September 2014

White Paper

Coping with 'Black Swan' Scenario by Black-Litterman Model in Multi-Asset

Portfolio manager John helps manage a mega-sized investment fund of $45 billion. The fund has no explicit liability profile to match and does not contain any investment that has a specific termination date. It is a partially illiquid fund, a small percentage of which is occasionally withdrawn based on the needs of its sponsors. Therefore, its investments cannot be wholly illiquid. The allocation of the fund can be modelled based on a U.S. multi-asset portfolio, which generating a desirable historical portfolio return of 10.16% and beta of 3.02 , as shown in **Figure 1**. John's major goal is to keep it generating steady return over the medium to long term even if the market may swing up and down in the shorter term.

US Multi-Asset Funds

Buy | Sell | Fields: Default | Calc. Horizon: 5 Year | Use Active Return | Include Cash | Use Weekly | Import | Export

Description			Position	Weight	P&L	Market	Risk	Return	
Product	Asset Class	Currency	Actual	Actual	Actual	Last Price	Volatility	Historical	6m Beta
American Funds Growth Fund of Ame...	Equity	USD	4,669,000,000.00	10.29%	0	46.42	10.75%	14.69%	4.27
DoubleLine Total Return Bond Fund	Fixed Inco...	USD	2,196,000,000.00	4.84%	0	10.95	2.89%	1.22%	0.18
Fortress Investment Group LLC	Hedge Funds	USD	2,214,000,000.00	4.88%	0	7.23	55.09%	35.50%	7.12
Franklin Templeton Hard Currency F...	Foreign Ex...	USD	1,856,000,000.00	4.09%	0	9.21	7.81%	2.77%	0.82
Goldman Sachs Commodity Strategy A	Commodity	USD	2,710,000,000.00	5.97%	0	5.36	14.40%	3.35%	0.91
PIMCO Total Return A	Fixed Inco...	USD	3,285,000,000.00	7.24%	0	10.89	5.28%	3.00%	0.65
PowerShares Global Listed Private Eq	Private Eq...	USD	2,346,000,000.00	5.17%	0	11.63	20.25%	12.76%	4.99
SPDR Gold Shares	Commodity	USD	1,086,570,000.00	2.39%	0	118.83	20.70%	5.41%	0.26
SPDR MSCI ACWI ex-US ETF	Equity	USD	3,676,000,000.00	8.10%	0	36.47	13.76%	4.06%	4.82
SPDR Russell Small Cap Completeness	Equity	USD	2,605,800,000.00	5.74%	0	85.80	13.51%	18.27%	4.89
SPDR S&P 500 ETF Trust	Equity	USD	10,029,500,000.00	22.10%	0	200.48	8.41%	13.74%	4.00
T. Rowe Price US Treasury Long-Term	Fixed Inco...	USD	3,780,000,000.00	8.33%	0	12.33	14.93%	6.43%	0.33
iShares Core Total US Bond Market ETF	Fixed Inco...	USD	3,059,280,000.00	6.74%	0	108.61	4.10%	3.00%	0.61
iShares Dow Jones US Real Estate (E...	Real Estate	USD	1,867,250,000.00	4.11%	0	71.48	9.15%	9.46%	4.92
Total			45,380,400,000.00	100.00%	0			10.16%	3.02

Figure 1: US Multi-Asset Funds

In order to reach the ultimate goal, John faces several significant limitations associated with the characteristics of a multi-asset portfolio. Firstly, the giant size of the portfolio makes it extremely difficult for him to calculate its alpha [4]. Secondly, significant changes to the portfolio usually generate high transaction costs, which offset the benefits from higher active returns [5]. In addition, hedging [6] is not a practical strategy in terms of managing such a large portfolio because a huge amount of shorts may crash the market artificially. Finally, there also exists a significant proportion of illiquid investments in the portfolio which are simply difficult to sell, for example iShares Dow Jones US Real Estate.

---

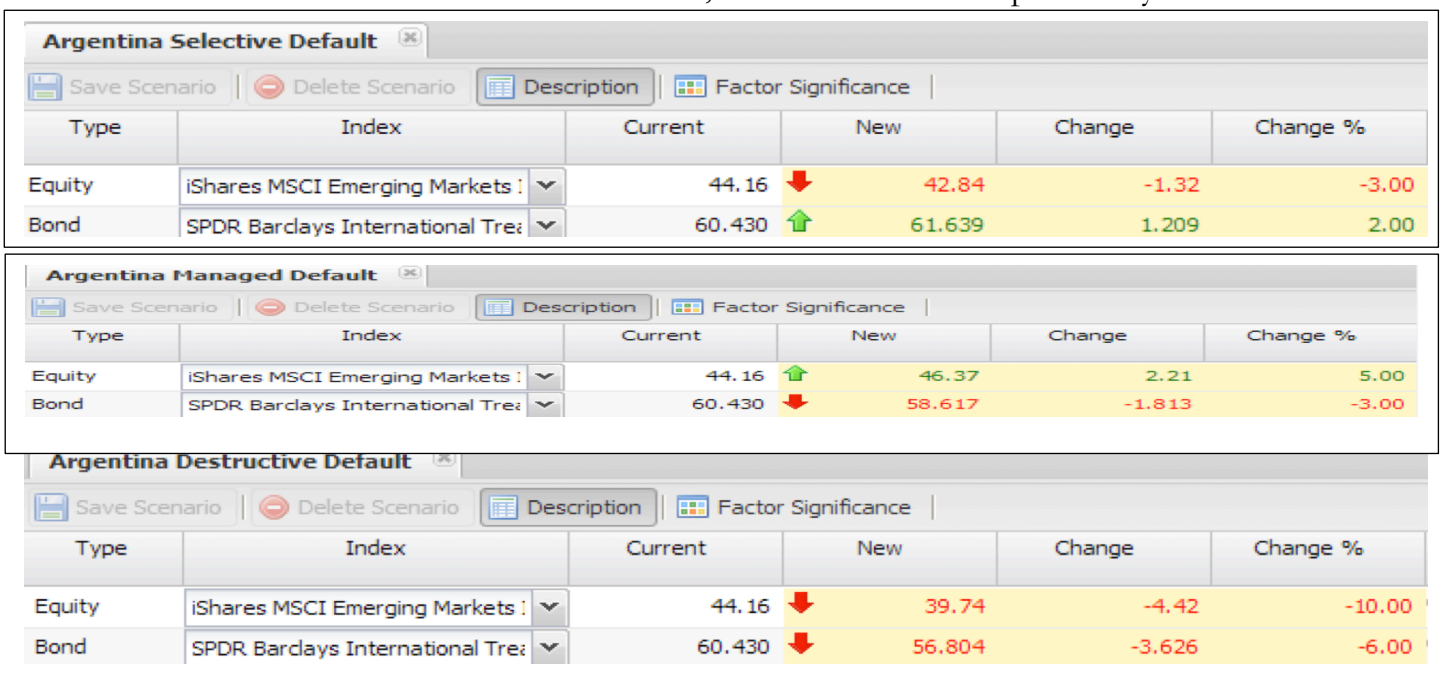
1 [2] A portflio with a proper asset allocation that produces maximul return with minimul risk
[3] Expense incured when buying and selling securities
[4] Alpha is a risk-adjusted measure of active return on an investment. A positive alpha of 1.0 means the fund has outperformed its benchmark index by 1%. Correspondingly, a similar negative alpha would indicate an underperformance of 1%.

Investment Problem – "Black Swan" Scenario [2]

After the financial crisis in 2008, the Board of the fund mandate to include "Black Swan" scenarios in its routine proactive portfolio optimization management because it is impossible to instantly make a large-scale change to a massive portfolio or put on a hedge right upon the time when another new financial crisis arrives. At the same time, the fund will not generate a helpful return if it is "permanently" positioned to avoid any risk. Below, John demonstrates how he managed to find the optimal asset allocation that incorporates the low-probability extreme events for the large multi-asset portfolio.

The Scenario: On July 30th 2014, Argentina claimed default again by not paying its bondholders following a 12-year battle after its last default. [See Appendix Sec I.1] John thinks that Argentina's default should trigger serious ripper effects in the global financial market and raise new potential risks that have significant impact on his portfolio's performance. In order to prevent the sponsors' wealth exposing to excessive risks and massive devaluation, an asset weight adjustment to rebalance it as an optimal portfolio is necessary under this new context.

John anticipates that there are three possible market scenarios about Argentina's default, dubbed as "Selective Default", "Managed Default" and "Destructive Default" scenario. [See Appendix Sec I.2] John's quantitative market view corresponding to each scenario are displayed in Figure 2.[See Appendix Sec I.3] He holds his mainstream view that Argentina is most likely to manage its default, although the devastating threats from the "Black Swan" scenarios, the "Selective Default" and the "destructive default" scenario, still remain in small possibility.



Naïve Modelling Strategy

John starts with the naïve approach of maximization of sharp ratio to solve the optimization the problem. [See Appendix Sec II.3, 4] First, he calculates its implied return under each view, as shown in Figure 4. Secondly, he calculates the optimal asset allocation independently for each view, as shown in Figure 3.

John identifies that naïve maximization sharp ratio model has some serious flaws when applied in practice. It is very sensitive to its inputs parameters and easily produces extreme corner solution containing many zeros. In addition, the asset allocation for the "managed default" scenario is dramatically different with those of the "black swan" scenarios; therefore, it is impossible to combine them together and get an asset allocation that incorporates both mainstream view and extreme views. Last, but the most important is that the overall portfolio weight change under each view is above 130%, which results in a huge amount of transaction

		Original	**Selective Default**		**Managed Default**		**Destructive Default**	
No.	Asset Name	%Weight	%Weight	% Abs Change	%Weight	% Abs Change	%Weight	% Abs Change
1	AGTHX-American Funds Growth Fund of Amer A	10.29%	**0.00%**	10.29%	**0.00%**	10.29%	**0.00%**	10.29%
2	DLTNX-DoubleLine Total Return Bond Fund	4.84%	**0.00%**	4.84%	**65.31%**	60.47%	**0.00%**	4.84%
3	FIG-Fortress Investment Group LLC	4.88%	**0.00%**	4.88%	**13.03%**	8.15%	**0.00%**	4.88%
4	ICPHX-Franklin Templeton Hard Currency FUND	4.09%	**5.08%**	0.99%	**0.00%**	4.09%	**0.00%**	4.09%
5	GSCAX-Goldman Sachs Commodity Strategy A	5.97%	**3.10%**	2.87%	**0.00%**	5.97%	**0.00%**	5.97%
6	PTTAX-PIMCO Total Return A	7.24%	**0.00%**	7.24%	**0.00%**	7.24%	**0.00%**	7.24%
7	PSP-PowerShares Global Listed Private Eq	5.17%	**0.00%**	5.17%	**1.57%**	3.60%	**0.00%**	5.17%
8	GLD-SPDR Gold Shares	2.39%	**15.63%**	13.24%	**0.00%**	2.39%	**0.00%**	2.39%
9	CWI-SPDR MSCI ACWI ex-US ETF	8.10%	**0.00%**	8.10%	**0.00%**	8.10%	**0.00%**	8.10%
10	RSCO-SPDR Russell Small Cap Completeness	5.74%	**0.00%**	5.74%	**20.08%**	14.34%	**0.00%**	5.74%
11	SPY-SPDR S&P 500 ETF Trust	22.10%	**0.00%**	22.10%	**0.00%**	22.10%	**26.05%**	3.95%
12	PRULX-T. Rowe Price US Treasury Long-Term	8.33%	**33.89%**	25.56%	**0.00%**	8.33%	**73.95%**	65.62%
13	AGG-iShares Core Total US Bond Market ETF	6.74%	**42.31%**	35.57%	**0.00%**	6.74%	**0.00%**	6.74%
14	IRY-iShares Dow Jones US Real Estate (ETF)	4.12%	**0.00%**	4.12%	**0.00%**	4.12%	**0.00%**	4.12%
	% Portfolio l Asset weight absolute Change			**150.70%**		**165.93%**		**139.14%**

Figure 3: Optimal Weight Allocation by Naïve Maximization of Sharp Ratio Approach for Three Views

0.00% 8.10%

US Multi-Asset Funds

Buy | Sell | Fields: Scenarios | Calc. Horizon: 5 Year | Use Active Return | Include Cash | Use Weekly | Import | Export

Description		Position	Weight	Return	Argentina Selective Default			Argentina Managed Default			Argentina Destructive Default		
Product	Asset Class	Actual	Actual	Historical	Pos Chg	Asst Chg	Scen Prc	Pos Chg	Asst Chg	Scen Prc	Pos Chg	Asst Chg	Scen Prc
American Funds Growth Fund of Ame...	Eq...	4,669,000,000.00	10.29%	14.69%	-0.33%	-3.16%	44.95	0.52%	5.10%	48.79	-0.39%	-3.78%	44.67
DoubleLine Total Return Bond Fund	Fix...	2,196,000,000.00	4.84%	1.22%	0.00%	0.03%	10.95	0.00%	-0.01%	10.95	-0.07%	-1.35%	10.80
Fortress Investment Group LLC	He...	2,214,000,000.00	4.88%	35.50%	-0.55%	-11.34%	6.41	0.88%	18.10%	8.54	-0.37%	-7.54%	6.68
Franklin Templeton Hard Currency F...	For...	1,856,000,000.00	4.09%	2.77%	0.01%	0.24%	9.23	-0.01%	-0.26%	9.19	-0.17%	-4.09%	8.83
Goldman Sachs Commodity Strategy A	Co...	2,710,000,000.00	5.97%	3.35%	-0.05%	-0.81%	5.32	0.08%	1.41%	5.44	-0.29%	-4.87%	5.10
PIMCO Total Return A	Fix...	3,285,000,000.00	7.24%	3.00%	0.04%	0.56%	10.95	-0.06%	-0.84%	10.80	-0.09%	-1.26%	10.75
PowerShares Global Listed Private Eq	Pri...	2,346,000,000.00	5.17%	12.76%	-0.25%	-4.82%	11.07	0.41%	7.88%	12.55	-0.52%	-10.07%	10.46
SPDR Gold Shares	Co...	1,086,570,000.00	2.39%	5.41%	0.09%	3.89%	123.45	-0.14%	-6.04%	111.66	-0.10%	-4.12%	113.94
SPDR MSCI ACWI ex-US ETF	Eq...	3,676,000,000.00	8.10%	4.06%	-0.27%	-3.31%	35.26	0.44%	5.44%	38.45	-0.66%	-8.19%	33.48
SPDR Russell Small Cap Completeness	Eq...	2,605,800,000.00	5.74%	18.27%	-0.23%	-4.04%	82.34	0.37%	6.49%	91.37	-0.26%	-4.57%	81.88
SPDR S&P 500 ETF Trust	Eq...	10,029,500,000.00	22.10%	13.74%	-0.63%	-2.84%	194.80	1.00%	4.54%	209.57	-0.49%	-2.23%	196.00
T. Rowe Price US Treasury Long-Term	Fix...	3,780,000,000.00	8.33%	6.43%	0.28%	3.37%	12.75	-0.46%	-5.47%	11.66	0.48%	5.82%	13.05
iShares Core Total US Bond Market ETF	Fix...	3,059,280,000.00	6.74%	3.00%	0.06%	0.91%	109.59	-0.10%	-1.44%	107.05	0.03%	0.38%	109.02
iShares Dow Jones US Real Estate (E...	Re...	1,867,250,000.00	4.11%	9.46%	-0.07%	-1.66%	70.29	0.11%	2.72%	73.43	-0.15%	-3.71%	68.83
Total		45,380,400,000.00	100.00%	10.16%	-1.89%			3.06%			-3.05%		

Figure 4: Implied Return under Three Views

Practical constrains

To overcome the flaws of the previous naïve approach, John takes several important practical constrains into his considerations.

First of all, the percentage of the portfolio's overall asset weight adjustment should be less than 1% over any month. Even moving 1% of the portfolio would amounts to moving $ 450 million in total, which is already a large quantity to buy/sell. Selling is not always possible when needed in reality because there may not be enough number of buyers in the market. Therefore, the fund may lose it ability to manage its exposure dynamically and even crash the market in the worst case.

Secondly, the market view should be consistent and stable for a significant period. In general, the Board expects that any strategy produced based on those market views should be stuck to for at least two or three years. When the investor uses the naïve approach, he is likely to jump around among three scenario since he has no super power to foresee which scenario will come true. It is an extreme bad idea because drastic modifications of view in the process may generate high market impact and transaction costs, which may totally offset the return an investor originally aims to achieve by modifying the portfolio.

Thirdly, a proper quantitative model should avoid produce corner solutions, dramatic changes and too much sensitivity to the parameters. In the naïve approach, the investor are mandatory to make a subjective prediction about the implied return under new context. Implied return usually changes a lot over month, which very dramatic different asset allocations. In this aspect, reverse optimization may produce more accurate and stable result under new context.[See Appendix] Thus the next challenge is that it is impossible to know whether the market portfolio is the best representation of the equilibrium view (an assumption of standard CAPM model). Since the fund is already so large, it is not unreasonable to suggest that the fund already

represents a good equilibrium view of what is investable.

Empowered Modelling Strategy – Black-LitterMan Model

In order to satisfy all the practical constraints set above, John attempts to apply Black-Litterman model, which gives him flexibility to combine multiple market views without completely leaving out Black Swan" scenarios. It produces a more precise implied return for the portfolio under that combined views, which results in more stable and accurate asset allocation after applied reverse optimization.

Single Scenario – Managed default

John first wants to find proper inputs to represent his view and tests how his mainstream view works in this model. Black Litterman model has four user inputs, including P, Q, confidence level and tau. [See Appendix Sec II.5] John sets P and Q based on his view under the managed default scenario and makes tau equal to 0.01 for testing. By fixing those three inputs, John compares the optimal asset allocation according to difference confidence levels, as shown in Figure 5. He believes the asset allocation with confidence level of 60% fits his view best, which has a reasonable total portfolio weight change of 0.58%. With fixed P, Q and confidence level = 60%, the model produces has a maximal portfolio weights change of 6.72% with tau =0.01, which is more desirable than 39.93% with tau =0.1 since John wants to control the overall weight adjustment at a relative low percentage.

CI	0%		20%		40%		60%		80%		95%		100%		95% (Tau =0.1)	
Asset	Weight	Abs Chg	Weight	Abs Chg	Weight	Abs Chg	Weight	Abs Chg	Weight	Abs Chg	Weight	Abs Chg	Weight	Abs Chg	Weight	Abs Chg
1	10.29%	0.00%	10.29%	0.00%	10.27%	0.02%	10.26%	0.03%	10.20%	0.09%	9.93%	0.36%	3.38%	6.91%	8.15%	2.14%
2	4.84%	0.00%	4.84%	0.00%	4.85%	0.01%	4.86%	0.02%	4.90%	0.06%	5.10%	0.26%	0.00%	4.84%	4.05%	0.79%
3	4.88%	0.00%	4.86%	0.02%	4.83%	0.05%	4.77%	0.11%	4.58%	0.30%	3.58%	1.30%	0.00%	4.88%	0.00%	4.88%
4	4.09%	0.00%	4.09%	0.00%	4.10%	0.01%	4.11%	0.02%	4.15%	0.06%	4.33%	0.24%	9.95%	5.86%	6.07%	1.98%
5	5.97%	0.00%	5.97%	0.00%	5.98%	0.01%	5.99%	0.02%	6.01%	0.04%	6.14%	0.17%	6.50%	0.53%	6.20%	0.23%
6	7.24%	0.00%	7.25%	0.01%	7.26%	0.02%	7.28%	0.04%	7.36%	0.12%	7.74%	0.50%	7.16%	0.08%	6.70%	0.54%
7	5.17%	0.00%	5.16%	0.01%	5.15%	0.02%	5.13%	0.04%	5.07%	0.10%	4.73%	0.44%	0.00%	5.17%	0.42%	4.75%
8	2.39%	0.00%	2.40%	0.01%	2.41%	0.02%	2.42%	0.03%	2.48%	0.09%	2.77%	0.38%	7.37%	4.98%	4.40%	2.01%
9	8.10%	0.00%	8.09%	0.01%	8.09%	0.01%	8.07%	0.03%	8.02%	0.08%	7.76%	0.34%	6.47%	1.63%	7.54%	0.56%
10	5.74%	0.00%	5.73%	0.01%	5.73%	0.01%	5.71%	0.03%	5.66%	0.08%	5.39%	0.35%	0.00%	5.74%	3.39%	2.35%
11	22.10%	0.00%	22.10%	0.00%	22.07%	0.03%	22.05%	0.05%	21.96%	0.14%	21.53%	0.57%	13.45%	8.65%	18.25%	3.85%
12	8.33%	0.00%	8.35%	0.02%	8.38%	0.05%	8.44%	0.11%	8.62%	0.29%	9.58%	1.25%	25.22%	16.89%	13.23%	4.90%
13	6.74%	0.00%	6.75%	0.01%	6.76%	0.02%	6.79%	0.05%	6.86%	0.12%	7.28%	0.54%	14.83%	8.09%	17.25%	10.51%
14	4.12%	0.00%	4.12%	0.00%	4.12%	0.00%	4.12%	0.00%	4.13%	0.01%	4.14%	0.02%	5.66%	1.54%	4.36%	0.24%
Total Portfolio Change	100.00%	0.00%	100.00%	0.10%	100.00%	0.28%	100.00%	**0.58%**	100.00%	1.58%	100.00%	6.72%	100.00%	75.80%	100.00%	39.73%

Figure 5 : Optimal Asset Weight Allocation by Black-Litterman Model with Different Confidence level (tau = 0.01)
Note: Last column is calculated with tau =0.1 and confidence level 95%

Multiple Scenarios

John incorporates the two extreme views into the mainstream view: "Selective default" view with a confidence level of 10% and "Destructive default" with a confidence level of 5%. Results are displayed in Figure 6.

By applying the black-litterman model, John has successfully coped with "black swan" scenarios in portfolio optimization with a return of 9.95% [See Appendix Sec II 5.e.2] and prepared his portfolio for the possible arrival of unexpected financial crisis in advance.

Asset	Weight	Abs Chg
1	10.25%	0.04%
2	4.87%	0.03%
3	4.75%	0.13%
4	4.11%	0.02%
5	5.99%	0.02%
6	7.29%	0.05%
7	5.12%	0.05%
8	2.43%	0.04%
9	8.06%	0.04%
10	5.71%	0.03%
11	22.05%	0.05%
12	8.46%	0.13%
13	6.79%	0.05%
14	4.12%	0.00%
Total Changes		0.68%

Figure 6 Optimal Weight Allocation under Combined Views by Black-Litterman Model

It also brings in additional advantages as follows:

- Cost Efficient - He only need to move 0.68%, less than 1% of the portfolio to rebalance the mega sized multi-asset portfolio, which significantly reduces transaction cost.
- Practical - The solution is not ideal but is practical to execute given the reality that part of the portfolio is illiquid
- Harmless - It avoids crashing the financial market given the pool of buyers in reality is limited
- Stable – The combined view including extreme events are more stable than a single view and can be held for a significant long time, which allows John to stick to his plan and makes time for him to make the adjustments in the following periods

HedgeSPA can help

The traditional way to implement black-litterman model on excel sheet is tedious and sophisticated. Without feeling irritated, John thinks that the process is efficiently simplified into several simple clicks when he uses HedgeSPA investment analytics platform to implement black-litterman model. Beyond that,

- The "Scenario Analysis Tool" on the platform provides him key information about up-to-date major market scenarios ,which help him quickly build up his own market views [See Appendix Sec I]
- Once P, Q, confidence level and tau are inputted, optimal weight allocation is automatically calculated without any human intervention, which dramatically improves the level of accuracy of the estimation.
- There are 5 additional objective functions that he can chooses to fit his different needs, such as maximization of alternative sharp ratio and minimization of variance.[See

Appendix

Section I

1. The Scenario Background
   In 2001, Argentina had the largest sovereign debt default in history at $93 billion. Between 2005 and 2010, the government restructured the debt and made settlements to pay back 33 cents on the dollar to its bondholders. However, $4 billion remains unsettled, and some "vulture funds" have been holding out for Argentina to pay back their debt at full.

2. Three Scenarios about Argentina's Default

	Selective Default	Managed default	Destructive Default
Action	Make a better offer to vulture fund before the expiration of RUFO[1]	Wait until everyone accepts the 33 cents offer after the expiration of RUFO	Never reach an settlement
Duration	Default ends before Dec 31st 2014	Default ends after Dec 31th 2014	Indefinite
Benefit	Restore reputation in bond market	Conserve its reserves and recovery in the long run	No payment need to be made
Risk	May result $29 billion tab, which depletes almost its entire foreigner currency reserve	Damage its reputation in bond market in the short run	Unable to get access to capital market and further economic instability
Probability	Low (0% -30%)	Medium (30%-70%)	Low (0%-30%)
Overall Assessment	Negative Scenario	Positive Scenario	Worst Scenario

Note: RUFO is short for 'Rights upon Future Offer' Clause on Argentina's bond contract, which gives right to bondholders to have the same benefit with each other as well as those 'vulture funds'.

3. Three Views about Changing of Market Factor under Each Scenario

	Market Factors	View 1 – Selective	View 2 – Managed	View 3 - Destructive
1.Asset	iShare MSCI Emerging Market	-3%	+5%	-10%
2.Equity	SPDR Barclay International Treasury	+2%	-3%	-6%
3.Forex	N/A	N/A	N/A	N/A
4.Commodity	N/A	N/A	N/A	N/A
Confidence Level	N/A	10%	60%	5%

Section II

Mathematical Modelling Setting and Result

1. Basic Modelling Setting
    a. This case study uses a 5 years' time series of historical data from Year1 to Year 2
    b. In Figure 1, annual historical return for each asset is calculated using forward 6 month rolling windows over the 5 years. For consistency, all the statistics will be computed over the same horizon, annualizing as and when appropriate.
    c. Risk-free asset return rate is set to be equal to 0.1% and is assumed to be constant.
2. Sharp Ratio

    $Sharp\ Ratio = \frac{E[Ra-Rrf]}{\sqrt{Var[Ra-Rrf]}}$, where Ra is the asset return, Rrf is the risk-free asset return and risk of the portfolio is measured by the variance of excessive return. The sharp ratio tells the investor if the higher return of the portfolio is due to better investment decision or coming from taking excess risk. A greater a portfolio's sharp ratio is, the better its risk-adjusted performance has been.

3. Portfolio Optimization and Inverse Optimization

    In the case study, portfolio optimization is aimed to find the optimal asset weight allocation that maximizes/minimizes a specific objective function with certain constraints, given the implied return for each scenario. Inverse portfolio optimization is otherwise aimed to calculate the implied return of each asset in the portfolio that maximizes/minimizes the objective function when we take certain weight allocation as market equilibrium.

4. Naïve Modelling Strategy

    In this case study, Naïve Modelling strategy is optimize portfolio with the objective function of maximization of sharp ratio under each scenario. The model is implemented in two steps.

    First Step: Calculate Implied Return under Each View

    The implied return under for each view is calculated by individually running regression of each asset' return against the market factor changes under each view as shown in Figure 2 and Appendix Sec I.3. The result is displayed in the "Asst Chg" column of Figure 4

    Second Step: Calculate the optimal weight allocation

    Find W= [w1, w2,⋯ w14], such that Maximizes $\frac{E[Ra-Rrf]}{\sqrt{Var[Ra-Rrf]}}$

    Constrained condition 1: w1+w2 ⋯+w14 = 1

    Constrained condition 2: $w_i \geq 0$ for i=1, 2,⋯ 14

    The Chart of Sharp ratio

Scenario	Original	Selective	Managed	Destructive
μ-portfolio return	10.16%	2.12%	3.78%	3.72%
σ-portfolio Std	10.11%	6.21%	8.39%	6.95%
Sharp Ratio	1.00	0.34	0.45	0.54

Note: The calculation process and result are attached in the excel file SharpRatioMax.xlsx

5. Empowered Modelling Strategy Approach - Black-Litterman Model
   a. Assumption: the current portfolio market capitalization weight (wMkt) is a good estimator to portfolio's equilibrium market asset weight.
   b. User Inputs:

P	Each column of matrix P representing an investor's view about assets' weighted implied return, which all the elements of the column sum up to 1
Q	A column vector , in which an element represents a total change of a portfolio's weighted implied return under a view
Confidence Level	A column vector, in which an element represents the investor's confidence in a view
Tau	A scalar for an investor to control the maximal change of the portfolio's by looking at the absolute portfolio weight change at 95% confidence level produced by Black-Litterman model

   c. The model is executed on Matlab in four steps.
      i. Compute P Matrix
      Each column of P matrix represents a view and is calculated independently in the same process. First, "Pos Chg" column in Figure 3 is produced by weighted the "Asst Chg". Then elements in the column are scaled to make sure that sum up to 100%.
      ii. Compute Implied Return Π
      Given wMkt of the current portfolio, use reverse optimization method to find implied return Π, such that it maximizes $\frac{E[Ra-Rrf]}{\sqrt{Var[Ra-Rrf]}}$. The close form solution for the optimization is,

      $\Pi = \frac{\Sigma * wMkt * (\mu^T * wMkt)}{wMkt^T * \Sigma * wMkt}$ with Σ is the covariance matrix of return of assets, μ is assets historical return.
      iii. Calculate a more powerful version of implied return E[R]
      $\mathbb{E}(R) = \Pi + \tau\Sigma P^T[\tau P\Sigma P^T]^{-1}(Q - P\Pi)$
      iv. Given E[R], get the optimal asset weight allocation that maximizes $\frac{E[Ra-Rrf]}{\sqrt{Var[Ra-Rrf]}}$. We uses the close form solution:
      $W_{optimal} = \frac{\Sigma^{-1} * E[R]}{I^T * \Sigma^{-1} * E[R]}$, I is an identical matrix that has the same size as E[R]
   d. Mathematical Property of the Model
      i. When the confidence level is 0% for every view, the optimal weight is equal to wMkt and the E[R] =Π. When confidence level is 0% in the setting of the single managed default view, the portfolio return is 10.16% which is consistent with the original portfolio return in Figure 1. When there is no additional views added, the model gives the original asset weight allocation. See Result at the chart below.

Optimal weight	Implied Return = Π	Portfolio Return
10.29%	13.59%	**10.16%**
4.84%	-0.29%	
4.88%	34.02%	
4.09%	5.51%	
5.97%	11.59%	
7.24%	1.22%	
5.17%	19.55%	
2.39%	2.09%	
8.10%	15.94%	
5.74%	15.83%	
22.10%	12.93%	
8.33%	-6.36%	
6.74%	-0.52%	
4.12%	13.59%	

ii. The model always converges to a specific weight allocation when the confidence level is 100% and thus picking a good estimation of confidence level is important. The weight allocation is shown below

Optimal Weight	Abs Chg
3.38%	6.91%
0.00%	4.84%
0.00%	4.88%
9.95%	5.86%
6.50%	0.53%
7.16%	0.08%
0.00%	5.17%
7.37%	4.98%
6.47%	1.63%
0.00%	5.74%
13.45%	8.65%
25.22%	16.89%
14.83%	8.09%
5.66%	1.54%
100.00%	75.80%

e. Additional resources about Three Cases used in the White Paper

1. Case I (a) (b) : Single View- Managed Default

a. Input

Q	**3.06%**
Confidence Level	**60%**
Tau case 1a	**0.01**
Tau case 1b	**0.1**

NO	Assets	**P-Managed**
1	AGTHX-American Funds Growth Fund of Amer A	**17.15%**
2	DLTNX-DoubleLine Total Return Bond Fund	**-0.02%**
3	FIG-Fortress Investment Group LLC	**28.87%**
4	ICPHX-Franklin Templeton Hard Currency FUND	**-0.35%**
5	GSCAX-Goldman Sachs Commodity Strategy A	**2.75%**
6	PTTAX-PIMCO Total Return A	**-1.99%**
7	PSP-PowerShares Global Listed Private Eq	**13.32%**
8	GLD-SPDR Gold Shares	**-4.72%**
9	CWI-SPDR MSCI ACWI ex-US ETF	**14.40%**
10	RSCO-SPDR Russell Small Cap Completeness	**12.18%**
11	SPY-SPDR S&P 500 ETF Trust	**32.80%**
12	PRULX-T. Rowe Price US Treasury Long-Term	**-14.89%**
13	AGG-iShares Core Total US Bond Market ETF	**-3.17%**
14	IRY-iShares Dow Jones US Real Estate (ETF)	**3.66%**
	Total	100.00%

b. Sharp Ratio Chart for Different Confidence Level

Confidence Level	**0%**	**20%**	**40%**	**60%**	**80%**	**95%**	**100%**	**95%(tau =0.1)**
$\boldsymbol{\mu}$	10.16%	10.13%	10.08%	9.98%	9.68%	8.22%	1.71%	3.35%
$\boldsymbol{\sigma}$	10.11%	10.10%	10.08%	10.05%	9.93%	9.34%	5.29%	6.46%
Sharp Ratio	1.00	1.0030	0.9997	0.9933	0.9745	0.8800	0.3229	0.5182

2. Case II Combined Views
    a. Inputs

NO	Assets	P - Selective	P –Managed	P - Destructive
1	AGTHX-American Funds Growth Fund of Amer A	**17.15%**	**17.25%**	**12.75%**
2	DLTNX-DoubleLine Total Return Bond Fund	**-0.02%**	**-0.08%**	**2.14%**
3	FIG-Fortress Investment Group LLC	**28.87%**	**29.35%**	**12.06%**
4	ICPHX-Franklin Templeton Hard Currency FUND	**-0.35%**	**-0.52%**	**5.48%**
5	GSCAX-Goldman Sachs Commodity Strategy A	**2.75%**	**2.56%**	**9.53%**
6	PTTAX-PIMCO Total Return A	**-1.99%**	**-2.15%**	**2.99%**
7	PSP-PowerShares Global Listed Private Eq	**13.32%**	**13.22%**	**17.06%**
8	GLD-SPDR Gold Shares	**-4.72%**	**-4.93%**	**3.23%**
9	CWI-SPDR MSCI ACWI ex-US ETF	**14.40%**	**14.22%**	**21.74%**
10	RSCO-SPDR Russell Small Cap Completeness	**12.18%**	**12.30%**	**8.60%**
11	SPY-SPDR S&P 500 ETF Trust	**32.80%**	**33.29%**	**16.15%**
12	PRULX-T. Rowe Price US Treasury Long-Term	**-14.89%**	**-14.89%**	**-15.89%**
13	AGG-iShares Core Total US Bond Market ETF	**-3.17%**	**-3.25%**	**-0.84%**
14	IRY-iShares Dow Jones US Real Estate (ETF)	**3.66%**	**3.63%**	**5.01%**
	Total	100.00%	100.00%	100%
	Q	**-1.89%**	**3.06%**	**-3.05%**
	Confidence Level	**10%**	**60%**	**5%**
	Tau		**0.01**	

Portfolio Return	9.95%
portfolio Std	10.03%
Sharp Ratio	0.99

6. Additional Objective functions
    - Minimization of Value at Risk
    - Minimization of Controlled Value at Risk at 95% percentile
    - Minimization of Portfolio Variance
    - Minimization of Maximum Drawdown
    - Maximization of Alternative Sharp Ratio