Agenda

1. Number systems → Importance
2. Binary and Decimal → Importance
3. Conversion $(\ )_x \rightarrow (\ )_{10} \rightarrow (\ )_y$
4. Bitwise operations → or (|)
   and (&)
   xor (^)
   not (~)

Roman number system:

$\boxed{\text{IV} \quad \text{V} \quad \text{VI}}$

$\boxed{\text{LXLVIII}}$

I → 1

V → 5

D → 500

L → 50

X → 10

Expand

$$1307 = 1 \quad 3 \quad 0 \quad 7$$

$$= \boxed{1000 + 300 + 7}$$

$$= 1 \times \boxed{10^3} + 3 \times \boxed{10^2} + 7 \times \boxed{10^0}$$

Power of 10 → Base

Decimal number system has base 10

$\boxed{0 - 9}$

10 digits

1. Base decides the weight / value of every position

2. Decide the number of digits the number system will have.

22

$(abcd)_{10} \Rightarrow a \times 10^3 + b \times 10^2 + c \times 10^1 + d \times 10^0$

$(1307)_{25} = 1 \times 25^3 + 3 \times 25^2 + 7 \times 25^0$

$= \boxed{\begin{matrix}625 \\ \times 25\end{matrix} + 375 + 7}$

decimal no. system

Octal Number System :

Base = 8

$(13)_8 = 8^1 \times 1 + 8^0 \times 3$

$= 8 + 3 = 11$

0
1
2
3
4
5
6
7
10

0 – 7

8 ?

$8^1 \times 1 + 0 \times 8^0 = 8 \quad (8)_{10}$

1326 ⇐ What are possible bases in which this number has been used.

7 ✓ 8 ✓ 9 ✓ 10 ✓

0 – 5

All $\boxed{\text{Base} \geq 7}$ ✓

~~6854~~

0 – 7

$8^4 \; 8^3 \; 8^2 \; 8^1 \; 8^0$

$(10001)_{10}$

$1 \times 8^4 + 1 \times 8^0$

0 - 9

Hexadecimal

Hexadecimal ⇒ Base ⇒ 16

16 digits

10 digits
0-9

6
A, B, C, D, E, F
10 11 15

$(1001)_{16} = 16^3 \times 1 + 16^0 \times 1$

$(AB)_{16} = (16^1 \times 10 + 16^0 \times 11)_{10}$

Why 10?

Humans Base-10?

⇓

10 fingers.

⇒

Computers

⇓

Electric signal.

0
1
2
3
4
5
.
.
9

1 ⇐ High voltage
0 ⇐ Low voltage

⇓

$\boxed{\text{Base-2}}$

⇓

* $\boxed{\text{Binary Number system}}$

0/1

$\overset{2^4}{1}\overset{2^3}{0}\overset{2^2}{0}\overset{2^1}{1}\overset{2^0}{0}$

$(10010)_2 = (\quad)_{10}$

$= 1 \times 2^4 + 1 \times 2^1 = 18$ ✓

$(abcd)_k = (\quad)_{10}$

$a \times k^3 + b \times k^2 + c \times k^1 + d \times k^0$

Convert any number system to Decimal

$k^3 \; k^2 \; k^1 \; k^0$

$(a \; b \; c \; d)_k$ ← 4 numbers

k^3

n digits

k^{n-1}

MSB → Most Significant

LSB → Least Significant

$3^4 \; 3^3 \; 3^2 \; 3^1 \; 3^0$

$(0 \, 2 \, 1 \, 0 \, 1)_3 =$

$0 \times 3^4 + \underbrace{2 \times 3^3}_{54} + \underbrace{1 \times 3^2}_{9} + \underbrace{1 \times 3^0}_{1}$

$= 64$

Decimal to Binary

$(1037)_{10} = a \times 2^{K} + b \times 2^{K-1} + c \times 2^{K-2} \ldots \; 2^0$

$a \to 0/1$

$\boxed{\text{Sum of powers of 2}}$

Per binary: 0/1

$(1011) = 1 \times 2^3 + 0 \times 2^2$

$(21)_{10} = a \times 2^4 + b \times 2^3 + c \times 2^2 + d \times 2^1 + e \times 2^0$

16, 8, 4, 2

$21 \% 2 \to$ odd $\to 1 \to e == 1$

$21 \% 2 \to$ even $\to 0 \to e == 0$

only odd factor

$\boxed{e = 21 \% 2}$

$21/2$

$= a \times 2^3 + b \times 2^2 + c \times 2^1 + d \times 2^0$

$(21/2) \% 2 \to$ even $\to d = 0$

$\to$ odd $\to d = 1$

2	21	1
2	10	0
2	5	1
2	2	0
2	1	1
	0	

$(21)_{10} = (10101)_2$

$21//2$

$(21//2)//2$

$1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 0\times 2^1 + \boxed{1\times 2^0}$

$1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 0\times 2^0$

$1\times 2^2 + 0\times 2^1 + 1\times 2^0$

$(35)_{10} = (100011)_2$

$\boxed{32 + 2 + 1}$

write a program to convert a decimal number to binary number representation.

A decimal system to octal

$(\quad)_{10} = (\quad)_8$

$(27)_{10} = a \times 8^2 + b \times 8^1 + c \times 8^0$

divisible by 8

$27 \% 8 = x$

8	27	3
8	3	3
	0	

$(33)_8 = 8^1 \times 3 + 8^0 \times 3$

$= 24 + 3$

$= 27$

2 → base

$(\quad)_k \to (\quad)_{10} \to (\quad)_y$

$(161)_8 = 1 \times 8^2 + 6 \times 8^1 + 1 \times 8^0$

100 10 8

64

decimal

$$\begin{array}{r} 396 \\ 44 \\ \hline 440 \end{array}$$

$$\begin{array}{r} 10110 \\ 10111 \\ \hline 111101 \end{array}$$

$= 2$

$= (10)_2$

(11)

$$\begin{array}{r} 3746 \\ 470 \\ \hline (4436)_8 \end{array}$$

$(11) \to (13)_8$

$$\begin{array}{r} 10 \\ \times 6 \\ \hline 60 \end{array}$$

78

(22)

26

$$\begin{array}{r} 731 \\ 731 \\ \hline 731 \end{array}$$

26 13

$$\begin{array}{r} 731 \\ \times 13 \\ \hline 26 \quad 13 \\ 731 \end{array}$$

$9 = 11$

$24 = 26$

Binary Number system

1. **Not Operation** (~) Unary

1 → 0

0 → 1

Toggles the bit

~(1001)

✓ (0110)

1	0
0	1

2. **And Operator** (&)

a & b

Truth Table

a	b	res
0	0	0
0	1	0
1	0	0
1	1	1

$(9)_{10}$ & $(11)_{10}$ ~ (1)

$(1001)_2$ $(1011)_2$

1001

9

(51)

$(10) \Rightarrow (1010)_2$

10 & 1

1010
0001

1010
0010

0 → Bit was off

%

0

ON

a & 1

ODD → Last bit ON → 1

EVEN → OFF → 0

10 Can you tell me if 2^1 bit is ON or OFF

1010
& 0010

2^1

OR

a	b	
0	0	0
0	1	1
1	0	1
1	1	1

15 | 16 ⇒ 31 ✓

01111
10000
11111 ⇒ 31

$2^5 - 1$

0
1
10
11
100
101
110
111

n bit number → how many decimal nos. can you represent

(_ _ _ _)

3 bit

0	0	0	→ 0
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	7

$\boxed{0 - 2^n - 1}$

$= 2^n$

13 → 1101

10 → 1000

1111 → 15

$\boxed{2^3 + 2^2 + 2^1 + 2^0} \Rightarrow 2^4 - 1$

$\boxed{a = 10}$ → 1010

$\boxed{a \mid 1}$

1010
0001
————
1011 = 11 ✓

2⁰

off

$\boxed{a = 9}$

$\boxed{1001}$

1001
0001
————
1001

9

odd → x → $\boxed{x}$ ✓

even → x → $\boxed{x+1}$ ✓

AND → Check if a bit is ON or OFF

OR → Switching ON any bit

1010
0100
————

XOR Operator

(^)

a	b	res
0	0	0
0	1	1 ←
1	0	1 ←
1	1	0

Both bits are sames → res = 0

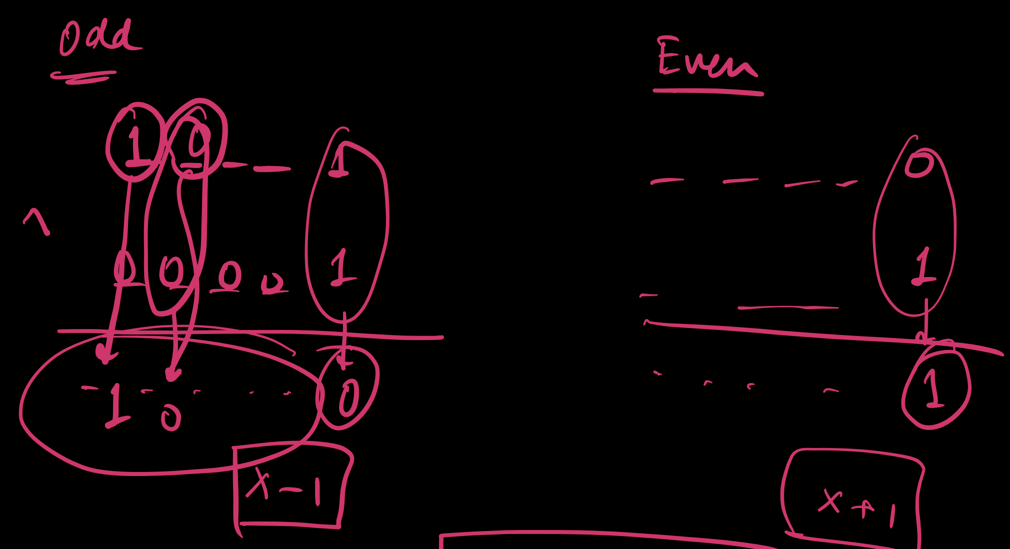
⇒ High if inputs are diff.

13 ^ 10

1101 1010

$$
\begin{array}{r}
1101 \\
1010 \\
\hline
0111
\end{array}
$$

= 7 ✓

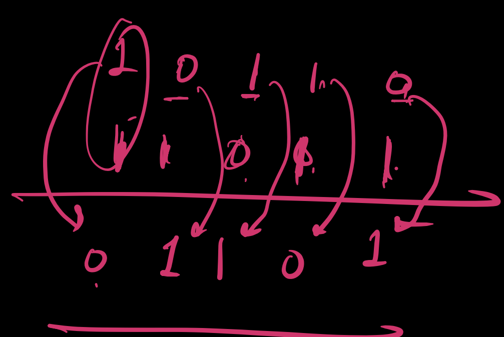


$1 \wedge 0 = 1$

$0 \wedge 0 = 0$

$\boxed{a \wedge 0 = a}$

$a \wedge 1 \rightarrow$ toggles the bit



$a \,\&\, b \rightarrow$ AND

$a \wedge b \rightarrow$ XOR

$a \mid b \rightarrow$ OR

$a \,\&\, b \,\&\, c = 0$

1 1 0

$a \,\&\, b \,\&\, c \,\&\, d = 0$

1 0 1 1

All bits have to be ON $= 1$

AND → ~~associative~~
→ ~~commutative~~

1011
0110
1010
0111

0010 = 2

$a \& b = b \& a$

$(a \& b) \& c = a \& (b \& c)$

OR

1011
0110
1001
1110

11111

$a | b$
$b | a$

$a | (b | c)$
$= (a | b) | c$

XOR

$1 \wedge 1 \wedge 0 \wedge 1$

$((1 \wedge 1) \wedge 0) \wedge 1$

$1 \wedge ((1 \wedge 0) \wedge 1)$

$= = 1$

commutative

$0 \wedge 0 =$
$1 \wedge 0$
$0 \wedge 1$
$1 \wedge 1$

order doesn't matter

$(a \wedge b) \wedge c \qquad a \wedge (b \wedge c)$ **Association**

$\boxed{(1 \wedge 0) \wedge 1} = \boxed{1 \wedge (0 \wedge 1)}$

$0 \qquad\qquad 0$

$0 \wedge 0 \wedge 0 \wedge 0 \wedge 0 \wedge 0 \wedge 0 = 0$

$0 \wedge 0 \wedge 1 = 1$

$0 \wedge 0 \wedge 1 \wedge 1 = 0$

Odd no. of 1's $\Rightarrow 1$

Even $\Rightarrow 0$

$10 \wedge 5 \wedge 4$

$$\begin{array}{r} 1010 \\ 0101 \\ 0100 \\ \hline 1011 \end{array} \Rightarrow 11$$

While using bitwise operators

$\Downarrow$

$\boxed{\text{Always consider each bit independent!}}$