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Fri 3 Jun 9:08 PM

Recap:

S : sample space

collection of all outcomes

Event: collection of any no. of outcomes

$E \subseteq S$

Dice

$S = \{1, 2, 3, 4, 5, 6\}$

4 → outcome

$E_1 = \{1, 2\}$

$E_2 = \{1\}$

$E_3 = \{1, 3, 5\}$

Coin toss twice

$S = \{HH, HT, TH, TT\}$

$E_1 = \{HH, HT, TH\}$

$E_2 = \{HH, TT\}$

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Recap: 1) $P[A|B] = \dfrac{P[A \cap B]}{P[B]}$ → conditional prob.

$P[A \cap B] = P[A|B]\,P[B]$

2) Independent events:

A and B are "independent"

$P[A|B] = P[A]$

$P[A \cap B] = P[A] \cdot P[B]$

mean the same

Obs and Pacing:

1) Pacing - we should slow down → for majority

2) Mismatch : prev and now

↳ assessments

→ go very slow

↓ more industry

3) Tutorials? ↑ ↓

60+Vs5

Monday

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Dice

$S = \{1, 2, 3, 4, 5, 6\}$

$E1 = \{1, 2\}$ $P[E_1] = \frac{2}{6} \rightarrow 2$

$E2 = \{1, 3, 5\}$ $P[E_2] = \frac{3}{6} \rightarrow 3$

Q $P[E1 \text{ "or" } E_2] = \frac{4}{6}$

$E_1 \cup E_2 = \{1, 2, 3, 5\}$ 4

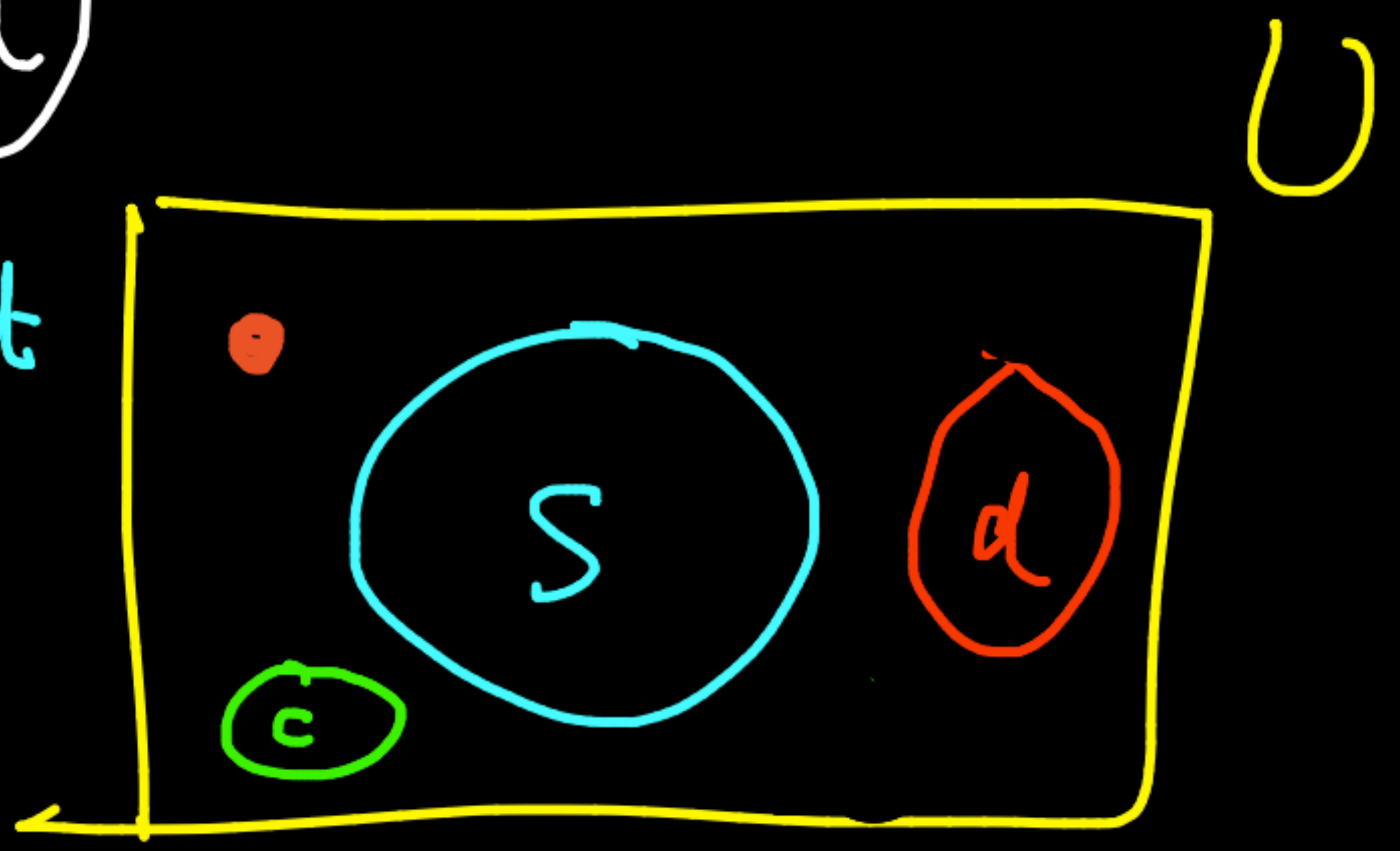
Why not $\frac{2}{6} + \frac{3}{6}$?

$E_1 \cap E_2 = \{1\}$

$$P[E_1 \cup E_2] = P[E_1] + P[E_2] - P[E_1 \cap E_2]$$

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

Mutually exclusive + Exhaustive (S, c, d)



U : all patient

S : survived → recovered. no sympt

c : case → symptoms, alive

ϕ

d : died

mutually exclusive → intersection is empty

$$S \cap d = \{\ \}$$

$$S \cap c = \{\ \}$$

$$c \cap d = \{\ \}$$

Exhaustive

$$S \cup c \cup d = U$$

union

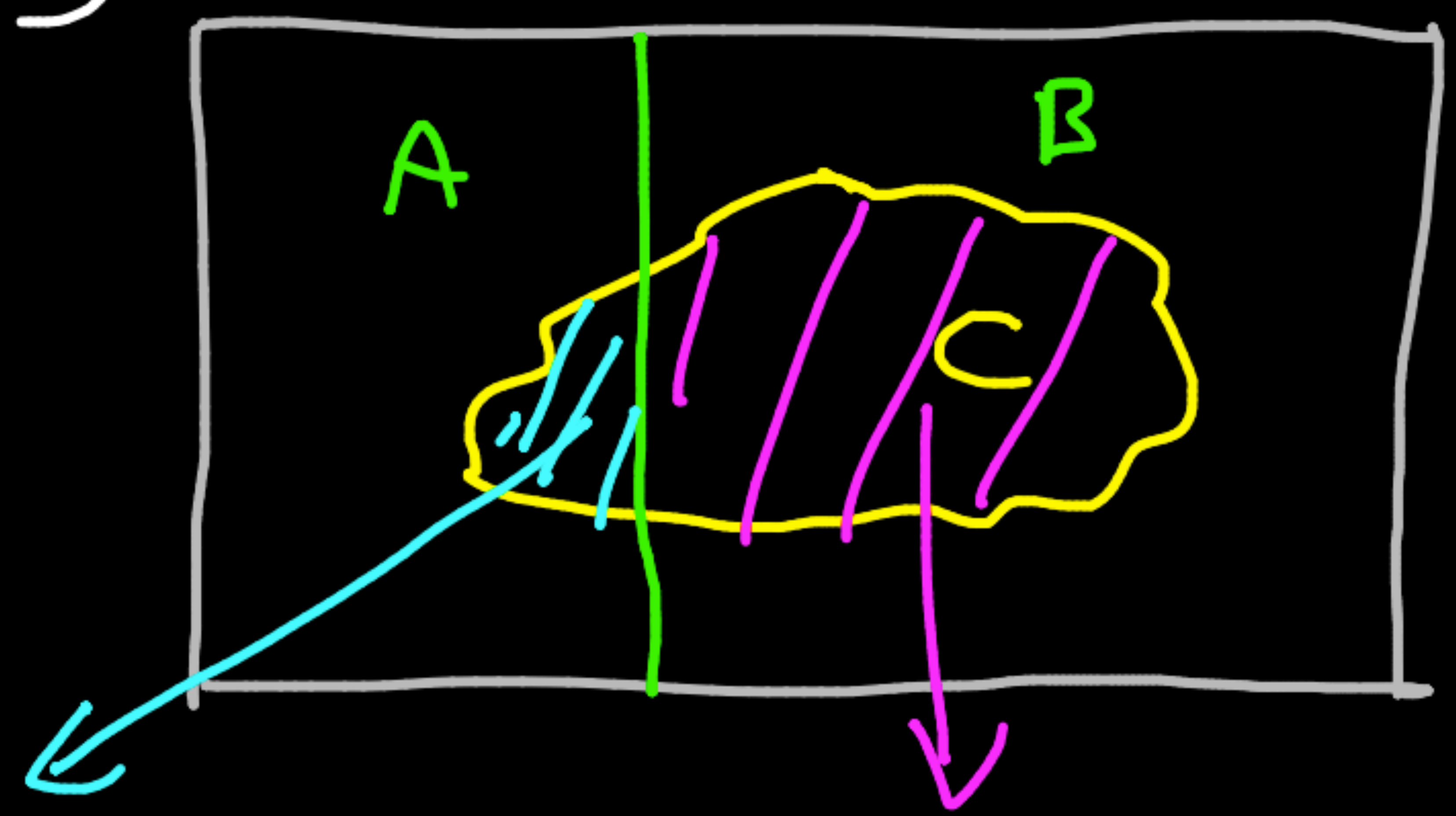
Universe

A & B

#MEE

$\boxed{10:40}$

S



$A \cap B = \{\ \}$ ϕ

$A \cup B = S$ → A & B are mut. excl. & exhausti.

S : whole cake

A :

B :

C : cream

$C_1 = C \cap A$

$C_2 = C \cap B$

$C = C_1 \cup C_2$

$C_1 \cap C_2 = \{\ \}$

$$P[C] = P[C_1] + P[C_2] - \overbrace{P[C_1 \cap C_2]}^{0}$$

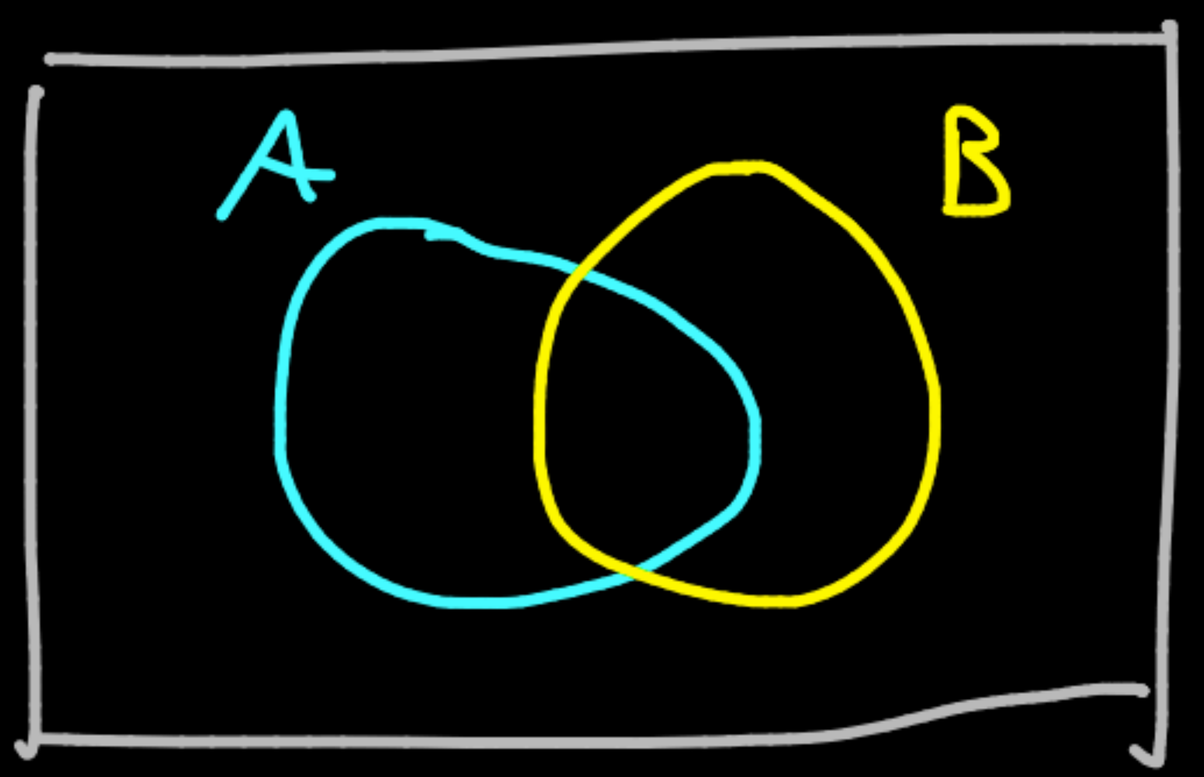
$$= P[C \cap A] + P[C \cap B]$$

$$= P[C \mid A]\, P[A] + P[C \mid B]\, P[B]$$

$$P[C \mid A] = \frac{P[C \cap A]}{P[A]}$$

$$P[C \cap A] = P[C \mid A]\, P[A]$$

A, A^c #MEE



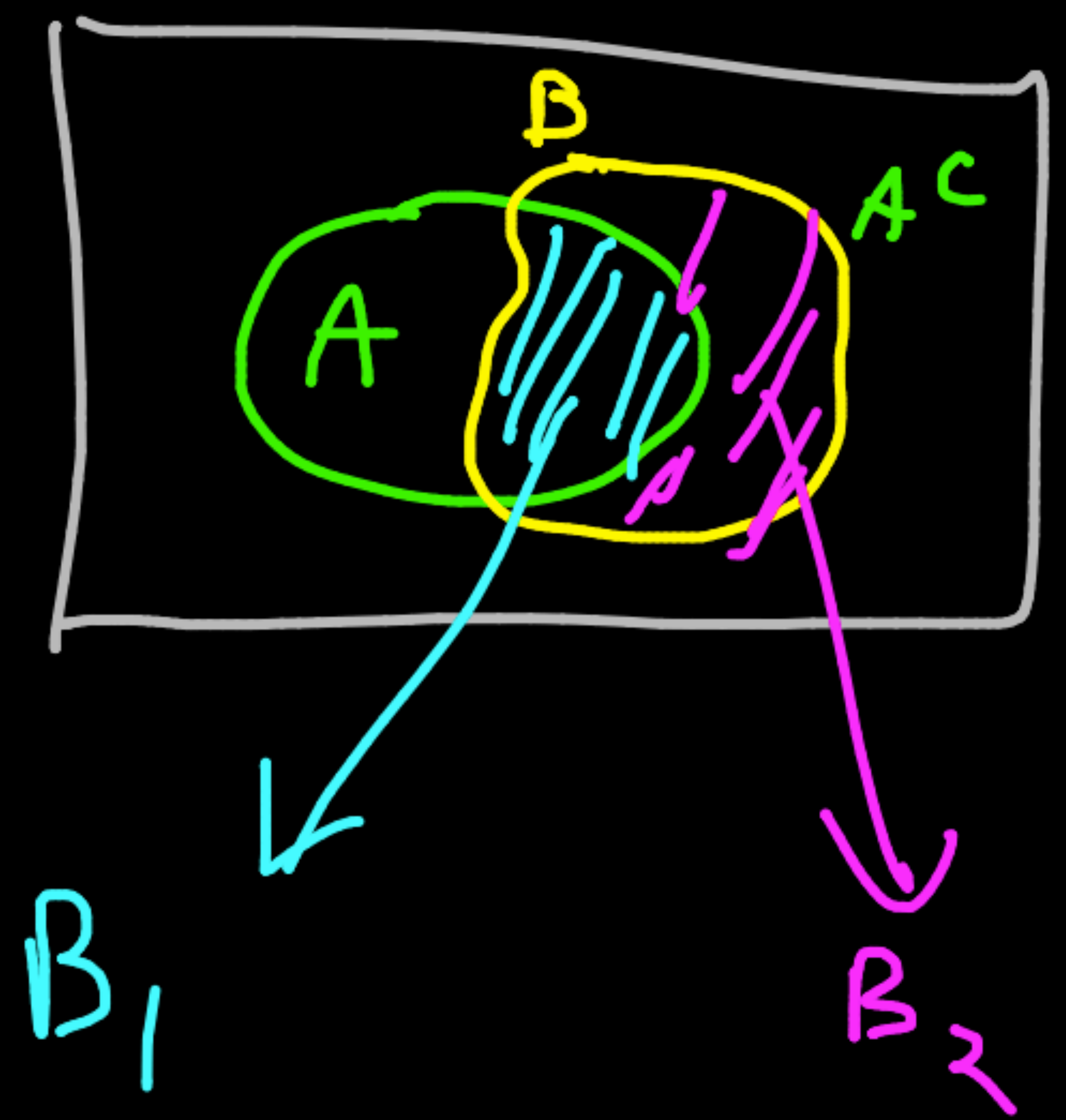
$$P[B/A] = \frac{P[B \cap A]}{P[A]}$$

$$P[B|A]P[A] = P[B \cap A]$$

$$P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{P[B|A]\,P[A]}{\boxed{P[B]}}$$

$$\boxed{P[B]} = P[B|A]\,P[A] + P[B|A^c]\,P[A^c]$$

S : universe/sample space



empty/null

$$B_1 \cap B_2 = \{\ \}$$

$$B_1 = B \cap A \qquad B_2 = B \cap A^c$$

$$B = B_1 \cup B_2$$

$$P[B] = P[B \cap A] + P[B \cap A^c]$$

$$= P(B|A)\, P[A] + P[B|A^c]\, P[A^c]$$

other wont work

needs #MEE

A & A^c are MEE

Baye's Theorem

$$P[A|B] = \frac{P[B|A]\;P[A]}{P[B|A]\,P[A] + P[B|A^c]\,P[A^c]}$$

H

true

Covid

$\{H, D\} \rightarrow$ people : healthy, disease

$\{-ve, +ve\} \rightarrow$ test

Ideally $H \rightarrow -ve$, $D \rightarrow +ve$

Errors (sometimes) $H \rightarrow +ve$, $D \rightarrow -ve$

(False +ve) (False -ve)

$$\left.\begin{array}{ll} P[H] = 0.9 & P[D] = 0.1 \\ P[-ve \mid H] = 0.75 & P[-ve \mid D] = 0.25 \\ P[+ve \mid H] = 0.25 & P[+ve \mid D] = 0.75 \end{array}\right\} \text{Data}$$

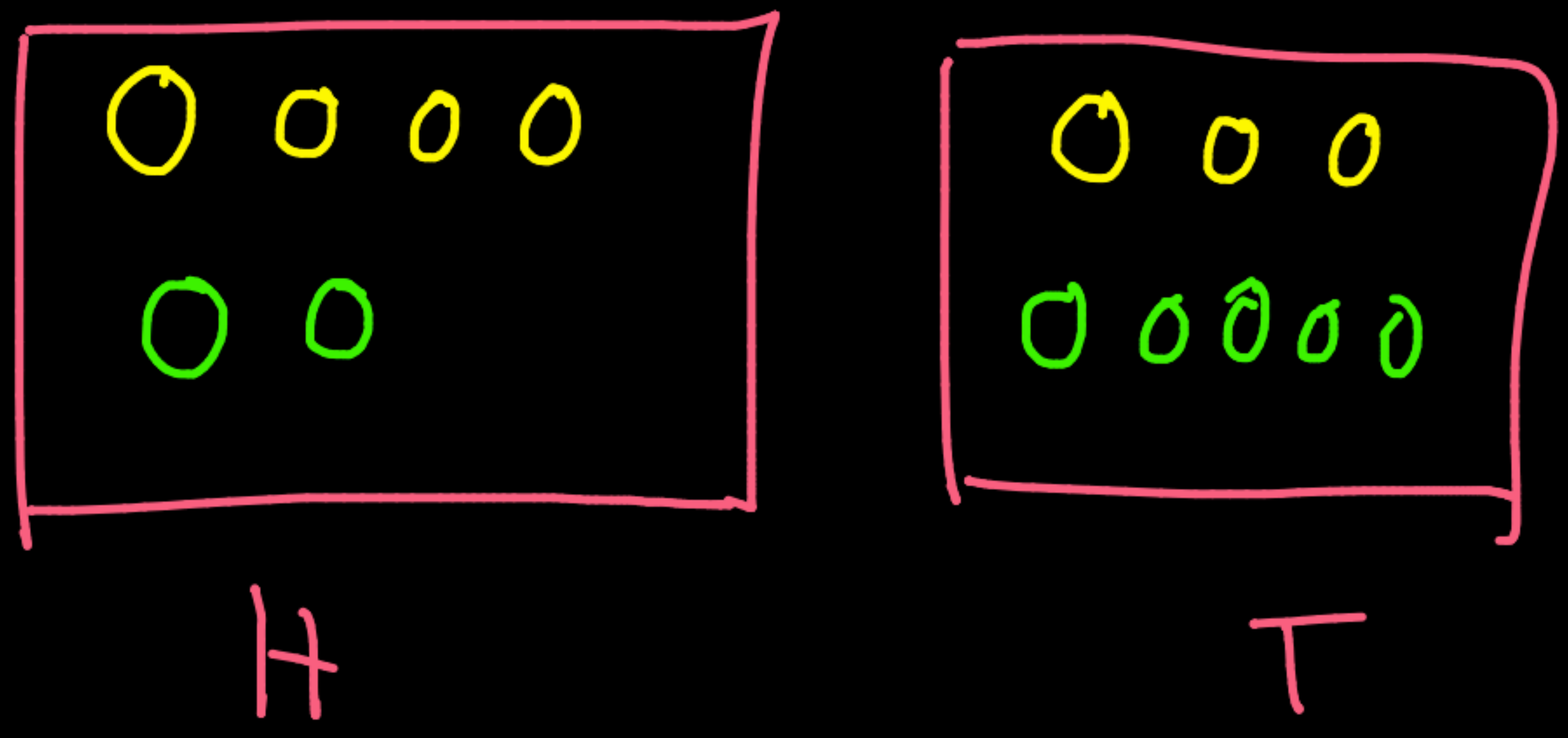
If we get a +ve, what is the probability that I am healthy

$$P[H \mid +ve] = \frac{P[+ve \mid H]\, P[H]}{P[+ve \mid H]\, P[H] + P[+ve \mid D]\, P[D]} = \frac{(0.25)(0.9)}{\underbrace{(0.25)(0.9) + (0.75)(0.1)}_{P(+ve) = 0.3}} = 0.75$$

Coin

2 boxes

4 yellow, 2 green



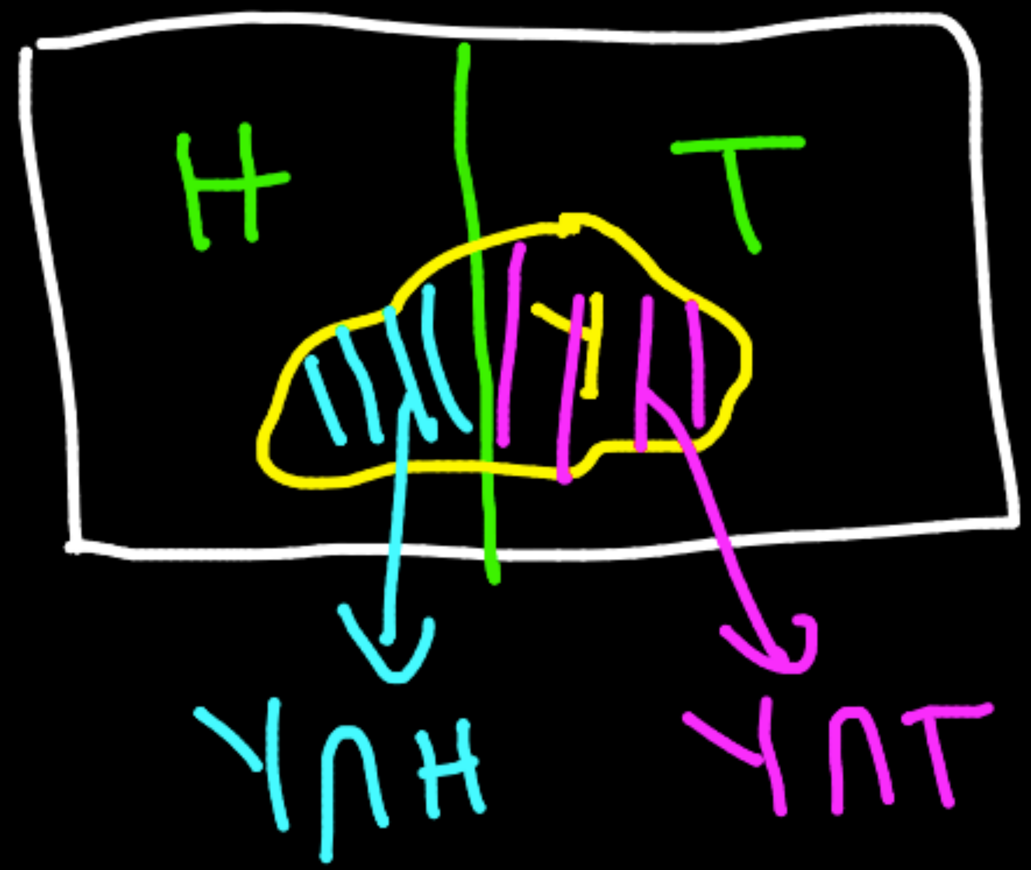
3 yellow, 5 green

$$P[Y|H] = \frac{4}{4+2}$$

$$P[Y|T] = \frac{3}{3+5}$$

If the ball ~~so~~ chosen is yellow, what is the prob. that $C = \{H\}$?

$$P[H|Y] = \frac{P[H \cap Y]}{P[Y]} = \frac{P[Y|H]\,P[H]}{P[Y]} = \frac{(P[Y|H]\,P[H])}{(P[Y|H]\,P[H]) + (P[Y|T]\,P[T])}$$



$$Y = (Y \cap H) \cup (Y \cap T)$$

$$P[Y] = P[Y \cap H] + P[Y \cap T] = P[Y|H]\,P[H] + P[Y|T]\,P[T]$$

Whatsapp

$X_1 \quad X_2 \quad X_3 \quad X_4$

How are

How are → ① you ② things ③ the

$P[\text{next word} \mid \text{How are}]$

$P[X_3 = \underline{w} \mid \{X_1 = \text{"How"}\} \cap \{X_2 = \text{"are"}\}]$

w → vocabulary

"you" → highest

"things" → 2nd highest

Random Variable

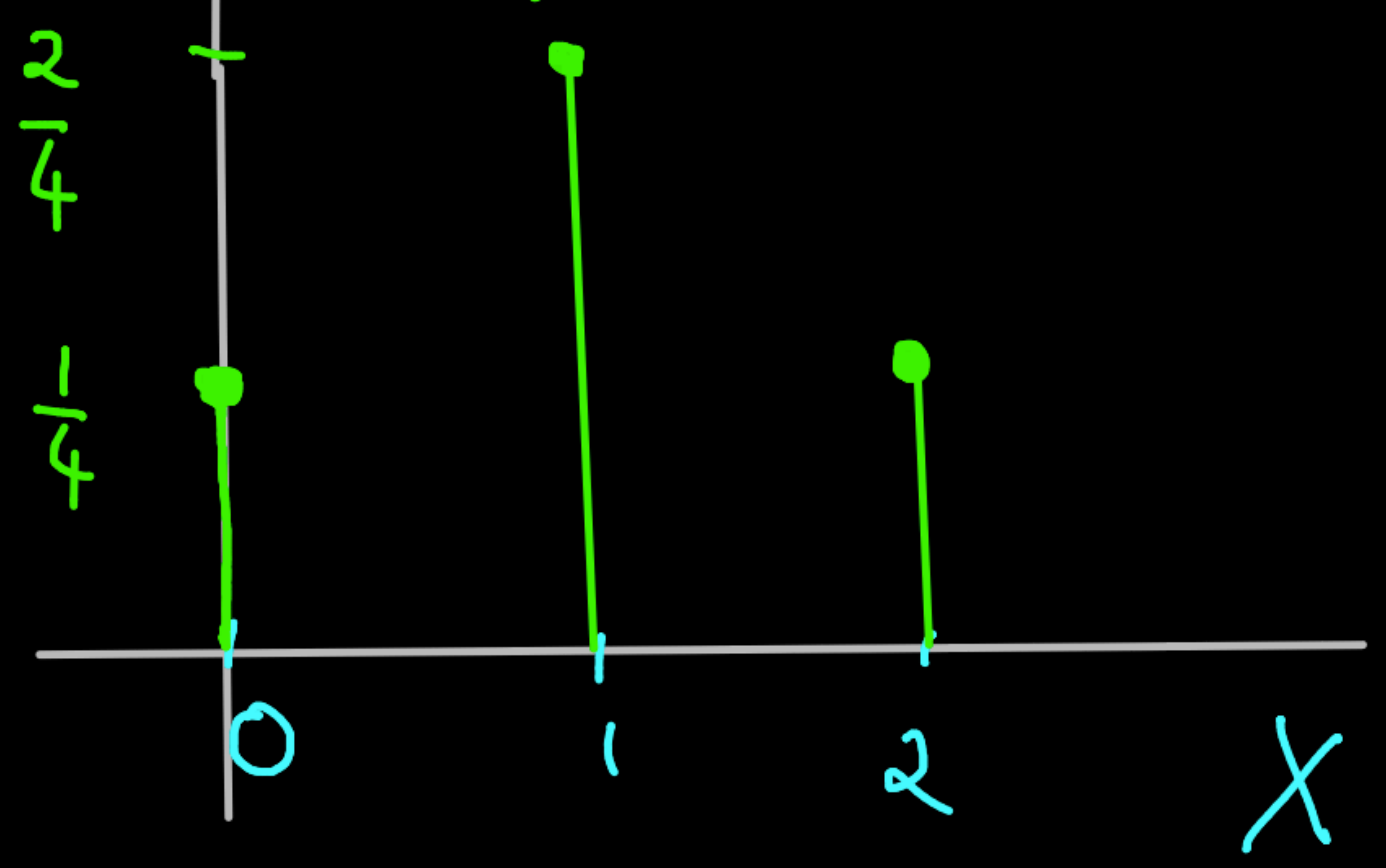
Coin toss twice $\quad S = \{HH, HT, TH, TT\}$

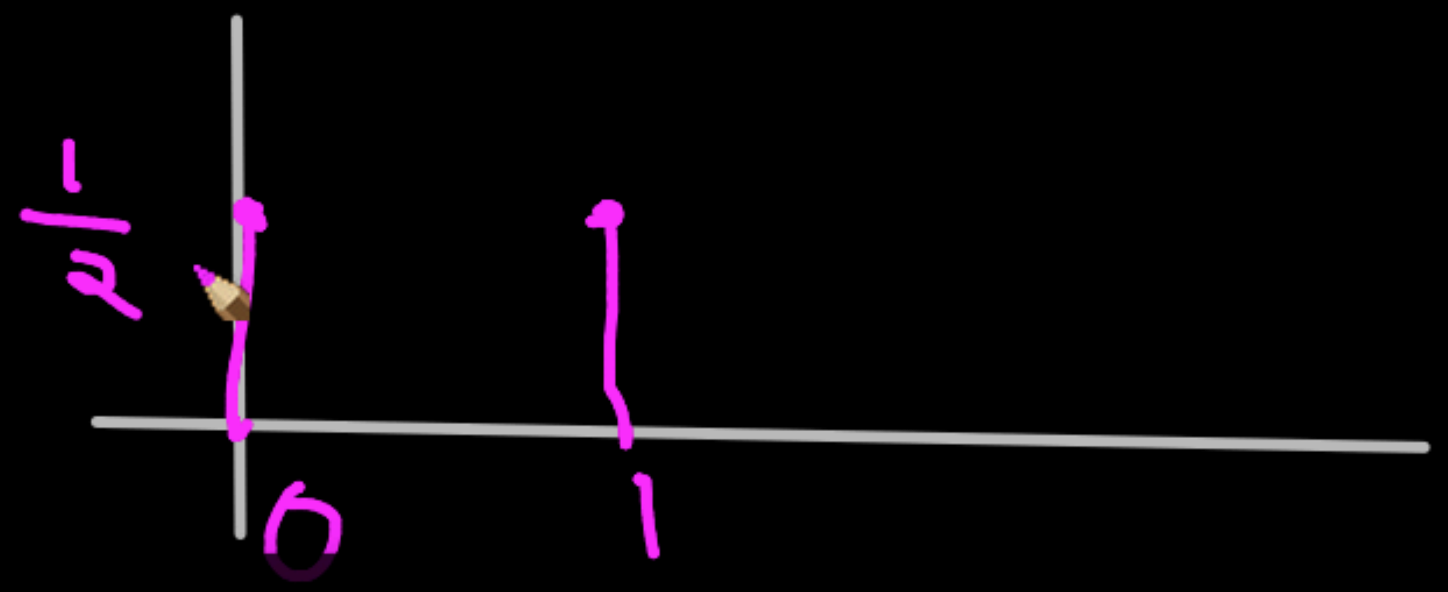
X = no. of heads $\rightarrow \{0, 1, 2\}$

$$X \rightarrow \begin{cases} 0 & \{TT\} & P[X=0] = P[\{TT\}] = \frac{1}{4} \\ 1 & \{HT, TH\} & P[X=1] = P[\{HT, TH\}] = \frac{2}{4} \\ 2 & \{HH\} & P[X=2] = P[\{HH\}] = \frac{1}{4} \end{cases}$$

$$Y \rightarrow \begin{cases} 0 & \{HH, TT\} & P[Y=0] = P[\{HH, TT\}] = \frac{1}{2} \\ 1 & \{HT, TH\} & P[Y=1] = P[\{HT, TH\}] = \frac{1}{2} \end{cases}$$

Probability mass function





Coin: Toss till I get first {H} $\longrightarrow$ experiment

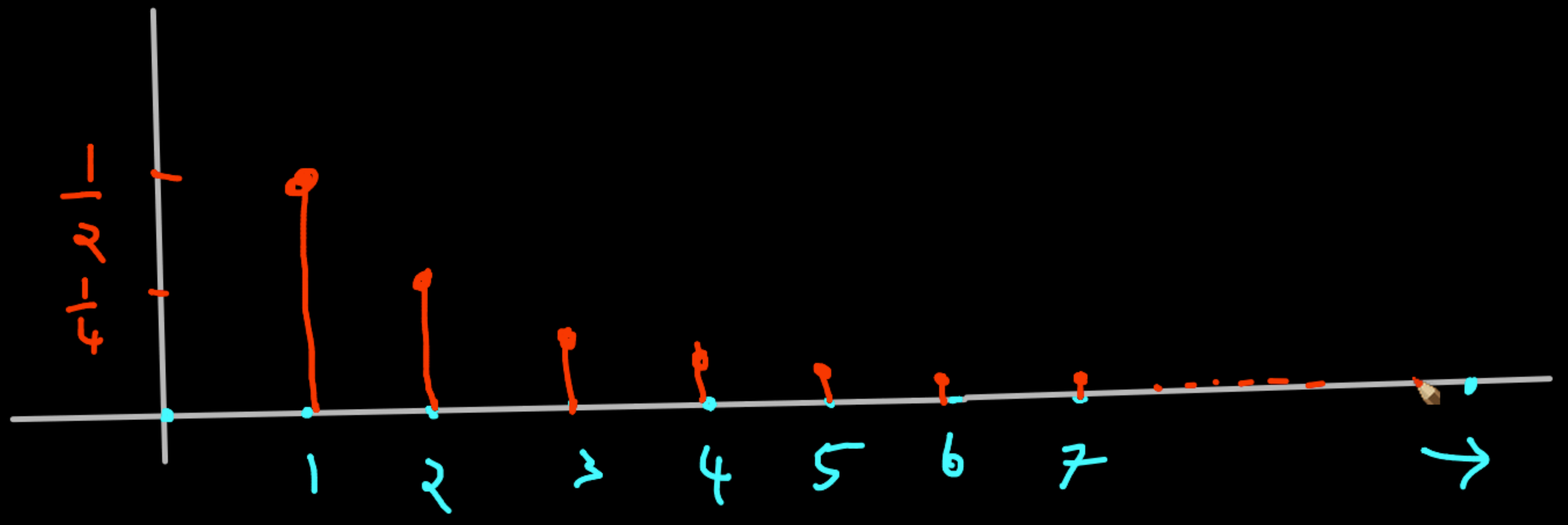
$$S = \{ \underbrace{H}_{X=1}, \underbrace{TH}_{X=2}, \underbrace{TTH}_{X=3}, \underbrace{TTTH}_{X=4}, \ldots \ldots, \ldots\ldots \}$$

X = number of tosses to get first {H}

$$P[X=5] = P[\{T \cap T \cap T \cap T \cap H\}] = P(T)P(T)P(T)P(T)P(H) = \frac{1}{2^5} = \frac{1}{32}$$

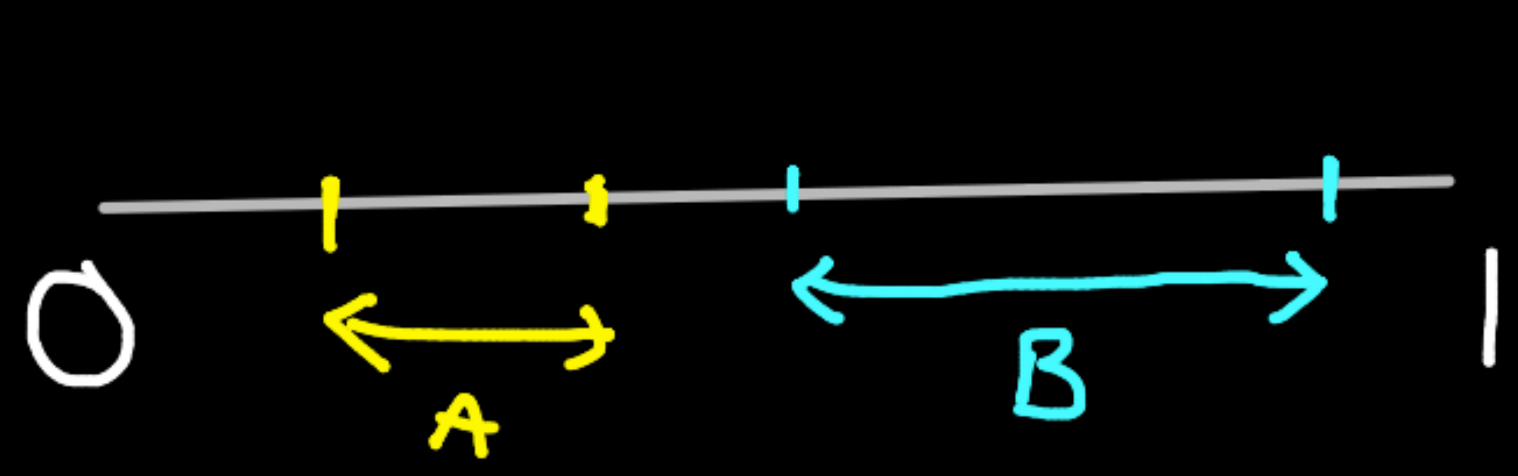
$$P[X=k] = \frac{1}{2^k}$$

$$P[X=1] = P(\{H\}) = \frac{1}{2}$$



Which one would you bet on?

$\underline{\underline{x}} \rightarrow$ point 1 touch



$$P[\underbrace{x \in A}_{\text{"lies in A"}}] < P[x \in B]$$

Claim :

If A and B are mutually exclusive, then they are **not** independent.

$$A \cap B = \{\ \} \longrightarrow P[A \cap B] = 0$$

$$P[A|B] = \frac{P[A \cap B]}{P[B]} = 0 \neq P[A]$$

$\downarrow$

not independent

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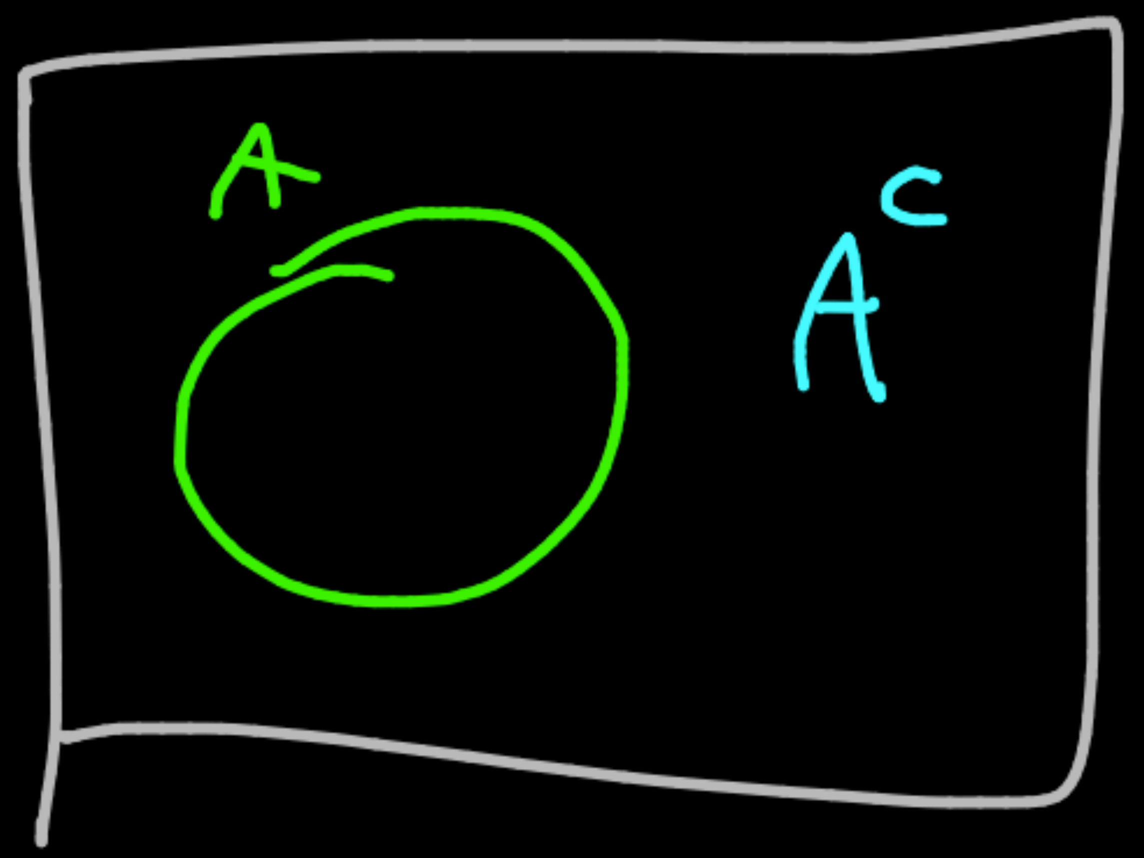
S



$$A \cup A^c = S$$

$$A \cap A^c = \{\} \rightarrow P(A \cap A^c) = 0$$

$$P(S) = 1$$

$$S = A \cup A^c$$

$$P(S) = P(A) + P(A^c) - \underbrace{P(A \cap A^c)}_{=0}$$

$$1 = P(A) + P(A^c)$$

Class is over : Advanced stuff

$\{1,2,3,4,5,6\}$

$A \rightarrow$ dice till he gets $\{6\}$ $\rightarrow$ N : number of tosses

$B \rightarrow$ toss a coin for every dice throw

Win 1 rupee every time he tosses $\{H\}$ $\rightarrow$ expected amount

$X \rightarrow$ amount he makes

$$P[X=k] = \sum_{n=1}^{\infty} \underbrace{P[X=k \mid N=n]} \underbrace{P[N=n]}_{\rightarrow \left(\frac{5}{6}\right)^{n-1} \frac{1}{6}}$$

$$= \sum_{n=k}^{\infty} {}^{n}C_{k} \left(\frac{1}{2}\right)^{n} \left(\frac{5}{6}\right)^{n-1} \frac{1}{6}$$

$A \rightarrow \quad 3, 4, 4, 1, 5, 2, 1, 6 \qquad N = 8$

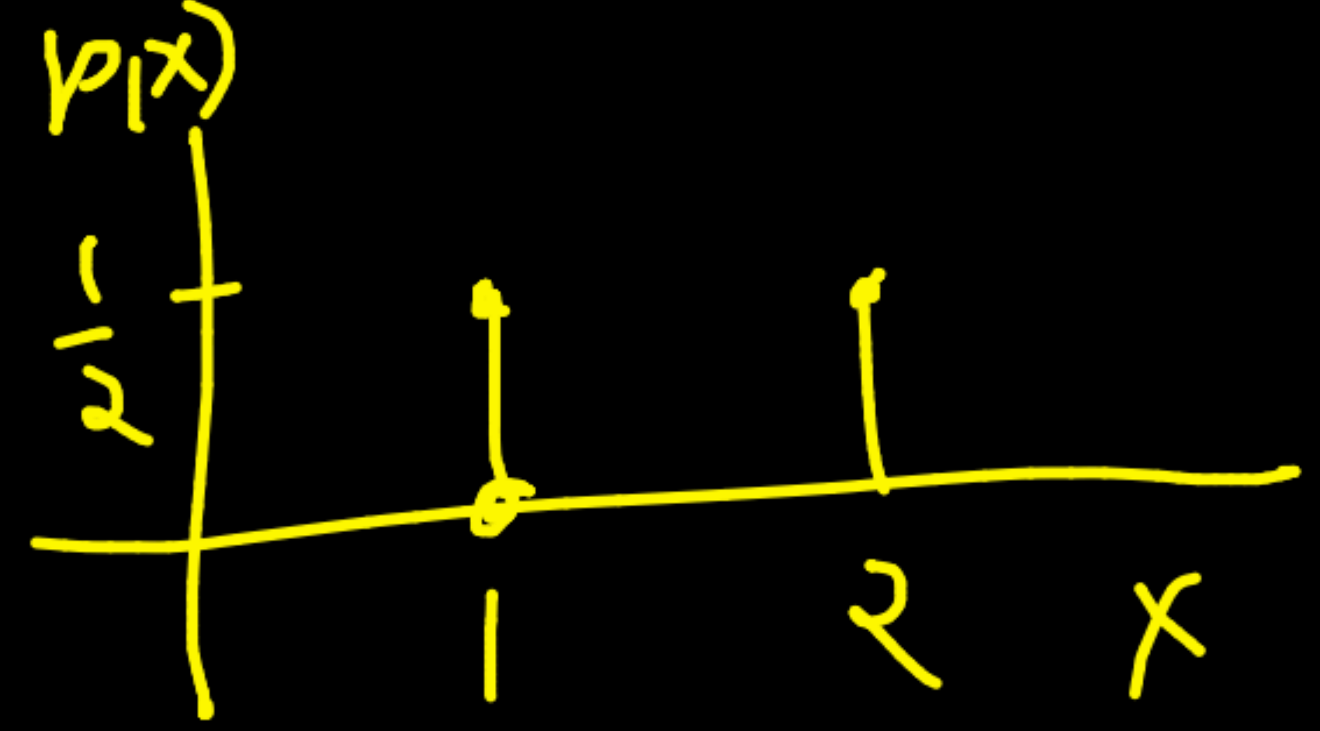
$B \rightarrow \quad H \; T \; T \; H \; H \; H \; T \; H$

$k = 5$ heads in 8 tosses $\qquad {}^{8}C_{5}$

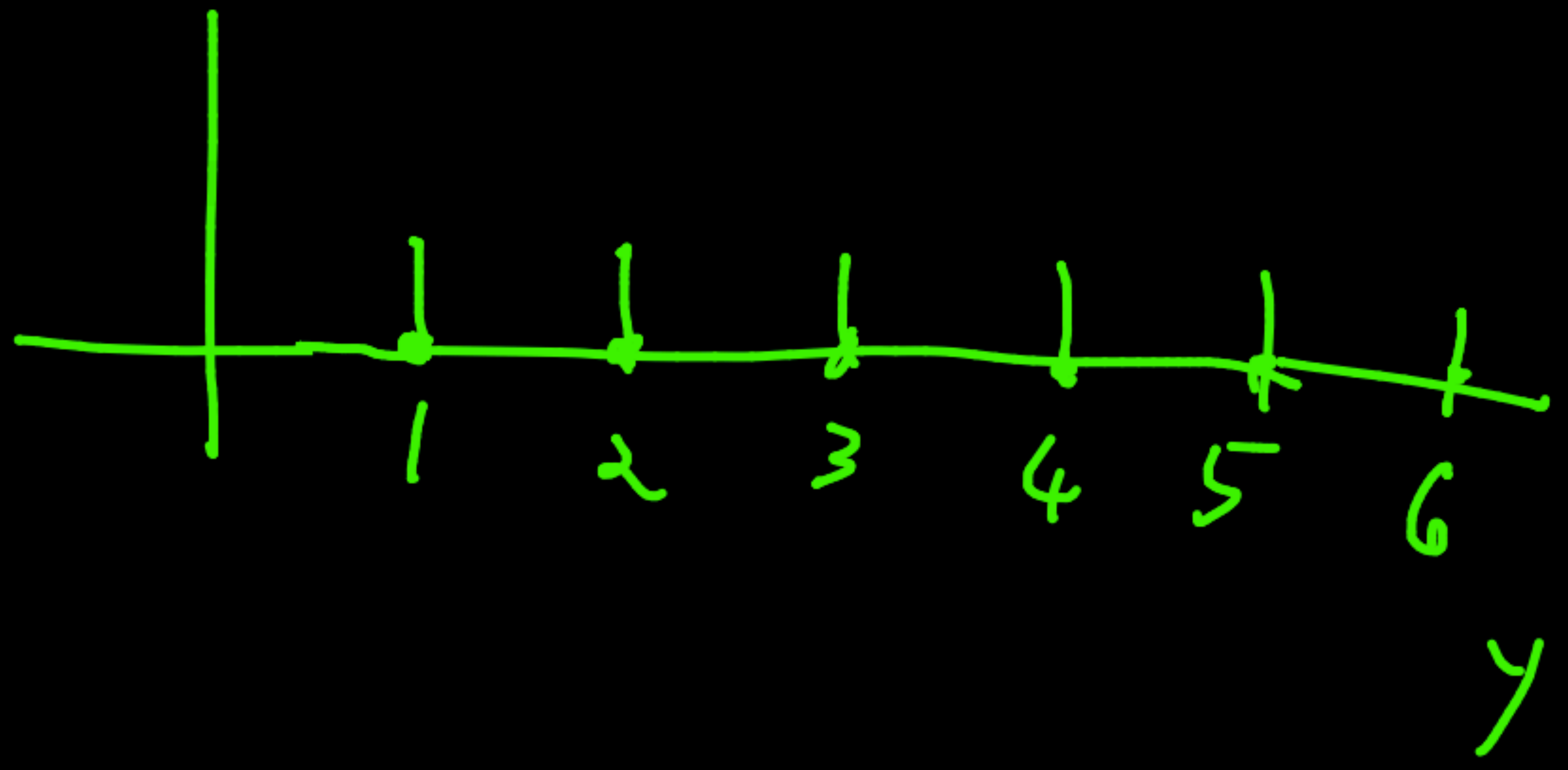
We have 1 coin → simulate a dice

$\{H, T\} \longrightarrow \{1, 2, 3, 4, 5, 6\}$

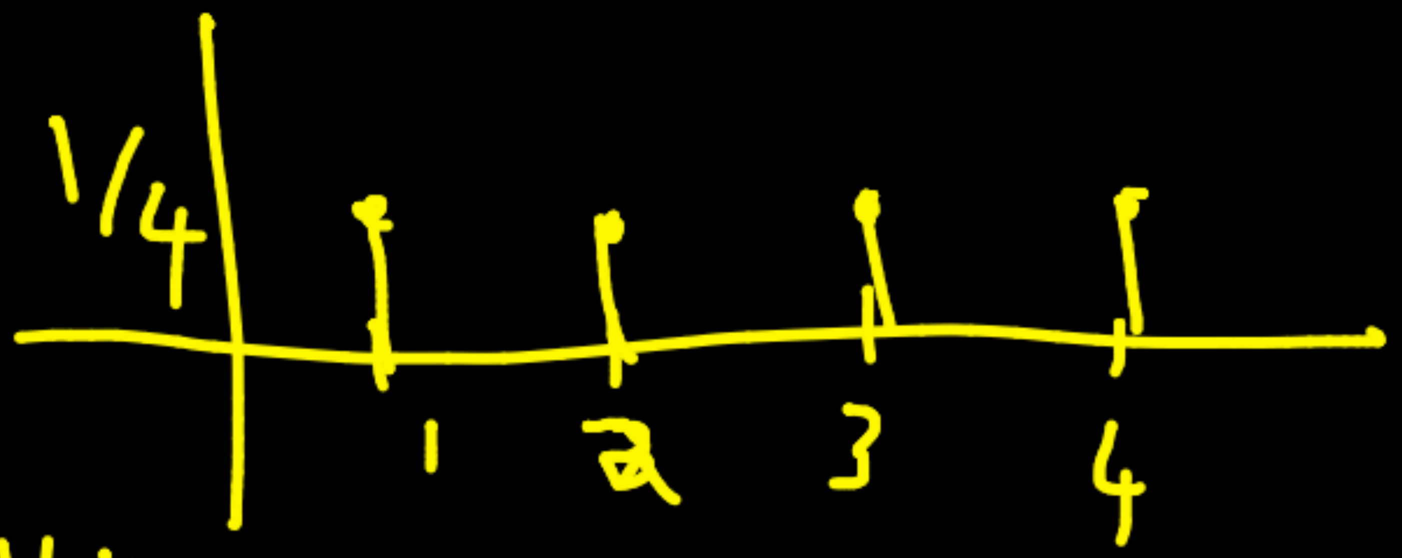
$X =$



$Y =$



$X1 = \{\underset{1}{HH}, \underset{2}{HT}, \underset{3}{TH}, \underset{4}{TT}\}$



$X2 = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$X2 =$

python3

Sat 4 Jun 12:15 AM

$X \in \{1, 2, 3, 4, 5, 6, 7, 8\}$

I can generate

$Y = \{1, 2, 3, 4, 5, 6\}$

I want this

```
While True:
    X = np.random.choice([1, 2, ... 8])
    if X <= 6
        Y = X
        break
```

→ indep every time

$$P[Y=2] = P[Y=2 \mid \quad] P[\quad] + P[Y=2 \mid \quad] P[\quad]$$

python3 Sat 4 Jun 12:28 AM

$$P[Y=2] = P[Y=2 \mid X \le 6]\, P[X \le 6] + \underline{P[Y=2 \mid X > 6]}\, P[X>6]$$

$$P[Y=2] = \frac{1}{\cancel{6}} \frac{\cancel{6}}{8} + P[Y=2]\, \frac{2}{8}$$

$$P[Y=2]\left(1 - \frac{2}{8}\right) = \frac{1}{8}$$

$$P[Y=2] = \frac{1}{6}$$

$$P[Y=2 \mid X > 6]$$