Question 1
A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

Total number of balls = 2 + 3 + 2 = 7

Total number of ways of selecting two balls,

$$n(S) = {}^{7}C_2 = \frac{7!}{5! \times 2!} = \frac{7 \times \overset{3}{\cancel{6}} \times \cancel{5}!}{\cancel{5}! \times \cancel{2} \times 1} = \mathbf{21}$$

Total number of blue balls = 2
Total number of ways of selecting two other than blue balls,

$$n(E) = {}^{5}C_2 = \frac{5!}{3! \times 2!} = \frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3}!}{\cancel{3}! \times \cancel{2} \times 1} = \mathbf{10}$$

Therefore, the Probability of getting none of balls drawn is blue,

$$P(E) = \frac{n(E)}{n(S)} = \frac{10}{21} = 0.4762$$

Question 2
At a car park there are 100 vehicles, 60 of which are Cars, 30 are Vans, and the remainder are Lorries. If every vehicle is equally likely to leave, find the probability of:
a) Van leaving first
b) Lorry leaving first
c) Car leaving second if either a Lorry or Van had left first

Total number of vehicles = n(S) = 100

(a) Number of Vans = n(E1)= 30

Probability of Van leaving first, $P(E1) = \frac{n(E1)}{n(S)} = \frac{30}{100} = \mathbf{0.3}$

(b) Number of Lorries = n(E2) = 10

Probability of Lorry leaving first, $P(E2) = \frac{n(E2)}{n(S)} = \frac{10}{100} = \mathbf{0.1}$

(c) Number of cars = n(E3) = 60
Number of vehicles left after either a Lorry or a Van has left = n(S3) = 99
Probability of Car leaving second after a Lorry or a Van,

$$P(E3) = \frac{n(E3)}{n(S3)} = \frac{60}{99} = \mathbf{0.6060}$$

Question 3

A jar contains 3 red marbles, 7 green marbles, and 10 white marbles. If 2 marbles are drawn from the jar at random, what is the probability that both marble are of same colour?

Total number of marbles = 3 + 7 + 10 = 20

Total number of selecting two marbles = n(S) = $^{20}C_2 = \dfrac{\overset{10}{\cancel{20}} \times 19 \times \cancel{18}!}{\cancel{18}! \times \cancel{2}!} = 190$

Number of ways of selecting both marbles are of same colour
= 2 red marbles out of 3 **OR** 2 green marbles out of 7 **OR** 2 white marbles out of 10

$$n(E) = {}^{3}C_2 + {}^{7}C_2 + {}^{10}C_2$$

$$= \left(\frac{3!}{1! \times 2!}\right) + \left(\frac{7!}{5! \times 2!}\right) + \left(\frac{10!}{8! \times 2!}\right)$$

$$= \left(\frac{3 \times \cancel{2}!}{1 \times \cancel{2}!}\right) + \left(\frac{7 \times \overset{3}{\cancel{6}} \times \cancel{5}!}{\cancel{5}! \times \cancel{2} \times 1}\right) + \left(\frac{\overset{5}{\cancel{10}} \times 9 \times \cancel{8}!}{\cancel{8}! \times \cancel{2} \times 1}\right)$$

$$= 3 + 21 + 45$$

$$= 69$$

Probability of both marble drawn are of same colour,

$$P(E) = \frac{69}{190} = \mathbf{0.3632}$$

Question 4

An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Total number of questions = 300 easy True/False
+ 200 difficult True/ False
+ 500 easy MCQ
+ 400 difficult MCQ
=**1400**

Number of easy questions = n(E) = 300 + 500

Probability of selecting an easy questions = $P(E) = \dfrac{800}{1400} = \dfrac{8}{14}$

Number of MCQs = n(M) = 500 + 400 = 900

Probability of selecting a MCQ = $P(M) = \dfrac{900}{1400} = \dfrac{9}{14}$

Number of easy MCQ = n(E$\cap$M) = 500

Probability of selecting easy MCQ = P(E$\cap$M) $= \frac{500}{1400} = \frac{5}{14}$

Probability of selecting an easy question **given that it is a MCQ**,

$$P(E|M) = \frac{P(E \cap M)}{P(M)} = \frac{\frac{5}{\cancel{14}}}{\frac{9}{\cancel{14}}} = \mathbf{\frac{5}{9}} = 0.5555$$

Question 5

Given that the two numbers appearing on throwing two dice are different. Find the proba bility of the event 'the sum of numbers on the dice is 4'.

Number of possible for unbiased dice = 36
Number of possible outcomes in which two dice will have same number = 6 viz.(1,1) (2,2) (3, 3) (4,4) (5,5) (6,6)

Therefore number of possible outcomes of required sample space = n(S) = 36-6 = 30

Number of ways of getting the sum of numbers on two dice = n(E)= 2 viz.(1,3) (3,1)

Probability of getting sum as 4 = P(E) $= \frac{2}{30} = \mathbf{\frac{1}{15}}$ = 0.067

Question 6

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, pre sent. However, the test also yields a false positive result for 0.5% of the healthy person test ed (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probabilit y that a person has the disease given that his test result is positive?

Let A be the event that population has the disease, P(A) = 0.1% = 0.0001
$\therefore$ P(A') = 1 – 0.1% = 0.999

Let E be the event that the test result is positive:

Probability that the result is positive given the person has disease = P(E|A) = 99% = 0.99
Probability that the result is positive given the person does not have disease = P(E|A') = 0.5% = 0.0005

Since the two events are mutually exclusive and exhaustive, we can use Baye's theorem.
Using Baye's theorem, the probability that the person has disease given that his result is positive

$$
\begin{aligned}
P(A|E) &= \frac{P(A)\cdot P(E|A)}{P(A)\cdot P(E|A)+P(A')\cdot P(E|A')}\\
&= \frac{0.99\times 0.001}{0.99\times 0.01+0.05\times 0.999}\\
&= \frac{0.00099}{0.005985}\\
&= 0.1654
\end{aligned}
$$

Question 7

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Total number of insured drivers = n(S) = 12000

Number of scooter drivers insured = n(Sc) = 2000

Probability of selecting a Scooter driver = $P(Sc) = \frac{n(Sc)}{n(S)} = \frac{2\cancel{000}}{12\cancel{000}} = \frac{2}{12}$

Number of Car drivers insured = n(C) = 4000

Probability of selecting a Car driver = $P(C) = \frac{n(C)}{n(S)} = \frac{4\cancel{000}}{12\cancel{000}} = \frac{4}{12}$

Number of Truck drivers insured = 6000

Probability of selecting a Truck driver = $P(T) = \frac{n(T)}{n(S)} = \frac{6\cancel{000}}{12\cancel{000}} = \frac{6}{12}$

Let A be the event that the person meets with an accident, then as per the given data
Probability of Scooter driver met with an accident = P(A|Sc) = 0.01
Probability of Car driver met with an accident = P(A|C) = 0.03
Probability of Truck driver met with an accident = P(A|T) = 0.15

To find the probability that the driver is a scooter driver, given that he met with an accident, P(Sc|A) = ?

Using Baye's Theorem,

$$
\begin{aligned}
P(Sc|A) &= \frac{P(Sc)\cdot P(A|Sc)}{P(Sc)\cdot P(A|Sc)+P(C)\cdot P(A|C)+P(T)\cdot P(A|T)}\\
&= \frac{\frac{2}{12}\times 0.01}{\left(\frac{2}{12}\times 0.01\right)+\left(\frac{4}{12}\times 0.03\right)+\left(\frac{6}{12}\times 0.15\right)}\\
&= \frac{2}{2+12+90} = \frac{2}{104} = \frac{1}{52} = \mathbf{0.01923}
\end{aligned}
$$

Question 8

Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Fur ther, if the first group wins, the probability of introducing a new product is 0.7 and the cor responding probability is 0.3 if the second group wins. Find the probability that the new p roduct introduced was by the second group.

Given :
Probability of first group wins = P(G1) = 0.6
Probability of second group wins = P(G2) = 0.4

Let N be the event of introducing a new product, then as per the given data
Probability of introducing a new product given that group1 wins = P(N|G1) = 0.7
Probability of introducing a new product given that group2 wins = P(N|G2) = 0.3

To find P(G2|N), i.e. probability that the new product introduced was by the second group
Using Baye's Theorem,

$$\begin{aligned} P(G2|N) &= \frac{P(G2)\cdot P(N\,|\,G2)}{P(G2)\cdot P(N\,|\,G2)+P(G1)\cdot P(N\,|\,G1)} \\ &= \frac{0.4\times 0.3}{(0.4\times 0.3)+(0.6\times 0.7)} \\ &= \frac{12}{12+42} \\ &= \frac{12}{54} \\ &= \frac{2}{9} = \mathbf{0.2222} \end{aligned}$$

Question 9

A manufacturer has three machine operators A, B and C. The first operator A produces 1 % defective items, whereas the other two operators B and C produce 5% and 7% defectiv e items respectively. A is on the job for 50% of the time, B is on the job for 30% of the tim e and C is on the job for 20% of the time. A defective item is produced, what is the probab ility that it was produced by A?

Given:
Probability of A producing defective items = P(A) = 0.01
Probability of B producing defective items = P(B) = 0.05
Probability of C producing defective items = P(C) = 0.07

Let D be the event of producing a defective item, then as per the given data
Probability of producing a defective item given that A is on duty = P(D|A) = 0.5
Probability of producing a defective item given that B is on duty = P(D|B) = 0.3
Probability of producing a defective item given that C is on duty = P(D|C) = 0.2

To find, P(A|D) i.e. the probability that defective item was produced by A
By using Baye's Theorem,

$$P(A|D) = \frac{P(A)\cdot P(D|A)}{P(A)\cdot P(D|A) + P(B)\cdot P(D|B) + P(B)\cdot P(D|B) + P(C)\cdot P(D|C)}$$

$$= \frac{0.01\times 0.5}{(0.01\times 0.5) + (0.05\times 0.3) + (0.07\times 0.2)}$$

$$= \frac{5}{5+15+14}$$

$$= \frac{5}{34} = \mathbf{0.1471}$$

Question 10

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are dra wn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Given :
Total number of cards in a pack = 52
Total number of diamond cards in a pack = 13
Let E be the event of card lost is a diamond card

$$\therefore P(E) = \frac{1}{4} \text{ and } P(E') = \frac{3}{4}$$

Number of ways of selecting two diamond cards $= {}^{13}C_2 = \frac{13!}{11!\times 2!} = \frac{13\times \overset{6}{\cancel{12}}\times \cancel{11}!}{\cancel{11}!\times \cancel{2}} = 78$

After a card is lost,
Number of ways of selecting two cadrs after a card is lost

$$= {}^{51}C_2 = \frac{51!}{49!\times 2!} = \frac{51\times \overset{25}{\cancel{50}}\times \cancel{49}!}{\cancel{49}!\times \cancel{2}} = 1275$$

Now number of ways of selecting two diamond cards when lost card is diamond =

$${}^{12}C_2 = \frac{12!}{10!\times 2!} = \frac{\overset{6}{\cancel{12}}\times 11\times \cancel{10}!}{\cancel{10}!\times \cancel{2}} = 66$$

Probability of selecting two diamond cards given that card lost is diamond,

$$P(A|E) = \frac{66}{1275}$$

Probability of selecting two diamond cards given that card lost is not diamond,

$$P(A|E') = \frac{78}{1275}$$

From Baye's Theorem,
To find the probability of lost card is diamond, given that two selected cards are diamonds

$$P(A \mid E') = \frac{P(E) \cdot P(A \mid E)}{P(E) \cdot P(A \mid E) + P(E') \cdot P(A \mid E')}$$

$$= \frac{\frac{66}{1275} \times \frac{1}{4}}{\left(\frac{66}{1275} \times \frac{1}{4}\right) + \left(\frac{78}{1275} \times \frac{3}{4}\right)}$$

$$= \frac{66}{66 + 234}$$

$$= \frac{66}{300}$$

$$= \mathbf{0.22}$$