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# -----
# 1. Imports
# -----
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

from sklearn.preprocessing import StandardScaler
from sklearn.decomposition import PCA
from sklearn.cluster import KMeans
from sklearn.metrics import silhouette_score

sns.set(style="whitegrid")
plt.rcParams["figure.figsize"] = (8, 5)

# -----
# 2. Load Dataset
# -----
df = pd.read_csv("data/Mall_Customers.csv") # change name if different
print(df.shape)
display(df.head())
print(df.info())

# We will use Annual Income & Spending Score for clustering
features = ['Annual Income (k$)', 'Spending Score (1-100)']
X = df[features].values

# -----
# 3. Scaling / Normalization
# -----
scaler = StandardScaler()
X_scaled = scaler.fit_transform(X)

# -----
# 4. Optional PCA (not really needed with 2 features,
#    but shown for general case)
# -----
pca = PCA(n_components=2)
X_pca = pca.fit_transform(X_scaled)

# -----
# 5. Elbow Method to choose K
# -----
inertias = []
K_range = range(1, 11)

for k in K_range:
    km = KMeans(n_clusters=k, random_state=42, n_init=10)
    km.fit(X_pca)
    inertias.append(km.inertia_)

plt.plot(K_range, inertias, marker='o')
plt.xlabel("Number of clusters K")
plt.ylabel("Inertia (Within-cluster SSE)")
plt.title("Elbow Method")
plt.show()

# -----
# 6. Silhouette Score for K >= 2
# -----
sil_scores = []
K_range2 = range(2, 11)

for k in K_range2:
    km = KMeans(n_clusters=k, random_state=42, n_init=10)
    labels = km.fit_predict(X_pca)
    score = silhouette_score(X_pca, labels)
    sil_scores.append(score)

plt.plot(K_range2, sil_scores, marker='o')
plt.xlabel("Number of clusters K")
plt.ylabel("Silhouette Score")
plt.title("Silhouette Score vs K")
plt.show()

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print("Silhouette scores:", dict(zip(K_range2, sil_scores)))

# Choose K (for Mall Customers usually K=5 works well;
# you can pick based on elbow + silhouette)
best_k = 5

# -----
# 7. Fit Final K-Means Model
# -----
kmeans = KMeans(n_clusters=best_k, random_state=42, n_init=10)
cluster_labels = kmeans.fit_predict(X_pca)

df['cluster'] = cluster_labels

print("\nCluster counts:")
print(df['cluster'].value_counts())

# -----
# 8. Visualize Clusters (2D)
# -----
plt.figure(figsize=(8,6))
scatter = plt.scatter(X_pca[:,0], X_pca[:,1],
                      c=cluster_labels, cmap='tab10', s=50)
plt.xlabel("PCA 1")
plt.ylabel("PCA 2")
plt.title(f"K-Means Clusters (K={best_k})")
plt.legend(*scatter.legend_elements(), title="Cluster")
plt.show()

# -----
# 9. Inspect Cluster Centroids (in original feature space)
# -----
centers_scaled = kmeans.cluster_centers_
centers_original = scaler.inverse_transform(pca.inverse_transform(centers_scaled))

centers_df = pd.DataFrame(centers_original, columns=features)
print("\nCluster Centers in original feature space:")
display(centers_df)

# -----
# 10. Silhouette Score for final model
# -----
final_silhouette = silhouette_score(X_pca, cluster_labels)
print("Final Silhouette Score:", final_silhouette)

print("\nTask 8 - K-Means clustering completed.")

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