

---

# Speed Control of an Electrical Drive

*Vitesse - Track A - Trinome 32*

---

JIN DING  
CARLOS BENÍTEZ ROSETY  
MARCOS ICHIRO SASAKI

TP Report  
Control Theory

19/09/2025 - 22/10/2025

### Abstract

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

# Contents

<b>1</b>	<b>Identification of the System Parameters</b>	<b>3</b>
1.1	Introduction . . . . .	3
1.2	Armature Resistance and Inductance: $R$ and $L$ . . . . .	4
1.3	Voltage Constant and Torque Constant: $K_e = K_c$ . . . . .	5
1.4	Coulomb Friction Torque and Viscous Friction Coefficient: $C_s$ and $f$ . . . . .	6
1.5	Moment of Inertia of the System: $J$ . . . . .	7
<b>2</b>	<b>Controller Design and Validation by Simulations</b>	<b>9</b>
2.1	Current Regulation . . . . .	9
2.1.1	State-Space Model of the Augmented System . . . . .	9
2.1.2	Characteristic Polynomial and Desired Poles . . . . .	10
2.1.3	Control Law by State Feedback . . . . .	11
2.1.4	Validation with the Synthesis Model . . . . .	11
2.2	Speed Regulation by a Controller with Integrator . . . . .	12
2.2.1	State-Space Model of the Augmented System . . . . .	12
2.2.2	LQ Control Design . . . . .	13
2.3	2.2.3 Simulation and Performance Evaluation . . . . .	14
2.3.1	Stability Margin Analysis . . . . .	14
2.4	Design of an Observer Capable of Reconstructing the Resistant Torque . . . . .	16
2.4.1	Augmented System and Observability . . . . .	16
2.4.2	Selection of Observer Dynamics . . . . .	16
2.4.3	Determination of the Observer Gain . . . . .	17
2.4.4	Determination of the Proportional Gain $K_{\Omega p}$ . . . . .	17
2.4.5	Closed-Loop Response of the Proportional Speed Controller . . . . .	18
2.4.6	Steady-State Error and Torque Compensation . . . . .	19
<b>3</b>	<b>Experimental Validation of the Controllers</b>	<b>20</b>
3.1	Current Regulation Experiment . . . . .	20
3.2	Speed Regulation Experiment . . . . .	21
3.3	Observer Implementation and Validation . . . . .	25
<b>4</b>	<b>Conclusion</b>	<b>28</b>
<b>5</b>	<b>Additional Examples</b>	<b>29</b>
<b>6</b>	<b>Useful Commands</b>	<b>29</b>

# 1 Identification of the System Parameters

## 1.1 Introduction

The objective of this practical work is to design and implement control strategies for the speed regulation of a direct current (DC) electrical drive system. Such systems are widely used in transportation, robotics, and industrial automation, where precise speed control ensures efficiency and reliability.

The study combines modeling, simulation, and experimentation. First, the electrical and mechanical parameters of the DC motor are identified using a platform composed of two coupled DC machines, sensors, and a DC/DC converter. Then, controllers are designed and validated in Matlab/Simulink an inner current loop by pole placement and an outer speed loop using a linear quadratic (LQ) approach. Finally, the controllers are tested on the physical setup to compare real and simulated performances.

Part 1 focuses on parameter identification, determining key characteristics such as resistance, inductance, torque and voltage constants, friction coefficients, and inertia. These parameters provide the foundation for accurate modeling and effective control design in the subsequent stages.

The equations governing the DC motor system are as follows:

$$U_m(t) = R \cdot I(t) + L \frac{dI(t)}{dt} + E(t) \quad (1)$$

$$E(t) = K_e \cdot \Omega(t) \quad (2)$$

$$C_e(t) = K_c \cdot I(t) \quad (3)$$

$$C_e(t) = C_r(t) + C_s + f \cdot \Omega(t) + J \frac{d\Omega(t)}{dt} \quad (4)$$

where:

- $R$ : armature resistance ( $\Omega$ )
- $L$ : armature inductance (H)
- $E$ : counter-electromotive force of the motor (V)
- $\Omega$ : rotational speed of the rotor ( $\text{rad} \cdot \text{s}^{-1}$ )
- $C_e$ : electromagnetic torque of the motor ( $\text{N} \cdot \text{m}$ )
- $K_e$ : voltage constant ( $\text{V} \cdot \text{s}$ )
- $K_c$ : torque constant ( $\text{N} \cdot \text{m} \cdot \text{A}^{-1}$ )
- $C_r$ : resistant torque ( $\text{N} \cdot \text{m}$ )
- $C_s$ : Coulomb friction torque ( $\text{N} \cdot \text{m}$ )
- $f$ : viscous friction coefficient ( $\text{N} \cdot \text{m} \cdot \text{s}$ )
- $J$ : moment of inertia of the shafts of the two machines ( $\text{kg} \cdot \text{m}^2$ )

## 1.2 Armature Resistance and Inductance: $R$ and $L$

The armature resistance ( $R$ ) and inductance ( $L$ ) characterize the electrical behavior of the DC motors armature circuit. The resistance  $R$  represents the opposition to the flow of current in the winding, while the inductance  $L$  reflects the ability of the winding to store energy in its magnetic field. These parameters directly influence the transient response of the current when a voltage is applied to the motor.

To determine  $R$  and  $L$ , the rotor is kept stationary so that no back electromotive force (EMF) is generated. A step voltage  $u_c(t)$  of 2.5 V is applied through the DC/DC converter, and the corresponding current response is observed in the Simulink oscilloscope. The experiment can be modeled as a simple first-order RL circuit governed by:

$$U_m(t) = RI(t) + L \frac{dI(t)}{dt}.$$

At steady state, the current reaches a constant value  $I_\infty$ , allowing the armature resistance to be calculated as:

$$R = \frac{U_m}{I_\infty}.$$

The inductance  $L$  is obtained from the electrical time constant  $\tau_e$  of the current response, given by:

$$\tau_e = \frac{L}{R} \implies L = R \cdot \tau_e.$$

The measured quantities during the experiment are summarized in Table 1.

Table 1: Measured and calculated values for  $R$  and  $L$

Quantity	Symbol	Measured Value	Unit
Applied voltage	$V$	6.5	V
Steady-state current	$I$	18.55	A
Electrical time constant	$\tau_e$	0.025	s
Calculated resistance	$R$	0.3504	$\Omega$
Calculated inductance	$L$	0.00876	H

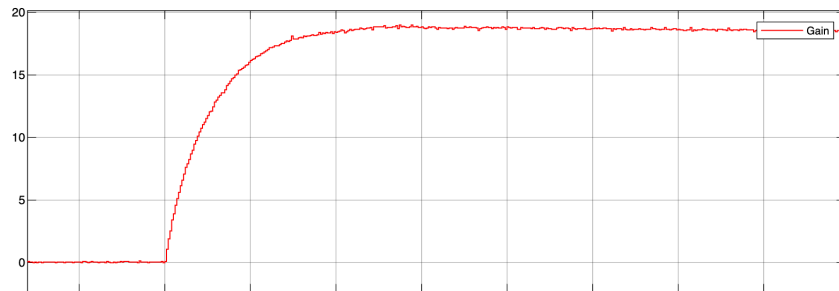


Figure 2: Current response  $I(t)$  to step voltage input, showing the transient behavior and steady-state value.

Thus, from the steady-state and transient measurements, the armature resistance and inductance of the DC machine were found to be:

$$R = 0.3504 \, \Omega, \quad L = 0.00876 \, \text{H}.$$

These values will be used in subsequent stages for the modeling and control design of the electrical drive system.

### 1.3 Voltage Constant and Torque Constant: $K_e = K_c$

In steady state, the armature voltage satisfies

$$U_m = E + R I = K_e \Omega + R I,$$

so the (backEMF) voltage constant  $K_e$  (equal to the torque constant  $K_c$  in SI units) can be obtained directly from averaged steady-state measurements as

$$K_e = \frac{\bar{U}_m - R \bar{I}}{\bar{\Omega}} \quad (\text{with } \bar{\Omega} \text{ in rad/s}).$$

With the rotor turning (stator energized), we applied a constant control  $u_c$  at the converter input and increased it in steps  $u_c = 0, 2, 3, 4 \, \text{V}$ , waiting for steady state each time. At steady state we recorded: (i) the average armature voltage  $U_m$  (voltmeter), (ii) the current sensor output  $V_I$  (converted to  $I$  via  $I = V_I/K_{\text{cap}I}$  with  $K_{\text{cap}I} = 0.1 \, \text{V/A}$ ), and (iii) the rotational speed in rpm (converted to  $\Omega$  via  $\Omega = \text{rpm} \cdot 2\pi/60$ ).

Table 2: Steady-state measurements and converted quantities.

$u_c$ (V)	$V_I$ (V)	rpm	$\Omega$ (rad/s)	$I = V_I/0.1$ (A)	$U_m$ (V)
0	0.0374	3.0184	0.316086	0.374	0
2	0.9976	57.1736	5.987205	9.976	5
3	1.1536	184.3761	19.307820	11.536	16
4	1.2517	317.6292	33.262052	12.517	27

Using the previously identified armature resistance

$$R = 0.350404313 \, \Omega,$$

and the averaged values from our measurements (already converted to SI units):

$$\bar{U}_m = 12 \, \text{V}, \quad \bar{I} = 0.860075 \, \text{A}, \quad \bar{\Omega} = 14.7182909 \, \text{rad/s} \quad (\text{from } 140.549325 \, \text{rpm}),$$

we obtain

$$K_e = \frac{12 - 0.350404313 \times 0.860075}{14.7182909} = 0.794835901 \, \text{V} \cdot \text{s/rad}.$$

Hence, in SI units,

$$K_e = K_c \approx 0.794835901 \, \text{V} \cdot \text{s/rad}.$$

## 1.4 Coulomb Friction Torque and Viscous Friction Coefficient: $C_s$ and $f$

In steady state (constant speed,  $\dot{\Omega} = 0$ ) and with zero external load torque ( $C_r = 0$ ), the DC motor torque balance reduces to

$$K_c I = f \Omega + C_s,$$

where  $K_c$  is the torque constant (equal to the backEMF constant in SI),  $f$  is the viscous friction coefficient, and  $C_s$  is the Coulomb (dry) friction torque.

Hence, plotting the measured current  $I$  versus angular speed  $\Omega$  and fitting the affine model

$$I = a \Omega + b$$

yields  $f = K_c a$  and  $C_s = K_c b$ .

Table 3: Steady-state points used for  $I\Omega$  identification.

$u_c$ (V)	$U_m$ (V)	rpm	$\Omega$ (rad/s)	$I$ (A)
2	5	57.1736	5.987205	0.9976
3	16	184.3761	19.307820	1.1536
4	27	317.6292	33.262052	1.2517
6	49	593.3711	62.137676	1.6074

A linear regression of  $I$  vs.  $\Omega$  gives (see plotted trendline):

$$I = \underbrace{0.0107}_a \Omega + \underbrace{0.9293}_b \text{ [A]},$$

with  $\Omega$  in rad/s.

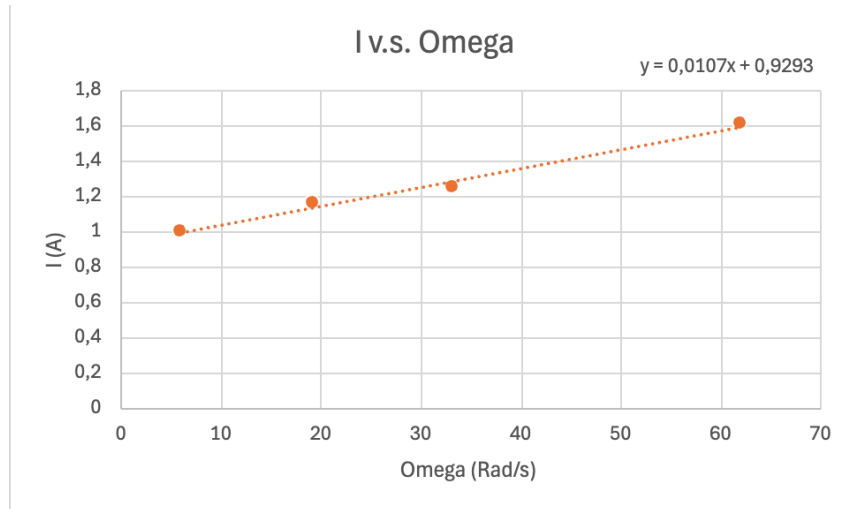


Figure 3: Linear regression of steady-state current  $I$  versus angular speed  $\Omega$ .

Using the previously identified  $K_c = K_e = 0.794835901 \text{ V s/rad} = \text{N m/A}$ , we obtain

$$f = K_c a = 0.794835901 \times 0.0107 = \mathbf{0.008504744 \text{ N m s}},$$

$$C_s = K_c b = 0.794835901 \times 0.9293 = \mathbf{0.738641003 \text{ N m.}}$$

These friction parameters will be used in the motor model and subsequent controller design/validation.

## 1.5 Moment of Inertia of the System: $J$

When the control signal is removed, the motor supply is cut and the rotor slows down under the effect of friction only. The dynamic equation governing the deceleration is:

$$J \dot{\Omega} + f \Omega + C_s = 0,$$

which can be written as:

$$\dot{\Omega} + \frac{1}{\tau} \Omega = -\frac{C_s}{J}, \quad \text{with } \tau = \frac{J}{f}.$$

The solution of this first-order differential equation is:

$$\Omega(t) = \left( \Omega_0 + \frac{C_s}{f} \right) e^{-t/\tau} - \frac{C_s}{f},$$

where  $\Omega_0$  is the speed at the moment the supply is interrupted.

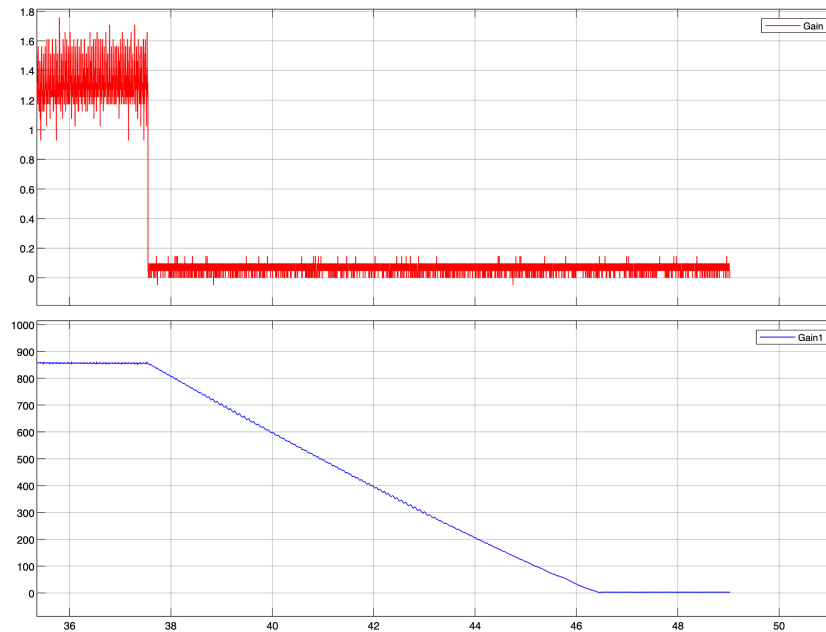


Figure 4: Angular velocity  $\Omega(t)$  decay (blue curve) when the control signal is removed, showing exponential deceleration due to friction.

From the experimental speed curve (see Figure 4), the motor exhibits an exponential decay in velocity until it comes to rest. By determining the time constant  $\tau$  from the slope of this decay, the moment of inertia is obtained as:

$$J = \tau f.$$



Using the identified viscous friction coefficient and the time constant estimated from the measured response, we find:

$$J = 0.1213266 \text{ kg} \cdot \text{m}^2.$$

This value will be used in the subsequent modeling and simulation stages to complete the characterization of the DC motors mechanical dynamics.

## 2 Controller Design and Validation by Simulations

### 2.1 Current Regulation

#### 2.1.1 State-Space Model of the Augmented System

The armature circuit of the DC motor is governed by:

$$U_m(t) = R I(t) + L \frac{dI(t)}{dt} + K_e \Omega(t)$$

and the converter is modeled as:

$$U_m(t) = k_h u_c(t), \quad k_h = 10$$

Hence, the current dynamics can be written as:

$$\frac{dI(t)}{dt} = -\frac{R}{L} I(t) + \frac{k_h}{L} u_c(t) - \frac{K_e}{L} \Omega(t)$$

To ensure zero steady-state error, an integral action is introduced through the integral of the current error:

$$x_{\varepsilon I}(t) = \int_0^t (V_{I^*}(\tau) - V_I(\tau)) d\tau$$

where the current sensor gives:

$$V_I = K_{\text{conv},I} I(t), \quad K_{\text{conv},I} = 0.1 \text{ V/A}$$

Thus:

$$\frac{dx_{\varepsilon I}(t)}{dt} = V_{I^*}(t) - V_I(t)$$

Define the augmented state vector and the output as:

$$x_I(t) = \begin{bmatrix} I(t) \\ x_{\varepsilon I}(t) \end{bmatrix}, \quad y_I(t) = V_I(t) = K_{\text{conv},I} I(t)$$

The augmented state-space model becomes:

$$\frac{d}{dt} \begin{bmatrix} I \\ x_{\varepsilon I} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{R}{L} & 0 \\ -K_{\text{conv},I} & 0 \end{bmatrix}}_{A_I} \begin{bmatrix} I \\ x_{\varepsilon I} \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{k_h}{L} \\ 0 \end{bmatrix}}_{B_I} u_c(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B'_I} V_{I^*}(t) + \underbrace{\begin{bmatrix} -\frac{K_e}{L} \\ 0 \end{bmatrix}}_{B''_I} \Omega(t)$$

The output equation is given by:

$$y_I(t) = \underbrace{\begin{bmatrix} K_{\text{conv},I} & 0 \end{bmatrix}}_{C_I} x_I(t)$$

**Summary of matrices:**

$$\begin{aligned} A_I &= \begin{bmatrix} -\frac{R}{L} & 0 \\ -K_{\text{conv},I} & 0 \end{bmatrix}, & B_I &= \begin{bmatrix} \frac{k_h}{L} \\ 0 \end{bmatrix}, \\ B'_I &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, & B''_I &= \begin{bmatrix} -\frac{K_e}{L} \\ 0 \end{bmatrix}, \\ C_I &= \begin{bmatrix} K_{\text{conv},I} & 0 \end{bmatrix} \end{aligned}$$

**Controllability:** The controllability matrix is:

$$\mathcal{C} = [B_I \quad A_I B_I] = \begin{bmatrix} \frac{k_h}{L} & -\frac{R k_h}{L^2} \\ 0 & -\frac{K_{\text{conv},I} k_h}{L} \end{bmatrix}$$

The determinant is:

$$\det(\mathcal{C}) = -\frac{K_{\text{conv},I} k_h^2}{L^2} \neq 0$$

Therefore, the augmented system is **controllable**.

### 2.1.2 Characteristic Polynomial and Desired Poles

The design requirements for the current control loop are:

- Settling time:  $T_s = 0.1$  s
- Overshoot:  $M_p < 10\%$

A damping ratio of  $\zeta_I = 0.7$  is chosen to satisfy the overshoot specification. From the second-order system relationship:

$$T_s \approx \frac{4}{\zeta_I \omega_{nI}}$$

the natural frequency is obtained as:

$$\omega_{nI} = \frac{4}{\zeta_I T_s} = \frac{4}{0.7 \times 0.1} \approx 57.1 \text{ rad/s}$$

The desired characteristic polynomial for the current loop is:

$$\lambda^2 + 2\zeta_I \omega_{nI} \lambda + \omega_{nI}^2 = 0$$

$$\lambda^2 + 80 \lambda + 3267 = 0$$

The corresponding desired eigenvalues (poles) are:

$$\lambda_{1,2} = -\zeta_I \omega_{nI} \pm j \omega_{nI} \sqrt{1 - \zeta_I^2} \approx -40.0 \pm j 40.8$$

In MATLAB, these values are obtained using:

```
zeta_I = 0.7; % Damping coefficient of the current loop
omega_nI = 4 / (0.7 * 0.1); % Natural frequency of the current loop
lambda_I = roots([1 2*zeta_I*omega_nI omega_nI^2]);
```

Thus, the desired complex conjugate poles ensuring the specified dynamic response are located at:

$$\lambda_{1,2} = -40.0 \pm j 40.8$$

### 2.1.3 Control Law by State Feedback

The control law for the current loop is defined as:

$$u_c(t) = -K_I x_I(t) = -[K_{I1} \ K_{I2}] \begin{bmatrix} I(t) \\ x_{\varepsilon I}(t) \end{bmatrix}$$

The gain vector  $K_I$  is determined by pole placement in order to obtain the desired dynamic performance, corresponding to the eigenvalues found in Question 2.1.b.

Using MATLAB, the computation is performed as:

```
K_I = place(A_I, B_I, lambda_I); % Find the controller gain for state feedback
e = eig(A_I - B_I*K_I);         % Verify the closed-loop poles
```

This function determines the feedback gain matrix  $K_I = [K_{I1} \ K_{I2}]$  such that the closed-loop system matrix

$$A_{cl} = A_I - B_I K_I$$

has the desired eigenvalues

$$\lambda_{1,2} = -40 \pm j 40.8$$

The resulting control law ensures that the current loop exhibits a fast and well-damped response, in accordance with the specified settling time ( $T_s = 0.1$  s) and overshoot ( $M_p < 10\%$ ).

### 2.1.4 Validation with the Synthesis Model

The closed-loop model of the current loop is validated using the state-space representation obtained in the previous steps. The closed-loop system matrix is given by:

$$A_{bf\_I} = A_I - B_I K_I$$

and the corresponding state-space model is defined in MATLAB as:

```
Abf_I = A_I - B_I*K_I;           % Closed-loop state matrix
Bbf_I = [0; 1];                 % Input matrix for current reference
Cbf_I = [1 0];                 % Output matrix (current measurement)
Dbf_I = 0;                     % Direct transmission term
sys_bf_I = ss(Abf_I, Bbf_I, Cbf_I, Dbf_I); % Closed-loop system

Iref = 20;                     % Current reference (A)
VRef = Iref * k_capI;          % Convert Iref to Vref using current sensor gain
opt = stepDataOptions('StepAmplitude', VRef);
[y_step_I, t_step_I, x_step_I] = step(sys_bf_I, opt); % Step response
u_step_I = -K_I * x_step_I';    % Control voltage (state feedback law)

% Plotting the current and voltage responses
figure(1)
subplot(2,1,1)
plot(t_step_I, y_step_I, 'b', 'LineWidth', 2)
grid on
ylabel('Current (A)')
```

```
title('Response of the current loop')

subplot(2,1,2)
plot(t_step_I, k_h*u_step_I, 'r', 'LineWidth', 2)
grid on
ylabel('Voltage (V)')
xlabel('Time (s)')
```

The resulting simulation plots are shown in Figures 5 and 6.

Figure 5: Step response of the current loop to a setpoint of 20 A.

Figure 6: Control voltage applied by the converter during the transient.

### Analysis of the results:

The step response of the current loop shows that:

- The current reaches its steady-state value of 20 A in approximately **0.1 s**.
- The overshoot is around **10%**, corresponding to the damping ratio  $\zeta_I = 0.7$ .
- The system exhibits a well-damped second-order behavior with no steady-state error.

The control voltage (bottom plot) shows a smooth transient reaching about **89 V** before settling near 6 V once the current reaches 20 A. This confirms that the designed state-feedback controller ensures a fast and stable current loop, satisfying the required specifications:

$$T_s \approx 0.1 \text{ s}, \quad M_p < 10\%.$$

Hence, the performance objectives for the current loop are successfully met.

## 2.2 Speed Regulation by a Controller with Integrator

### 2.2.1 State-Space Model of the Augmented System

Assuming an ideal current loop, the speed dynamics of the DC motor are given by:

$$J \frac{d\Omega(t)}{dt} = K_c V_I^*(t) - C_r(t) - f \Omega(t)$$

The speed measurement is expressed as:

$$V_\Omega(t) = K_{\text{cap},\Omega} \Omega(t)$$

To ensure zero steady-state error and disturbance rejection, an integral of the speed error is introduced:

$$x_{\varepsilon\Omega}(t) = \int_0^t (V_{\Omega^*}(\tau) - V_\Omega(\tau)) d\tau$$

The state vector and output are defined as:

$$x_\Omega(t) = \begin{bmatrix} \Omega(t) \\ x_{\varepsilon\Omega}(t) \end{bmatrix}, \quad y_\Omega(t) = V_\Omega(t) = K_{\text{cap},\Omega} \Omega(t)$$

The augmented state-space model becomes:

$$\frac{d}{dt} \begin{bmatrix} \Omega \\ x_{\varepsilon\Omega} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{f}{J} & 0 \\ -K_{\text{cap},\Omega} & 0 \end{bmatrix}}_{A_\Omega} \begin{bmatrix} \Omega \\ x_{\varepsilon\Omega} \end{bmatrix} + \underbrace{\begin{bmatrix} K_c \\ J \\ 0 \end{bmatrix}}_{B_\Omega} V_I^*(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B'_\Omega} V_{\Omega^*}(t) + \underbrace{\begin{bmatrix} -\frac{1}{J} \\ 0 \end{bmatrix}}_{B''_\Omega} C_r(t)$$

and

$$y_\Omega(t) = \underbrace{\begin{bmatrix} K_{\text{cap},\Omega} & 0 \end{bmatrix}}_{C_\Omega} x_\Omega(t)$$

**Controllability:**

$$C_\Omega = [B_\Omega \quad A_\Omega B_\Omega] = \begin{bmatrix} \frac{K_c}{J} & -\frac{f K_c}{J^2} \\ 0 & -\frac{K_{\text{cap},\Omega} K_c}{J} \end{bmatrix}$$

with

$$\det(C_\Omega) = -\frac{K_{\text{cap},\Omega} K_c^2}{J^2} \neq 0$$

Therefore, the augmented system is **controllable**.

### 2.2.2 LQ Control Design

The control law is defined as:

$$V_I^*(t) = -K_\Omega x_\Omega(t) = -[K_{\Omega 1} \quad K_{\Omega 2}] \begin{bmatrix} \Omega(t) \\ x_{\varepsilon\Omega}(t) \end{bmatrix}$$

The feedback gain  $K_\Omega$  is obtained using the Linear Quadratic (LQ) approach, which minimizes the cost function:

$$J(t) = \int_0^\infty (x_\Omega^T Q x_\Omega + r (V_I^*(t))^2) dt$$

where  $Q = \text{diag}(q_1, q_2)$  and  $r$  are the weighting factors.

A stabilizing solution exists if  $(A_\Omega, B_\Omega)$  is controllable (verified in Question 2.2.a) and if  $Q$  is positive semi-definite and  $r > 0$ .

**Normalization of the state variables:**

- $\Omega_{\text{max}} = 1500 \text{ rpm} = 1500 \times 2\pi/60$
- $V_{W_{\text{max}}} = \Omega_{\text{max}} \times K_{\text{cap},\Omega}$
- $x_{\varepsilon\Omega, \text{max}} = 0.5 V_{W_{\text{max}}} \times 0.2$
- $U_{\text{max}} = 4 \text{ V}$

These limits are used to normalize the state and control variables prior to tuning the weighting matrices.

### MATLAB implementation:

```
%% Question 2.2.b : LQ approach

% Normalization of the state variables
omega_max = 1500*2*pi/60;           % Maximum speed (rad/s)
VW_max = omega_max*K_capW;          % Max speed sensor voltage
q_wmax = 0.5*VW_max*0.2;            % Max integral of the error
U_max = 4;                          % Max control voltage (V)

% Scaling matrices
Qn_lq = [1/omega_max 0; 0 1/q_wmax];
Rn_lq = 1/U_max;

% Base weighting matrices
Q_tilda_lq = [100 0; 0 200];
R_tilda_lq = 1;

% Normalized LQ matrices
Q_lq = Qn_lq * Q_tilda_lq * Qn_lq;
R_lq = Rn_lq * R_tilda_lq * Rn_lq;

% Compute LQ gain
K_W = lqr(A_W, B_W, Q_lq, R_lq);
```

The function `lqr()` returns the optimal feedback gain vector  $K_{\Omega} = [0.417 \quad -69.13]$  that minimizes  $J(t)$  and ensures a stable closed-loop behavior.

### Effect of weighting factors:

- Increasing  $q_1$  emphasizes faster speed tracking but may cause larger control efforts.
- Increasing  $q_2$  improves disturbance rejection (faster integral action).
- Increasing  $r$  reduces the control voltage amplitude but slows down the response.

A suitable balance between  $Q$  and  $r$  results in a settling time close to 2 s and an overshoot below 10%, as required by the design specifications.

## 2.3 2.2.3 Simulation and Performance Evaluation

Simulate the closed-loop system for different setpoints (e.g., 300, 500, and 1200 rpm). Discuss the effect of load torque disturbances and verify that the specifications (settling time = 2 s, overshoot < 10%) are achieved.

### 2.3.1 Stability Margin Analysis

The stability of the designed LQ speed controller can be analyzed by examining the open-loop transfer function of the system defined by:

$$\dot{x}_{\Omega}(t) = A_{\Omega}x_{\Omega}(t) + B_{\Omega}u(t), \quad y(t) = K_{\Omega}x_{\Omega}(t)$$

The open-loop system is implemented in MATLAB and converted to a transfer function form to determine the gain and phase margins using the `margin()` function.

#### MATLAB implementation:

```
% Question 2.2.d : Stability margin analysis

C_bo = K_W;                % Controller acts as output matrix
D_bo = 0;                  % No direct transmission term
sys_bo = ss(A_W, B_W, C_bo, D_bo); % Open-loop state-space model
G_bo = tf(sys_bo);         % Convert to transfer function

figure(3)
margin(G_bo);              % Bode plot with margins
grid on

[GM, PM, Wcg, Wcp] = margin(G_bo);
fprintf('\n--- Stability margins ---\n');
fprintf('Gain margin = %.2f dB\n', 20*log10(GM));
fprintf('Phase margin = %.2f°\n', PM);
fprintf('Gain crossover frequency = %.2f rad/s\n', Wcg);
fprintf('Phase crossover frequency = %.2f rad/s\n', Wcp);
```

#### Obtained results:

- Gain margin:  $\infty$  dB
- Phase margin:  $73.31^\circ$
- Gain crossover frequency: not defined (system always stable)
- Phase crossover frequency: 28.54 rad/s

Figure 7: Bode diagram of the open-loop speed control system.

#### Interpretation of results:

- The infinite gain margin indicates that the system remains stable for all practical gain variations the open-loop gain does not cross 0 dB.
- The phase margin of  $73.3^\circ$  demonstrates a high robustness and well-damped transient response.
- The crossover frequency of 28.5 rad/s defines the approximate bandwidth of the speed loop, ensuring sufficient speed-tracking capability without excessive oscillations.

In conclusion, the LQ-designed controller provides a highly stable and robust speed control system, satisfying the required performance criteria for both stability and dynamic behavior.



## 2.4 Design of an Observer Capable of Reconstructing the Resistant Torque

### 2.4.1 Augmented System and Observability

The augmented state-space model of the DC machine used for the observer design is given by:

$$\frac{dx_a(t)}{dt} = A_a x_a(t) + B_a u_c(t), \quad y_a(t) = C_a x_a(t)$$

where the state vector and output are:

$$x_a = \begin{bmatrix} I \\ \Omega \\ C_r \end{bmatrix}, \quad y_a = V_I$$

The matrices of the augmented system are defined as:

$$A_a = \begin{bmatrix} -\frac{R}{L} & -\frac{K_c}{L} & 0 \\ \frac{K_c}{J} & -\frac{f}{J} & -\frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix}, \quad B_a = \begin{bmatrix} \frac{k_h}{L} \\ 0 \\ 0 \end{bmatrix}, \quad C_a = [K_{\text{cap}I} \quad 0 \quad 0], \quad D_a = \mathbf{0}_{3 \times 1}$$

**MATLAB implementation:**

```
A_a = [-R/L  -Kc/L  0;
        Kc/J  -f/J  -1/J;
        0      0      0];
```

```
B_a = [kh/L; 0; 0];
```

```
C_a = [K_capI 0 0];
```

```
D_a = zeros(3,1);
```

```
QO_a = obsv(A_a, C_a); % Observability matrix
rank(QO_a) % Check the observability rank
```

The observability matrix has full rank ( $\text{rank}(QO_a) = 3$ ), confirming that the system is **observable**.

### 2.4.2 Selection of Observer Dynamics

According to the design requirements, the observer must be approximately three times faster than the speed control loop. The damping ratio and crossover frequency are set as:

$$\zeta_{n\Omega} = 0.7, \quad \omega_{n\Omega} = \frac{3 \times 4}{0.7 \times 0.1}$$

The desired observer eigenvalues are then computed from the characteristic polynomial:

$$\lambda^2 + 2\zeta_{n\Omega}\omega_{n\Omega}\lambda + \omega_{n\Omega}^2 = 0$$

and an additional third eigenvalue is added, chosen to be twice as fast as the previous ones.

**MATLAB implementation:**

```
zeta_nW = 0.7; % Damping coefficient
omega_nW = 3*4/(0.7*0.1); % Crossover frequency
lambda_W = roots([1, 2*zeta_nW*omega_nW, omega_nW^2]);
lambda_W = [lambda_W; sum(lambda_W)]; % Add a faster 3rd pole
lambda_obs = lambda_W; % Observer eigenvalues
```

These eigenvalues ensure that the observer converges rapidly to the actual system states while maintaining numerical stability.

### 2.4.3 Determination of the Observer Gain

The observer gain  $K_{\text{obs}}$  is computed by pole placement, ensuring the desired dynamic performance. Using the transposed system matrices, we find:

$$K_{\text{obs}} = \text{place}(A'_a, C'_a, \lambda_{\text{obs}})'$$

**MATLAB implementation:**

```
K_obs = place(A_a', C_a', lambda_obs); % Observer gain (transpose form)
K_obs = K_obs'; % Convert to column vector
```

The resulting observer equations are:

$$\frac{d\hat{x}_a(t)}{dt} = (A_a - K_{\text{obs}}C_a)\hat{x}_a(t) + B_a u_c(t) + K_{\text{obs}}y_a(t)$$

The corresponding matrices of the observer are:

$$\begin{aligned} A_{\text{obs}} &= A_a - K_{\text{obs}}C_a \\ B_{\text{obs}} &= [B_a \ K_{\text{obs}}] \\ C_{\text{obs}} &= I_{3 \times 3} \\ D_{\text{obs}} &= \mathbf{0}_{3 \times 2} \end{aligned}$$

**MATLAB implementation:**

```
A_obs = A_a - K_obs*C_a;
B_obs = [B_a, K_obs];
C_obs = eye(3);
D_obs = zeros(3,2);
```

### 2.4.4 Determination of the Proportional Gain $K_{\Omega p}$

To design the proportional speed controller, we start from the dynamic equation of the motor:

$$J \frac{d\Omega(t)}{dt} = K_c I^*(t) - C_r(t) - f \Omega(t)$$

Assuming an ideal current loop ( $I^*(t) = K_{\Omega p}(V_{\Omega}^*(t) - V_{\Omega}(t))$ ), we can express the system as:

$$\frac{d\Omega(t)}{dt} = -\frac{f}{J}\Omega(t) + \frac{K_c}{J}I^*(t)$$

Replacing  $I^*(t)$  by its control law gives a first-order model with the proportional gain  $K_{\Omega p}$  as the tuning parameter.

**MATLAB implementation:**

```
% Question 2.4.a
lambda_Wp = -160; % Desired closed-loop eigenvalue
K_Wp = place(-f/J, Kc/(K_capI*J), lambda_Wp); % Proportional gain computation
```

The chosen eigenvalue  $\lambda_{Wp} = -160$  provides a good trade-off between response speed and stability. The resulting proportional gain ensures an adequately damped transient without oscillation.

### 2.4.5 Closed-Loop Response of the Proportional Speed Controller

The closed-loop dynamics of the speed loop without integral action are expressed as:

$$\frac{d\Omega(t)}{dt} = \left( -\frac{f}{J} - \frac{K_{\Omega p} K_{cap, \Omega} K_c}{J K_{cap, I}} \right) \Omega(t) + \frac{K_{\Omega p} K_c}{J K_{cap, I}} V_{\Omega}^*(t)$$

**MATLAB implementation:**

```
% Question 2.4.b : Closed-loop simulation
A_bfW = -f/J - K_Wp * K_capW * Kc / (K_capI * J);
B_bfW = K_Wp * Kc / (K_capI * J);
C_bfW = 1;
D_bfW = 0;
sys_bf_KW = ss(A_bfW, B_bfW, C_bfW, D_bfW);

VWref_amp = K_capW * 300 * pi / 30; % Speed reference amplitude (300 rpm)
opt = stepDataOptions('StepAmplitude', VWref_amp);
[y_step_KW, t_step_KW, x_step_KW] = step(sys_bf_KW, opt);
u_step_KW = K_Wp * (VWref_amp - x_step_KW * K_capW); % Control voltage

figure(4)
subplot(2,1,1)
plot(t_step_KW, y_step_KW * 30/pi, 'b', 'LineWidth', 2)
ylabel('Speed (r/min)')
title('Response of the speed loop without integral action')
grid on

subplot(2,1,2)
plot(t_step_KW, 10*u_step_KW, 'r', 'LineWidth', 2)
ylabel('Current (A)')
xlabel('Time (s)')
grid on
```

Figure 8: Step response of the speed loop with proportional control only.

**Interpretation of results:**

- The speed increases smoothly to the reference of 300 r/min with no overshoot.
- The current decreases exponentially as the speed rises, consistent with motor dynamics.

- The system behaves as a first-order response with a short settling time (less than 0.7 s).

The proportional control ensures a stable response, but a small steady-state error remains due to the lack of integral action.

### 2.4.6 Steady-State Error and Torque Compensation

In steady state, the error can be analyzed from the equilibrium condition of the closed-loop equation:

$$0 = \left( -\frac{f}{J} - \frac{K_{\Omega p} K_{\text{cap}, \Omega} K_c}{J K_{\text{cap}, I}} \right) \Omega_{\text{ss}} + \frac{K_{\Omega p} K_c}{J K_{\text{cap}, I}} V_{\Omega}^*$$

which yields:

$$\Omega_{\text{ss}} = \frac{K_{\Omega p} K_c / K_{\text{cap}, I}}{f + K_{\Omega p} K_{\text{cap}, \Omega} K_c / K_{\text{cap}, I}} V_{\Omega}^*$$

**MATLAB implementation:**

```
% Question 2.4.c : Calculation of the steady-state error
Omega_ss = dcgain(sys_bf_KW) * VWref_amp;    % Steady-state speed (rad/s)
Val_finale = Omega_ss * 30/pi;                % Convert to rpm
erreur = 300 - Val_finale;                    % Steady-state error (rpm)
```

**Interpretation:**

- The proportional control leaves a small steady-state error proportional to the load torque  $C_r$ .
- Increasing  $K_{\Omega p}$  reduces this error, but too high a value may cause instability.
- To eliminate the effect of  $C_r$ , a static compensation term proportional to the estimated torque  $\hat{C}_r$  can be introduced:

$$V_{\Omega}^* = K_{\Omega p}(\Omega^* - \Omega) + \frac{K_c}{K_{\text{cap}, I}} \hat{C}_r$$

**Conclusion:** The proportional controller alone ensures fast dynamics and good damping but cannot eliminate steady-state error. Introducing a torque compensation term or an integral action in the speed loop is required for perfect tracking.

## 3 Experimental Validation of the Controllers

This section reports the laboratory validation of the controllers designed in Part 2. Several Simulink configurations were used successively (current loop, speed loop, and finally the observer). Due to limited laboratory time, **only the final implementation corresponding to the observer (Part 3.3) was saved as a representative Simulink diagram**. However, **all measurement results** obtained at each step were recorded and are presented below to assess the performance of every loop. Earlier configurations for the current and speed control followed the same structure as the final one (inner current loop, outer speed loop, voltage saturation, and acquisition blocks).

*Practical constraints.* For hardware safety, the armature current was limited to 20 A, voltage commands were saturated at  $\pm 9$  V, and step amplitudes were increased gradually to avoid excessive mechanical stress and large current transients.

### 3.1 Current Regulation Experiment

The experimental results of the current control loop are shown in Figure 9. The upper plot displays the armature current (*courant*) and the lower plot shows the corresponding motor speed (*vitesse*). Several step changes were applied to the current reference, increasing successively from approximately 1 A to 2 A, 3 A and finally 10 A. The current follows the reference with a fast transient and negligible overshoot, demonstrating a well-tuned inner current loop. Small fluctuations in the measured signal are mainly due to sensor noise. The motor speed evolves consistently with the current variations, rising up to about 1000 rpm and then decreasing when the current is reduced or reversed. This confirms the expected electromechanical coupling between torque and speed. At the end of the experiment, both current and speed return smoothly to zero when the converter command is disabled.

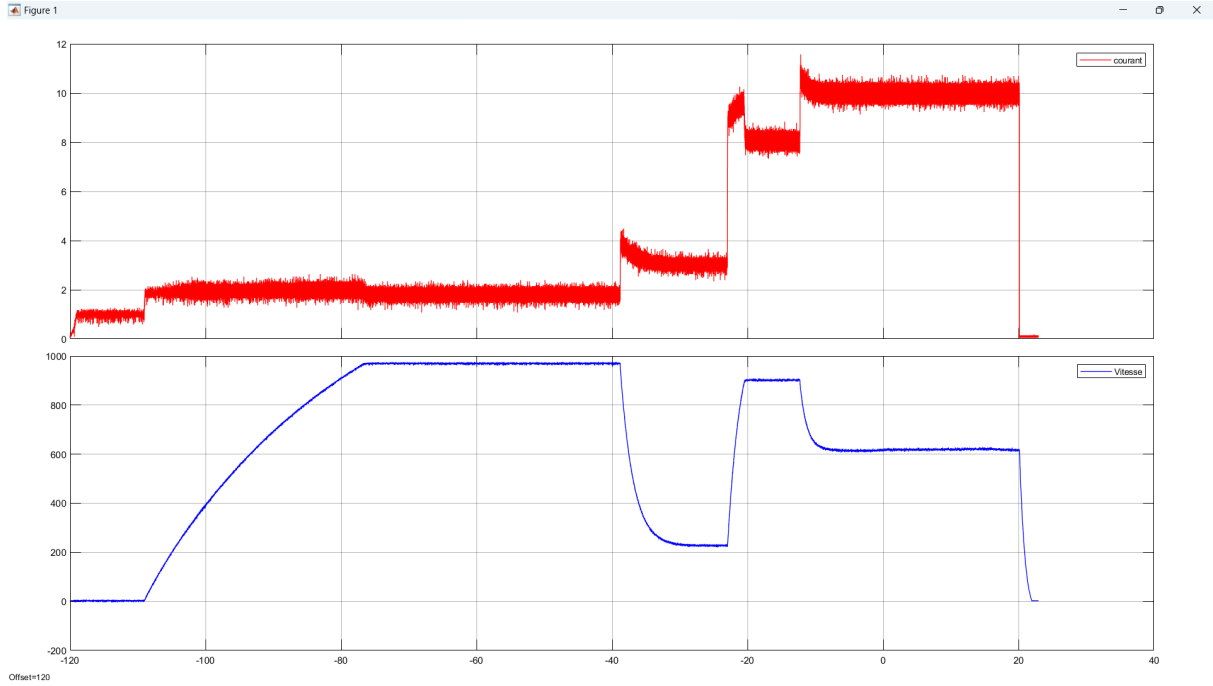


Figure 9: Measured current (top, in A) and speed (bottom, in rpm) during the current regulation test. The current loop shows fast and stable response with negligible overshoot.

### 3.2 Speed Regulation Experiment

The experimental validation of the speed controller was performed following the four test scenarios defined in Question 2.2.c. For each case, the converter operated in “Automatic” mode, and the speed reference  $\Omega^*(t)$  was imposed through the control panel. The measured motor speed (*vitesse*) and armature current (*courant*) were recorded simultaneously.

- **Case 1** :  $C_r(t) = 0$ ,  $\Omega^*(t)$  changes from 0 to 300 rpm at  $t = 1$  s.
- **Case 2** :  $C_r(t) = 0$ ,  $\Omega^*(t)$  changes from 0 to 500 rpm at  $t = 1$  s.
- **Case 3** :  $C_r(t) = 0$ ,  $\Omega^*(t)$  changes from 0 to 1200 rpm at  $t = 1$  s.
- **Case 4** :  $\Omega^*(t)$  changes from 0 to 300 rpm at  $t = 1$  s, while the resistant torque  $C_r(t)$  increases from 0 to 5 Nm at  $t = 10$  s.

The measured responses are summarized in Figures 10–13. In all cases, the speed regulation shows a well-damped transient with limited overshoot and negligible steady-state error. For moderate speed commands (300 rpm and 500 rpm), the current peaks remain within the admissible range (below 20 A), confirming that the current limiter effectively constrains the torque demand. When the reference is increased to 1200 rpm, the system still converges but the transient becomes slower, illustrating the saturation effect of the voltage converter.

When a disturbance torque of 5 Nm is applied at  $t = 10$  s, a brief speed drop is observed, immediately compensated by the integral action of the speed controller. The steady-state speed returns to its nominal value within less than one second, demonstrating the robustness of the closed-loop regulation against load disturbances.

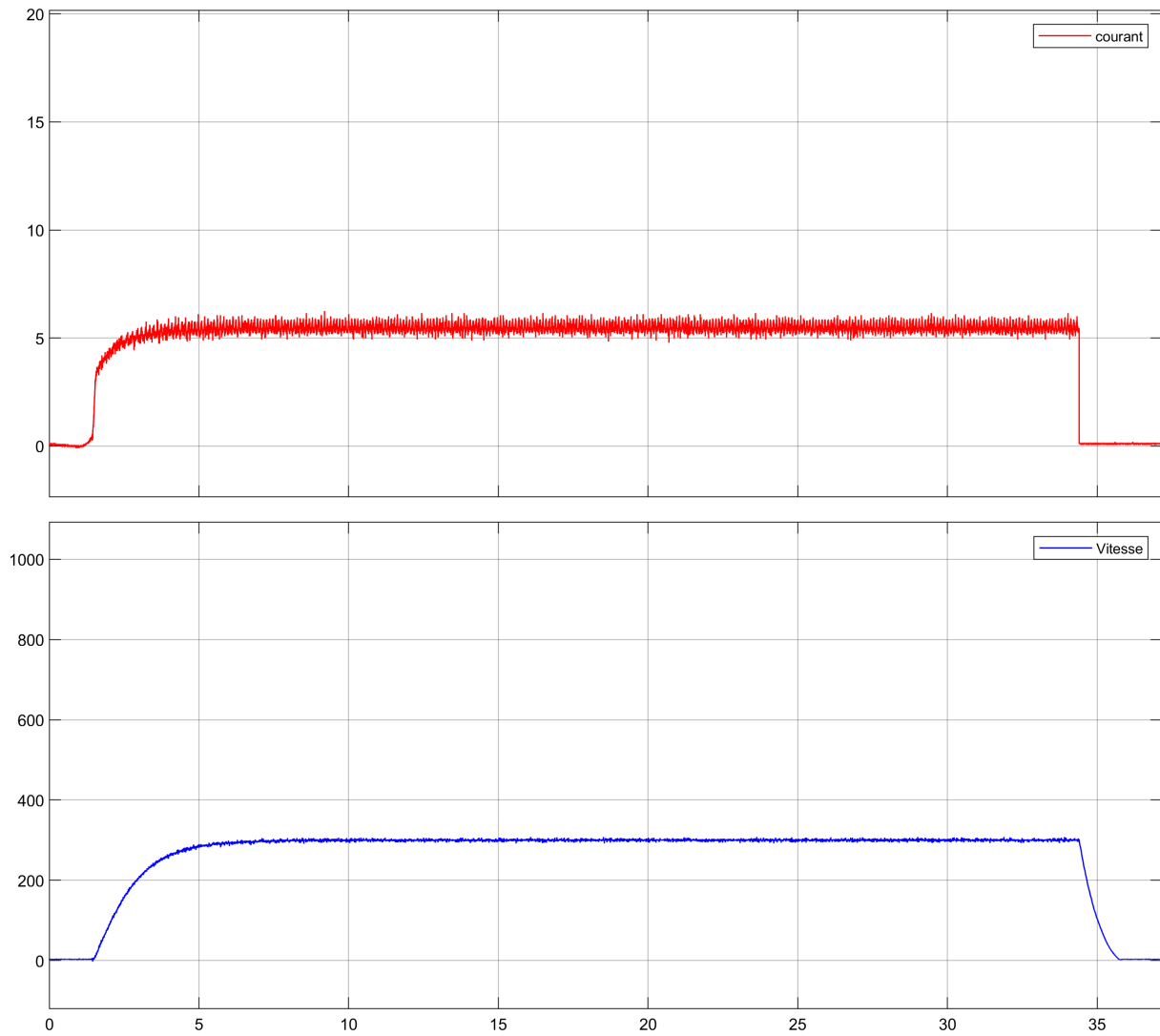


Figure 10: Speed loop — measured response for a  $0 \rightarrow 300$  rpm reference step.

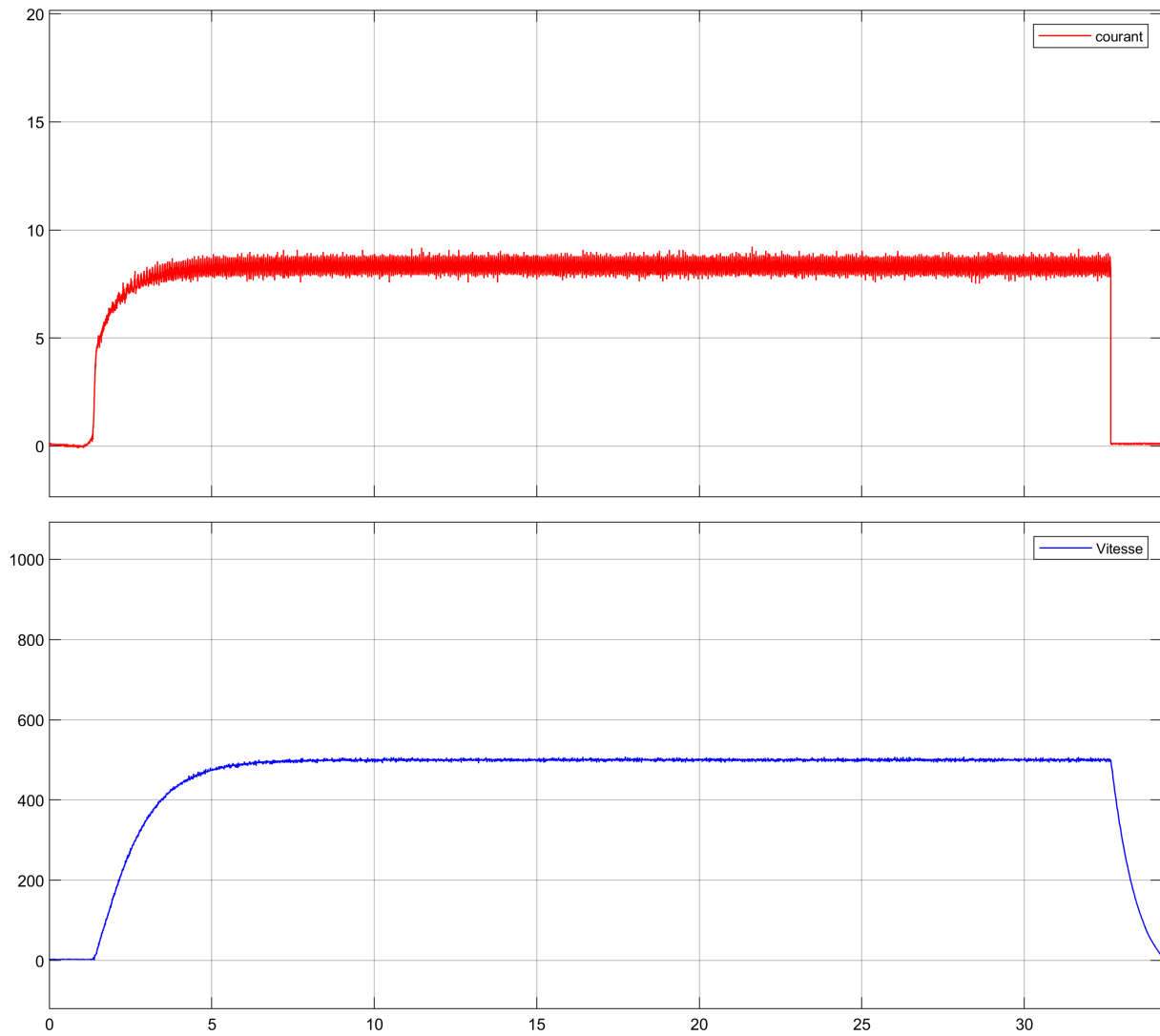


Figure 11: Speed loop — measured response for a  $0 \rightarrow 500$  rpm reference step.



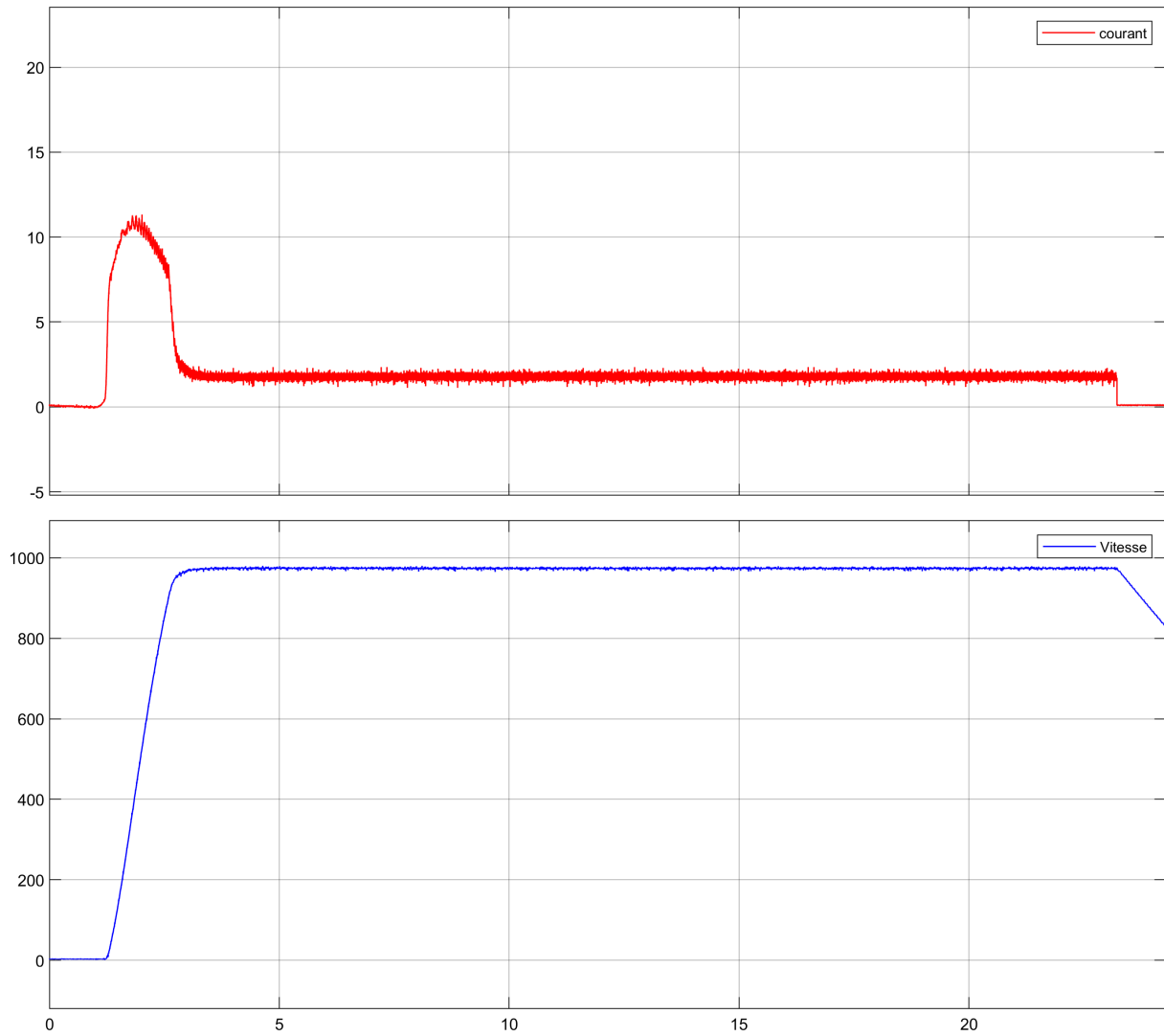


Figure 12: Speed loop — measured response for a  $0 \rightarrow 1200$  rpm reference step, showing converter saturation effects.

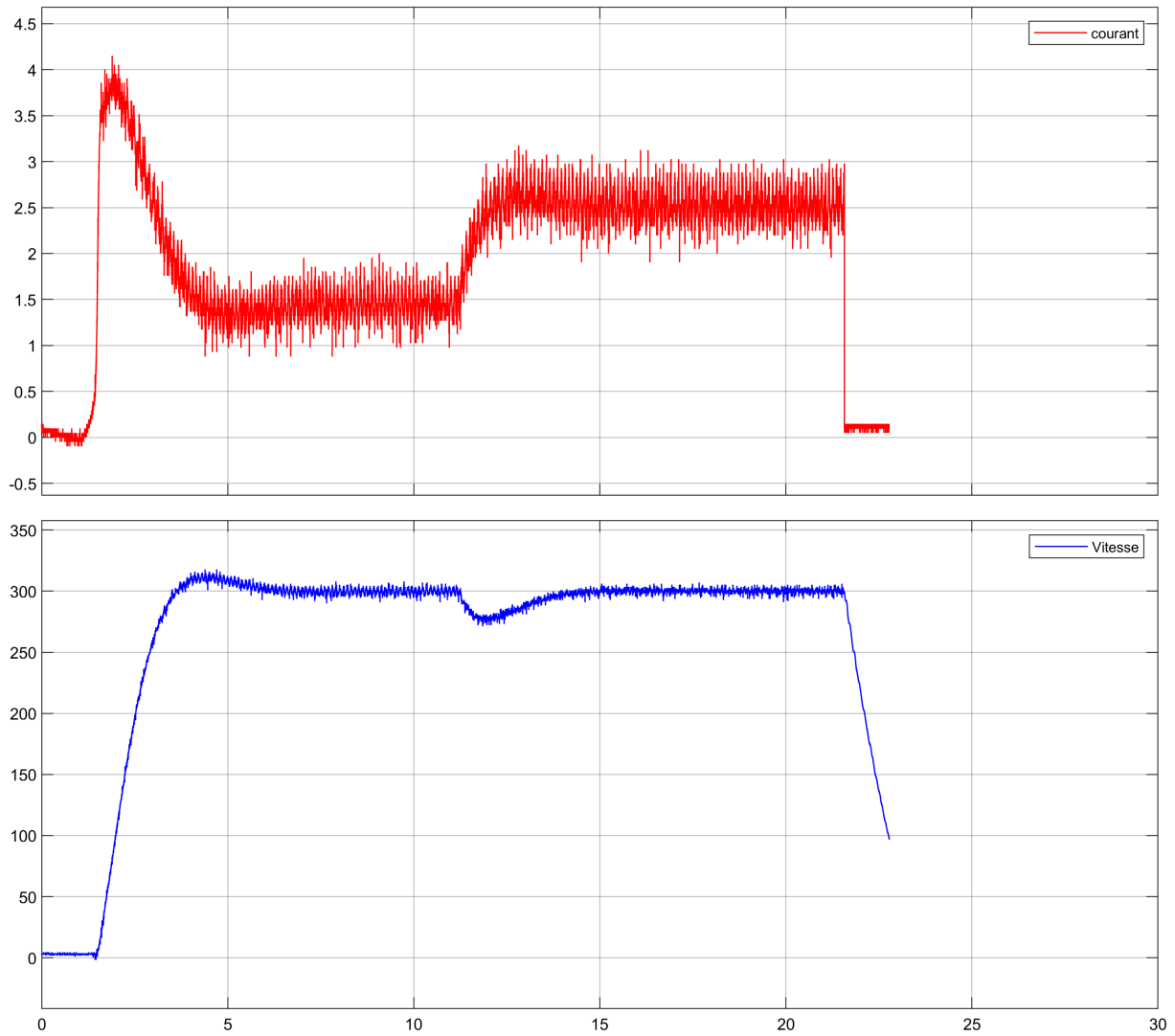


Figure 13: Speed loop — disturbance rejection test: a 5 Nm torque is applied at  $t = 10$  s. The controller compensates the perturbation and restores the nominal speed.

### 3.3 Observer Implementation and Validation

Finally, the observer developed in Section 2.3 was implemented to estimate the states  $[\hat{I}, \hat{\Omega}, \hat{C}_r]$ . The **final Simulink configuration** used for all experiments is shown in Figure 14; it summarizes the complete control architecture (inner current loop, outer speed loop, observer, saturation, and I/O blocks).

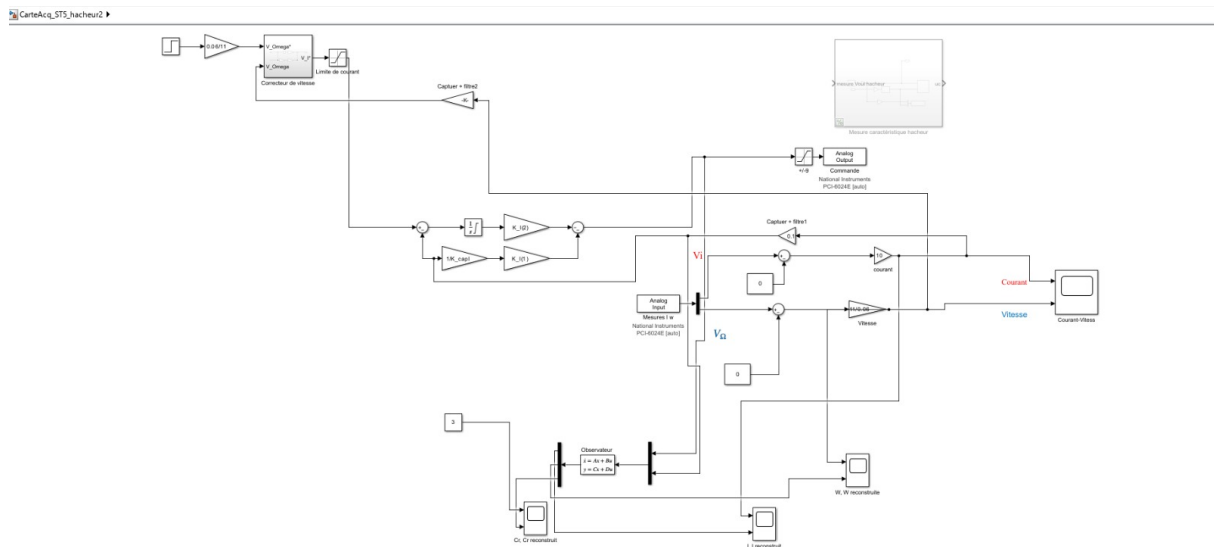


Figure 14: Final Simulink implementation used on the bench (saved during Part 3.3). Earlier configurations for the current and speed loops followed the same structure.

The current estimate converged rapidly and accurately. The speed estimate showed small mismatches and high-frequency noise, mainly due to sensor noise and converter nonlinearities. When using  $\hat{C}_r$  for proportional disturbance compensation, the closed loop became sensitive to this noise and could lose stability. Therefore, compensation based on  $\hat{C}_r$  was not retained on the bench without additional filtering or gain scheduling. Figure 15 illustrates the convergence of the estimated variables towards their measured counterparts.

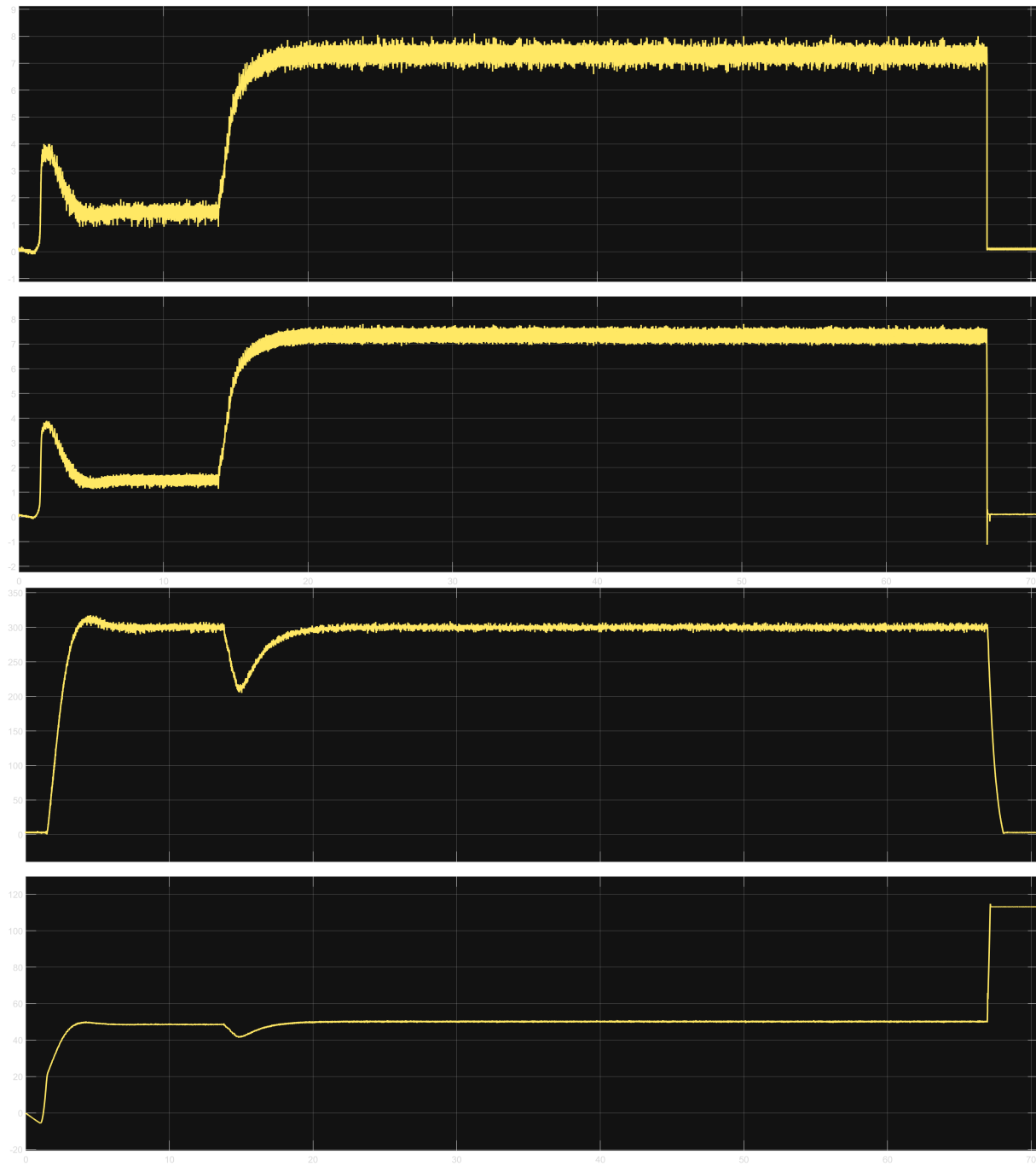


Figure 15: Observer validation — comparison between measured and estimated variables (top: current , bottom: speed).

## 4 Conclusion

The experimental validation confirmed that the control structures developed in simulation work effectively on the real DC drive, with minor adjustments to account for hardware and safety constraints. The current and speed control loops both met the steady-state and dynamic specifications, demonstrating good robustness against disturbances.

The observer-based approach was successful in simulation and partially in the laboratory. The current estimation was accurate, but the torque estimation was affected by measurement noise, leading to instability when used for disturbance compensation. This highlights the practical challenges of implementing sensorless control on real hardware.

Overall, this practical work successfully connected theoretical modeling, simulation, and experimental implementation. It demonstrated the fundamental principles of cascaded control and observer design, and showed how these techniques can be adapted for real-world electrical drives, where nonlinearity, friction, and converter effects must be carefully handled to ensure system stability and precision.

## 5 Additional Examples

## 6 Useful Commands

Here are some useful commands:



Figure 16: Figure caption

Here, I cite the image 16

$$\rho + \Delta = 42 \tag{5}$$

Equation 5 is cited here.