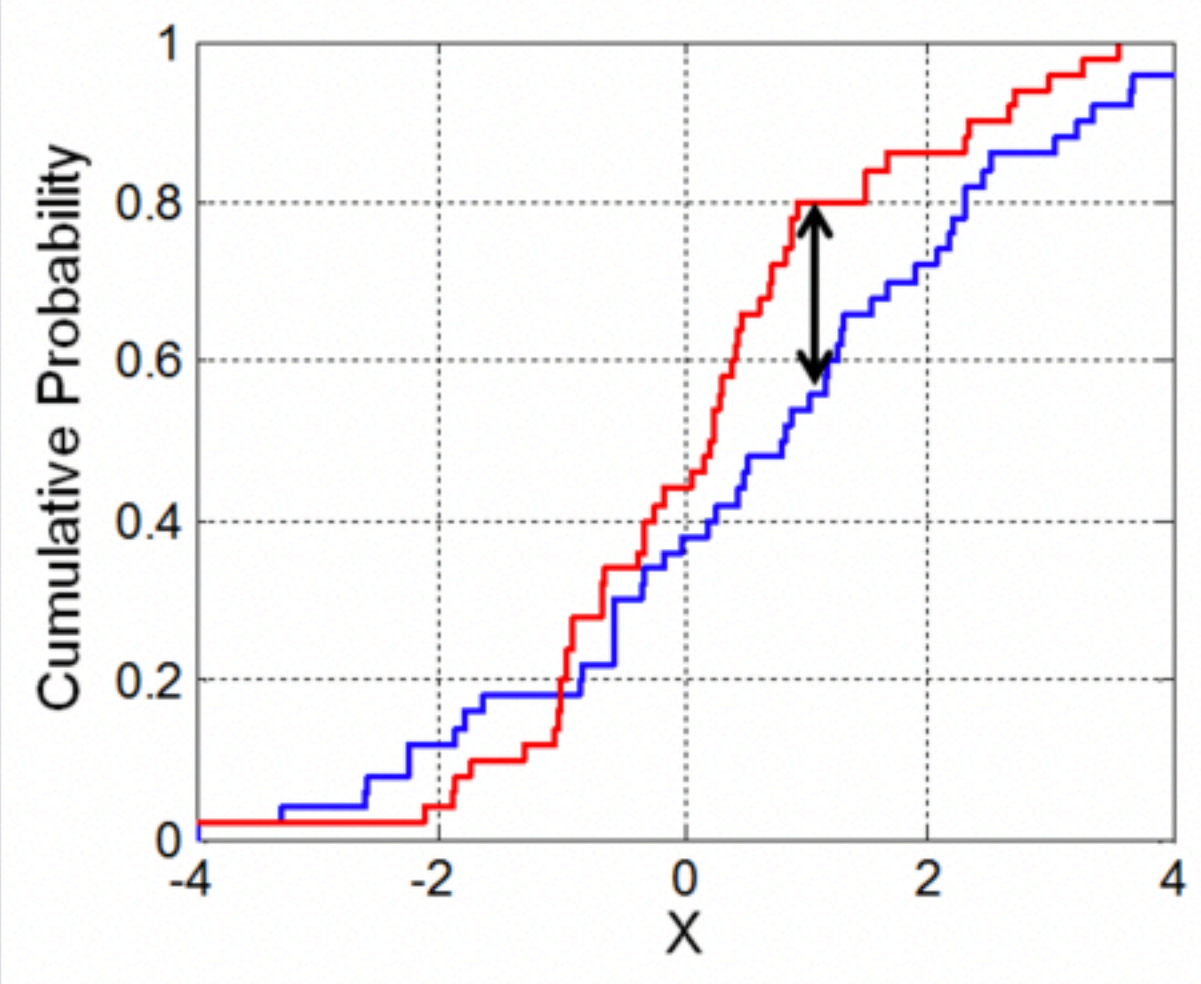


Illustration of the two-sample Kolmogorov–Smirnov statistic. Red and blue lines each correspond to an empirical distribution function, and the black arrow is the two-sample KS statistic.

The Kolmogorov–Smirnov test may also be used to test whether two underlying one-dimensional probability distributions differ. In this case, the Kolmogorov–Smirnov statistic is

$$D_{n,m} = \sup_x |F_{1,n}(x) - F_{2,m}(x)|,$$

where $F_{1,n}$ and $F_{2,m}$ are the empirical distribution functions of the first and the second sample respectively, and $\sup$ is the supremum function.

For large samples, the null hypothesis is rejected at level α if

$$D_{n,m} > c(\alpha)\sqrt{\frac{n+m}{n \cdot m}}.$$

Where n and m are the sizes of first and second sample respectively. The value of $c(\alpha)$ is given in the table below for the most common levels of α

α	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001
$c(\alpha)$	1.073	1.138	1.224	1.358	1.48	1.628	1.731	1.949

and in general[19] by

$$c(\alpha) = \sqrt{-\ln\left(\tfrac{\alpha}{2}\right) \cdot \tfrac{1}{2}},$$

so that the condition reads

$$D_{n,m} > \sqrt{-\ln\left(\tfrac{\alpha}{2}\right) \cdot \tfrac{1+\frac{m}{n}}{2m}}.$$

Here, again, the larger the ... ple sizes (e.g. $m = n$), the minimal bound scales in the size of either of the samples acc...

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$c(\alpha)$	1.073	1.138	1.224	1.358	1.48	1.628	1.731	1.949

and in general[19] by

$$c(\alpha) = \sqrt{-\ln\left(\tfrac{\alpha}{2}\right) \cdot \tfrac{1}{2}},$$

CDF of K-dist

$\alpha = 5\%$

so that the condition reads

$$D_{n,m} > \sqrt{-\ln\left(\tfrac{\alpha}{2}\right) \cdot \frac{1+\frac{m}{n}}{2m}}.$$

→ accept H_a & reject H_0

Test statistic

Here, again, the larger the sample sizes, the more sensitive the minimal bound: For a given ratio of sample sizes (e.g. $m = n$), the minimal bound scales in the size of either of the samples according to its inverse square root.

Illustration of the two-sample Kolmogorov–Smirnov statistic. Red and blue lines each correspond to an empirical distribution function, and the black arrow is the two-sample KS statistic.

Note that the two-sample test checks whether the two data samples come from the same distribution. This does not specify what that common distribution is (e.g. whether it's normal or not normal). Again, tables of critical values have been published. A shortcoming of the univariate Kolmogorov–Smirnov test is that it is not very powerful because it is devised to be sensitive against all possible types of differences between two distribution functions. Some argue[20][21] that the Cucconi test, originally proposed for simultaneously comparing location and scale, can be much more powerful than the Kolmogorov–Smirnov test when comparing two distribution functions.

Setting confidence limits for the shape of a distribution function [edit]

Main article: Dvoretzky–Kiefer–Wolfowitz inequality

While the Kolmogorov–Smirnov test is usually used to test whether a given $F(x)$ is the underlying probability distribution of $F_n(x)$, the procedure may be inverted to give confidence limits on $F(x)$ itself. If one chooses a critical value of the test statistic D_α such that $P(D_n > D_\alpha) = \alpha$, then a band of width $\pm D_\alpha$ around $F_n(x)$ will entirely contain $F(x)$ with probability $1 - \alpha$.

The Kolmogorov-

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```
9.53931977,  9.02602273,  6.79374185,  8.59715131,  8.37747338,
8.78161815,  6.78716383,  8.28473394,  8.20283798, 12.50518811,
10.19772574,  8.93758457,  8.9540311 ,  8.28927558,  6.28935098,
7.69447559,  9.66777701, 10.33898342,  8.71199578,  5.12781581,
9.70954569,  9.13685031,  7.28989718,  8.0868909 ,  7.42937556,
7.31356749,  9.92345816,  8.60211814,  9.33228465,  8.14132658,
6.17871495, 10.28358242,  7.31898597,  7.95085527,  6.20331719,
9.19119762,  6.98600628,  7.05314883, 10.57921482,  6.83637574,
7.86199283,  8.23350975,  5.87625665,  7.78945364,  8.83612492]
```

```
d1 = np.array(r1)
d2 = np.array(r2)
```

```
n1 = len(d1)
n2 = len(d2)
```

```
n1
```

100

```
n2
```

120

```
n1
```

```
100
```

```
n2
```

```
120
```

Max gap

1.5×10^{-5}

```
#2-sample KS Test
stats.ks_2samp(d1, d2)
```

```
KstestResult(statistic=0.32333333333333333, pvalue=1.5163387982131127e-05)
```

p-value = 0.00001516 < 0.001 = 0.1% = alpha

=> Reject H0 (r1 and r2 same distribution)

=> Accept Ha (the distributions of r1 and r2 are different)

```
plt.grid()
a = plt.hist(d1, bins=100, cumulative=True, label='CDF', density=True, histtype='step')
b = plt.hist(d2, bins=100, cumulative=True, label='CDF', density=True, histtype='step')
plt.show()
```

p-value = 0.00001516 < 0.001 = 0.1% = alpha

=> Reject H0 (r1 and r2 same distribution)

=> Accept Ha (the distributions of r1 and r2 are different)

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b = plt.hist(d2, bins=100, cumulative=True, label='CDF', density=True, histtype='step')
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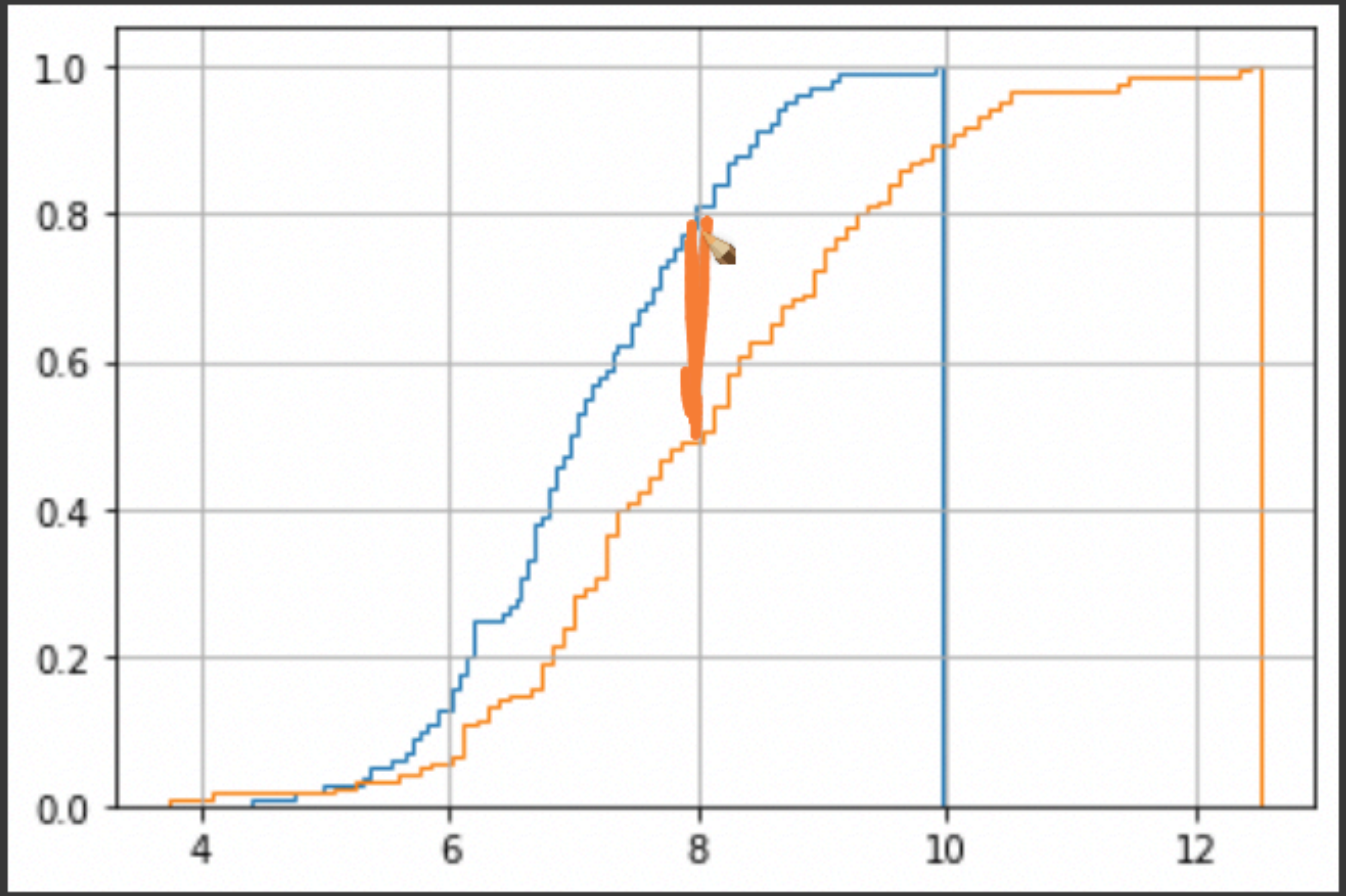
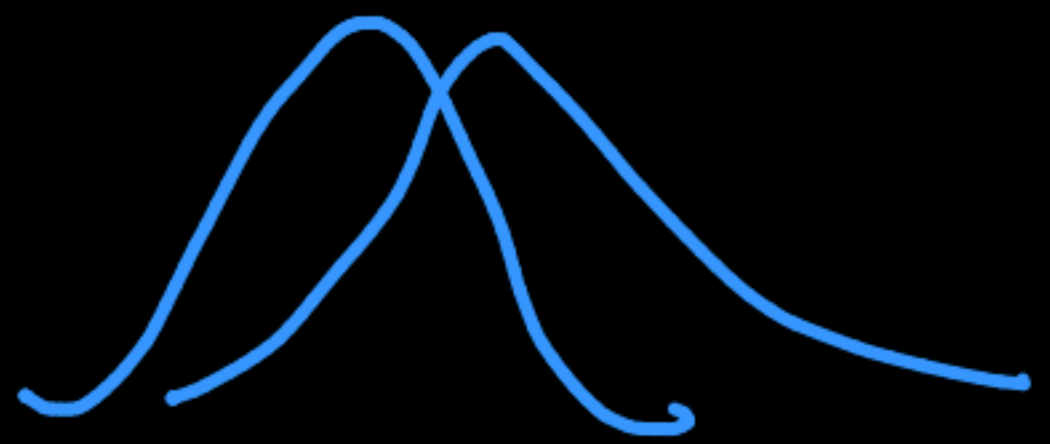
```



KL-div → metric / measure



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$\boxed{\geq 100}$

n & m

KS-test general purpose

$x_1 \dots x_n$

$N(\mu, \sigma)$

$x_1 \dots x_n$ $y_1 \dots y_m$

$x_1 \dots x_n$

Stricter tests

Specialized tests

Anderson-Darling test of Normality

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Q

Z / T-test (CLT assumptions)

KS-test

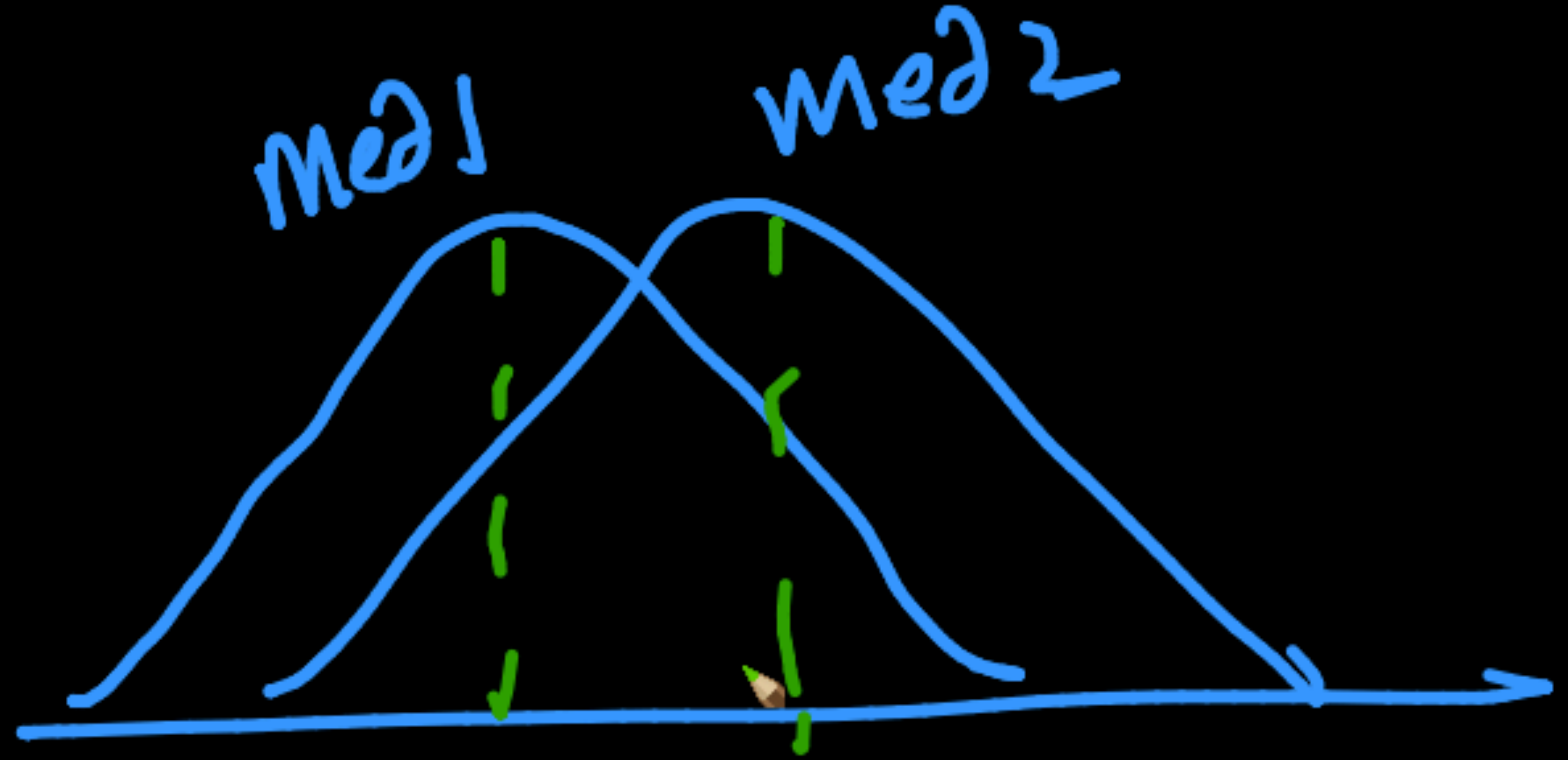
param vs Non-param technique

some

no assumptions about the populations or distb

$\emptyset$

$M1 \sim M2$



p-val

Q

$x_1 \; x_2 \; \ldots \; x_{200} \sim \text{expo}(\lambda)$

QQ-plot

KS-test $\boxed{\text{p-val}}$ ✓

n_1 & $n_2 \geq 30$ → Z - proportions Test

Retail-website

A: old website → $\dfrac{conv_1}{visits_1} = \boxed{p_1}$ n_1 → conv. rate

B: new website → $\dfrac{conv2}{visits_2} = \boxed{p_2}$ n_2

observed proportion

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$\hat{p}_1$ & $\hat{p}_2$: sample observed proportions n_1 & n_2 obs

Task: Compare the population proportions

p_1 p_2

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① $H_0: \boxed{p_1 = p_2}$

$H_a: p_1 \neq p_2$

$$\hat{p}_1 = \frac{\text{conv}_1}{\text{visits}_1 = n_1}$$

$$\hat{p}_2 = \frac{\text{conv}_2}{\text{visits}_2 = n_2}$$

② $$T_{obs} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{p} = \frac{\text{conv}_1 + \text{conv}_2}{\text{visits}_1 + \text{visits}_2}$$

aggregate con. rate

$X \sim Ber(p)$

$Var(X) = p(1-p)$

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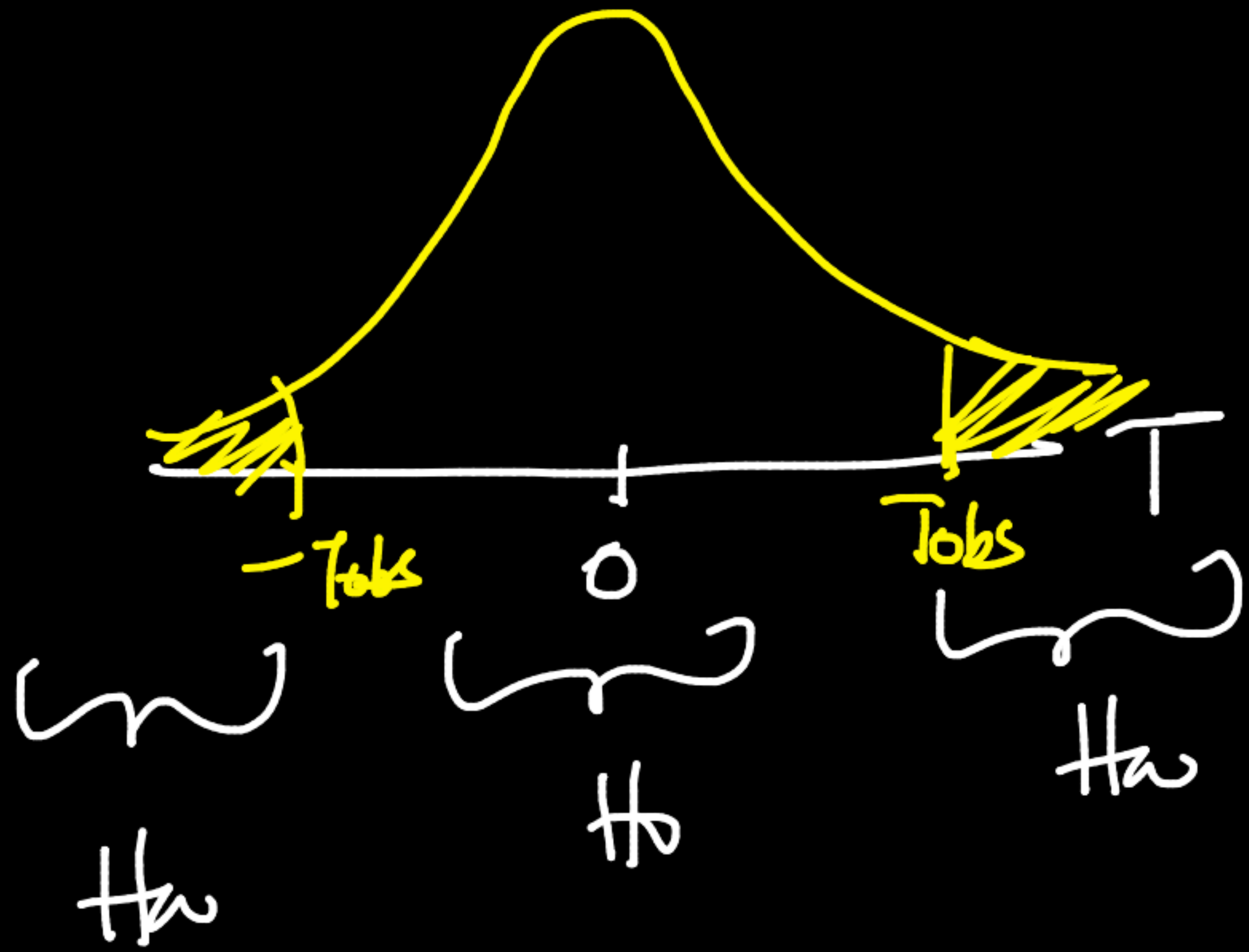
$T \sim Z(0,1)$

2-sided

p-val

$\alpha = 5\%$

p-val vs α



will assess whether or not a sample from a population represents the true proportion from the entire population.

Critical Value Approach

1-Sample Z-prop test

The steps to perform a test of proportion using the critical value approval are as follows:

1. State the null hypothesis H_0 and the alternative hypothesis H_A.
2. Calculate the test statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Ho: $\hat{p} = p_0 = 0.1 = 10\%$

where p_0 is the null hypothesized proportion i.e., when $H_0 : p = p_0$

3. Determine the critical region.
4. Make a decision. Determine if the test statistic falls in the critical region. If it does, reject the null hypothesis. If it does not, do not reject the null hypothesis.

Example S.6.1

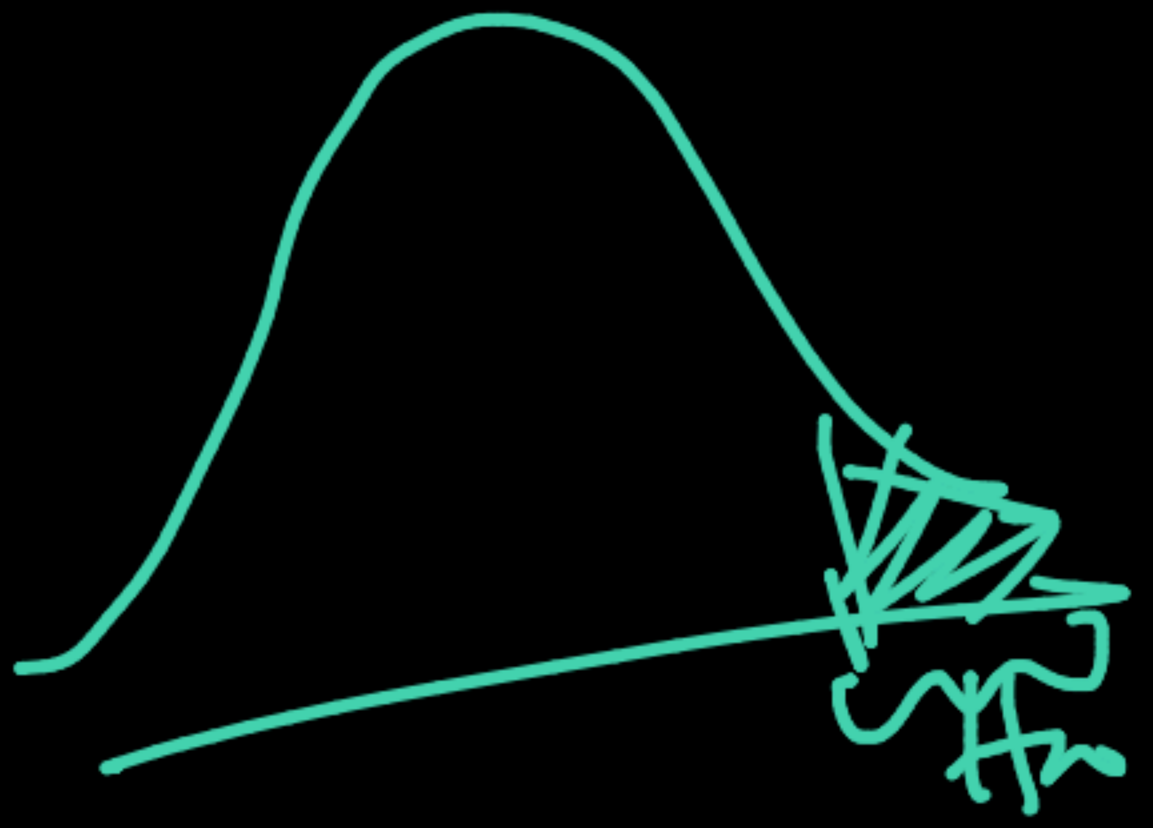
Newborn babies are more likely to be boys than girls. A random sample found 13,173 boys were born

$$\boxed{H_0: p_1 = p_2}$$

$H_a: p_1 \neq p_2 \rightarrow$ 2-sided

also have

$H_a: p_1 > p_2$



$H_a: p_1 < p_2 \rightarrow$ left tailed test

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χ^2-test

Test of independence

Youtube

Biz:

regular → premium

✓ Q does gender matter in converting from reg to premium

47 / 48

Biz:

discrete / discrete

Does premium-sub depend on gender or not?

premium-sub: 1, 0

gender: M, F

Data:

Contigency Table

Gender ↓ \ P →	0	1
F	346	149
M	339	165

$P(P=0 \text{ and } G=F)$

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(1) H_0: Gender is indep of Premium

→ Gender doesn't impact premium sub..

H_a: Gender impacts premium sub

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Obs:

G \ P	0	1	
F	346 (O_1)	149 (O_2)	495
M	339 (O_3)	165 (O_4)	504
	685	314	$\boxed{999}$

$$p(\text{non-sub}) = \frac{685}{999}$$

poem

$$p(\text{sub}) = \frac{314}{999}$$

poem

G \ P	0	1
F	E_1 339	E_2 153
M	E_3 345	E_4 158
	684	313

Under H_0:

Expected

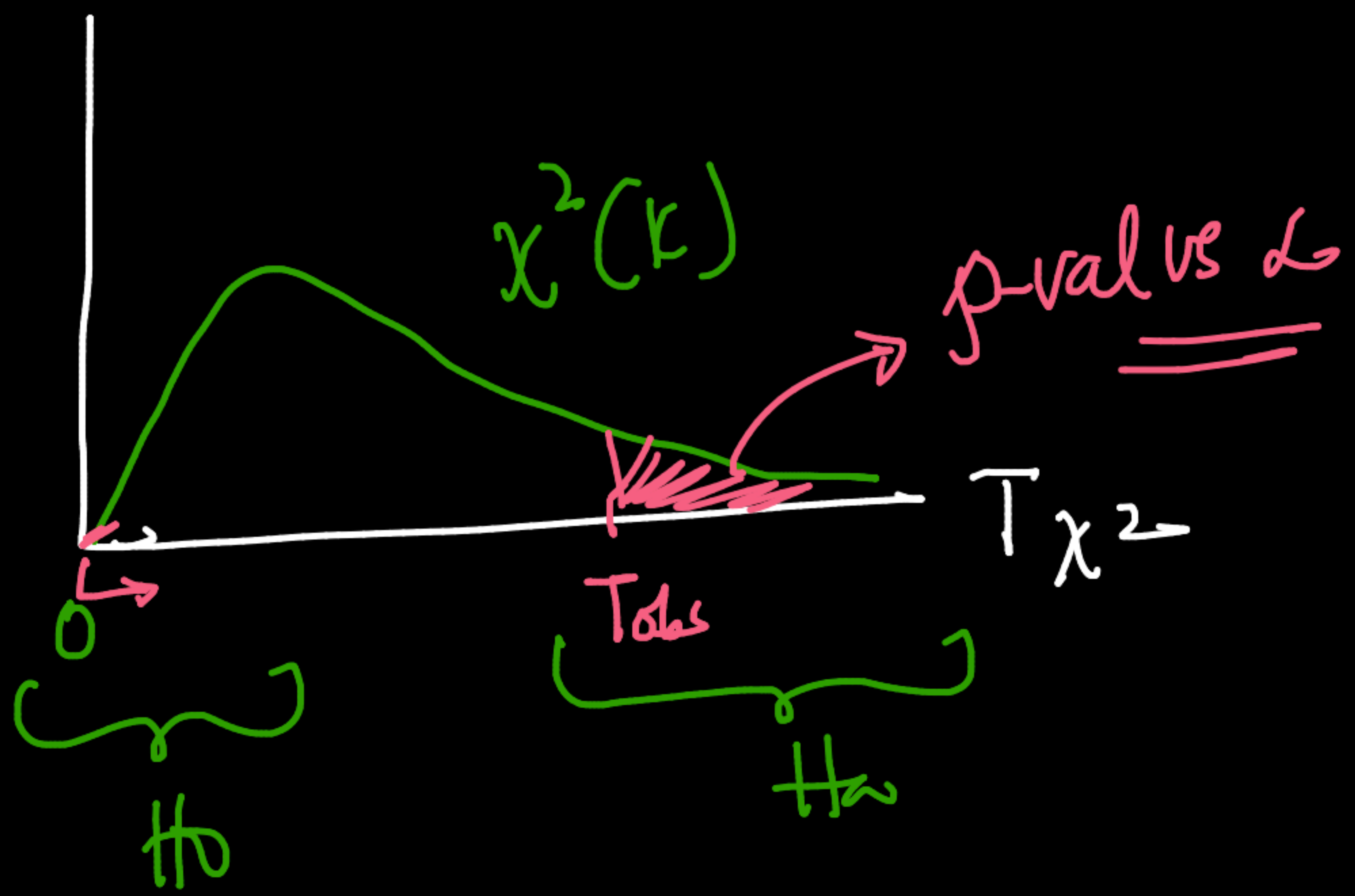
$$\frac{495 \times 685}{999}$$

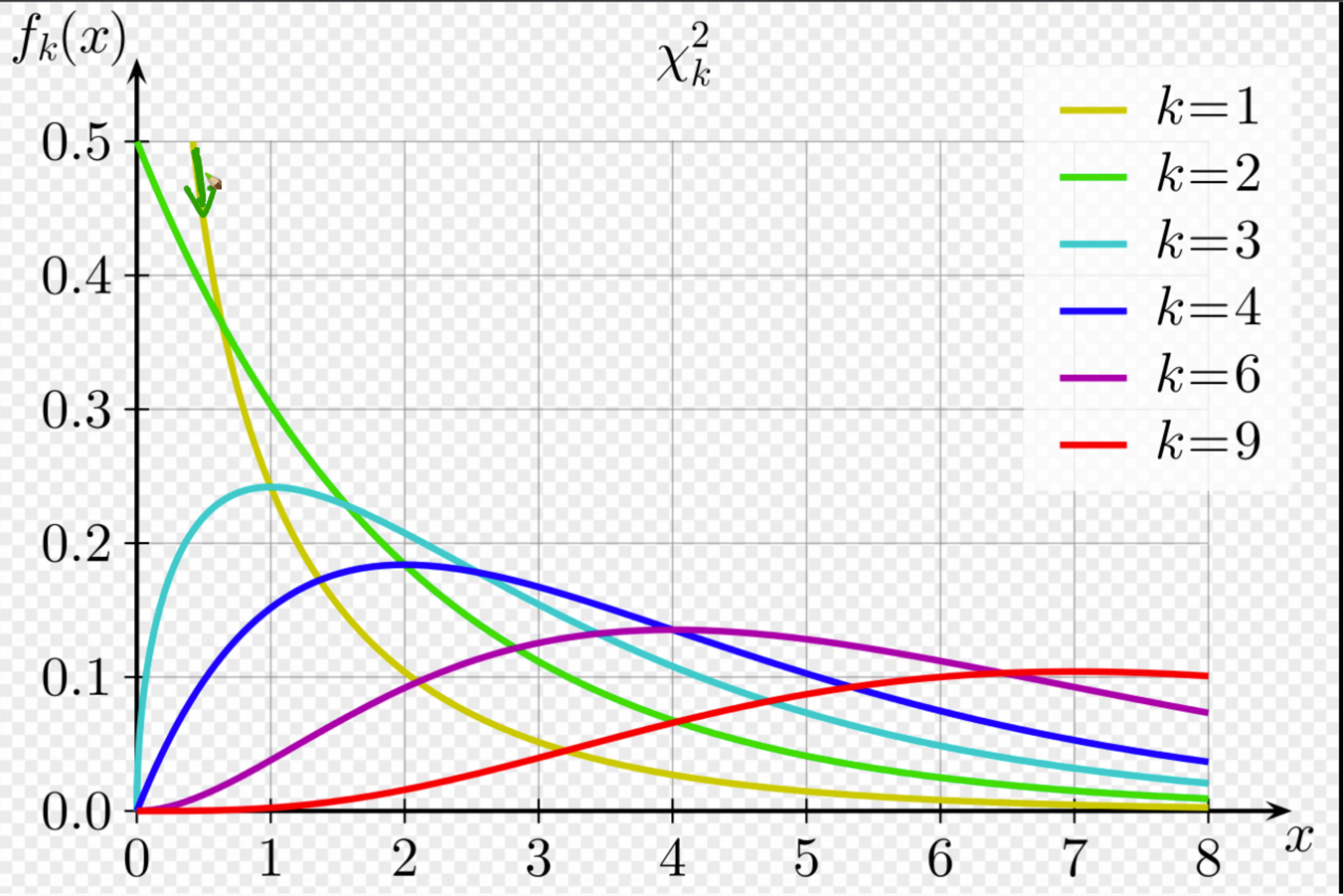
2

$$T_{\chi^2} = \sum_{i=1}^{\#\text{cells}} \frac{(O_i - \overset{H_0}{E_i})^2}{E_i} \rightarrow \text{logical}$$

$$T_{\chi^2} \sim \chi^2(k)$$

$$k = (n_{rows}-1)(n_{col}-1)$$





Plot of the chi-square distribution for values of k = {1, 2, 3, 4, 6, 9}. Accurate plotcurves. Labels are embedded in Computer-Modern font.

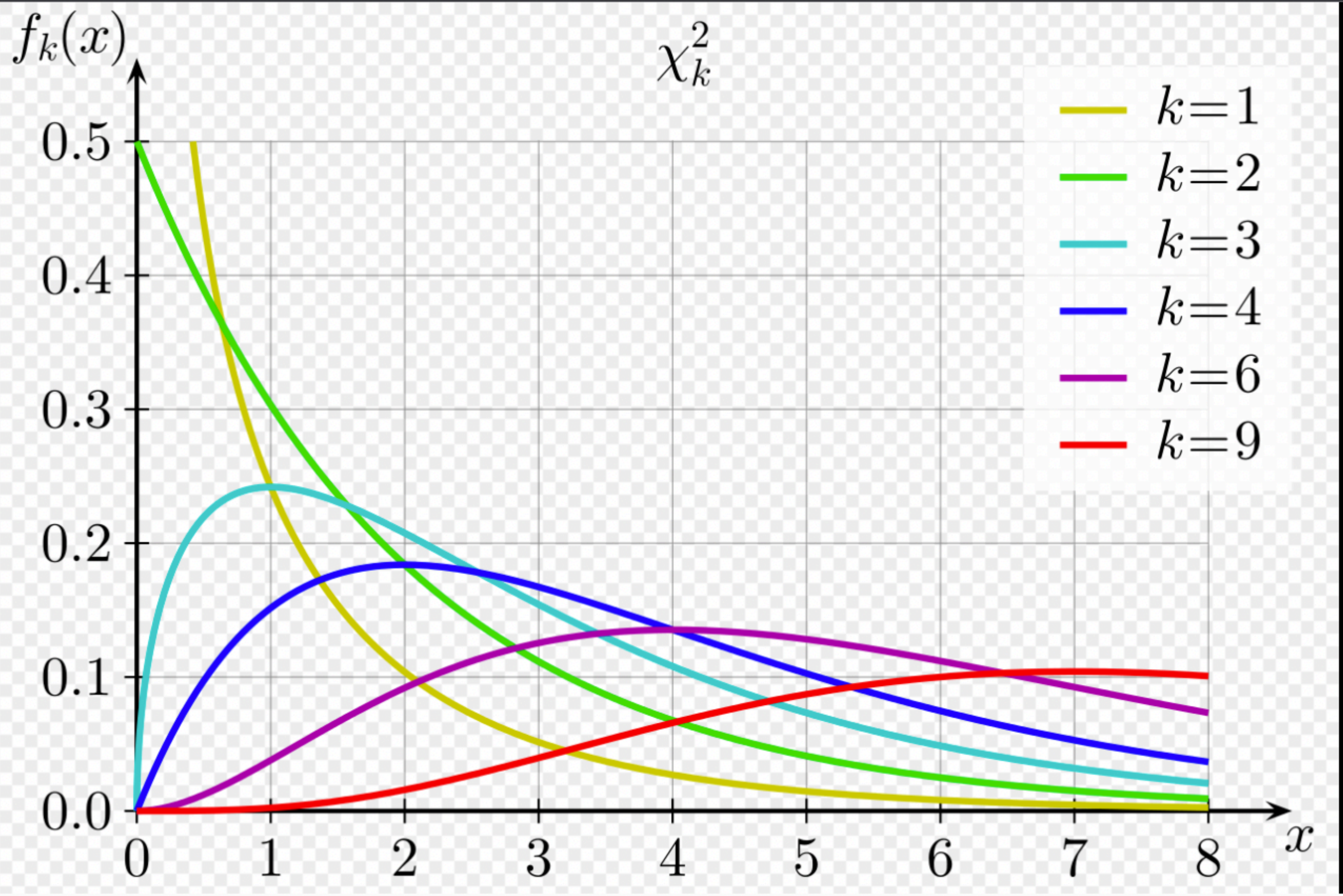
More details

Geek3 - Own work

Q

$\tilde{H}_0$: G is indep of P

$$\text{Test-statistic} = \sum_{i=1}^{\#\text{cells}} \frac{(O_i - E_i)^2}{E_i} \longrightarrow \chi^2(k)$$



Plot of the chi-square distribution for values of k = {1, 2, 3, 4, 6, 9}. Accurate plotcurves. Labels are embedded in Computer-Modern font.

More details

Geek3 - Own work

Gender vs Prem.

$p_1 = p_2$ or not

V-good Z-prop. test

Female $\rightarrow$ $\boxed{\hat{p}_1} = \dfrac{\#\text{prem}}{\#\text{total fem}}$

Males $\rightarrow$ $\hat{p}_2 = \dfrac{\#\text{prem}}{\#\text{total males}}$

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Gender \ Sub	Free	$1 pm	$10 p.M
F	✓	✓	✓
M			
ND			

1 χ^2 test

Discouraged

Multiple

z-prop.

Tests

(next class)

p-hacking

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Q

χ^2 test → Gender impacts perm sub

ⓐ p_F p_M

95 c.I of ...

95% c.I of

ⓑ $H_0: p_F = p_M$

$H_a: p_F > p_M$

Z-prop test

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χ^2-Test $\rightarrow$ Non-param Tests

KS-Test

Q1

Sample - Means

μ_1 rem $\rightarrow$ m_1

$m_1 - m_2$

placebo $\rightarrow$ m_2

μ_2

Z / T-test

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$

$\mu_1 \geq \mu_2$

$\mu_1 \leq \mu_2$

$H_0: \mu_1 \neq \mu_2$

disb of T under H_0

Let

If $T = \mu_1 - \mu_2$

disb of T under $\underbrace{\mu_1 \neq \mu_2}$

H_0 ✓

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$T = M_1 - M_2$

disb of T under Ho

rem placebo

$H_0: \mu_1 \geq \mu_2$

$= \mu_1 - \mu_2 \geq 0$

placebo is better

$H_a: \mu_1 - \mu_2 \leq 0$

rem placebo

Rem.. is better

disb of T under Ho

$H_0: \mu_1 \leq \mu_2$

$\Rightarrow \mu_1 - \mu_2 \leq 0$

$H_a: \mu_1 - \mu_2 > 0$

63 / 63

Q

χ^2-test

H_0: G is indep of Pre

$$\begin{cases} H_0: p_F = p_M \\ H_a: p_F > p_M \end{cases}$$

$$T = \sum_{i=1}^{\#\text{cells}} \frac{(O_i - E_i)^2}{E_i}$$

$\sim \chi^2(k)$

T under H_0 $\not\sim$ $\chi^2(k)$

Z-test :

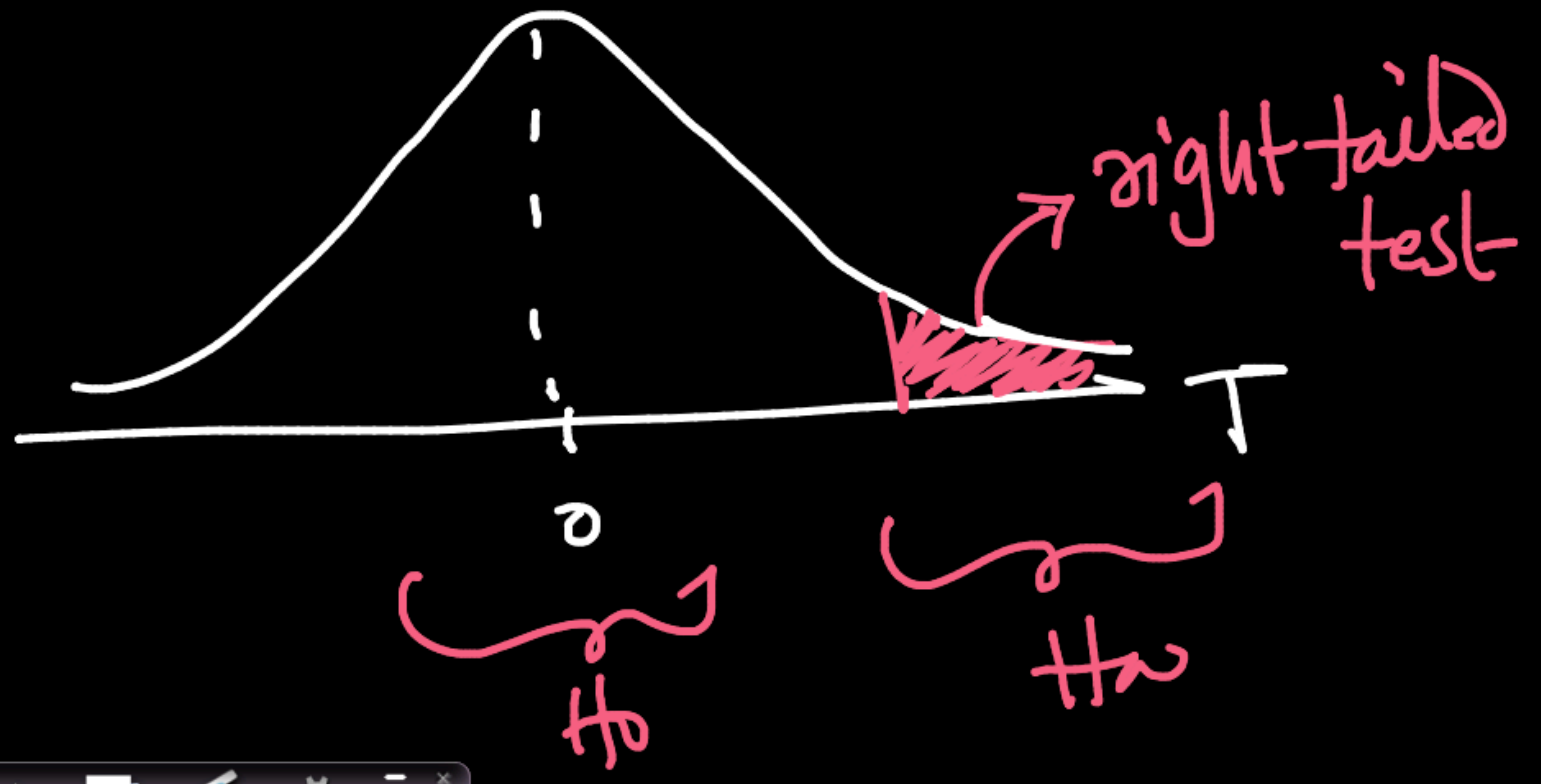
$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$

$$T = \frac{\overbrace{M_1 - M_2}^{\text{default}}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim Z(0,1)$$

If

$$T = \frac{M_2 - M_1}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \rightarrow \text{left-tailed}$$



Q

cont r.V

cont r.V

H	W

Is ω dependent on H?

Cannot use χ^2-test

Pearson CC
&
Spearman rank CC

χ^2-test

Cont r.V

150-160 160-170 170-180 180+

H → H_1 H_2 H_3 H_4

W →

discr. r.V

100 x 50

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Q

Compare means → Z-test (σ_1 & σ_2 or n_1 & n_2 are large)

→ T-test ✓

Compare prop → Z-prop. test

Test for indep → χ^2-test (Cat. r.v)

distb-sim → KS-test or AD-test (normality)

68 / 68

Q

[8.5 8.5]

$X = [8.5 \text{ - - - - } 8.5]$

$Y = [8, 8 \text{ - - - } 8]$

$Z = [1, 1 \text{ - - - } 1]$

C.1m X

C.1m Y [8; 8]

C.1m Z [1, 1]

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Compare means

med 1 vs Med 2 → T test

med 1 vs Med 2 vs Med 3 . . .

ANOVA

assmp:

rec times are all gaussian dis

70 / 70

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Q

mean of bern = p ✓

Bernoulli(p)

$x_1 \quad x_2 \quad x_3 \quad \dots \quad x_n$ n obs.

$1 \quad 0 \quad 0 \quad 1 \quad \dots \quad 1$

$\underline{1} \quad \underline{2}$ $n = 100$

$X \sim$

$x_i \in \{0, 1\}$

Sample mean

$$\hat{p} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{20}{100} = 0.2$$

B.S + $\boxed{\text{CLT}}$ ✓ → Sample means $\sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$

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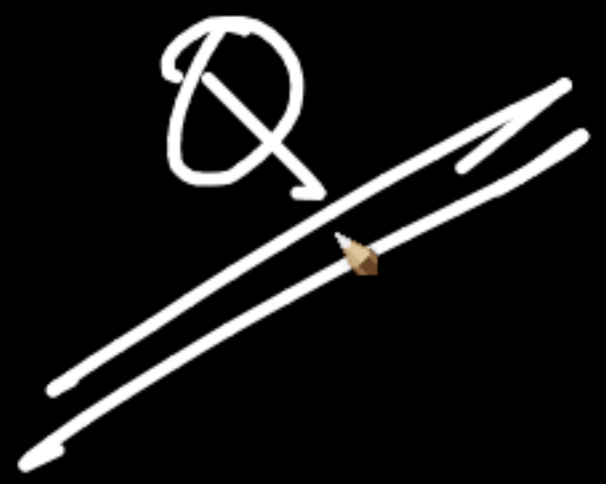
$X \sim b$



```
>>> rng = np.random.default_rng()
```

$x_1, x_2 \dots x_n$

```
>>> x = np.linspace(-15, 15, 9)
>>> stats.ks_1samp(x, stats.norm.cdf)
(0.44435602715924361, 0.038850142705171065)
```

```
>>> stats.ks_1samp(stats.norm.rvs(size=100, random_state=rng),
...                stats.norm.cdf)
KstestResult(statistic=0.165471391799..., pvalue=0.007331283245...)
```

Test against one-sided alternative hypothesis

Shift distribution to larger values, so that `` CDF(x) < norm.cdf(x)``:

```
>>> x = stats.norm.rvs(loc=0.2, size=100, random_state=rng)
>>> stats.ks_1samp(x, stats.norm.cdf, alternative='less')
KstestResult(statistic=0.100203351482..., pvalue=0.125544644447...)
```

Reject null hypothesis in favor of alternative hypothesis: less

```
>>> stats.ks_1samp(x, stats.norm.cdf, alternative='greater')
0.920581859791...)
```

```
>>> x = np.linspace(-15, 15, 9)
>>> stats.ks_1samp(x, stats.norm.cdf)
(0.44435602715924361, 0.038850142705171065)
```

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```

Reject null hypothesis in favor of alternative hypothesis: less

```
>>> stats.ks_1samp(x, stats.norm.cdf, alternative='greater')
0.920581859791...)
```

75 / 75

Q

$x_1 \dots x_n$

C.I on +P99 $\longrightarrow$ B.S $\longrightarrow$ ..middle 95%

2.5 97.5

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KStest_Ttest.ipynb - Colaborat | Kolmogorov–Smirnov test - Wi | Bernoulli distribution - Wikiped | S.6 Test of Proportion | STAT | 10.5 Hypothesis Testing for Tw | t test test for proportions - Go | scipy.stats.ks_1samp — SciPy

docs.scipy.org/doc/scipy/reference/generated/scipy.stats.ks_1samp.html

Getting started User Guide **API reference** Development Release notes

RELEASE 1.8.1

scipy.stats.ks_1samp

`scipy.stats.ks_1samp(x, cdf, args=(), alternative='two-sided', mode='auto')` [source]

Performs the one-sample Kolmogorov-Smirnov test for goodness of fit.

This test compares the underlying distribution F(x) of a sample against a given continuous distribution G(x). See Notes for a description of the available null and alternative hypotheses.

Parameters:

x : *array_like*

a 1-D array of observations of iid random variables.

cdf : *callable*

callable used to calculate the cdf.

args : *tuple, sequence, optional*

Distribution parameters, used with *cdf*.

alternative : *{'two-sided', 'less', 'greater'}, optional*

Defines the null and alternative hypotheses. Default is 'two-sided'. Please

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mode : *{'auto', 'exact', 'approx', 'asymp'}, optional*

Defines the distribution used for calculating the p-value. The following