~~Advanced~~ More Distributions:

✓ (Q) - which disb to use where? → Bernoulli ✓, Binomial ✓, Gaussian ✓

✓ - why do we need disb?

loos of disb

- Poisson: # visitors to my website in next 1hr
- Exponential: Time between two goals
- Geometric: # failures before 1st offer letter
- ✓ log-normal: Prices of amazon-products

1 / 4

Ops:

Ques → tab

Chat → interactivity ; Y/N

{ $\boxed{\text{concept}}$ → Q&A → . . .

(Q) which disb to use where?

$$X : \begin{cases} \overset{p}{1} & \text{if a learner attends a class} \\ \underset{1-p}{0} & \text{o/w} \end{cases}$$

Bernoulli

$\boxed{\text{conditions for a disb to arise}}$ ① * 2 (e.g) # heads in $\boxed{10}$ tosses

H ← p

T

Binomial → n indep bernoulli trails

p prob of success in each trail

Count the # success

$p_1 \quad p_2 \quad \cdots \quad p_n$

Chrome File Edit View History Bookmarks Profiles Tab Window Help

(e.g) $\boxed{W:}$ baby - weights → $\overbrace{w_1, w_2 \ldots\ldots w_n}$

[Gaussian/Normal]

② Statistical tools → QQ-plot (visual)

→ KS - test (later)

⋮

[later]

5 / 5

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X: discrete r.V

PMF

$P(X=i)$ $\forall i$

Y: Continous r.V $\longrightarrow$ PDF

$P(X=x)$

6 / 6

why do we need distributions?

Physicists

e.g: baby-weights $\sim N(\mu = 2.1\text{kg}, \sigma = 0.7\text{kg})$

① ②

"succint" Models

represent the pop.

summarize

to compute prob.. or metrics

insights

$b_i = 4\text{kg}$

$p(2.0 \leq B \leq 3.0)$

Poisson Distribution

```
[ ] # Goals in World cup soccer matches
    import numpy as np
    from scipy.stats import poisson
    import matplotlib.pyplot as plt
    import pandas as pd
    import seaborn as sns
```

$G = 0, 1, 2, 3, 4, 5, \ldots$

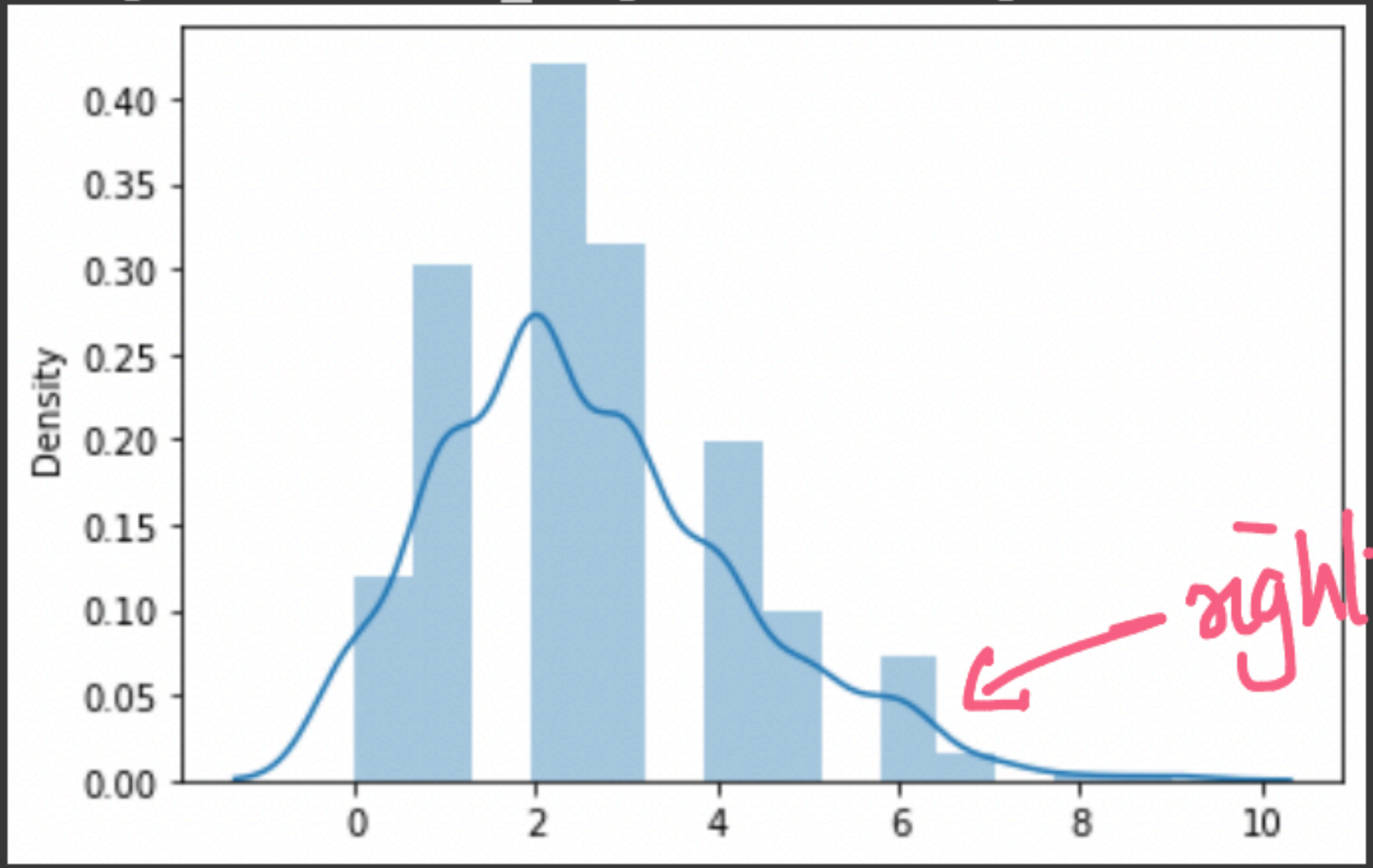
discrete r.v

```
[ ] G = [3, 4, 1, 4, 3, 4, 0, 1, 6, 3, 5, 0, 4, 2, 0, 0, 6, 3, 1, 2, 6, 2,
         1, 3, 4, 3, 2, 4, 0, 1, 1, 2, 3, 4, 3, 2, 2, 4, 2, 2, 3, 4, 3, 4,
         5, 3, 3, 0, 4, 2, 0, 5, 2, 0, 1, 5, 2, 4, 1, 5, 3, 2, 2, 3, 6, 3,
         1, 4, 3, 4, 2, 0, 4, 3, 7, 1, 3, 2, 3, 1, 3, 3, 2, 2, 2, 4, 2, 6,
         4, 2, 3, 2, 2, 2, 4, 5, 2, 4, 6, 3, 3, 2, 2, 2, 3, 2, 1, 2, 4, 2,
         2, 3, 1, 2, 1, 4, 1, 5, 6, 0, 2, 3, 0, 4, 2, 0, 2, 4, 0, 2, 5, 3,
         4, 3, 0, 2, 5, 1, 4, 2, 4, 4, 2, 0, 5, 2, 6, 4, 4, 1, 1, 6, 2, 3,
         0, 2, 3, 4, 2, 4, 1, 4, 2, 1, 1, 1, 2, 3, 2, 2, 0, 1, 5, 2, 2, 4,
         2, 1, 3, 5, 2, 1, 2, 5, 2, 3, 3, 2, 2, 5, 2, 3, 2, 2, 1, 2, 2, 5,
         4, 2, 6, 3, 1, 1, 2, 4, 0, 1, 3, 6, 1, 2, 2, 3, 7, 2, 1, 5, 1, 1,
         2, 3, 6, 2, 2, 3, 3, 3, 1, 2, 3, 3, 2, 1, 1, 4, 3, 2, 1, 1, 4, 2,
         3, 3, 2, 1, 0, 3, 1, 1, 1, 5, 2, 1, 0, 1, 3, 2, 3, 3, 1, 2, 2, 4,
         1, 7, 2, 3, 5, 1, 4, 3, 4, 1, 5, 4, 3, 1, 1, 4, 0, 1, 2, 3, 5, 3,
         5, 1, 1, 2, 1, 2, 3, 1, 2, 1, 5, 7, 1, 3, 6, 2, 3, 1, 3, 2, 2, 1,
         5, 2, 2, 1, 2, 4, 3, 3
```

```
4, 2]
```

```
sns.distplot(G) # not gaussian for sure
```

```
/usr/local/lib/python3.7/dist-packages/seaborn/distributions.py:2619: FutureWarning: `distplot` is a deprecated func
  warnings.warn(msg, FutureWarning)
&lt;matplotlib.axes._subplots.AxesSubplot at 0x7fa7b49391d0&gt;
```



right skew

X Gaussian : cont. r.v

```
m = print(np.mean(G))
```

```
2.576
```

```
4, 2]
```

```
sns.distplot(G) # not gaussian for sure
```

```
/usr/local/lib/python3.7/dist-packages/seaborn/distributions.py:2619: FutureWarning: `distplot` is a deprecated func
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```

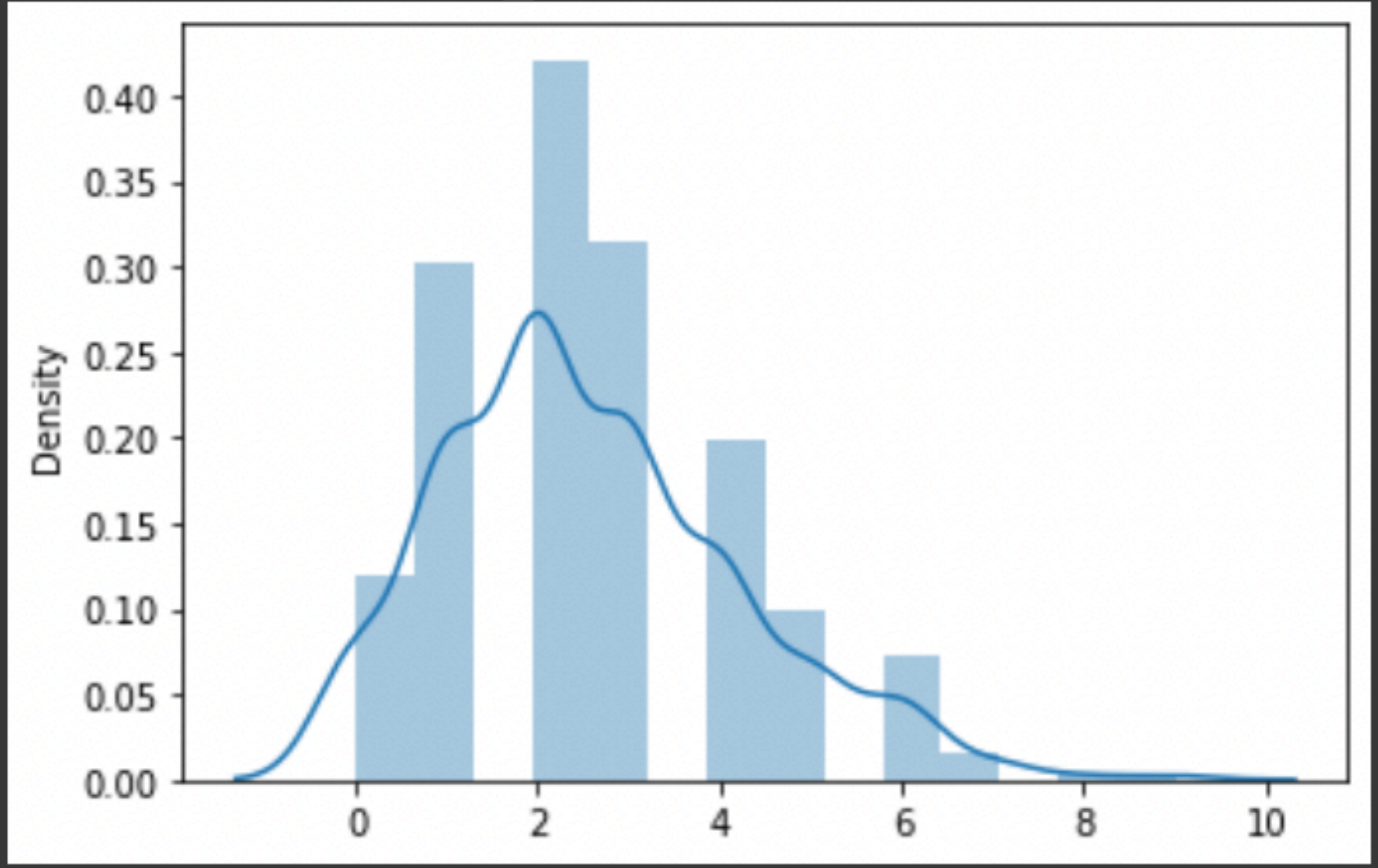

(~750)

```
m = print(np.mean(G))
```

```
2.576
```

```
[ ]          4, 2]
```

```
sns.distplot(G) # not gaussian for sure
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```
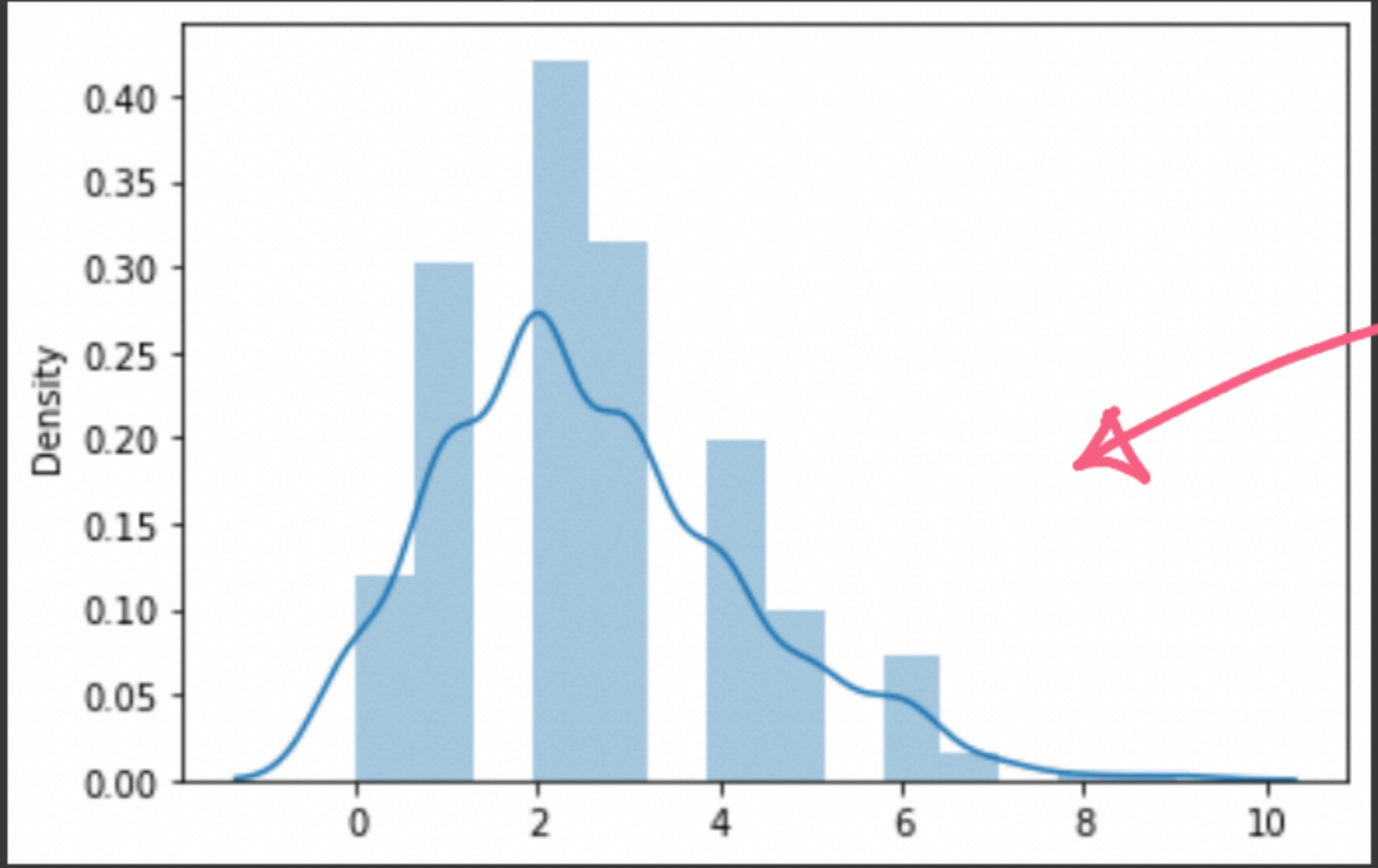



100's of disb

```
[ ] m = print(np.mean(G))
```

```
2.576
```

```python
# Goals in World cup soccer matches
import numpy as np
from scipy.stats import poisson
import matplotlib.pyplot as plt
import pandas as pd
import seaborn as sns
```

# of events in unit time

goal

90min

```python
G = [3, 4, 1, 4, 3, 4, 0, 1, 6, 3, 5, 0, 4, 2, 0, 0, 6, 3, 1, 2, 6, 2,
     1, 3, 4, 3, 2, 4, 0, 1, 1, 2, 3, 4, 3, 2, 2, 4, 2, 2, 3, 4, 3, 4,
     5, 3, 3, 0, 4, 2, 0, 5, 2, 0, 1, 5, 2, 4, 1, 5, 3, 2, 2, 3, 6, 3,
     1, 4, 3, 4, 2, 0, 4, 3, 7, 1, 3, 2, 3, 1, 3, 3, 2, 2, 2, 4, 2, 6,
     4, 2, 3, 2, 2, 2, 4, 5, 2, 4, 6, 3, 3, 2, 2, 2, 3, 2, 1, 2, 4, 2,
     2, 3, 1, 2, 1, 4, 1, 5, 6, 0, 2, 3, 0, 4, 2, 0, 2, 4, 0, 2, 5, 3,
     4, 3, 0, 2, 5, 1, 4, 2, 4, 4, 2, 0, 5, 2, 6, 4, 4, 1, 1, 6, 2, 3,
     0, 2, 3, 4, 2, 4, 1, 4, 2, 1, 1, 1, 2, 3, 2, 2, 0, 1, 5, 2, 2, 4,
     2, 1, 3, 5, 2, 1, 2, 5, 2, 3, 3, 2, 2, 5, 2, 3, 2, 2, 1, 2, 2, 5,
     4, 2, 6, 3, 1, 1, 2, 4, 0, 1, 3, 6, 1, 2, 2, 3, 7, 2, 1, 5, 1, 1,
     2, 3, 6, 2, 2, 3, 3, 3, 1, 2, 3, 3, 2, 1, 1, 4, 3, 2, 1, 1, 4, 2,
     3, 3, 2, 1, 0, 3, 1, 1, 1, 5, 2, 1, 0, 1, 3, 2, 3, 3, 1, 2, 2, 4,
     1, 7, 2, 3, 5, 1, 4, 3, 4, 1, 5, 4, 3, 1, 1, 4, 0, 1, 2, 3, 5, 3,
     5, 1, 1, 2, 1, 2, 3, 1, 2, 1, 5, 7, 1, 3, 6, 2, 3, 1, 3, 2, 2, 1,
     5, 2, 2, 1, 2, 4, 3, 3, 5, 2, 3, 6, 7, 1, 7, 3, 2, 2, 1, 3, 0, 2,
     3, 2, 1, 4, 5, 1, 3, 1, 2, 2, 3, 2, 3, 2, 6, 3, 1, 0, 5, 1, 3, 1,
     2, 2, 0, 6, 1, 0, 1, 1, 3, 3, 5, 0, 2, 3, 4, 2, 3, 3, 1, 5, 1, 1,
     4, 5, 3, 2, 2, 2, 1, 2
```

90 min Goals;

vistors to website in 1hr — Poisson-Distb

X = numbers of events per unit time

Conditions:

1. underlying constant mean rate of occurence (λ)
2. events must be indep
3. no simultaneous events

µsec gap

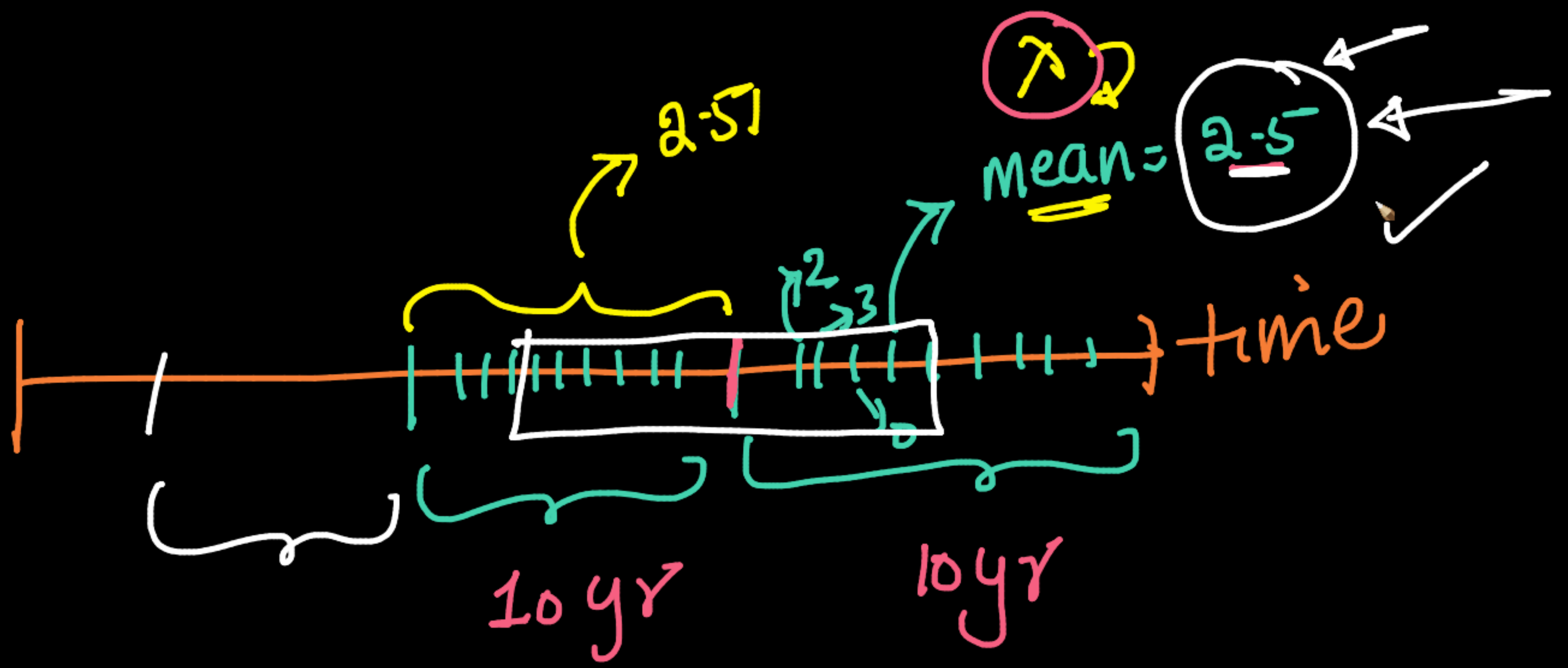
Q ~~# visitors~~ events to a restaurant in 1hr during {weekdays b/w 7-9pm}

X = # groups

Conditions

1. Constant rate (λ) of events
2. events are indep
3. no sim- events

→ Family / Group



$$\rightarrow \quad P(A, B) = p(A)\, p(B)$$

Goals:

$$\boxed{F = G\,\frac{m_1 M_2}{r^2}}$$

param

$X \sim \text{Poisson}(\lambda)$

Mean

real-num

discrete r.v

$N(\mu, \sigma)$

Binomial (n, p)

Binomial

math-der (limits)

discrete

$X \sim \text{Poisson}(\lambda)$

PMF: $P(X=k) = \boxed{\dfrac{\lambda^k e^{-\lambda}}{k!}}$, $k = 0, 1, 2, \ldots$

$k \to 0, 1, 2, \ldots$

CDF: $P(X \le k) = \sum_{i=0}^{k} P(X=i) = e^{-\lambda} \sum_{i=0}^{k} \frac{\lambda^i}{i!}$

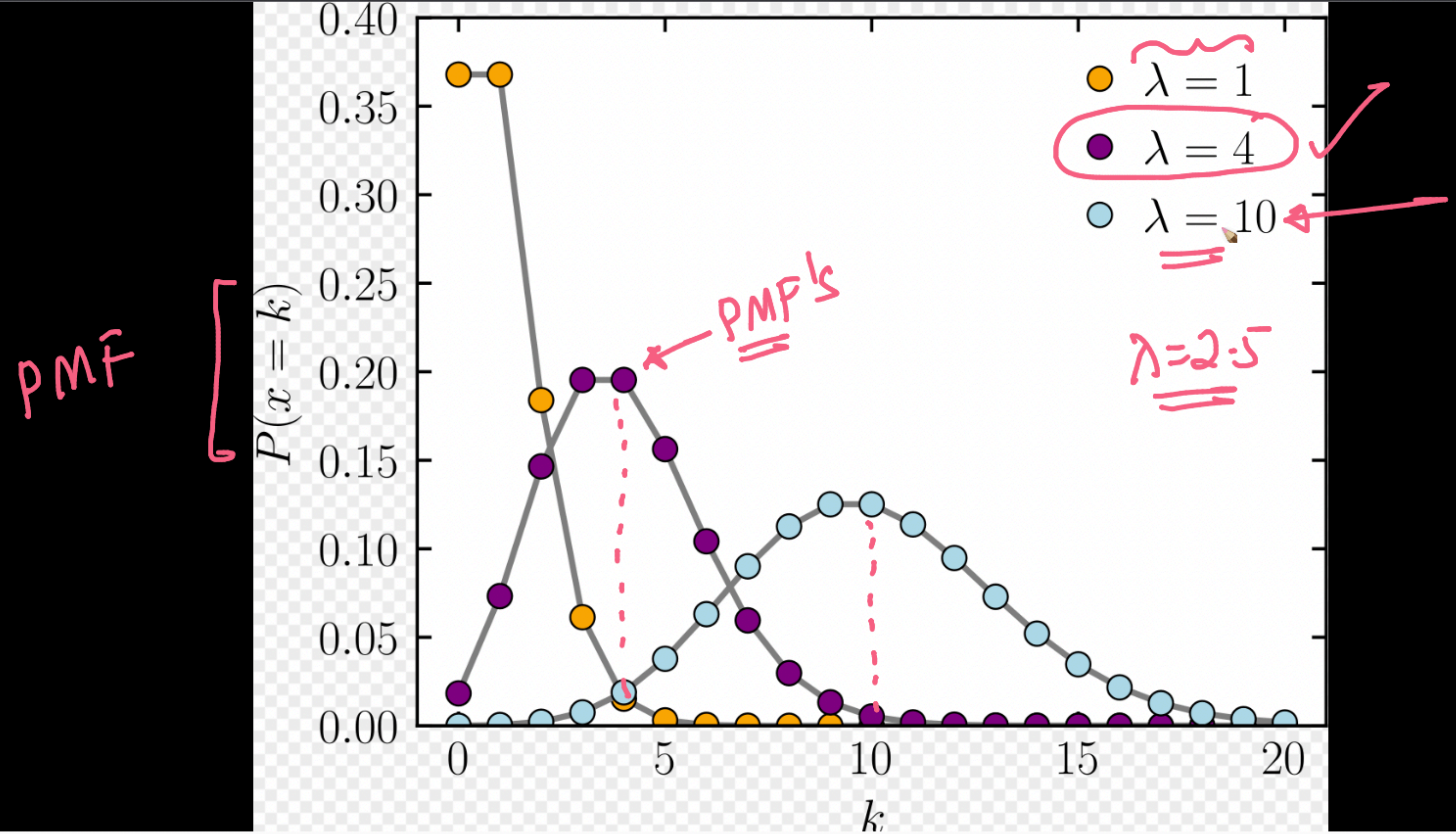
18 / 19

1 2

$$\text{Goals} \longrightarrow \text{Poisson}(\lambda = 2.5)$$

$$1 - P(G \le 9) = P(G \ge 10) = ? \quad \sum_{i=10}^{\infty} e^{-\lambda} \frac{\lambda^{i}}{i!}$$

$$P(G = 0) = \checkmark$$

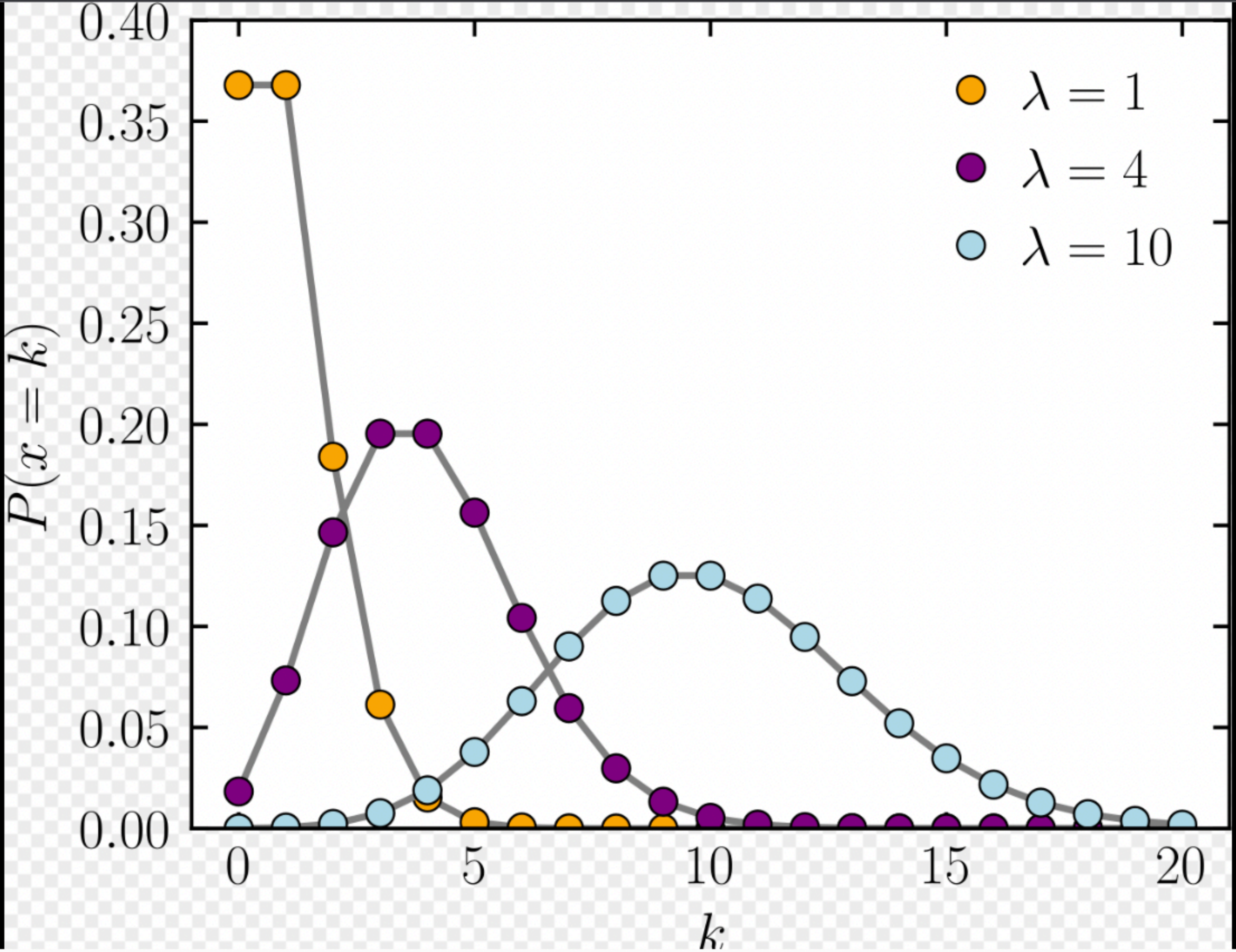


Plot of the probability mass function for the Poisson distribution.

More details

Skbkekas - Own work

20 / 20



Plot of the probability mass function for the Poisson distribution.

More details

Skbkekas - Own work

The probability for 0 to 6 overflow floods in a 100-year period.

María Dolores Ugarte and colleagues report that the average number of goals in a World Cup soccer match is approximately 2.5 and the Poisson model is appropriate.[15] Because the average event rate is 2.5 goals per match, $\lambda = 2.5$.

$$P(k \text{ goals in a match}) = \frac{2.5^k e^{-2.5}}{k!}$$

$$P(k = 0 \text{ goals in a match}) = \frac{2.5^0 e^{-2.5}}{0!} = \frac{e^{-2.5}}{1} \approx 0.082$$

$$P(k = 1 \text{ goal in a match}) = \frac{2.5^1 e^{-2.5}}{1!} = \frac{2.5e^{-2.5}}{1} \approx 0.205$$

$$P(k = 2 \text{ goals in a match}) = \frac{2.5^2 e^{-2.5}}{2!} = \frac{6.25e^{-2.5}}{2} \approx 0.257$$

k	P(k goals in a World Cup soccer match)
0	0.082
1	0.205
2	0.257
3	0.213
4	0.133
5	0.067
6	0.028
7	0.010

The probability for 0 to 7 goals in a match.

Once in an interval events: The special case of $\lambda = 1$ and $k = 0$ [edit]

Suppose that astronomers estimate that large meteorites (above a certain size) hit the earth on average once every 100 years ($\lambda = 1$ event per 100 years), and that the number of meteorite hits follows a Poisson distribution. What is the probability of $k = 0$ meteorite hits in the next 100 years?

$$P(k = 0 \text{ meteorites hit in next 100 years}) = \frac{1^0 e^{-1}}{0!} = \frac{1}{e} \approx 0.37$$

Under these assumptions, the pro … 37. The remaining 1 − 0.37 = 0.63 is the probability of 1, 2, 3, or more large meteorite hits in the next 100 … every 100 years ($\lambda = 1$). The probability of no overflow

The probability for 0 to 7 goals in a match.

Once in an interval events: The special case of $\lambda = 1$ and $k = 0$ [edit]

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large

$$P(k = 0 \text{ meteorites hit in next 100 years}) = \frac{1^0 e^{-1}}{0!} = \frac{1}{e} \approx 0.37$$

Under these assumptions, the probability that no large meteorites hit the earth in the next 100 years is roughly 0.37. The remaining 1 – 0.37 = 0.63 is the probability of 1, 2, 3, or more large meteorite hits in the next 100 years. In an example above, an overflow flood occurred once every 100 years (λ = 1). The probability of no overflow floods in 100 years was roughly 0.37, by the same calculation.

In general, if an event occurs on average once per interval (λ = 1), and the events follow a Poisson distribution, then *P*(0 events in next interval) = 0.37. In addition, *P*(exactly one event in next interval) = 0.37, as shown in the table for overflow floods.

Examples that violate the Poisson assumptions [edit]

The number of students who arrive at the student union per minute will likely not follow a Poisson distribution, because the rate is not constant (low rate during class time, high rate between class times) and the arrivals of individual students are not independent (students tend to come in groups).

The number of magnitude 5 earthquakes per year in a country may not follow a Poisson distribution if one large earthquake increases the probability of aftershocks of similar magnitude.

Examples in which at least one event is guaranteed are not Poisson distributed; but may be modeled using a zero-truncated Poisson distribution.

Count distributions in which the number of intervals with zero events is higher than predicted by a Poisson model may be modeled using a zero-inflated model.

Properties [edit]

Descriptive statistics [edit]

- The expected value and varian
- The coefficient of variation is $\lambda^{-1/2}$ while the index of dispersion is 1

23 / 23

Under these assumptions, the probability that no large meteorites hit the earth in the next 100 years is roughly 0.37. The remaining 1 – 0.37 = 0.63 is the probability of 1, 2, 3, or more large meteorite hits in the next 100 years. In an example above, an overflow flood occurred once every 100 years ($\lambda = 1$). The probability of no overflow floods in 100 years was roughly 0.37, by the same calculation.

In general, if an event occurs on average once per interval ($\lambda = 1$), and the events follow a Poisson distribution, then P(0 events in next interval) = 0.37. In addition, P(exactly one event in next interval) = 0.37, as shown in the table for overflow floods.

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Count distributions in which the number of intervals with zero events is higher than predicted by a Poisson model may be modeled using a zero-inflated model.

Properties [edit]

Descriptive statistics [edit]

- The expected value and variance of a Poisson-distributed random variable are both equal to λ.
- The coefficient of variation is $\lambda^{-1/2}$, while the index of dispersion is 1.[6]:163
- The mean absolute deviation about the mean is[6]:163

$$\mathrm{E}[|X-\lambda|] = \frac{2\lambda^{\lfloor\lambda\rfloor+1}e^{-\lambda}}{\lfloor\lambda\rfloor!}.$$

- The mode of a Poisson-distributed random variable with non-integer λ is equal to $\lfloor\lambda\rfloor$, which is the largest integer less than or equal to λ. This is also written as floor(λ). When λ is a positive integer, the modes are λ and λ – 1.
- All of the cumulants of the Poi[...] the Poisson distribution is λ^n.
- The expected value of a Poisson process is sometimes decomposed into the product of intensity and exposure (or more generally expressed as the integral of an

Occurrence and applications [edit]

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Applications of the Poisson distribution can be found in many fields including:[46]

- count data in general
- Telecommunication example: telephone calls arriving in a system.
- Astronomy example: photons arriving at a telescope.
- Chemistry example: the molar mass distribution of a living polymerization.[47]
- Biology example: the number of mutations on a strand of DNA per unit length.
- Management example: customers arriving at a counter or call centre.
- Finance and insurance example: number of losses or claims occurring in a given period of time.
- Earthquake seismology example: an asymptotic Poisson model of seismic risk for large earthquakes.[48]
- Radioactivity example: number of decays in a given time interval in a radioactive sample.
- Optics example: the number of photons emitted in a single laser pulse. This is a major vulnerability to most Quantum key distribution protocols known as Photon Number Splitting (PNS).

The Poisson distribution arises in connection with Poisson processes. It applies to various phenomena of discrete properties (that is, those that may happen 0, 1, 2, 3, … times during a given period of time or in a given area) whenever the probability of the phenomenon happening is constant in time or space. Examples of events that may be modelled as a Poisson distribution include:

- The number of soldiers killed by horse-kicks each year in each corps in the Prussian cavalry. This example was used in a book by Ladislaus Bortkiewicz (1868–1931).[10]:23-25
- The number of yeast cells used when brewing Guinness beer. This example was used by William Sealy Gosset (1876–1937).[49][50]
- The number of phone calls arriving at a call centre within a minute. This example was described by A.K. Erlang (1878–1929).[51]
- Internet traffic.
- The number of goals in sports involving two competing teams.

Occurrence and applications [edit]

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- The number of phone calls arriving at a call centre within a minute. This example was described by A.K. Erlang (1878–1929).[51]
- Internet traffic.
- The number of goals in sports involving two competing teams.

PMF

$$E(X) = \sum_{i=0}^{\infty} i \cdot P(X=i)$$

$$P(X=i) = \frac{e^{-\lambda}\lambda^{i}}{i!}$$

$\boxed{\lambda}$

$$Var(X) = E(X^2) - (E(X))^2$$

λ^2

Poisson process

1. Const rate (λ) of events
2. indep. events
3. no sim. events

X : # events per unit time

↳ Poisson dist

Questions : " $\boxed{\text{END}}$: - - - - - - - - - "

after 11:30

29 / 30

Exponential Disb

Conditions

Poisson-process

time interval (Δt)

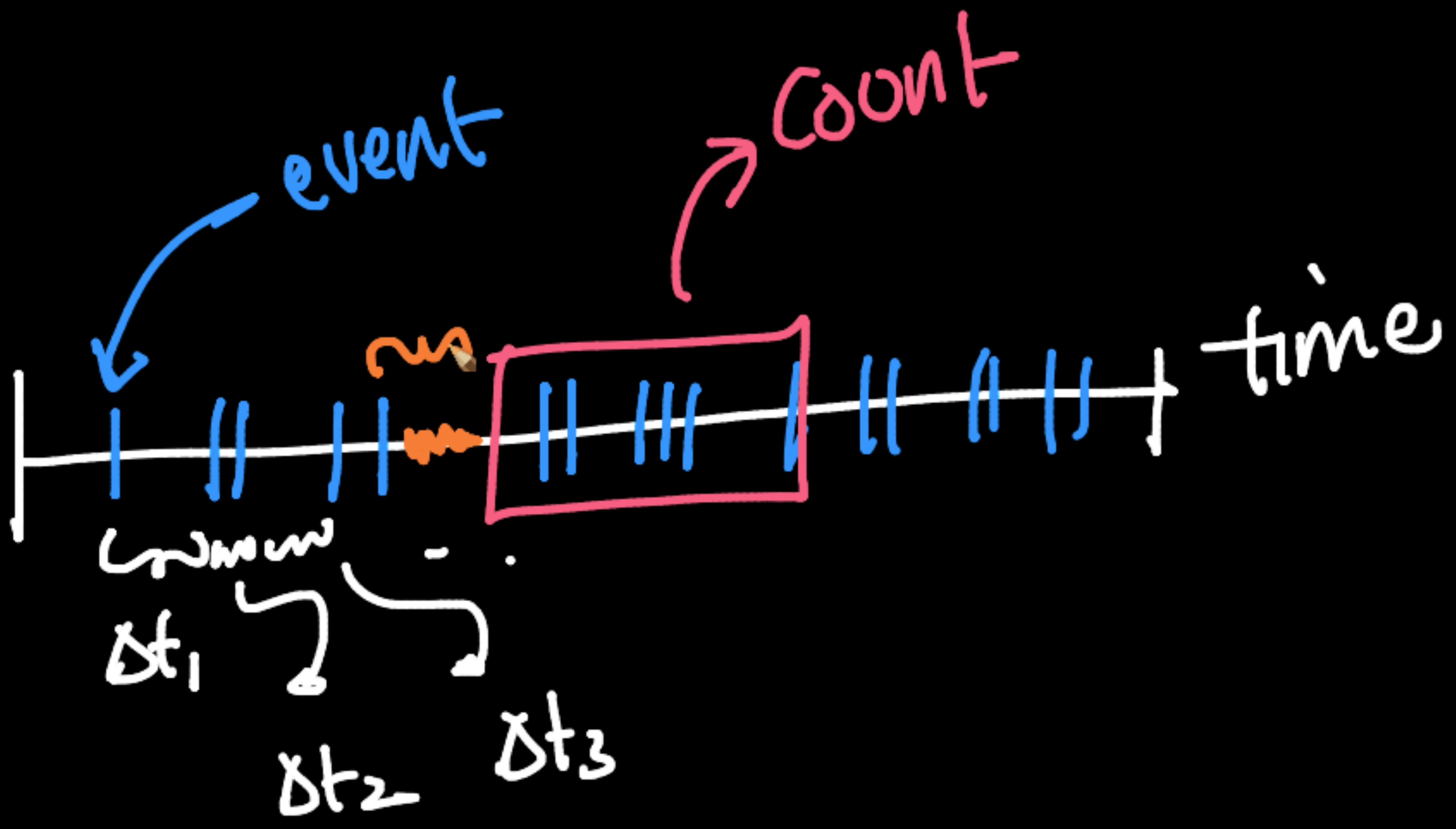
between 2 events

Goals/visits

Goals in WC- soccer match

$G = \#$ goals per match $= \text{Poisson}(\lambda = 2.5)$

Δt between goals $= \text{expo}(\lambda = 2.5)$



$$\Delta t_i \sim \text{expo}(\lambda)$$

Δt

$X \sim expo(\lambda)$ → rate of the poisson process

continous

PDF : $\lambda e^{-\lambda x}$ $x \geq 0$ (?)

Δt

CDF : $1 - e^{-\lambda x}$ $x \geq 0$ (?)

$P(X = 30\ min)$

8 References
9 External links

	λ
Mode	0
Variance	$\frac{1}{\lambda^2}$
Skewness	2
Ex. kurtosis	6
Entropy	$1 - \ln \lambda$
MGF	$\frac{\lambda}{\lambda - t}$, for $t < \lambda$
CF	$\frac{\lambda}{\lambda - it}$
Fisher information	$\frac{1}{\lambda^2}$
Kullback-Leibler divergence	$\ln \frac{\lambda_0}{\lambda} + \frac{\lambda}{\lambda_0} - 1$

Definitions

$$\lambda e^{-\lambda \cdot 0} = \lambda$$

Probability density function

The probability density function (pdf) of an exponential distribution is

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0, \\ 0 & x < 0. \end{cases}$$

Here $\lambda > 0$ is the parameter of the distribution, often called the *rate parameter*. The distribution is supported on the interval $[0, \infty)$. If a random variable X has this distribution, we write $X \sim \mathrm{Exp}(\lambda)$.

The exponential distribution exhibits infinite divisibility.

Cumulative distribution function

The cumulative distribution function is given by

$$F(x;\lambda) = \begin{cases} 1 - e^{-\lambda x} & x \ge 0, \\ 0 & x < 0. \end{cases}$$

Alternative parametrization

The exponential distribution is sometimes parametrized in terms of the scale parameter $\beta = 1/\lambda$, which is also the mean:

$$f(x;\beta) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & x \ge 0, \\ 0 & x < 0. \end{cases} \qquad F(x;\beta) = \begin{cases} 1 - e^{-x/\beta} & x \ge 0, \\ 0 & x < 0. \end{cases}$$

Properties

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Random article
About Wikipedia
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Donate

Contribute
Help
Learn to edit
Community portal
Recent changes
Upload file

Tools
What links here
Related changes
Special pages
Permanent link
Page information
Cite this page
Wikidata item

Exponential distribution

$$P(X=x) = \lambda e^{-\lambda x}$$

From Wikipedia, the free encyclopedia

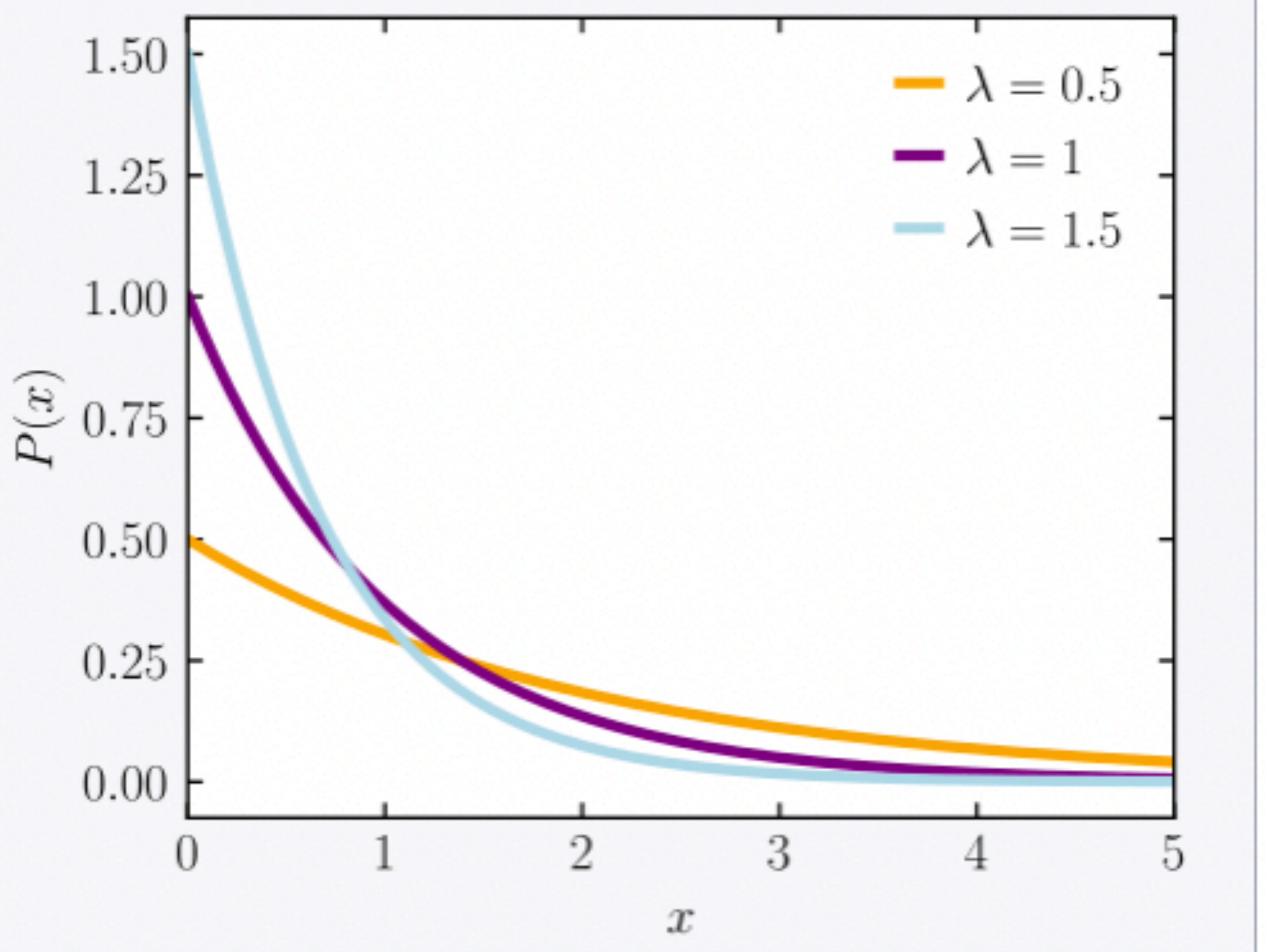
Not to be confused with the exponential family of probability distributions.

In probability theory and statistics, the **exponential distribution** is the probability distribution of the time between events in a Poisson point process, i.e., a process in which events occur continuously and independently at a constant average rate. It is a particular case of the gamma distribution. It is the continuous analogue of the geometric distribution, and it has the key property of being memoryless. In addition to being used for the analysis of Poisson point processes it is found in various other contexts.

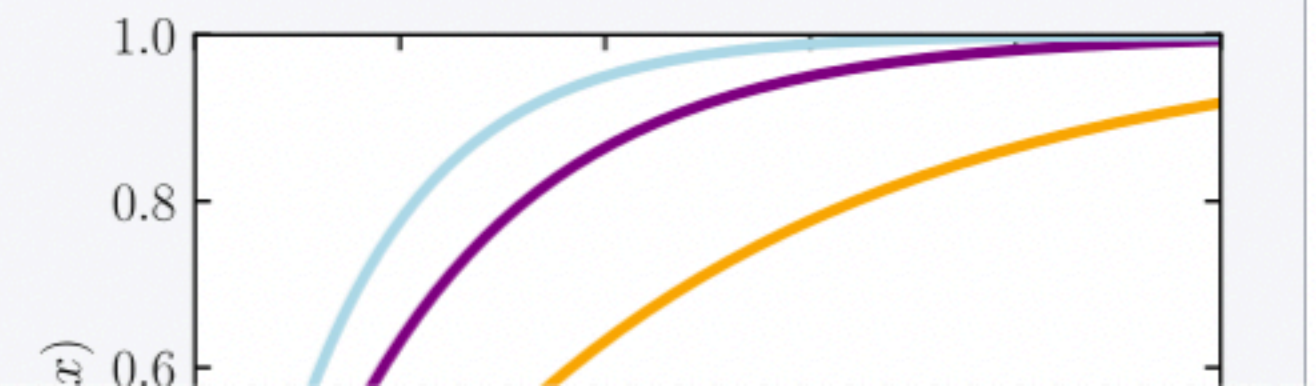
The exponential distribution is not the same as the class of exponential families of distributions, which is a large class of probability distributions that includes the exponential distribution as one of its members, but also includes the normal distribution, binomial distribution, gamma distribution, Poisson, and many others.

Exponential

Probability density function



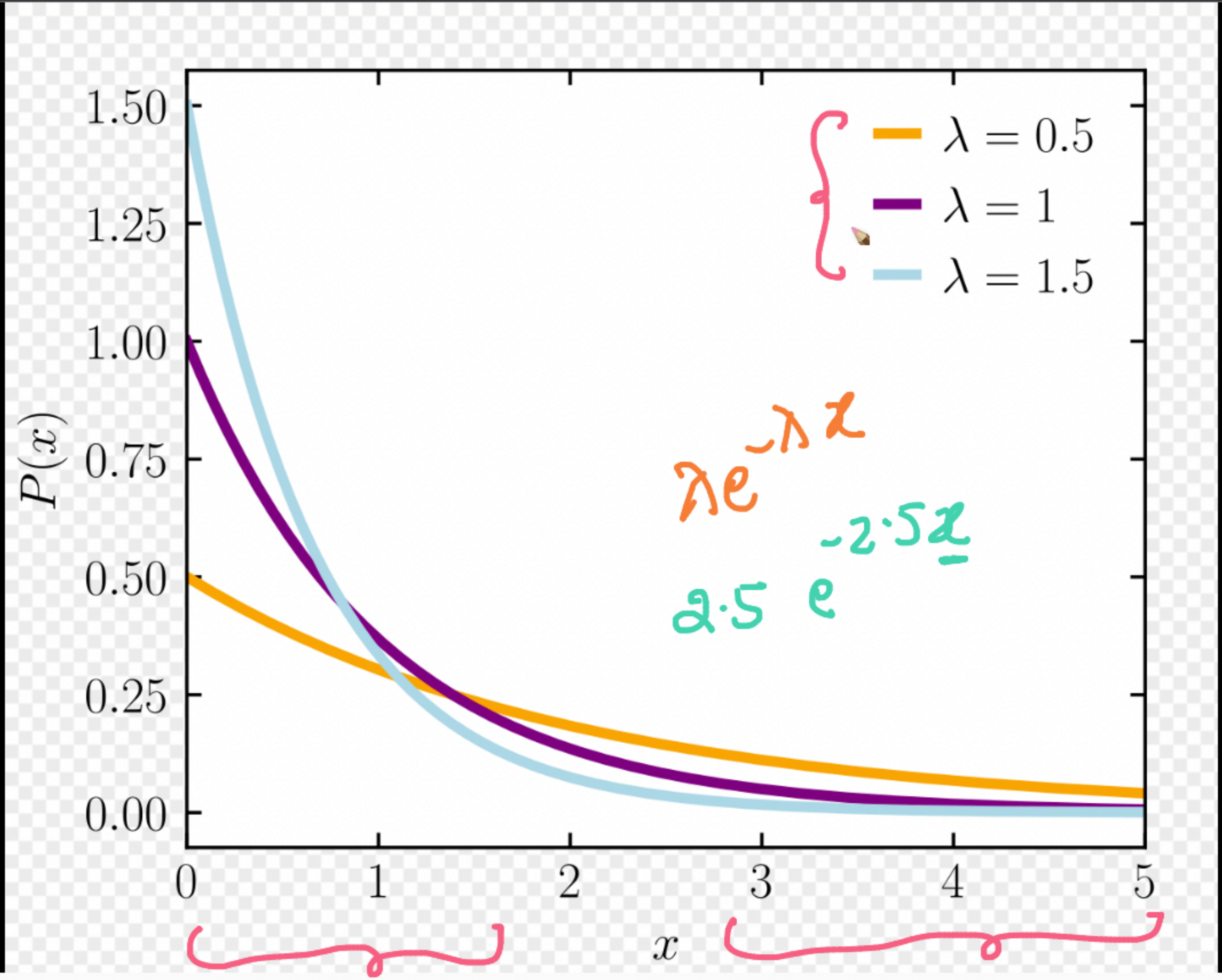
Cumulative distribution function



Contents [hide]

1 Definitions
1.1 Probability density function
1.2 Cumulative distribution function
1.3 Alternative parametrization
2 Properties
2.1 Mean, variance, moments, and median
2.2 Memorylessness
2.3 Quantiles
2.4 Kullback–Leibler divergen
2.5 Maximum entropy distribution

35 / 36



plot of the probability density function of the exponential distribution

More details

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Applications of the Poisson distribution can be found in many fields including:[46]

- count data in general
- Telecommunication example: telephone calls arriving in a system.
- Astronomy example: photons arriving at a telescope.
- Chemistry example: the molar mass distribution of a living polymerization.[47]
- Biology example: the number of mutations on a strand of DNA per unit length.
- Management example: customers arriving at a counter or call centre.
- Finance and insurance example: number of losses or claims occurring in a given period of time.
- Earthquake seismology example: an asymptotic Poisson model of seismic risk for large earthquakes.[48]
- Radioactivity example: number of decays in a given time interval in a radioactive sample.
- Optics example: the number of photons emitted in a single laser pulse. This is a major vulnerability to most Quantum key distribution protocols known as Photon Number Splitting (PNS).

λ

#calls $\leftarrow$ Poisson(λ)

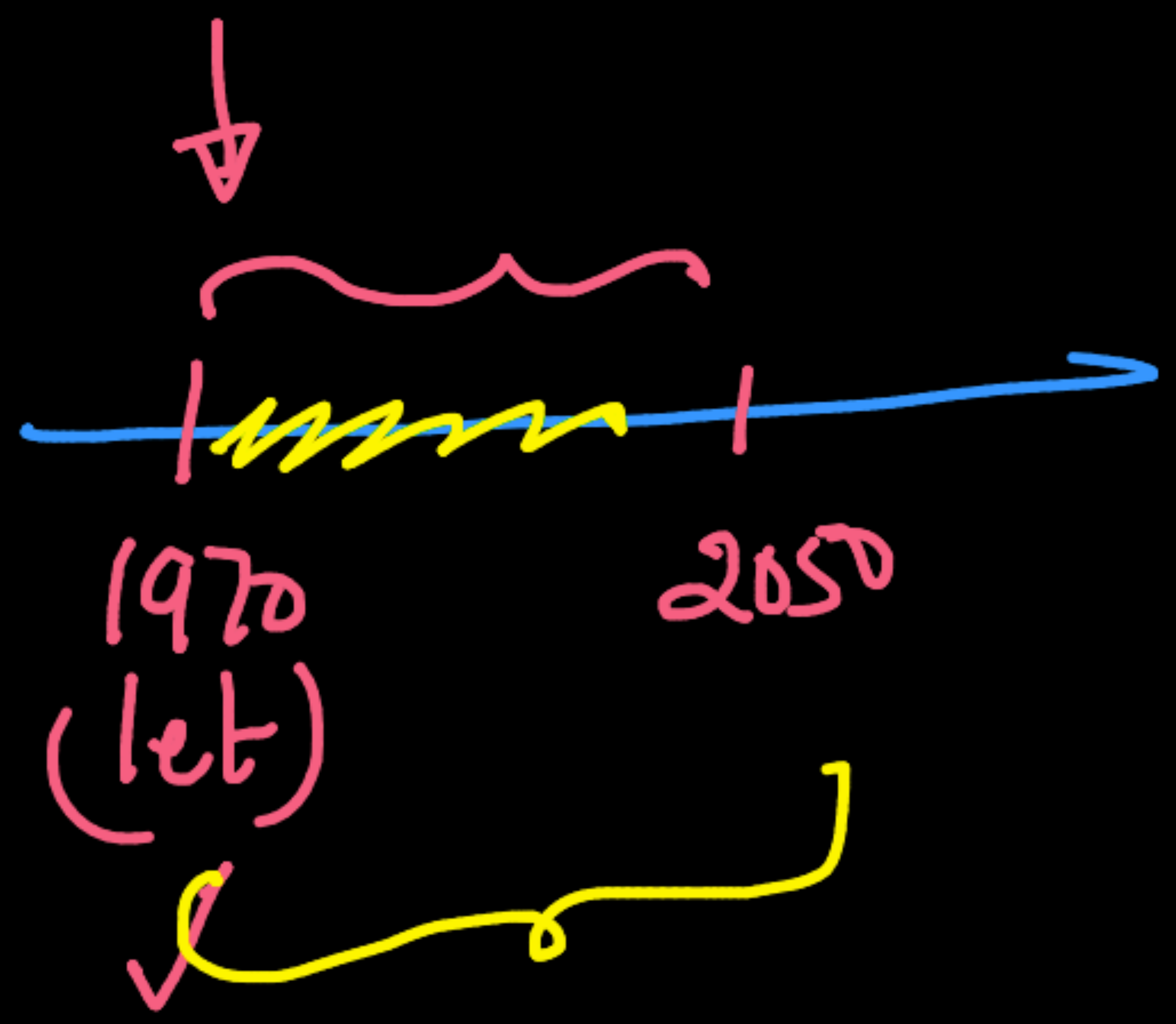
$\Delta t \leftarrow$ expo(λ)

The Poisson distribution arises in connection with Poisson processes. It applies to various phenomena of discrete properties (that is, those that may happen 0, 1, 2, 3, … times during a given period of time or in a given area) whenever the probability of the phenomenon happening is constant in time or space. Examples of events that may be modelled as a Poisson distribution include:

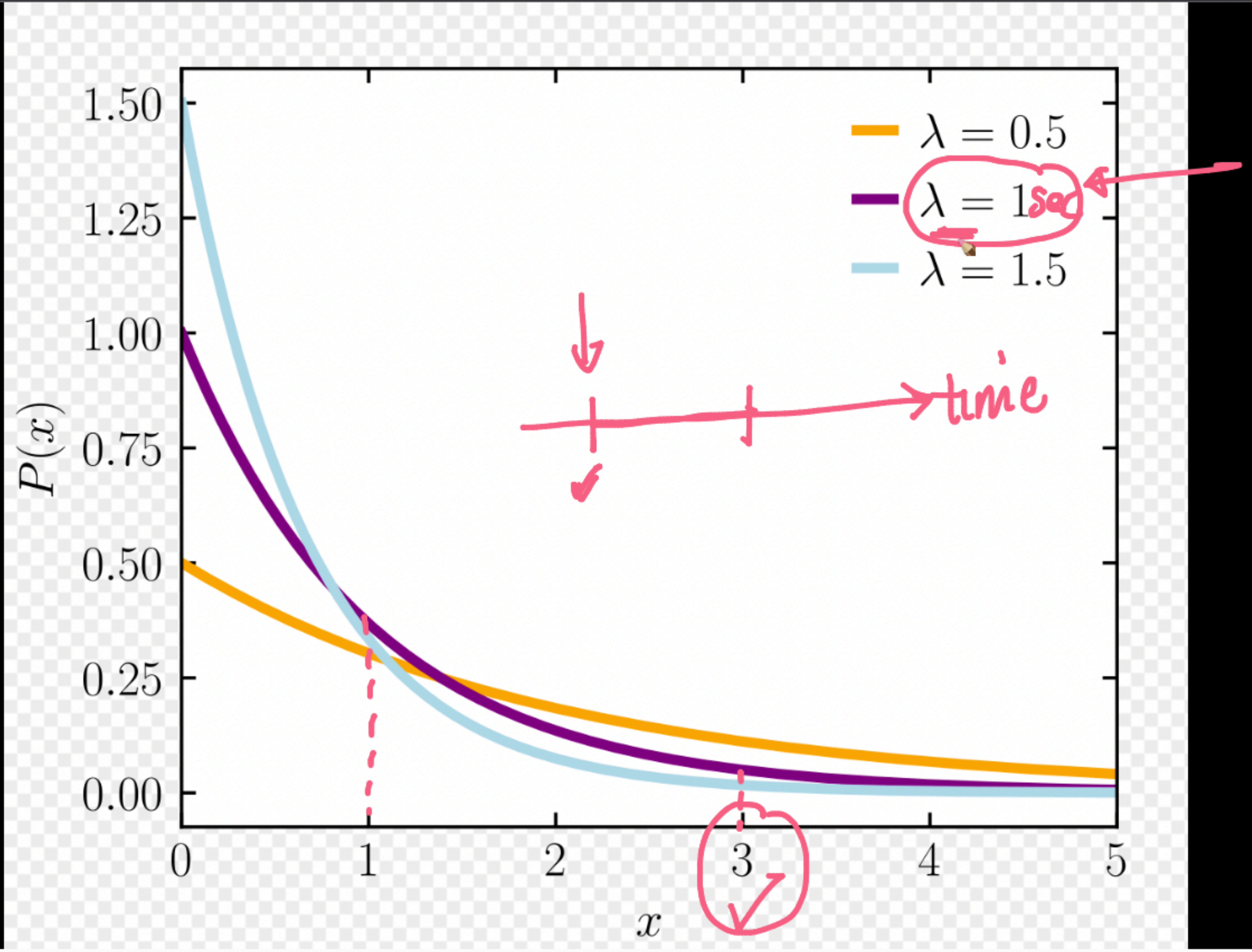
- The number of soldiers killed by horse-kicks each year in each corps in the Prussian cavalry. This example was used in a book by Ladislaus Bortkiewicz (1868–1931).[10]:23-25
- The number of yeast cells used when brewing Guinness beer. This example was used by William Sealy Gosset (1876–1937).[49][50]
- The number of phone calls arriving at a call centre within a minute. This example was described by A.K. Erlang (1878–1929).[51]
- Internet traffic.
- The number of goals in sports involving two competing teams.[52]
- The number of deaths per year in a given age group.
- The number of jumps in a stock price in a given time interval.
- Under an assumption of homogeneity, the number of times a web server is accessed per minute.
- The number of mutations in a
- The proportion of cells that will be infected at a given multiplicity of infection.

Poisson process: large meteorites $\lambda = 1$ per 100 yrs

\# large-meteorite hits $\leftarrow$ Poisson$(\lambda = 1)$

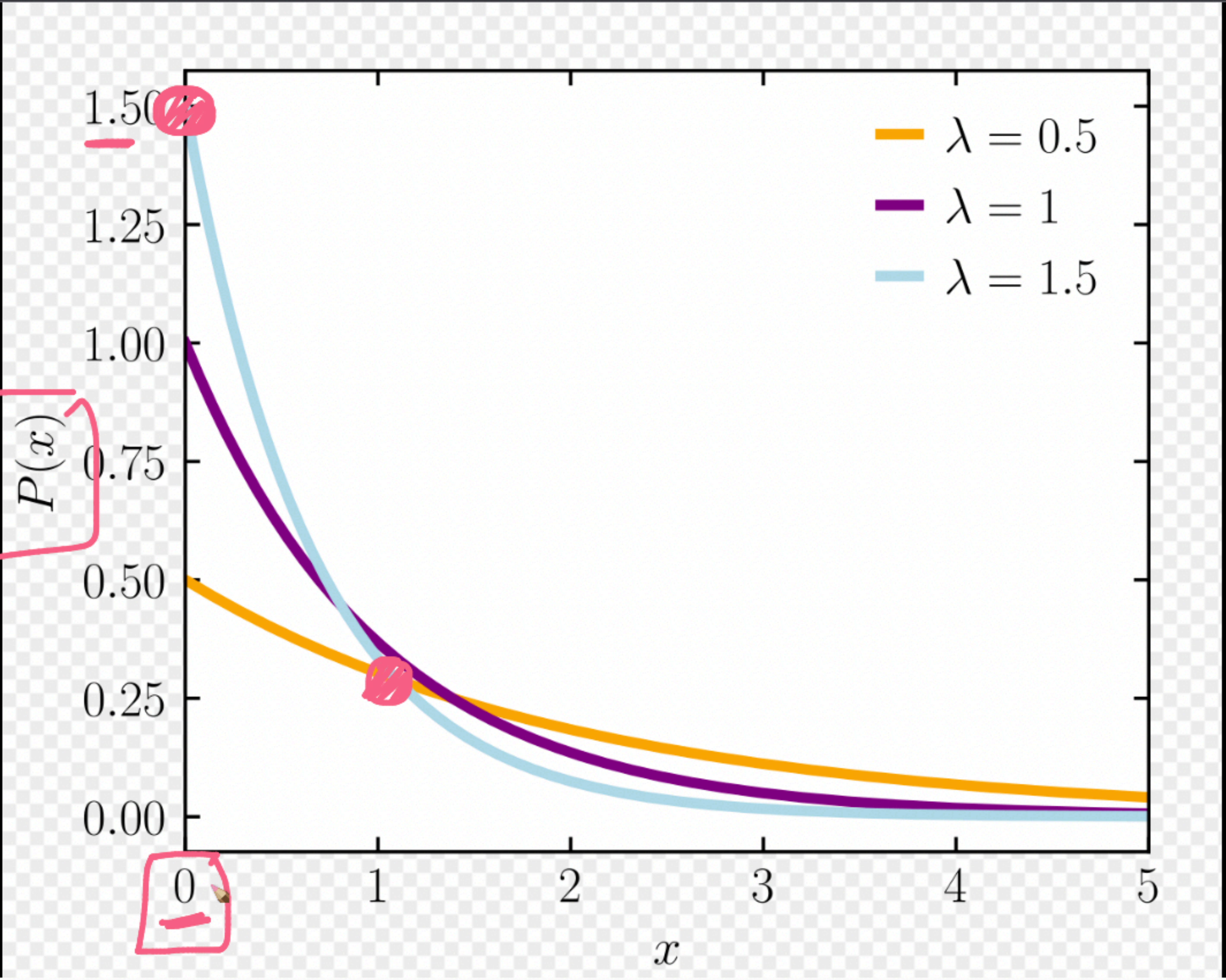


$P(\Delta t \leq 80)$: expo-distb



plot of the probability density function of the exponential distribution

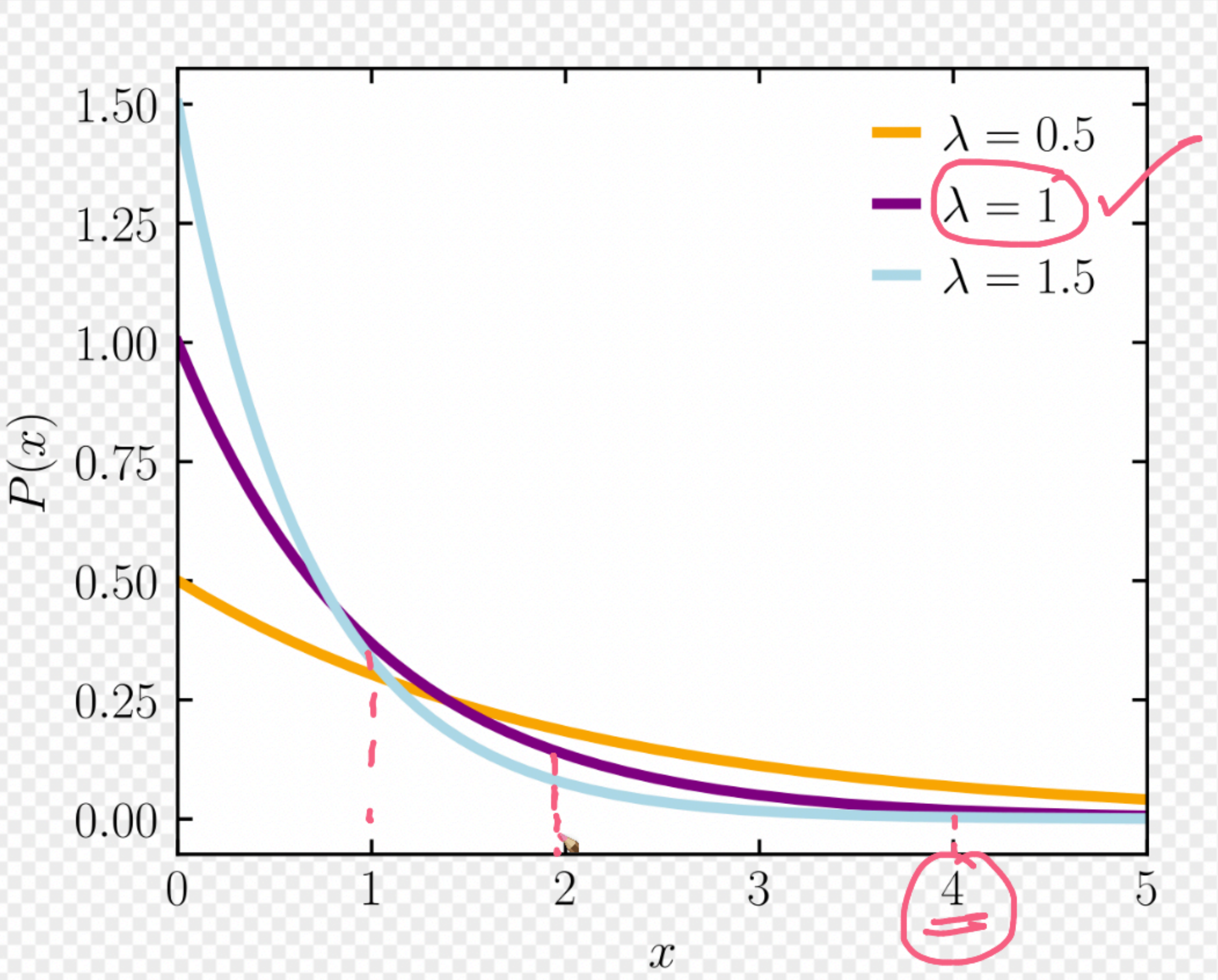
PDF



plot of the probability density function of the exponential distribution

More details

Newystats - Own work



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CDF

↓

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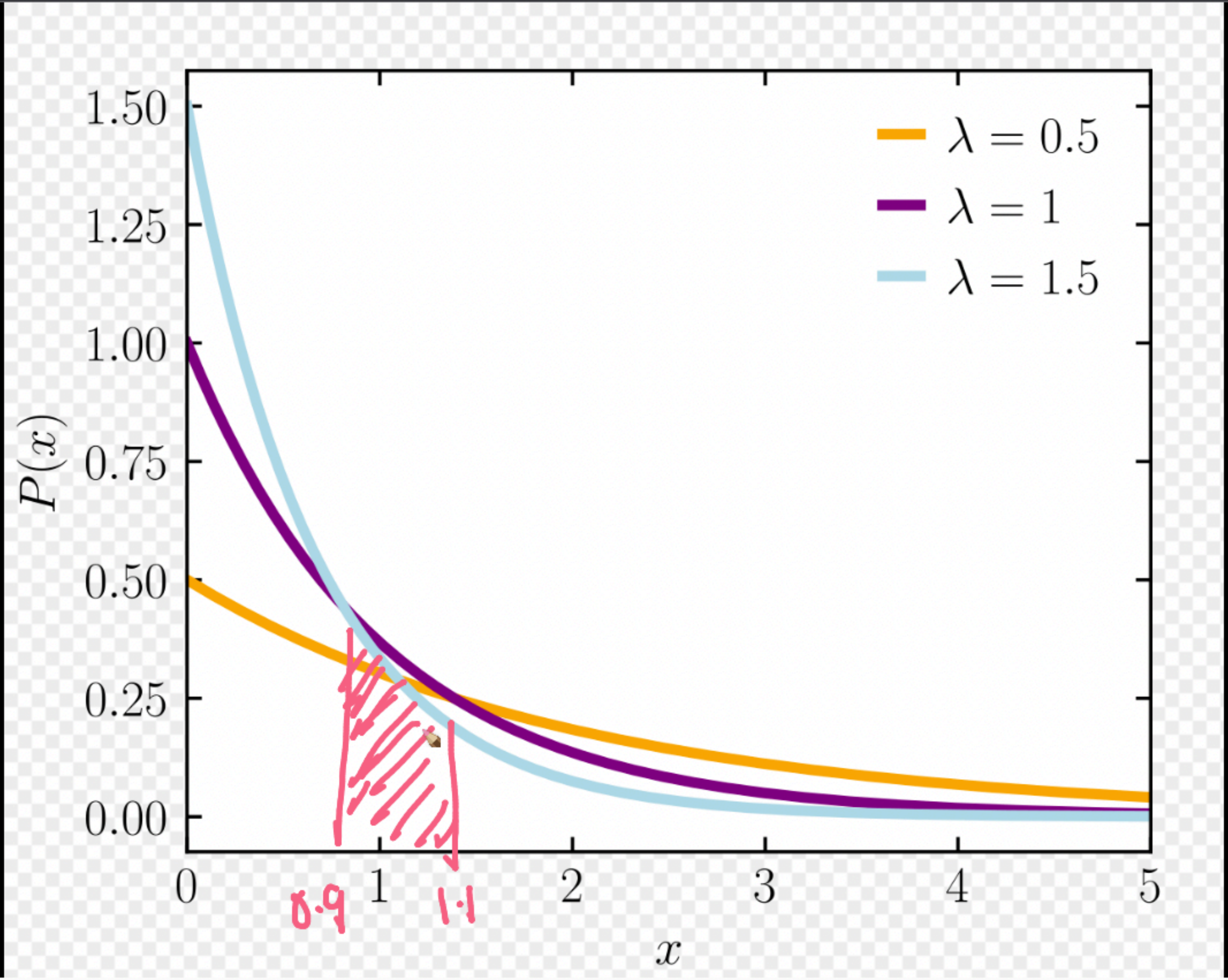

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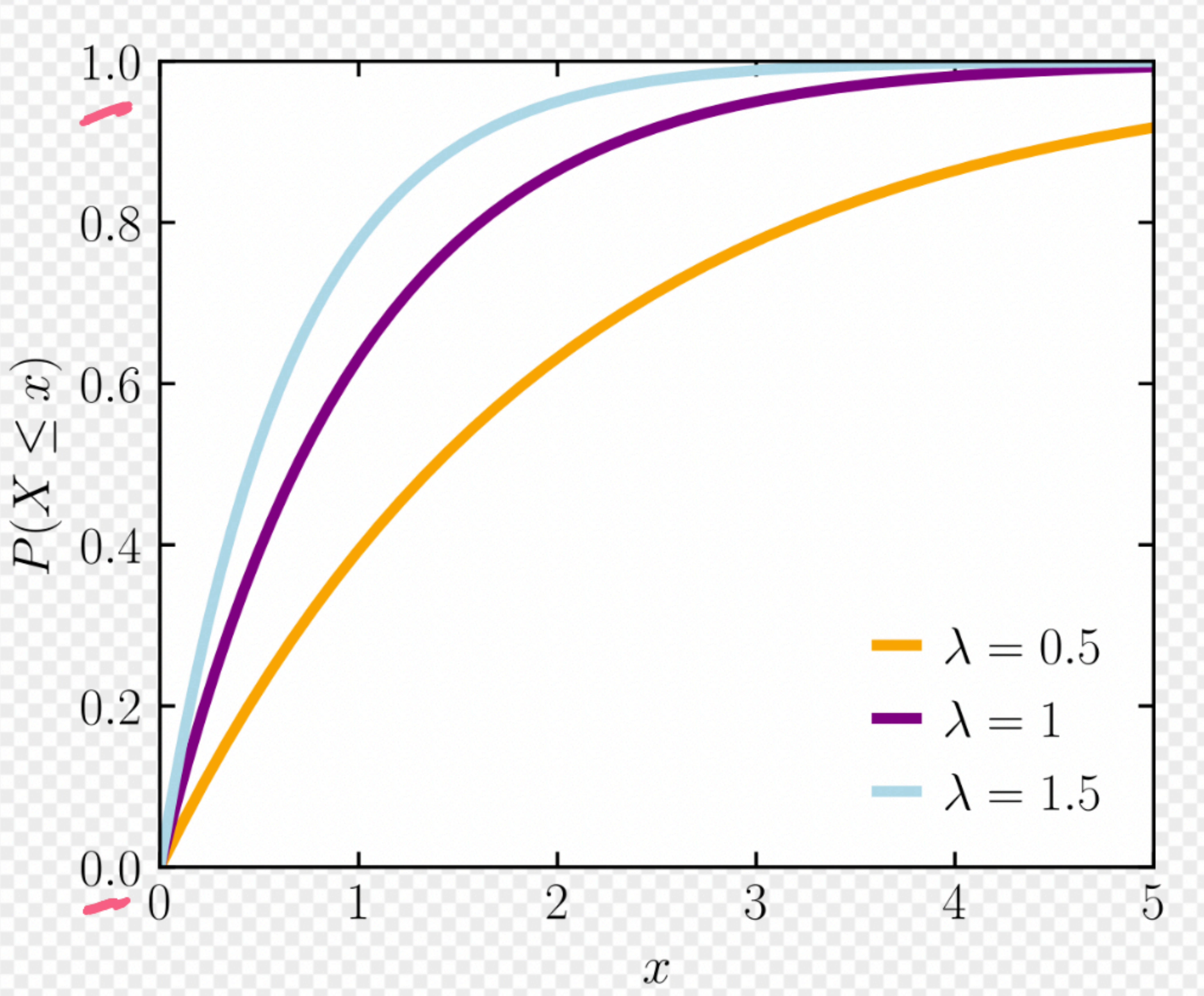
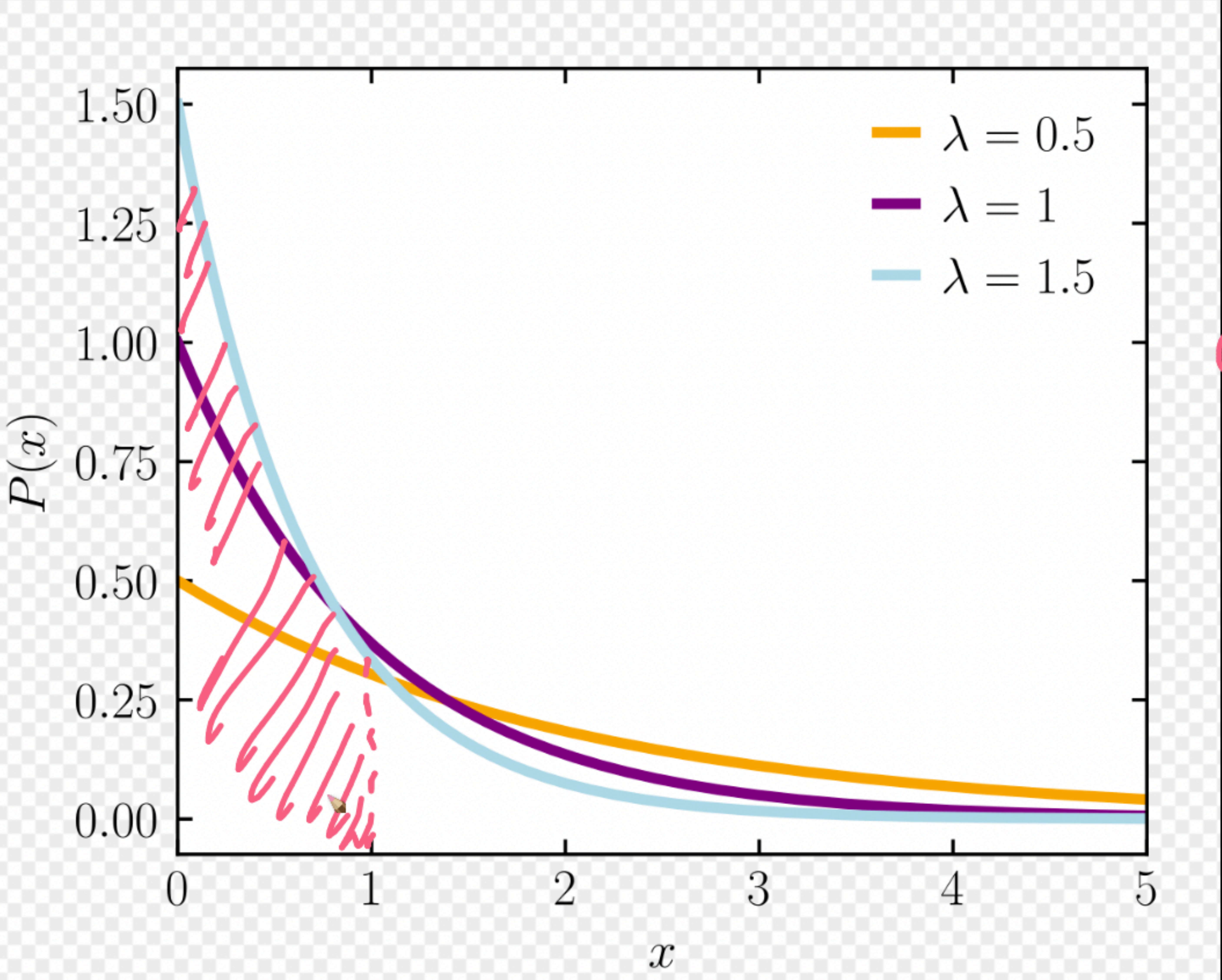


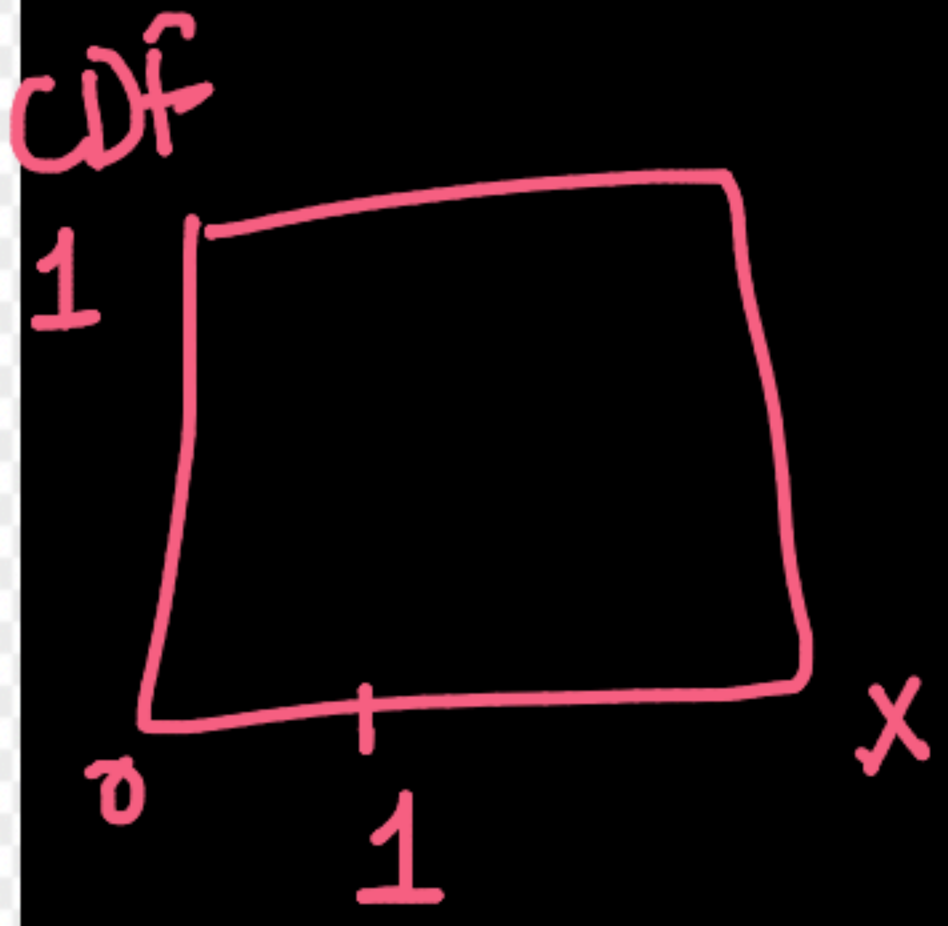
Cumulative distribution function

More details

Skbkekas - Own work

en.wikipedia.org/wiki/Exponential_distribution#/media/File:Exponential_probability_density.svg





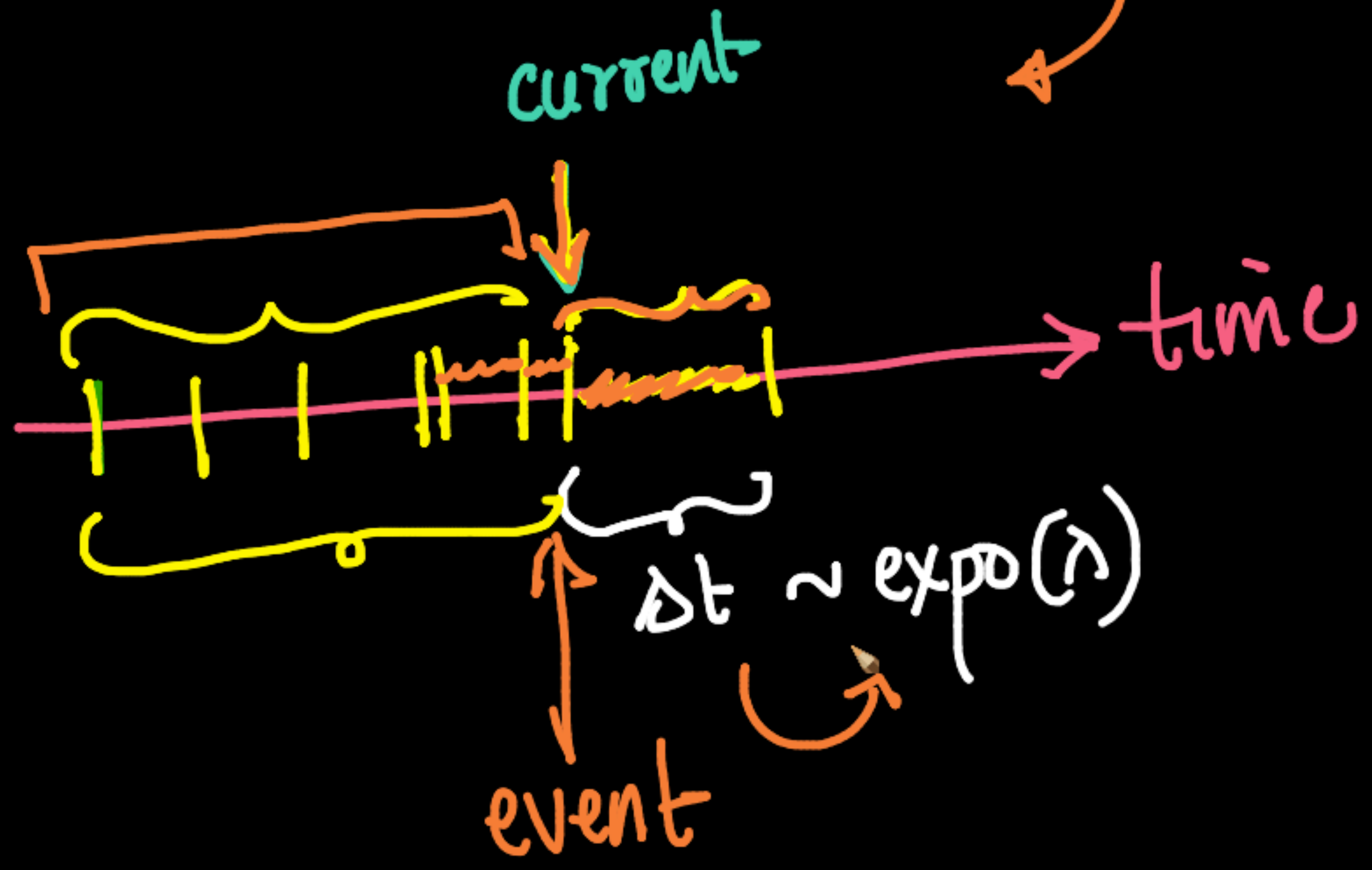
$P(X \le 1)$

plot of the probability density function of the exponential distribution

More details

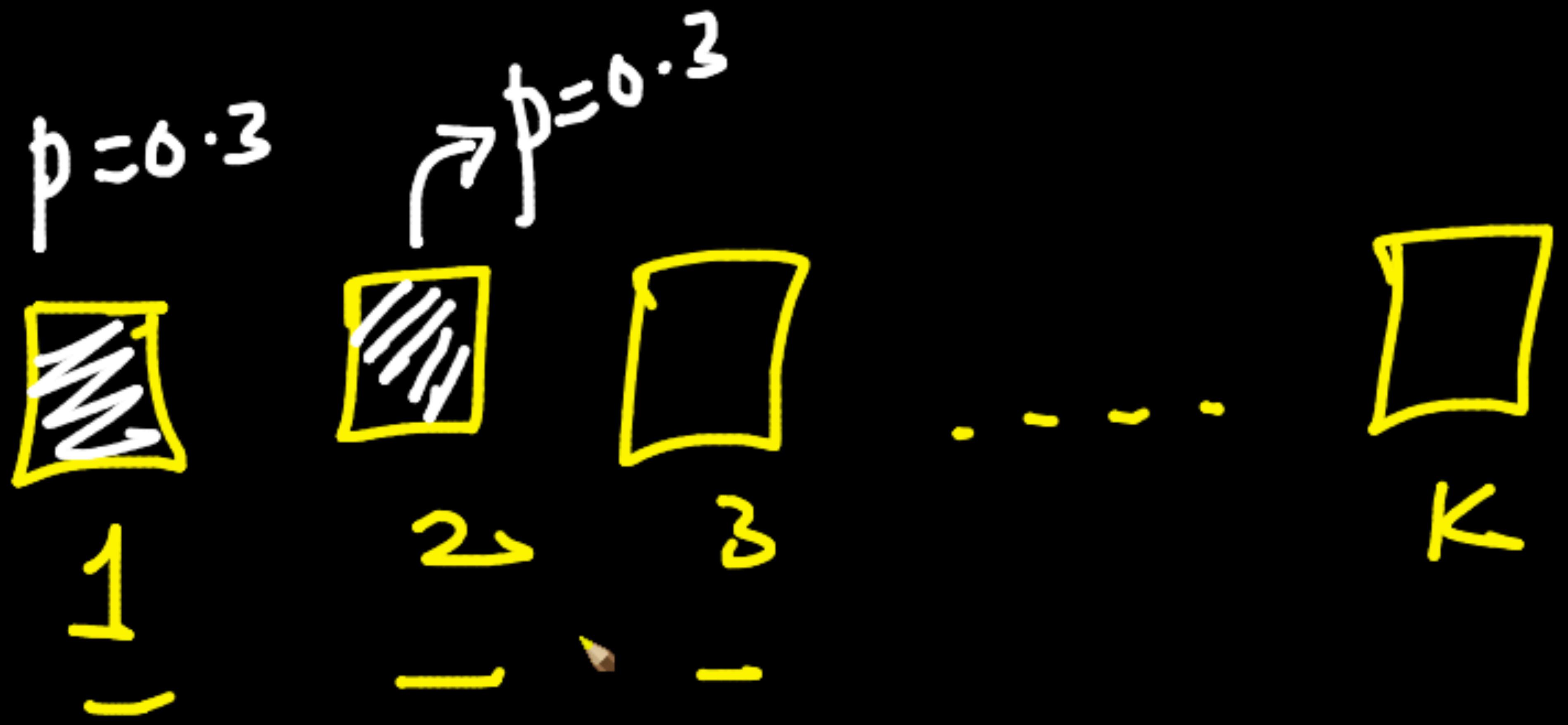
Newystats - Own work

Exponential $\rightarrow$ memoryless



Geometric Disb

jobs



p of success : bernouli trail

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Geom. distb

X = number of failures before a success in a series of bernouli trails of same p

Bin: # successes in n indep bernouli trails of same p

48 / 52

Conditions

(1) Seq. of indep bernouli trails → S, F

(2) same p of success in each trail

(e.g) # tosses before you observe 1st head success $\rightarrow p = 0.3$

X = # failures before a success

$P(X=5)$ = prob. of 5 failures before a success

$$\overbrace{\underset{f}{(1-p)}\;\underset{f}{(1-p)}\;\underset{f}{(1-p)}\;\underset{f}{(1-p)}\;\underset{f}{(1-p)}}\;\underset{s}{p}$$

$$= (1-p)^5 p^1$$

$$P(ABCDE) = P(A)P(B)P(C)\dots$$

Geom-distb

(examples)

PMF: $P(X=k) = (1-p)^{k} p^{1}$

CDF:- $P(X \le k) = \sum_{i=1}^{k} (1-p)^{i} p^{1}$

$$E(X) = \sum_{i=0}^{\infty} i \cdot (1-p)^{i} p^{j}$$

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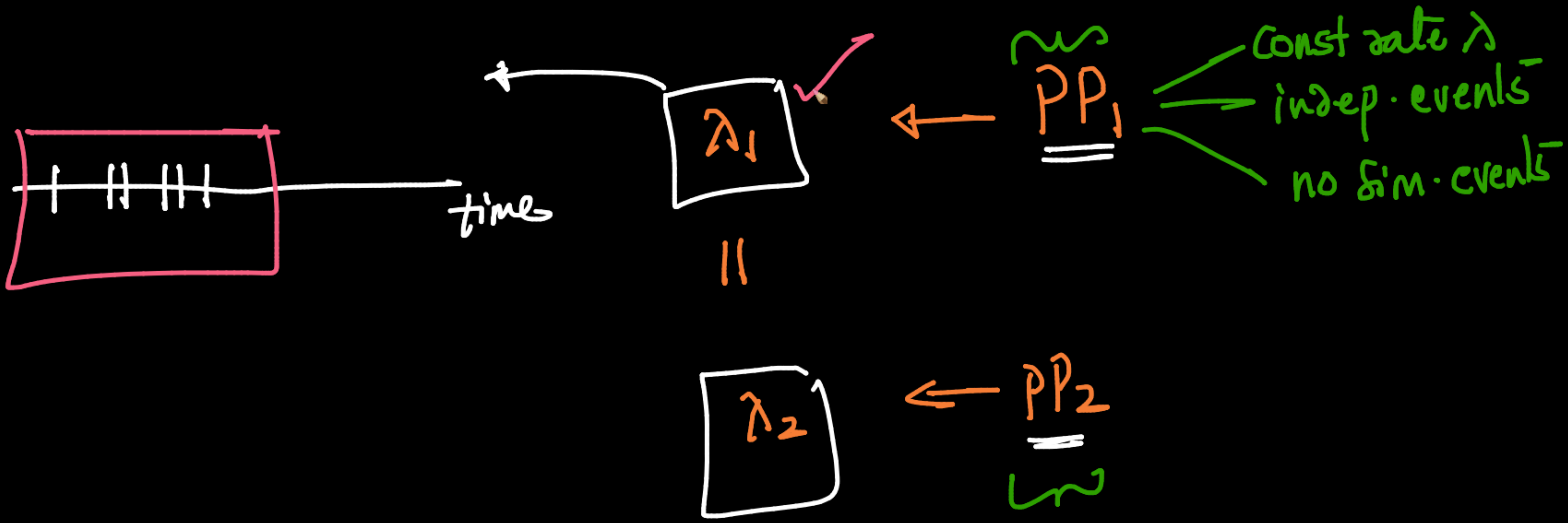
restaurent!

X = # visitors in 1hr ....

Group A 4 friends

53 / 57


time
λ1
PP1
const rate λ
indep. events
no sim. events
λ2
PP2

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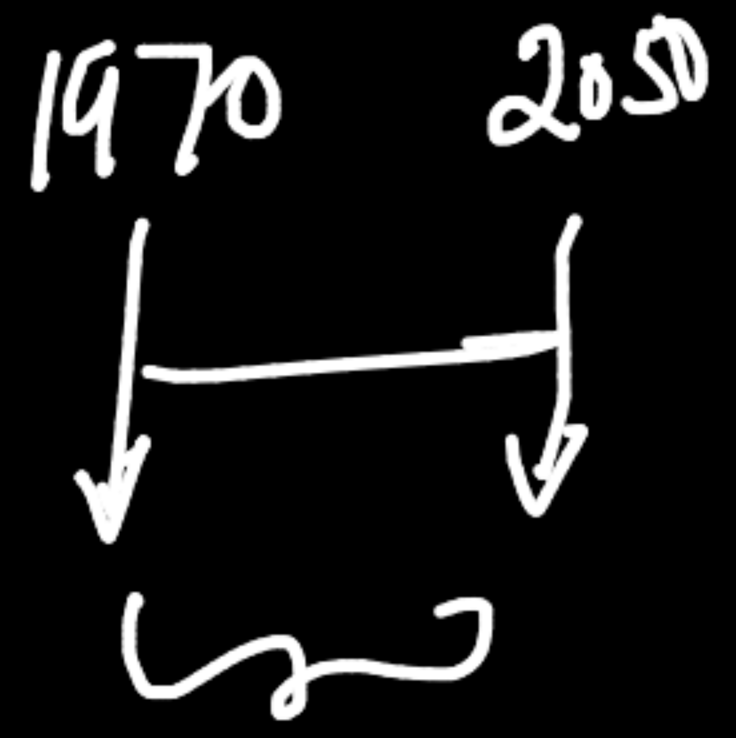
57 / 57



$P(\Delta t \le 80) \rightarrow$ PDF of expo. dist

Poisson-process :- $\lambda = 1$ per 100 yrs

1 event 100 yrs

$\Downarrow$

0.8 event 80 yrs

$\lambda =$

$P(X=2)$

$P(X=2) \rightarrow$ Poisson dist

$\{ x_1, x_2 \ldots\ldots x_n = 75$

95% C.I of median

Uber del. times

$\downarrow$

{ bootstrapping :- re

Sample: $\overbrace{x_1 \ x_2 \ \cdots \ x_n}$

$\downarrow$

sample-median = 35.4

population

{ 95% C.I of median (BS)

$\downarrow$

[34 36.5]

Pop:

BS on sample

95% C.I of Median

$S_1^n \rightarrow M_1$

$S_2^n \rightarrow M_2$

$S_3^n \rightarrow M_3$

$\vdots \qquad \vdots$

$S_r^n \rightarrow M_r$

2.5th percentile

97.5th percentile

$\underline{\underline{n=10}}$

$$\left.\begin{array}{l} S_1^n \\ S_2^n \\ \vdots \\ S_{10}^n \end{array}\right\} \longrightarrow \text{Unim}$$

1000

10n. samples

$$\boxed{S}$$

$$\downarrow$$

BS

$$= p\Sigma_{j=0}^{\infty}(1-p)^j$$
$$= p\frac{1}{1-(1-p)}$$
$$= 1$$

Can now compute $E(X)$:

$$\begin{aligned}
E(X) &= \Sigma_{k=1}^{\infty} k\cdot(1-p)^{k-1}p \\
&= p\left[\Sigma_{k=1}^{\infty}(1-p)^{k-1} + \Sigma_{k=2}^{\infty}(1-p)^{k-1} + \right. \\
&\qquad \left. \Sigma_{k=3}^{\infty}(1-p)^{k-1} + \cdots\right] \\
&= p[(1/p) + (1-p)/p + (1-p)^2/p + \cdots] \\
&= 1 + (1-p) + (1-p)^2 + \cdots \\
&= 1/p
\end{aligned}$$

So, for example, if the success probability p is $1/3$, it will take on average 3 trials to get a success.

- All this computation for a result that was intuitively clear all along . . .

Compare these two distributior

- Distribution 1:

$$\Pr(49) = \Pr(51) = 1$$

- Distribution 2: $\Pr(0) = \Pr($

Both have the same expectation less "dispersed" than the secon *dispersion.*

- One measure of dispersion i the mean, on average.

Given a random variable X, (how far the value of s is from th tation) of X. Define the *varia*

$$\mathrm{Var(X) = E((X - E(X))^2) = \Sigma}$$

The *standard deviation* of X i