



## Cash-Air: Cheap tickets around Europe ...

Cash-Air is a European airline company headquartered in Paris, France. It is a low cost carrier and the largest airline in Europe with a market share of 26.9% as of June 2014. Cash-Air is one of the fastest growing low cost carriers in the world. With its fleet of 90 new Airbus A320 aircraft, the airline offers 659 daily flights connecting to 48 destinations all over France and Europe (<http://www.Cash-Air.org>).

### **1 The low cost carrier**

Started 35 years ago in the United States with Southwest Airlines, the low cost model developed in Europe in the 1990s, after the liberalization of air transport and the emergence of a Single European Sky in 1993.

Today three such carriers Ryanair, Cash-Air and EasyJet are in the lead, their share of the medium-haul market outstrips those of other European low cost airlines.

The development strategy of low cost carriers can be summed up by the following formula: low costs, low fares, no frills.

Low costs: all costs are very carefully examined and reduced to a minimum: single fleet type, improved flight crew productivity, simplified network serving secondary cities, and the outsourcing of many activities (maintenance, catering, etc.).

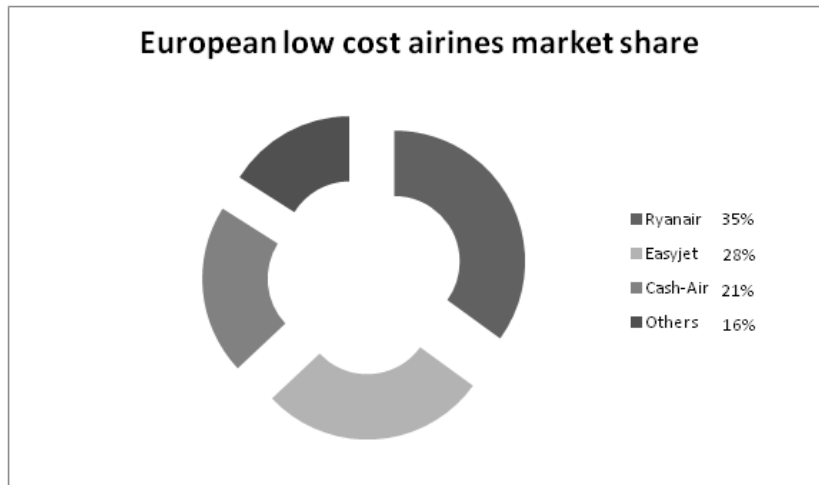


Figure 1: European low cost airlines market share

Low fares: low costs enable the carrier to offer very low fares with a highly simplified pricing structure.

No frills: inflight service is virtually non-existent, so the number of flight attendants on board corresponds to the minimum regulatory requirement. The relationship between the airline and the customer is limited to the contract of carriage. Each service is billed to the customer.

Faced with competition from the low cost model, traditional European airlines such as Air France KLM have also reworked its medium-haul product to compete with low cost carriers, reduce its operating costs and meet the changing needs of passengers. The company decided to cooperate with the well-known low cost company transavia.com with a participation of 60%.

## 2 Company Background

Cash-Air was set up in early 2004 by Virginia Routh, a Franco-American businesswoman who graduated from ECP in 1986. Cash-Air placed a firm order for 10 Airbus A320-200 aircraft in March 2004 with plans to commence operations in mid-2005. Cash-Air took delivery of its first Airbus A320-200 aircraft in June 2005, nearly one year after placing the order, and commenced

operations in 6 July 2005 with a service from Paris to Nice. By the end of 2005, the company had seven aircraft. Cash-Air was expanding rapidly and therefore making solid profits. It has become a leading Domestic and International European Airline which always offers cheap, affordable, on time flights.



To be competitive with Ryanair and Easyjet, the company incorporates the best hardware, software, interface design & personnel from around the world. The Cash-Air team uses all of these resources to design processes and rules that are safe and simple which in turn ensures a uniquely smooth customer experience at fares that are always affordable.

### **3 Airline Planning Process**

Cash-Air must perfectly master its planning process which is typically divided into several stages mainly aircraft fleeting, aircraft routing and crew pairing. In the process, the output of fleeting becomes the input to aircraft maintenance routing, the output of aircraft routing (also called Tail Assignment) is in turn the part of the input to crew pairing.

#### **3.1 Timetable Construction**

The first step in the airline planning process is to model the market behavior to construct a timetable. The timetable contains information about the time and place of the arrival and departure of each flight.



Figure 2: Route Map of Cash-Air in Europe

### 3.2 Fleet Assignment

The next step is to decide which aircraft type (fleet) is going to operate each flight in the constructed timetable. This makes it possible to actually operate the timetable and insures there are no imbalances between the flights. It also allows sufficient turn time.

### 3.3 Crew Pairing

Each crew member must get an individual work schedule (called a roster) well in advance of the day of operation.

### 3.4 Crew Rostering

The crew planning step is very complex when it comes to rules and regulations. There are a lot of safety and union rules that must be fulfilled by each crew roster, such as restricting the working time and ensuring enough rest time.

### 3.5 Aircraft routing

Routes for the individual aircraft are constructed, ensuring that each aircraft gets enough maintenance, and that all operational constraints are fulfilled.

## 4 Assigning Aircraft to Flights

Christian Leroy, the head of aircraft routing department in Cash-Air airline company, thinks it is about time to develop a more compact model for the problem of Tail Assignment considered until this day as the most heavy and priced task in term of resources consumption. Given a flight timetable, the aircraft assignment problem is to determine which aircraft should fly to which Origin-Destination pairs (flights). The objective is to minimize affecting costs while satisfying constraints related to the flights operations. Christian believes that it should be possible to work and investigate in order to define a more compact model to elicit this problem. To study the feasibility of his proposition, Christian organizes a meeting and invites Patrick Lefevre, Optimization Solutions Architect and PhD on Operation Research from Ecole Centrale Paris. Since he has a strong confidence in his advanced skills and experience. Thomas Wang who recently graduated from Ecole Centrale Paris is also invited to the meeting. He has been working in this company for one year. In collaboration with Patrick, Thomas acquired good skills and reputation, working successfully on different projects. Christian thinks that it is high time to get them involved and start the reorganization study of the aircraft routing.

Christian: *‘You know after the interesting study made by all the directors of our company, it is obviously clear that the most time consuming and costly task of the optimization process is Tail Assignment process. It is certainly our job and for a long time I have been wondering if we have done our best and I am not sure that the classical model that we are using to assign aircraft to flights is the best way to deal with the problem of tail assignment. We should be able to improve our algorithms*

*and find another innovative and compact model as soon as possible. I believe we should make some changes right to way. '*

Patrick: *'It's true that's what I am thinking of since we got the last meeting report. We need to rethink our existing model and try to define a new mathematical model.'*

Thomas: *'Yes, I am familiar with such models. It is similar to a problem we have encountered in our M.S. program. We need to use a decision variable with three indexes. Let our binary decision variables  $x_{ijt}$  be 1 if flight  $j$  should follow immediately after flight  $i$  and these flights should be operated by aircraft  $t$ , and 0 otherwise. I can send to you interesting papers resuming the advances in this field in terms of mathematical models (see bibliography). For constraints, I have no idea about the problematic because I am not working on it.'*

Patrick: *'Well, this is what we are using right now, we considered a linear integer problem with same binary decision variable  $x_{ijt}$ . Considering that we have a limited version of the current Optimization Software in terms of integer variables we can't create the schedule of the whole month and we resort every time to the heuristic local search. We need to reduce the number of variables and limit our binary variable to  $x_{ij}$  with just keeping the index  $i$  to the flight  $i$  and index  $j$  for the aircraft  $j$ . Do you think it will be possible ?'*

Christian: *'Excellent idea! that's my opinion too. Last night I was considering that option. I tried to bring modifications to our model but I found out that we need to rethink about modelling our constraints, they are no longer available.'*

Thomas: *'Okay! Can you tell me more about the required constraints ?'*

Patrick: *'There are several constraints that should be respected and they are classified in three:*

**C1:** *Minimum allowed time (called also turn time) between the arrival and departure of an aircraft, needed for activities of cleaning, refuelling, changing crew and passengers.*

*C2: Aircraft must depart from an initial position, that means the first flight operated by an aircraft must have as a departure airport the initial position of the aircraft*

*C3: Flights are with no ferry that means that we don't allow a flight without passengers or even commercial flights. For one aircraft, the arrival airport for a flight should be the same as the departure flight operated by the same aircraft.'*

Christian: *'Great! You also need to make sure that every flight must have one aircraft assigned to it. As you have all the elements about the problematic, why don't you work on it a bit. Let us schedule another meeting, would three weeks be enough to develop this idea ?'*

Patrick: *'We will do our best.'*

After the meeting Patrick and Thomas were a bit worried because restructuring the mathematical model using the new decision variables is not an easy task to be done over night. However, as Christian always expects quick and efficient results, they should start immediately investigating the issue. They first collected some documents and papers related to aircraft routing and studied the current situation (see bibliography). To help them testing their model, Christian sent a relatively simple set data (see tables below) representing a sample of a timetable for three days composed of 30 flights and they were asked to develop a schedule of three days using every day just 3 aircraft. They quickly realized that, even being "compact", the model would have too many binary variables and they remembered that large integer models would cause computational difficulties.

Day 1				
Id	Departure Airport	Arrival Airport	Departure Time	Arrival Time
F1	PAR	BER	07h25	09h10
F2	BER	NCE	09h45	11h50
F3	BER	NCE	10h15	12h20
F4	NCE	PAR	12h25	13h50
F5	PAR	BCN	14h25	16h10
F6	BER	PAR	08h10	09h55
F7	PAR	BCN	11h00	12h30
F8	BCN	NCE	13h15	14h30
F9	NCE	PAR	13h05	14h30
F10	PAR	BER	15h15	17h00

Day 2				
Id	Departure Airport	Arrival Airport	Departure Time	Arrival Time
F11	BCN	PAR	06h15	07h10
F12	BER	PAR	09h15	11h00
F13	PAR	MAD	07h45	09h55
F14	MAD	PAR	10h35	12h45
F15	PAR	BCN	11h30	13h15
F16	BCN	PAR	13h45	15h00
F17	PAR	BER	15h45	17h30
F18	NCE	PAR	10h15	11h40
F19	PAR	BCN	12h15	14h10
F20	BCN	BER	14h40	17h20



Day 3				
Id	Departure Airport	Arrival Airport	Departure Time	Arrival Time
F21	BER	PAR	07h45	09h30
F22	PAR	NCE	10h00	11h25
F23	PAR	LYN	09h55	10h55
F24	NCE	PAR	12h00	13h25
F25	BER	LYN	08h05	09h55
F26	LYN	BER	11h40	13h30
F27	BER	PAR	14h00	15h45
F28	LYN	BER	10h35	12h25
F29	BER	BCN	13h00	15h40
F30	BCN	PAR	16h10	18h05

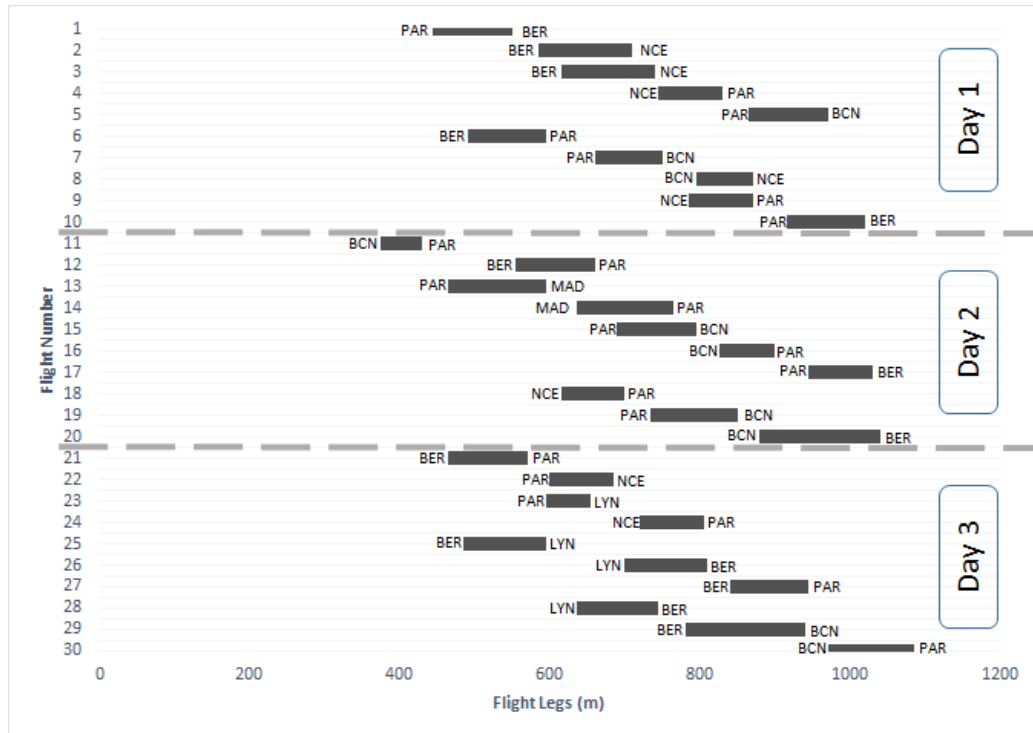


Figure 3: Gantt chart for flights

Id	Initial position of the aircraft
A1	PAR
A2	BER
A3	BER

Table 1: Cost of assigning flight  $i$  to aircraft  $j$

ID	Flights	Aircraft1	Aircraft2	Aircraft3
F1	PAR-BER	4505	4505	4505
F2	BER-NCE	5074	5079	5097
F3	BER-NCE	5074	5079	5097
F4	NCE-PAR	4270	4265	4302
F5	PAR-BCN	4471	4469	4501
F6	BER-PAR	4505	4505	4505
F7	PAR-BCN	4471	4469	4501
F8	BCN-NCE	3755	3744	3794
F9	NCE-PAR	4270	4265	4302
F10	PAR-BER	4505	4505	4505
F11	BCN-PAR	4471	4469	4501
F12	BER-PAR	4505	4505	4505
F13	PAR-MAD	4919	4922	4943
F14	MAD-PAR	4919	4922	4943
F15	PAR-BCN	4471	4469	4501
F16	BCN-PAR	4471	4469	4501
F17	PAR-BER	4505	4505	4505
F18	NCE-PAR	4270	4265	4302
F19	PAR-BCN	4471	4469	4501
F20	BCN-BER	6071	6088	6082
F21	BER-PAR	4505	4505	4505

ID	Flights	Aircraft1	Aircraft2	Aircraft3
F22	PAR-NCE	4270	4265	4302
F23	PAR-LYN	3376	3361	3419
F24	NCE-PAR	4270	4265	4302
F25	BER-LYN	5045	5050	5068
F26	LYN-BER	5045	5050	5068
F27	BER-PAR	4505	4505	4505
F28	LYN-BER	5045	5050	5068
F29	BER-BCN	6071	6088	6082
F30	BCN-PAR	4471	4469	4501

Patrick and Thomas discussed how to decompose the problem and find the right schedule. They were able to use a genius method allowing to produce a 3-day Schedule easily. After testing their proposition using Cplex 12.4 and OPL, they were pleasantly surprised to see how much they accomplished in three weeks. They explained the procedure and show the results to Christian.

Christian: *‘Smashing for a start. It gives me some idea about the possible extensions. I’m really proud of you guys. Before starting to implement the new mathematical model. I am curious to know if our model is robust or not. You have to test it on real data from our client (see [http://www.lgi.ecp.fr/mousseau/Cours/AD-AR/pmwiki-2.1.27/pmwiki.php / Main /Projet2014](http://www.lgi.ecp.fr/mousseau/Cours/AD-AR/pmwiki-2.1.27/pmwiki.php/Main/Projet2014)). You have to find a bright idea to represent results in a simple way so that we can easily see which is the route of each aircraft ’.*

Thomas: *‘OK. The report will be ready next week’.*

Christian: *‘Thank you guys! Thanks to you. We have not only improved our optimization model but also built a compact one !’.*

After testing the new formulation for few months, Christian was surprised by the efficiency of the model. He is very proud of the reorganization project of the aircraft routing. After several years working in aircraft routing department, Christian knows that even if the aircraft routing model is compact and efficient, several unexpected perturbing events can occur during the execution of the optimal flight plan. These perturbations lead to a situation in which the initial optimal flight plan is not anymore optimal, and even sometimes unfeasible. Therefore, it should be able to quickly adapt or repair the plan in operation to take into account events making it anymore or in partly inapplicable.

Christian checked the historic of carried out repair interventions and identified the most frequent scenarios. He called Patrick and Thomas to study the collected events and prepare in advance repair models to handle them.

Christian: *‘Hello guys, we are here today to study repair models for Tail Assignment problem. As you know formulating a compact mathematical model for tail assignment is not enough because in the domain of airlines planning various unexpected events can occur making the current plan not feasible. I want to anticipate the appearance of such scenarios and start the study of repair models’.*

Patrick: *‘I am with you. For the personal experience you have gained in this field, Have you any idea about the most frequent perturbing events occurred ?’*

Christian: *‘Of course, I have already checked all repair interventions passed and I identified three scenarios :*

- 1. Scenario 1: One or more flights are delayed because of various unexpected maintenance actions at specific given time horizon.*
- 2. Scenario 2: Prohibition to an aircraft landing in a particular airport during a specific period for unavailability of adequate maintenance materials for this airplane.*

3. *Scenario 3: Cancellation of one or more flights due to the closure of airports for weather conditions.*

’.

Thomas: ‘ *I have noticed all these scenarios but I have a question. While repairing a flight plan, we usually want to avoid to change too much the flight plan : the less we change the initial plan, the better it is in terms of operating the "repaired" plan. So we will change our objective and we will no more have a single objective but at least two* ’.

Christian: ‘ *Good remark Thomas! Effectively, we have henceforth two objectives to minimize which are :*

1. *The assignment costs same as the tail assignment model.*
2. *The number of impacted airplanes. Generally we don't want to change a lot the road of airplanes. We want to maintain an assignment close enough to the initial planning.*

*You have to notice also that all the assignments that are operated before the date of the unexpected event should not change* ’.

Patrick: ‘ *We should first model all these scenarios and criteria as additional constraints and decision variables* ’.

After presenting all formulations to Christian. Patrick and Thomas discussed how to obtain different solutions. They checked their books and noted that a solution is efficient if there is no other feasible solution that is at least as good in every objective and strictly better in at least one objective. They recalled from their graduate studies that they had to make sure to obtain efficient solutions. They decided to use a method called "  $\varepsilon$ -constraints" that would put an upper bound ( $\varepsilon$ ) on the impacted airplanes objective and minimize the assignment cost. By changing the upper bound value, they would obtain several solutions.

They obtained several efficient solutions and were pleasantly surprised to see how much they accomplished in few days. They explained what they did and showed their results to Christian.

Christian: *‘These look good for a start. It gives me some idea about the possible solutions. Before making a decision, I would like to see more solutions though. Would it make sense to obtain all efficient solutions? Would there be too many?’*

Thomas: *‘I think there could be too many in general. There may not be too many for this problem, though. We can try and see’.*

Christian: *‘I will also welcome any comments on the solutions that would facilitate the decision. Perhaps you can plot the solutions you have obtained’.*

Patrick: *‘We intended to prepare a plot but ran out of time. We will do it next time. Hopefully, we will not have time constraints as we have now!’*

Christian: *‘I understand. I will not give you a deadline this time. You let me know when you are ready’.*

Leaving the meeting, Patrick and Thomas were not sure if they were better off with or without the deadline. They knew that they would work even harder now. It occurred to them that Christian also knew it.

## References

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