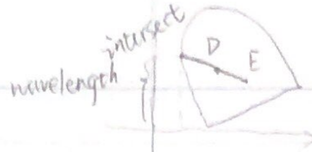
CSCI 576 HW2



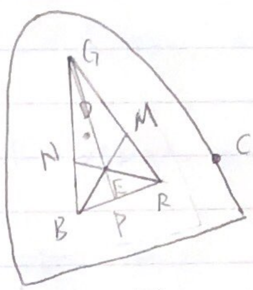
Q1

① we take a line from E to D, the line will have an intersection with this graph, and the y-coordinate of that intersection is wavelength of D

② No, it don't. There is not all colors have a dominate wavelength
For example, assume there is a point F at the bottom of the graph, and we take a line from F to D, we find an intersection with this graph on a straight line. Therefore, this point F doesn't have a dominate wavelength.

③ we take a line from C to E, the line will have an intersection with this graph, let's call this intersection Z, this color Z is complimentary to the color C.

④



$$R' = \frac{R}{R+G+B}, \quad G' = \frac{G}{R+G+B} \quad B' = \frac{B}{R+G+B}$$

connect RE and intersect with GB at point N
connect BE and intersect with RG at point M
connect GE and intersect with BR at point P
Thus, the line N → E and E → M are colors in RGB color space map to the equiluminous point E upon projection into the chromaticity space.

Q2.

① $x = \dfrac{X}{X+Y+Z}$ $y = \dfrac{Y}{X+Y+Z}$ $z = \dfrac{Z}{X+Y+Z}$

$$Z = \left[\frac{(1-x-y)}{y} \right] \cdot Y$$

$$Z = (1-x-y) \cdot \frac{Y}{y}$$

$$Z = \left(1 - \frac{X}{X+Y+Z} - \frac{Y}{(X+Y+Z)}\right) \cdot (X+Y+Z)$$

$$Z = (X+Y+Z) - X - Y$$

$$Z = Z$$

∴ Left = Right, thus proved.

② Yes, this algorithm will work effectively

③ The cartoon image will be performed better by this algorithm.

④ We can choose furthest color instead of nearest color.

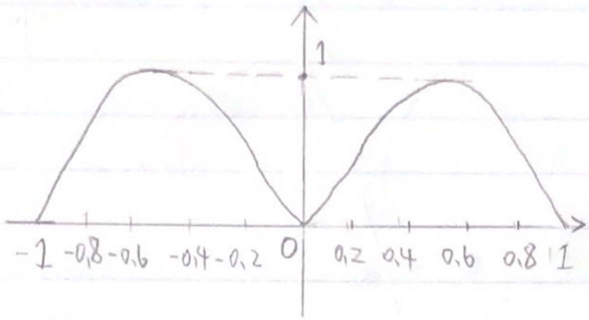
Q3.

① Entropy Function:

$$\sum P(i)\log_2 P(i) = -P(X)\log_2 P(X) - P(Y)\log_2 P(Y)$$

$$= -x^2\log_2 x^2 + x^2\log_2(1-x^2) - \log_2(1-x^2)$$

②



when $x=0$ or $x=1$
entropy become minimum.

③ Let take the derivate of the above formula

$$f(x) = -x^2\log_2 x^2 + x^2\log_2(1-x^2) - \log_2(1-x^2)$$

$$f(x) = -x^2\log_2 x^2 - (1-x^2)\log_2(1-x^2)$$

$$-f(x) = x^2\log_2 x^2 + (1-x^2)\log_2(1-x^2)$$

$$-f'(x) = 2x\log_2 x^2 + x^2\frac{2x}{x^2\ln 2} - 2x\log_2(1-x^2) - 2x\frac{(1-x^2)}{(1-x^2)\ln 2}$$

$$-f'(x) = 2x\log_2 x^2 + \frac{2x}{\ln 2} - 2x\log_2(1-x^2) - \frac{2x}{\ln 2}$$

$$-f'(x) = 2x(\log_2 x^2 - \log_2(1-x^2))$$

Let make $-f'(x)=0$, then we get $\begin{cases} x_1 = \frac{\sqrt{2}}{2} \\ x_2 = 0 \end{cases}$

$$\begin{cases} f'(x) > 0, & \text{when } 0 < x < \frac{\sqrt{2}}{2} \\ f'(x) < 0, & \text{when } \frac{\sqrt{2}}{2} < x < 1 \end{cases}$$

when $x = \frac{\sqrt{2}}{2}$, entropy is maximum, which is $f(\frac{\sqrt{2}}{2}) = 1$

④ According to calculation above, when $x=0$ or $x=1$
entropy is minimum.

Q4

① 22 24 24 28 28 28 25 26 26 26 21 21 20 20
22 24 24 24 23 24 20 16 10 10 8 11 6 9
9 12 15 19

② $2^5 = 32$, thus $2^5 \times \log_2 2^5 = 160$ bits

③ Differences: 2, 0, 4, 0, 0, -3, 1, 0, 0, -5, 0, -1, 0
2, 2, 0, 0, -1, 1, -4, -4, -6, 0, -2, 3, -5,
3, 0, 3, 3, 4.

$\begin{cases} \text{maximum difference} = 4 \\ \text{minimum difference} = -6 \end{cases}$

We need $\log_2 16 = 4$ bits per signal.
Thus, $31 \times 4 = 124$ bits

④ Compression ratio is $\frac{124}{160} = 0.775 = 77.5\%$

No.
DATE / /

⑤

Symbol	Huffman Code		
-1.4	11111		5
-1.3	11110		5
-1	11101		5
-0.8	11100		5
-0.3	11011		5
0.1	11010		5
0.2	11001		5
0.3	11000	$\Rightarrow$	5
0.7	01111		5
1	01110		5
-1.1	0110		8
-0.5	0101		8
0.6	0100		8
0.4	101		9
0.8	100		9
0	00		18

Sum = 110 bits

Thus, we need 110 bits to encode the sequence.

⑥ Compression Ratio $= \frac{110}{160} = 0.6875 = 68.75\%$