Question 1

Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover?

The possible outcomes are (1) Patient dies & (2) Patient doesn't dies.

Therefore this is a Binomial Distribution problem.

Let X = number of patients who recover

By Binomial Distribution,

$$P(X) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

Here, number of selected patients $n = 6$, $x = 4$

75 % die i.e. $q = 0.75$

$p = 1 - q = 0.25$

$$\begin{aligned} P(X) &= {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x} \\ &= {}^{6}C_{4} \cdot p^{4} \cdot q^{6-4} \\ &= \left(\frac{6 \times 5 \times \cancel{4}!}{2! \times \cancel{4}!}\right) \times (0.25)^{4} \times (0.75)^{2} \\ &= 15 \times 2.1973 \times 10^{-3} \\ &= 0.03295 \end{aligned}$$

$\therefore$ The probability that of 6 randomly selected patients, 4 will recover is 0.03295

R CODE :

```
dbinom(x=4, size=6, prob=0.25)
 [1] 0.03295898
```

Question 2

A manufacturer of metal pistons finds that on the average, 12% of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain (a) no more than 2 rejects? (b) at least 2 rejects?

The possible outcomes are : Success = rejected, Failure = not rejected
Therefore this Binomial Distribution

$$P(X) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

Let X = number of rejected pistons
Given $n = 10$, $p = 0.12$, $q = 0.88$

(a) no more than 2 rejects which means, x = 0, 1, 2

At $x = 0$,

$$\begin{aligned} P(X) &= {}^{10}C_{0} \times (0.12)^{10} \times (0.88)^{10-0} \\ &= 0.2785 \end{aligned}$$

At $x = 1$,

$$\begin{aligned} P(X) &= {}^{10}C_1 \times (0.12)^{10} \times (0.88)^{10-1} \\ &= 0.37977 \end{aligned}$$

At x = 2,

$$\begin{aligned} P(X) &= {}^{10}C_2 \times (0.12)^{10} \times (0.88)^{10-2} \\ &= 0.23304 \end{aligned}$$

So, the probability of getting no more than 2 projects,
P(X) = 0.2785 + 0.37977 + 0.23304
= 0.89131

R CODE :

```
pbinom(q=2, size=10, prob=0.12, lower.tail = T)
 [1] 0.8913182
```

(b) at least 2 rejects which means x = 2, 3, 4, 5, 6, 7, 8, 9, 10

The probability of getting atleast 2 rejects,
= P (X ≥ 2)
= 1 – P(X < 2)
= 1 – [P(X = 0) + P(X = 1)]
= 1 – (0.2785 + 0.37977)
= 0.34173

R CODE :

```
pbinom(q=1, size=10, prob=0.12, lower.tail = F)
 [1] 0.341725
```

Question 3

A man was able to complete 3 files a day on an average. Find the probability that he can complete 5 files the next day.

Average = 3 files a day
Therefore this problem follows Poisson's Distribution,

$$P(X) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

Given, x = 5, λ = 3
Probability of completing 5 files next day

$$\begin{aligned} &= \frac{(2.71828)^{-3} \cdot 3^{5}}{5!} \\ &= \frac{0.049787 \times 243}{120} \\ &= 0.1008 \end{aligned}$$

R CODE :

```
dpois(x=5,lambda=3)
 [1] 0.1008188
```

Question 4
The average number of homes sold by the Acme Realty Company is 2 homes per day. What is the probability that exactly 3 homes will be sold tomorrow?

Average = 2 houses sold per day
Therefore this problem follows Poisson's Distribution,

$$\mathrm{P(X)} = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

Given, x = 3, λ = 2
Probability of selling 3 homes next day,

$$= \frac{e^{-2} \cdot 2^{3}}{3!}$$
$$= \frac{0.049787 \times 8}{6}$$
$$= 0.180447$$

R CODE :
```
dpois(x=3,lambda=2)
[1] 0.180447
```

Question 5
The average speed of a car is 65 kmph with a standard deviation of 4. Find the probability that the speed is less than 60 kmph.

This problem follows Normal Distribution,
Mean = μ = 65
Standard Deviation = σ = 4
Expected Value = X = 60

$$\mathrm{Z} = \frac{\mathrm{X} - \mu}{\sigma}$$
$$= \frac{60 - 65}{4}$$
$$= -1.25$$

From the table,
Required probability =[area to the left of z = -1.25]= 0.1056

R CODE :
```
pnorm(q=60, mean=65, sd=4)
 [1] 0.1056498
```

Question 6

The average score of a statistics test for a class is 85 and standard deviation is 10. Find the probability of a random score falling between 75 and 95.

This problem follows Normal Distribution,
Mean = μ = 85
Standard Deviation = σ = 10
Expected Value = X = 75,.. 95

For X = 75,

$$Z = \frac{X-\mu}{\sigma} = \frac{75-85}{10} = -1$$

For X = 95,

$$Z = \frac{X-\mu}{\sigma} = \frac{95-85}{10} = 1$$

Required Probability, P(-1 < Z < 1) = [area to the left of z=1] – [area to the left of z=-1]
= 0.8413 – 0.1587
= 0.6826

R CODE :

```
pnorm(q=95, mean=85, sd=10) - pnorm(q=75, mean=85, sd=10)
 [1] 0.6826895
```

Question 7

Suppose we know that the birth weight of babies is normally distributed with mean 3500g and standard deviation 500g. What is the probability that a baby is born that weighs less than 3100g?

This problem follows Normal Distribution,
$X \sim N(\mu, \sigma^2)$
Mean = μ = 3500
Standard Deviation = σ = 500
Expected Value = X < 3100

$$Z = \frac{X-\mu}{\sigma} = \frac{3100-3500}{500} = -0.8$$

Required Probability = P(Z<0.8)= [area the left of z=-0.8]
= 1 – [area to the left of z=0.8]
= 1 – 0.7881
= 0.2119

R CODE :

```
pnorm(q=3100, mean=3500, sd=500)
 [1] 0.2118554
```