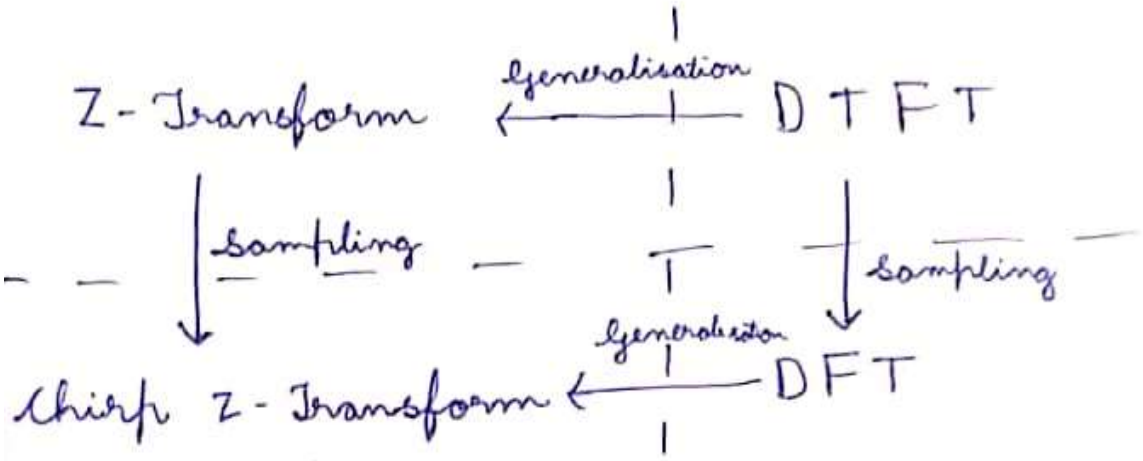
Team 17: Chirp z- transform and its application

- We plan to find out the chirp z- transform of discrete time signals in our project, derive the formula, discuss applications, and write a matlab code.

Basic idea about chirp z-transform:

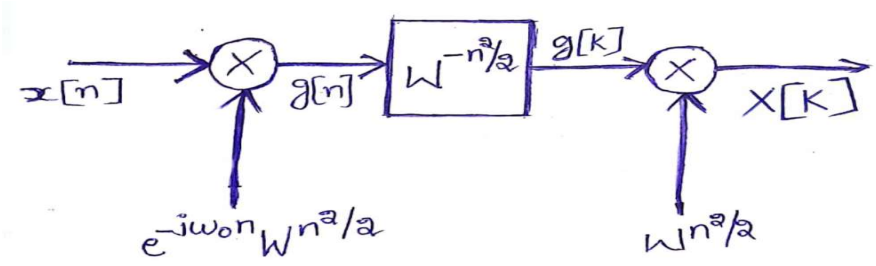
- We are familiar with the discrete Fourier transform (DFT). It is nothing but sampling the discrete time Fourier transform (DTFT) which are on a unit circle (X(e^jw)).
- The z transform gives continuous points which maybe spiral arcs. Sampling these points which are not specifically on a circle but in a spiral contour gives us the Chirp z transform.



- The chirp z-transform can be thought of a generalisation of a DFT.
- The **formula** associated with the chirp z-transform is as follows

$$X(e^{j(\omega_0+k\Delta\omega)}) = \sum_{n=0}^{N-1} x[n]e^{-j(\omega_0+k\Delta\omega)n}, \quad k = 0, 1, \ldots, M-1$$

- **Derivation** of the tranform is understood from the following block diagram



Block diagram of chirp Z-transform Algorithm

- The sequence $W^{-n\wedge 2/2}$ is like a complex exponential sequence with increasing frequency. In Radar systems, such signals are called **Chirp signals** and so we get the name chirp z- transform.
- FFT(fast fourier transform) which is a quick implementation of DFT is extremely important in digital processing but it is done in a very restricted contour i.e. circle. So chirp z-transform comes in very handy as we can calculate it for a wide range of spiral contours with a reasonably good time complexity **$(N+M)\log_2(M+N)$** which we will prove.
- There are many **applications** discussed in the paper but we have planned to deal with only one application which is **enhancement of poles** for use in spectral analysis.
- We have also planned to write a **matlab code** that calculates the chirp z-transform for a given signal. We might directly use the inbuilt function czt().

References:

1) The given paper Rabiner(1969)
2) https://krex.k-state.edu/dspace/bitstream/handle/2097/7844/LD2668R41972S43.pdf
   A study of chirp z- transform and its applications-Shilling, Alan
3) https://apps.dtic.mil/dtic/tr/fulltext/u2/a107739.pdf
   Enhancement of Poles in Spectral Analysis- Fung-I Tseng; Tapan K. Sarkar