10. Test for a Number to be Prime:

Let p be a given number and let n be the smallest counting number such that $n^2 \geq p$.

(i) We know that $(12)^2 > 137$.
Prime numbers less than 12 are 2, 3, 5, 7, 11.
Clearly, none of them divides 137.
$\therefore$ 137 is a prime number.

V. Important Formulae:

(i) $(a + b)^2 = a^2 + b^2 + 2ab$

(ii) $(a - b)^2 = a^2 + b^2 - 2ab$

(iii) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(iv) $(a + b)^2 - (a - b)^2 = 4ab$

(v) $(a + b)^3 = a^3 + b^3 + 3ab\ (a + b)$

(vi) $(a - b)^3 = a^3 - b^3 - 3ab\ (a - b)$

(vii) $a^2 - b^2 = (a + b)\ (a - b)$

($viii$) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(ix) $a^3 + b^3 = (a + b)\ (a^2 + b^2 - ab)$

(x) $a^3 - b^3 = (a - b)\ (a^2 + b^2 + ab)$

(xi) $a^3 + b^3 + c^3 - 3abc = (a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca)$

(xii) If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

1. Divisibility By 2:

A number is divisible by 2 if its unit digit is any of 0, 2, 4, 6, 8.

Ex. 58694 is divisible by 2, while 86945 is not divisible by 2.

2. Divisibility By 3:

A number is divisible by 3 only when the sum of its digits is divisible by 3.

Ex. (i) In the number 695421, the sum of digits = 27, which is divisible by 3.
$\therefore$ 695421 is divisible by 3.

(ii) In the number 948653, the sum of digits = 35, which is not divisible by 3.
$\therefore$ 948653 is not divisible by 3.

3. Divisibility By 9:

A number is divisible by 9 only when the sum of its digits is divisible by 9.

Ex. (i) In the number 246591, the sum of digits = 27, which is divisible by 9.
$\therefore$ 246591 is divisible by 9.

(ii) In the number 734519, the sum of digits = 29, which is not divisible by 9.
$\therefore$ 734519 is not divisible by 9.

4. Divisibility By 4:

A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

Ex. (i) 6879376 is divisible by 4, since 76 is divisible by 4.

(ii) 496138 is not divisible by 4, since 38 is not divisible by 4.

5. Divisibility By 8:

A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

6. **Divisibility By 10:**

A number is divisible by 10 only when its unit digit is 0.

Ex. (*i*) 7849320 is divisible by 10, since its unit digit is 0.

(*ii*) 678405 is not divisible by 10, since its unit digit is not 0.

7. **Divisibility By 5:**

A number is divisible by 5 only when its unit digit is 0 or 5.

Ex. (*i*) Each of the numbers 76895 and 68790 is divisible by 5.

8. **Divisibility By 11:**

A number is divisible by 11 if the difference between the sum of its digits at odd places and the sum of its digits at even places is either 0 or a number divisible by 11.

Ex. (*i*) Consider the number 29435417.

(Sum of its digits at odd places) – (Sum of its digits at even places)

$= (7 + 4 + 3 + 9) - (1 + 5 + 4 + 2) = (23 - 12) = 11$, which is divisible by 11.

$\therefore$ 29435417 is divisible by 11.

(*ii*) Consider the number 57463822.

(Sum of its digits at odd places) – (Sum of its digits at even places)

$= (2 + 8 + 6 + 7) - (2 + 3 + 4 + 5) = (23 - 14) = 9$, which is not divisible by 11.

$\therefore$ 57463822 is not divisible by 11.

9. **Divisibility By 25:**

A number is divisible by 25 if the number formed by its last two digits is either 00 or divisible by 25.

Ex. (*i*) In the number 63875, the number formed by last 2 digits, namely 75 is divisible by 25.

$\therefore$ 63875 is divisible by 25.

(*ii*) In the number 96445, the number formed by last 2 digits, namely 45 is not divisible by 25.

$\therefore$ 96445 is not divisible by 25.

10. **Divisibility By 7 or 13:**

Divide the number into groups of 3 digits (starting from right) and find the difference between the sum of the numbers in odd and even places. If the difference is 0 or divisible by 7 or 13 (as the case may be), it is divisible by 7 or 13.

11. **Divisibility By 16:**

A number is divisible by 16, if the number formed by its last 4 digits is divisible by 16.

Ex. (*i*) In the number 463776, the number formed by last 4 digits, namely 3776, is divisible by 16.

$\therefore$ 463776 is divisible by 16.

(*ii*) In the number 895684, the number formed by last 4 digits, namely 5684, is not divisible by 16.

$\therefore$ 895684 is not divisible by 16.

12. **Divisibility By 6:** *A number is divisible by 6, if it is divisible by both 2 and 3.*
13. **Divisibility By 12:** *A number is divisible by 12, if it is divisible by both 3 and 4.*
14. **Divisibility By 15:** *A number is divisible by 15, if it is divisible by both 3 and 5.*
15. **Divisibility By 18:** *A number is divisible by 18, if it is divisible by both 2 and 9.*
16. **Divisibility By 14:** *A number is divisible by 14, if it is divisible by both 2 and 7.*
17. **Divisibility By 24:** *A given number is divisible by 24, if it is divisible by both 3 and 8.*
18. **Divisibility By 40:** *A given number is divisible by 40, if it is divisible by both 5 and 8.*
19. **Divisibility By 80:** *A given number is divisible by 80, if it is divisible by both 5 and 16.*

IX. Multiplication BY Short cut Methods

1. Multiplication By Distributive Law:

(*i*) $a \times (b + c) = a \times b + a \times c$ (*ii*) $a \times (b - c) = a \times b - a \times c$

Ex. (*i*) $567958 \times 99999 = 567958 \times (100000 - 1) = 567958 \times 100000 - 567958 \times 1$
$= (56795800000) - 567958) = 56795232042.$

(*ii*) $978 \times 184 + 978 \times 816 = 978 \times (184 + 816) = 978 \times 1000 = 978000.$

2. Multiplication of a Number By 5^n: Put n zeros to the right of the multiplicand and divide the number so formed by 2^n.

Ex. $975436 \times 625 = 975436 \times 5^4 = \dfrac{9754360000}{16} = 609647500.$

X. Division Algorithm or Euclidean Algorithm

If we divide a given number by another number, then:

Dividend = (Divisor × Quotient) + Remainder

3. **Multiplication of Decimal Fractions:** Multiply the given numbers considering them without the decimal point. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.

Suppose we have to find the product $(.2 \times .02 \times .002)$.

Now, $2 \times 2 \times 2 = 8$. Sum of decimal places $= (1 + 2 + 3) = 6$.

$\therefore \quad .2 \times .02 \times .002 = .000008.$

4. **Dividing a Decimal Fraction By a Counting Number:** Divide the given number without considering the decimal point, by the given counting number. Now, in the quotient, put the decimal point to give as many places of decimal as there are in the dividend.

Suppose we have to find the quotient $(0.0204 \div 17)$. Now, $204 \div 17 = 12$.

Dividend contains 4 places of decimal. So, $0.0204 \div 17 = 0.0012$.

I. 'BODMAS' Rule: This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression.

Here, '*B*' stands for 'Bracket', '*O*' for 'of', '*D*' for 'Division', '*M*' for 'Multiplication', '*A*' for 'Addition' and '*S*' for 'Subtraction'.

Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order ( ), { } and [ ].

After removing the brackets, we must use the following operations strictly in the order:

(*i*) of (*ii*) Division (*iii*) Multiplication (*iv*) Addition (*v*) Subtraction

II. Modulus of a Real Number: Modulus of a real number a is defined as

$$|a| = \begin{cases} a, \text{ if } a > 0 \\ -a, \text{ if } a < 0 \end{cases}$$

Thus, $|5| = 5$ and $|-5| = -(-5) = 5$.

III. Virnaculum (or Bar): When an expression contains Virnaculum, before applying the 'BODMAS' rule, we simplify the expression under the Virnaculum.

IV. Some Important Formulae:

(*i*) $(a + b)^2 = (a^2 + b^2 + 2ab)$ (*ii*) $(a - b)^2 = (a^2 + b^2 - 2ab)$

(*iii*) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$ (*iv*) $(a + b)^2 - (a - b)^2 = 4ab$

(*v*) $(a^2 - b^2) = (a + b)(a - b)$ (*vi*) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(*vii*) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ (*viii*) $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

(*ix*) $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

(*x*) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(*xi*) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

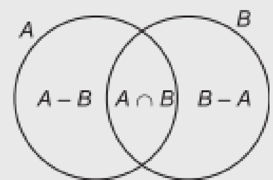
V. For any two sets A and B, we have:

(*i*) $n(A - B) + n(A \cap B) = n(A)$

(*ii*) $n(B - A) + n(A \cap B) = n(B)$

(*iii*) $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

(*iv*) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.



I. Average $= \left(\dfrac{\text{Sum of observations}}{\text{Number of observations}}\right)$

II. Suppose a man covers a certain distance at x kmph and an equal distance at y kmph. Then, the average speed during the whole journey is $\left(\dfrac{2xy}{x + y}\right)$ kmph. office 365 off

I. Logarithm: If a is a positive real number, other than 1 and $a^m = x$, then we write: $m = \log_a x$ and we say that the value of log x to the base a is m.

Example:

(*i*) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$ (*ii*) $3^4 = 81 \Rightarrow \log_3 81 = 4$

(*ii*) $2^{-3} = \frac{1}{8} \Rightarrow \log_2 \frac{1}{8} = -3$ (*iv*) $(.1)^2 = .01 \Rightarrow \log_{(.1)} \cdot 01 = 2$

II. Properties of Logarithms:

1. $\log_a (xy) = \log_a x + \log_a y$
2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_x x = 1$
4. $\log_a 1 = 0$
5. $\log_a (x^p) = p (\log_a x)$
6. $\log_a x = \frac{1}{\log_x a}$
7. $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a}$
8. $a^{\log_a x} = x$
9. $x^{\log_a y} = y^{\log_a x}$
10. $\log_{a^q} x^p = \frac{p}{q} \log_a x$

I. Concept of Percentage: By a certain ***percent***, we mean that many hundredths. Thus, **x percent means x** hundredths, written as x%.

***To express x% as a fraction*:** We have, $x\% = \frac{x}{100}$.

Thus, $20\% = \frac{20}{100} = \frac{1}{5}$; $48\% = \frac{48}{100} = \frac{12}{25}$, etc.

Thus, $\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = 25\%; 0.6 = \frac{6}{10} = \frac{3}{5} = \left(\frac{3}{5} \times 100\right)\% = 60\%$.

II. If a certain value p increases by x%, then increased value = $(100 + x)\%$ of p.

If a certain value p decreases by x%, then decreased value = $(100 - x)\%$ of p.

III. If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is

$$\left[\frac{R}{(100 + R)} \times 100\right]\%$$

If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is

$$\left[\frac{R}{(100 - R)} \times 100\right]\%$$

IV. Results on Population: Let the population of a town be P now and suppose it increases at the rate of R% per annum, then:

1. Population after n years $= P\left(1 + \frac{R}{100}\right)^n$.

2. Population n years ago $= \frac{P}{\left(1 + \frac{R}{100}\right)^n}$.

V. Results on Depreciation: Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then:

1. Value of the machine after n years $= P\left(1-\frac{R}{100}\right)^n$.

2. Value of the machine n years ago $= \frac{P}{\left(1-\frac{R}{100}\right)^n}$.

VI. If A is R% more than B, then B is less than A by $\left[\frac{R}{(100+R)}\times 100\right]\%$.

If A is R% less than B, then B is more than A by $\left[\frac{R}{(100-R)}\times 100\right]\%$.

IV. Gain% $= \left(\frac{\text{Gain}\times 100}{\text{C.P.}}\right)$

V. Loss% $= \left(\frac{\text{Loss}\times 100}{\text{C.P.}}\right)$

VI. S.P. $= \frac{(100+\text{Gain}\%)}{100}\times \text{C.P.}$

VII. S.P. $= \frac{(100-\text{Loss}\%)}{100}\times \text{C.P.}$

VIII. C.P. $= \frac{100}{(100+\text{Gain}\%)}\times \text{S.P.}$

IX. C.P. $= \frac{100}{(100-\text{Loss}\%)}\times \text{S.P.}$

X. If an article is sold at a gain of say, 35%, then S.P. = 135% of C.P.

XI. If an article is sold at a loss of say, 35%, then S.P. = 65% of C.P.

XII. When a person sells two similar items, one at a gain of say, x%, and the other at a loss of x%, then the seller always incurs a loss given by:

$$\text{Loss}\% = \left(\frac{\text{Common Loss and Gain}\%}{10}\right)^2 = \left(\frac{x}{10}\right)^2.$$

XIII. If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain}\% = \left[\frac{\text{Error}}{(\text{True Value})-(\text{Error})}\times 100\right]\%.$$

XIV. If a trader professes to sell his goods at a profit of x% but uses false weight which is y% less than the actual weight, then

$$\text{Gain }\% = \left\{\left(\frac{x+y}{100-y}\right)\times 100\right\}\%$$

XV. If a trader professes to sell his goods at a loss of x% but uses false weight which is y% less than the actual weight, then

$$\text{Gain or Loss }\% = \left\{\left(\frac{y-x}{100-y}\right)\times 100\right\}\%$$

I. Ratio: The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.

In the ratio $a : b$, we call a as the **first term** or **antecedent** and b, the **second term** or **consequent**.

Ex. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Ex. 4 : 5 = 8 : 10 = 12 : 15 etc. Also, 4 : 6 = 2 : 3.

II. Proportion: *The equality of two ratios is called proportion.*

If $a : b = c : d$, we write, $a : b :: c : d$ and we say that a, b, c, d are in proportion.

Here a and d are called **extremes**, while b and c are called **mean terms**.

Product of means = Product of extremes.

Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

III. (*i*) **Fourth Proportional:** If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

(*ii*) **Third Proportional:** If $a : b = b : c$, then c is called the third proportional to a and b.

(*iii*) **Mean Proportional:** Mean proportional between a and b is $\sqrt{ab}$.

IV. (*i*) **Comparison of Ratios:** We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

(*ii*) **Compounded Ratio:**

The compounded ratio of the ratios $(a : b), (c : d), (e : f)$ is $(ace : bdf)$.

V. (*i*) **Duplicate ratio** of $(a : b)$ is $(a^2 : b^2)$.

(*ii*) **Sub-duplicate ratio** of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

(*iii*) **Triplicate ratio** of $(a : b)$ is $(a^3 : b^3)$.

(*iv*) **Sub-triplicate ratio** of $(a : b)$ is $\left(a^{\frac{1}{3}} : b^{\frac{1}{3}}\right)$.

(*v*) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. (componendo and dividendo)

VI. Variation:

(*i*) We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$.

(*ii*) We say that x is inversely proportional to y, if $xy = k$ for some constant k and we write, $x \propto \frac{1}{y}$.

VII. Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid in the final mixture = $\left[x\left(1-\frac{y}{x}\right)^n\right]$ units.

I. Partnership: When two or more than two persons run a business jointly, they are called partners and the deal is known as partnership.

II. Ratio of Division of Gains:

***(i)* Simple Partnership:** *A simple partnership is the one in which the capitals of all the partners are invested for the same time.*

In this partnership, the gain or loss is distributed among the partners in the ratio of their investments. Suppose A and B invest ₹ x and ₹ y respectively for a year in a business, then at the end of the year: (A's share of profit) : (B's share of profit) = $x : y$.

***(ii)* Compound Partnership:** *A compound partnership is the one in which the capitals of the partners are invested for different time periods.*

In this partnership, the equivalent capitals are calculated for a unit of time by taking (capital × number of units of time). Now, gain or loss is divided in the ratio of these capitals.

Suppose A invests ₹ x for p months and B invests ₹ y for q months, then

(A's share of profit) : (B's share of profit) = $xp : yq$.

III. Working and Sleeping Partners: *A partner who manages the business is known as a working partner and the one who simply invests the money is a sleeping partner.*

I. Inlet: *A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an* ***inlet****.*

Outlet: *A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an* ***outlet****.*

II. (*i*) If a pipe can fill a tank in x hours, then part filled in 1 hour = $\frac{1}{x}$.

(*ii*) If a pipe can empty a full tank in y hours, then part emptied in 1 hour = $\frac{1}{y}$.

(*iii*) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$), then on opening both the pipes, the net part filled in 1 hour = $\left(\frac{1}{x}-\frac{1}{y}\right)$.

(*iv*) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $x > y$), then on opening both the pipes, the net part emptied in 1 hour = $\left(\frac{1}{y}-\frac{1}{x}\right)$.

I. If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$.

II. If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in n days.

II. If A is thrice as good a workman as B, then :

Ratio of work done by A and B = 3 : 1.

Ratio of times taken by A and B to finish a work = 1 : 3.

I. Speed = $\left(\frac{\text{Distance}}{\text{Time}}\right)$, Time = $\left(\frac{\text{Distance}}{\text{Speed}}\right)$, Distance = (Speed × Time)

II. x km/hr = $\left(x \times \frac{5}{18}\right)$ m/sec

III. x m/sec = $\left(x \times \frac{18}{5}\right)$ km/hr

IV. If the ratio of the speeds of A and B is $a : b$, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or $b : a$.

V. Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr. Then, the average speed during the whole journey is $\left(\frac{2xy}{x+y}\right)$ km/hr.

VI. Suppose two men are moving in the same direction at u m/s and v m/s respectively, where $u > v$, then their relative speed = $(u - v)$ m/s.

VII. Suppose two men are moving in opposite directions at u m/s and v m/s respectively, then their relative speed = $(u + v)$ m/s.

VIII. If two persons A and B start at the same time in opposite directions from two points and after passing each other they complete the journeys in a and b hours respectively, then

A's speed: B's speed = $\sqrt{b} : \sqrt{a}$.

I. In water, the direction along the stream is called ***downstream***. And, the direction against the stream is called ***upstream***.

II. If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr, then :

Speed downstream = $(u + v)$ km/hr

Speed upstream = $(u - v)$ km/hr

III. If the speed downstream is a km/hr and the speed upstream is b km/hr, then:

Speed in still water = $\frac{1}{2}(a+b)$ km/hr

Rate of stream = $\frac{1}{2}(a-b)$ km/hr

IV. Suppose a man can swim in still water at the rate of u km/hr, the speed of current/stream is v km/hr and the man wishes to cross the stream (of width x metres) straight along its width, then time taken to cross the river is the same as time taken to swim x metres at u km/hr.

[**Note :** This is because the stream sways the man such that both the distance and the effective velocity increase and the time taken to cross the river remains unaffected.]

V. A man can swim directly across a stream of width x km in t hours when there is no current and in t' hours when there is a current. Then, the rate of the current is

$$\left(x\sqrt{\frac{1}{t^2}-\frac{1}{t'^2}}\right) \text{km/hr.}$$

I. a km/hr $= \left(a \times \frac{5}{18}\right)$ m/s. **II.** $\boxed{a \text{ m/s} = \left(a \times \frac{18}{5}\right) \text{km/hr.}}$

III. Time taken by a train of length l metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover l metres.

IV. Time taken by a train of length l metres to pass a stationary object of length b metres is the time taken by the train to cover $(l + b)$ metres.

V. Suppose two trains or two bodies are moving in the same direction at u m/s and v m/s, where $u > v$, then their relative speed = $(u - v)$ m/s.

VI. Suppose two trains or two bodies are moving in opposite directions at u m/s and v m/s, then their relative speed = $(u + v)$ m/s.

VII. If two trains of length a metres and b metres are moving in opposite directions at u m/s and v m/s, then time taken by the trains to cross each other = $\frac{(a+b)}{(u+v)}$ sec.

VIII. If two trains of length a metres and b metres are moving in the same direction at u m/s and v m/s, then the time taken by the faster train to cross the slower train = $\frac{(a+b)}{(u-v)}$ sec.

IX. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then **(*A's speed*) : (*B's speed*) = $(\sqrt{b} : \sqrt{a})$.**

III. Simple Interest (S.I.): If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called ***simple interest.***

Let Principal = P, Rate = R% per annum (p.a.) and Time = T years.

Then, (i) S.I. = $\left(\frac{P \times R \times T}{100}\right)$.

Let Principal = P, Rate = R% per annum, Time = n years.

I. When interest is compounded Annually:

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

II. When interest is compounded Half-yearly:

$$\text{Amount} = P\left[1+\frac{(R/2)}{100}\right]^{2n}$$

III. When interest is compounded Quarterly:

$$\text{Amount} = P\left[1+\frac{(R/4)}{100}\right]^{4n}$$

IV. When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{2}{5}R}{100}\right)$$

V. When rates are different for different years, say R_1%, R_2%, R_3% for 1st, 2nd and 3rd year respectively.

$$\text{Then, Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right).$$

VI. Present worth of ₹ x due n years hence is given by:

$$\text{Present Worth} = \frac{x}{\left(1+\frac{R}{100}\right)^n}.$$

3. **Pythagoras' Theorem:** In a right-angled triangle,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$$

4. The line joining the mid-point of a side of a triangle to the opposite vertex is called the **median.**
5. The point where the three medians of a triangle meet, is called **centroid.** The centroid divides each of the medians in the ratio 2 : 1.

II. Results on Quadrilaterals:

1. The diagonals of a parallelogram bisect each other.
2. Each diagonal of a parallelogram divides it into two triangles of the same area.
3. The diagonals of a rectangle are equal and bisect each other.
4. The diagonals of a square are equal and bisect each other at right angles.
5. The diagonals of a rhombus are unequal and bisect each other at right angles.
6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
7. Of all the parallelograms of given sides, the parallelogram which is a rectangle has the greatest area.
8. The line joining the mid-points of the non-parallel sides of a trapezium is parallel to each of the parallel sides and equal to half of their sum.
9. The line joining the mid-points of the diagonals of a trapezium is parallel to each of the parallel sides and equal to half of their difference.

I. 1. Area of a rectangle = (Length × Breadth).

$\therefore$ Length = $\left(\dfrac{\text{Area}}{\text{Breadth}}\right)$ and Breath = $\left(\dfrac{\text{Area}}{\text{Lengdth}}\right)$

2. Perimeter of a rectangle = 2 (Length + Breadth).

II. Area of a square = $(\text{side})^2 = \dfrac{1}{2}(\text{diagonal})^2$.

III. Area of 4 walls of a room = 2 (Length + Breadth) × Height.

IV. 1. Area of a triangle = $\dfrac{1}{2} \times \text{Base} \times \text{Height}$.

2. Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$,

where a, b, c are the sides of the triangle and $s = \dfrac{1}{2}(a+b+c)$

3. Area of an equilateral triangle = $\dfrac{\sqrt{3}}{4} \times (\text{side})^2$.

4. Area of a triangle = $\dfrac{1}{2} ab \sin\theta$, where a and b are the lengths of any two sides of the triangle and θ is the angle between them.

5. Radius of incircle of an equilateral triangle of side a = $\dfrac{a}{2\sqrt{3}}$.

6. Radius of circumcircle of an equilateral triangle of side a = $\dfrac{a}{\sqrt{3}}$.

7. Radius of incircle of a triangle of area Δ and semi-perimeter s = $\dfrac{\Delta}{s}$.

8. Radius of circumcircle of a triangle = $\dfrac{\text{Product of sides}}{4\Delta}$.

V. 1. Area of a parallelogram = (Base × Height).

2. Area of a rhombus = $\dfrac{1}{2} \times (\text{Product of diagonals})$.

3. Area of a trapezium = $\dfrac{1}{2} \times (\text{sum of parallel sides}) \times (\text{distance between them})$

VI. 1. Area of a circle = πR^2, where R is the radius

2. Circumference of a circle = $2\pi R$

3. Length of an arc = $\dfrac{2\pi R\theta}{360}$, where θ is the central angle

4. Area of a sector = $\dfrac{1}{2}(\text{arc length} \times R) = \dfrac{\pi R^2\theta}{360}$.

VII. 1. Area of a semi-circle = $\dfrac{\pi R^2}{2}$.

2. Circumference of semi-circle = πR.

3. Perimeter of a semi-circle = $\pi R + 2R$.

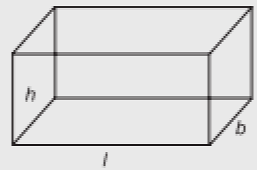
VIII. 1. Area of a regular polygon of N sides, with a as the length of each side = $\dfrac{a^2 N}{4\tan\left(\dfrac{180}{N}\right)}$.

2. Area of a regular hexagon of side a = $\dfrac{3\sqrt{3}}{2}a^2$.

3. Area of a regular pentagon of side a = $1.72\,a^2$.

I. Cuboid

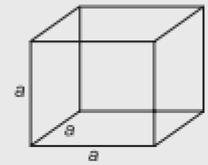
Let length = l, breadth = b and height = h units. Then,



1. **Volume** = $(l \times b \times h)$ cubic units.
2. **Surface area** = $2\,(lb + bh + lh)$ sq. units.
3. **Diagonal** = $\sqrt{l^2 + b^2 + h^2}$ units.

II. Cube

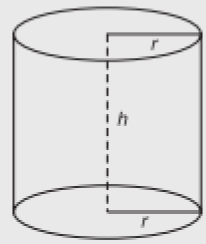
Let each edge of a cube be of length a. Then,



1. **Volume** = a^3 cubic units.
2. **Surface area** = $6a^2$ sq. units.
3. **Diagonal** = $\sqrt{3}\,a$ units.

III. Cylinder

Let radius of base = r and Height (or length) = h. Then,

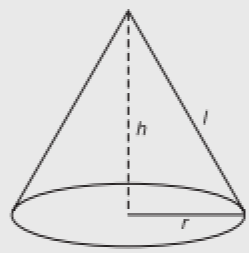


1. **Volume** = $(\pi r^2 h)$ cubic units.
2. **Curved surface area** = $(2\pi r h)$ sq. units.
3. **Total surface area** = $(2\pi r h + 2\pi r^2)$ sq. units
   $= 2\pi r\,(h + r)$ sq. units.

IV. Cone

Let radius of base = r and Height = h. Then,

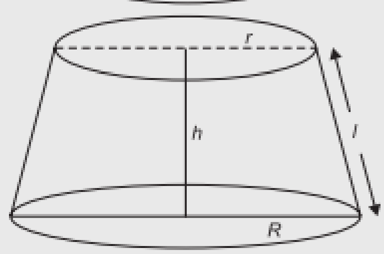
1. **Slant height,** $l = \sqrt{h^2 + r^2}$ units.
2. **Volume** = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.
3. **Curved surface area** = $(\pi r l)$ sq. units.
4. **Total surface area** = $(\pi r l + \pi r^2)$ sq. units.



V. Frustum of a Cone

When a cone is cut by a plane parallel to the base of the cone then the portion between the plane and the base is called the frustum of the cone.

Let radius of base = R, radius of top = r, and height = h. Then,

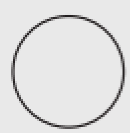


1. **Volume** = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$ cubic units.
2. **Slant height,** $l = \sqrt{(R-r)^2 + h^2}$ units.
3. **Lateral (or curved) surface area** = $\pi . l\,(R + r)$ sq. units.
4. **Total surface area** = $\pi\,[R^2 + r^2 + 1\,(R + r)]$ sq. units.

VI. Sphere

Let the radius of the sphere be r. Then,

1. **Volume** = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.
2. **Surface area** = $(4\pi r^2)$ sq. units.



VII. Hemisphere

Let the radius of a hemisphere be r. Then,

1. **Volume** = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.
2. **Curved surface area** = $(2\pi r^2)$ sq. units.
3. **Total surface area** = $(3\pi r^2)$ sq. units.

VIII. Pyramid

1. **Volume** = $\frac{1}{3} \times \text{area of base} \times \text{height}$.
2. **Whole surface area** = Area of base + Area of each of the lateral faces

Remember : 1 litre = 1000 cm^3.

Race : *A contest of speed in running, riding, driving, sailing or rowing is called a race.*

Race Course : *The ground or path on which contests are made is called a race course.*

Starting Point : *The point from which a race begins is known as a starting point.*

Winning Point or Goal : *The point set to bound a race is called a winning point or a goal.*

Winner : *The person who first reaches the winning point is called a winner.*

Dead Heat Race : *If all the persons contesting a race reach the goal exactly at the same time, then the race is said to be a dead heat race.*

Start : Suppose A and B are two contestants in a race. If before the start of the race, A is at the starting point and B is ahead of A by 12 metres, then we say that 'A gives B, a start of 12 metres'.

To cover a race of 100 metres in this case, A will have to cover 100 metres while B will have to cover only (100 – 12) = 88 metres.

In a 100 m race, 'A can give B 12 m' or 'A can give B a start of 12 m' or 'A beats B by 12 m' means that while A runs 100 m, B runs (100 – 12) = 88 m.

Games : 'A game of 100, means that the person among the contestants who scores 100 points first is the winner'.

If A scores 100 points while B scores only 80 points, then we say that 'A can give B 20 points'.

We are supposed to find the day of the week on a given date.

For this, we use the concept of *odd days.*

I. Odd Days: In a given period, the number of days more than the complete weeks are called *odd days.*

II. Leap Year:

(*i*) Every year divisible by 4 is a leap year, if it is not a century.

(*ii*) Every 4th century is a leap year and no other century is a leap year.

Note : A leap year has 366 days.

Examples:

(*i*) Each of the years 1948, 2004, 1676 etc. is a leap year.

(*ii*) Each of the years 400, 800, 1200, 1600, 2000 etc. is a leap year.

(*iii*) None of the years 2001, 2002, 2003, 2005, 1800, 2100 is a leap year.

III. Ordinary Year:

The year which is not a leap year is called an ordinary year. An ordinary year has 365 days.

IV. Counting of Odd Days:

(*i*) 1 ordinary year = 365 days = (52 weeks + 1day).

$\therefore$ *1 ordinary year has 1 odd day.*

(*ii*) 1 leap year = 366 days = (52 weeks + 2 days).

$\therefore$ *1 leap year has 2 odd days.*

(*iii*) 100 years = 76 ordinary years + 24 leap years

$= (76 \times 1 + 24 \times 2)$ odd days = 124 odd days

= (17 weeks + 5 days) $\equiv$ 5 odd days.

$\therefore$ Number of odd days in 100 years = 5

Number of odd days in 200 years = $(5 \times 2) \equiv 3$ odd days

Number of odd days in 300 years = $(5 \times 3) \equiv 1$ odd day.

Number of odd days in 400 years = $(5 \times 4 + 1) \equiv 0$ odd day.

Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years, etc. has 0 odd days.

V. Day of the Week Related to Odd Days:

No. of days	0	1	2	3	4	5	6
Day	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.

Too Fast and Too Slow : *If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast.*

On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow.

Both the hands of a clock are together after every $65\frac{5}{11}$ min. So, if both the hands are meeting after an interval less than $65\frac{5}{11}$ min, the clock is running fast and if they meet after an interval greater than $65\frac{5}{11}$ min, the clock is running slow.

Interchange of Hands: Whenever the hands of the clock interchange positions (i.e., the minute hand takes the place of hour hand and the hour hand and takes the place of minute hand), the sum of the angles traced by hour hand and minute hand is 360°.

Suppose this happens after x minutes.

Angle traced by minute hand in x min = $(6x)°$.

Angle traced by hour hand in x min = $(0.5x)°$.

$\therefore\ 0.5x + 6x = 360 \Leftrightarrow 6.5x = 360 \Leftrightarrow x = \frac{3600}{65} = 55\frac{5}{13}$.

Thus, the hands of a clock interchange positions after every $55\frac{5}{13}$ minutes.

I. Factorial n: *Let* n *be a positive integer. Then, factorial* n *is denoted by* $\underline{|n}$ *or* $n!$, *defined as* $\underline{|n} = n(n-1)(n-2)(n-3)\ldots..4\cdot3\cdot2\cdot1$.

Ex. (*i*) $\underline{|4} = (4 \times 3 \times 2 \times 1) = 24$. (*ii*) $\underline{|5} = (5 \times 4 \times 3 \times 2 \times 1) = 120$.

Note: We define, $\underline{|0} = 1$.

II. (*i*) **Permutations:** *The different arrangements of a given number of things by taking some or all at a time are called permutations.*

Ex.1. All permutations or arrangements made with the letters *a, b, c* by taking two at a time are (*ab, ba, ac, ca, bc, cb*).

Ex.2. All permutations made with the letters *a, b, c* by taking 3 at a time are (*abc, acb, bac, bca, cab, cba*).

(*ii*) ***Number of Permutations of n things, taking r at a time is given by:***

$$^nP_r = n(n-1)(n-2)\ldots..(n-r+1) = \frac{\underline{|n}}{\underline{|n-r}}.$$

Ex. (*i*) $^8P_2 = (8 \times 7) = 56$. (*ii*) $^7P_3 = (7 \times 6 \times 5) = 210$.

(*iii*) ***Number of all permutations of n things, taking all at a time is*** $\underline{|n}$.

(*iv*) If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of rth kind such that $(p_1 + p_2 + \ldots + p_r) = n$, then **number of permutations** $= \dfrac{\underline{|n}}{\underline{|p_1}\cdot\underline{|p_2}\cdot\underline{|p_3}\cdot\ldots..\underline{|p_r}}$.

III. (*i*) **Combinations:** *Each of the different groups or selections which can be formed by taking some or all at a time, is called a combination.*

Ex. 1. *Out of three boys A, B, C we want to select two.*

The possible selections are (AB, BC, CA).

Note that AB and BA represent the same combination.

Ex. 2. *The only combination of three letters A, B, C taken all at a time is* ***ABC.***

Ex. 3. *Various groups of two out of 4 persons A, B, C, D are AB, AC, AD, BC, BD, CD.*

Important Note: AB and BA are two different permutations.

But, they represent the same combination.

(*ii*) ***Number of all combinations of n things, taken r at a time, is***

$$^nC_r = \frac{\underline{|n}}{(\underline{|r})(\underline{|n-r})} = \frac{n(n-1)(n-2)\ldots..\text{ to } r \text{ factors}}{\underline{|r}}$$

(*iii*) $^nC_n = 1$ and $^nC_0 = 1$ (*iv*) $^nC_r = {}^nC_{(n-r)}$

Ex. (*i*) $^8C_3 = \dfrac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$. (*ii*) $^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_3 = \dfrac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560$.

I. Experiment: An operation which can produce some well-defined outcomes is called an experiment.

II. Random Experiment: An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

Examples of Performing a Random Experiment:

(*i*) Rolling an unbiased dice

(*ii*) Tossing a fair coin

(*iii*) Drawing a card from a pack of well-shuffled cards

(*iv*) Picking up a ball of certain colour from a bag containing balls of different colours

Details :

(*i*) When we throw a coin, then either a Head (H) or a Tail (T) appears.

(*ii*) A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively.
When we throw a die, the outcome is the number that appears on its upper face.

(*iii*) A pack of cards has 52 cards.
It has 13 cards of each suit, namely **Spades, Clubs, Hearts** and **Diamonds.**
Cards of spades and clubs are **black cards.**
Cards of hearts and diamonds are **red cards.**
There are 4 honours of each suit.
These are **Aces, Kings, Queens** and **Jacks.**
These are called **face cards.**

III. Sample Space: When we perform an experiment, then the set S of all possible outcomes is called the **Sample Space.**

Examples of Sample Spaces:

(*i*) In tossing a coin, S = {H, T}.

(*ii*) If two coins are tossed, then S = {HH, HT, TH, TT}.

(*iii*) In rolling a dice, we have, S = {1, 2, 3, 4, 5, 6}.

IV. Event : Any subset of a sample space is called an event.

V. Probability of Occurrence of an Event:

Let S be the sample space and let E be an event. Then, $E \subseteq S$.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)}.$$

VI. Results on Probability:

(*i*) $P(S) = 1$ (*ii*) $0 \leq P(E) \leq 1$ (*iii*) $P(\phi) = 0$

(*iv*) For any events A and B, we have :
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(*v*) If $\bar{A}$ denotes (not-A), then $P(\bar{A}) = 1 - P(A)$.

IMPORTANT CONCEPTS

Suppose a man has to pay ₹ 156 after 4 years and the rate of interest is 14% per annum. Clearly, ₹ 100 at 14% will amount to ₹ 156 in 4 years. So, the payment of ₹ 100 now will clear off the debt of ₹ 156 due 4 years hence. We say that :

Sum due = ₹ 156 due 4 years hence;

Present Worth (P.W.) = ₹ 100;

True Discount (T.D.) = ₹ (156 – 100) = ₹ 56 = (Sum due) – (P.W.).

We define : ***T.D. = Interest on P.W.***

Amount = (P.W.) + (T.D.).

Interest is reckoned on P.W. and true discount is reckoned on the amount.

IMPORTANT FORMULAE

Let rate = R% per annum and Time = T years. Then,

I. $\text{P.W.} = \dfrac{100 \times \text{Amount}}{100 + (R \times T)} = \dfrac{100 \times \text{T.D.}}{R \times T}$

II. $\text{T.D.} = \dfrac{(\text{P.W.}) \times R \times T}{100} = \dfrac{\text{Amount} \times R \times T}{100 + (R \times T)}$

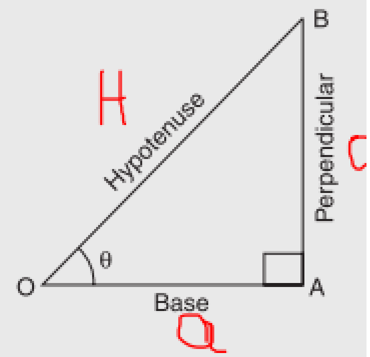
III. $\text{Sum} = \dfrac{(\text{S.I.}) \times (\text{T.D.})}{(\text{S.I.}) - (\text{T.D.})}$

IV. (S.I.) – (T.D.) = S.I. on T.D.

V. When the sum is put at compound interest, then $\text{P.W.} = \dfrac{\text{Amount}}{\left(1 + \dfrac{R}{100}\right)^T}$.

I. We already know that:

In a right angled ΔOAB, where $\angle BOA = \theta$,



(*i*) $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{AB}{OB}$; $\dfrac{o}{h}$

(*ii*) $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{OA}{OB}$; $\dfrac{a}{h}$

(*iii*) $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{AB}{OA}$; $\dfrac{o}{a}$

(*iv*) $\text{cosec}\,\theta = \dfrac{1}{\sin\theta} = \dfrac{OB}{AB}$;

(*v*) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{OB}{OA}$;

(*vi*) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{OA}{AB}$.

II. Trigonometrical Identities:

(*i*) $\sin^2\theta + \cos^2\theta = 1.$ (*ii*) $1 + \tan^2\theta = \sec^2\theta.$ (*iii*) $1 + \cot^2\theta = \text{cosec}^2\,\theta.$

III. Values of T-ratios:

θ	$0°$	$(\pi/6)$ $30°$	$(\pi/4)$ $45°$	$(\pi/3)$ $60°$	$(\pi/2)$ $90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

IV. Angle of Elevation: Suppose a man from a point O looks up at an object