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CT-3

1) $\theta(t) = \theta(0) + 2\pi k_f \int_0^t m(\tau)\,d\tau$

$A = 550°$ (from figure)

$\Rightarrow A = \frac{550}{180} \cdot \pi$

$\approx 3\pi$ (Mentioned in the question)

$3\pi \sin(2\pi f t) = \theta(t)$

0.2 ms is one time period

$\Rightarrow f = \frac{1}{0.2 \times 10^{-3}} = 5\text{ kHz}$

$\theta(t) = 3\pi \sin(2\pi f t) = 2\pi k_f \int_0^t m(\tau)\,d\tau$

$\Rightarrow \frac{d\left(\frac{3}{2k_f} \sin(2\pi f t)\right)}{dt} = m(t) \times \frac{dt}{dt}$

$\Rightarrow \boxed{m(t) = \cancel{\frac{3}{2k_f} \sin(2\pi f t)}\ \frac{3\pi f}{k_f} \cos(2\pi f t)}$

b) Clearly, message bandwidth $\boxed{B = f}$

a) We know $\Delta f = k_f\, m(t)$

& $\Delta f_{max} = k_f \max_t |m(t)|$

$\Rightarrow \Delta f_{max} = k_f \times \frac{3\pi f}{k_f} = 3\pi f$

We know that $\boxed{\beta = \frac{\Delta f_{max}}{B}}$

$$\Rightarrow \beta = \frac{3\pi f}{f}$$

$$\Rightarrow \boxed{\beta = 3\pi}$$

Another simple way without doing all this calculation is by directly recognising that $\theta(t) = \beta \sin(2\pi f_m t)$

c) Larsons formula says that the ~~boon~~ bandwidth signal is sum of Narrow band and wideband FM

$$\therefore \boxed{B_{FM} \approx 2B + 2\Delta f_{max} = 2B(\beta + 1)}$$

$$\Rightarrow B_{FM} = 2 \times 5 \times 10^3 (3\pi + 1)$$

$$= (30\pi + 10) \text{ KHz}$$

$$\boxed{B_{FM} = 104.247 \text{ KHz}}$$

2) $M(f) = \begin{cases} j2\pi f & |f| \leq 1 \\ 0 & \text{otherwise.} \end{cases}$

$K_f = 1$

$U_{FM}(t) = A\cos(2\pi f_c t + \phi(t))$

We know that $\phi(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$

Applying Fourier transform,

$$\phi(f) = 2\pi k_f \times FT\left\{ \int_0^t m(\tau)\,d\tau \right\}$$

We know that if $u(t) = \int_{-\infty}^{t} x(t)\,dt = x(t) * v(t)$

$$U(f) = X(f)V(f)$$

$$= \frac{X(f)}{j2\pi f} + \pi\delta(f)$$

Here, we get $\int_0^t m(\tau)\,d\tau = \frac{M(f)}{j2\pi f} + 0$

$$\Rightarrow \phi(f) = 2\pi k_f \times \frac{M(f)}{j2\pi f}$$

We know that $M(f) = j2\pi f\, I_{[-1,1]}(f)$

$$\Rightarrow \phi(f) = 2\pi k_f\, I_{[-1,1]}(f)$$

$$k_f = 1$$

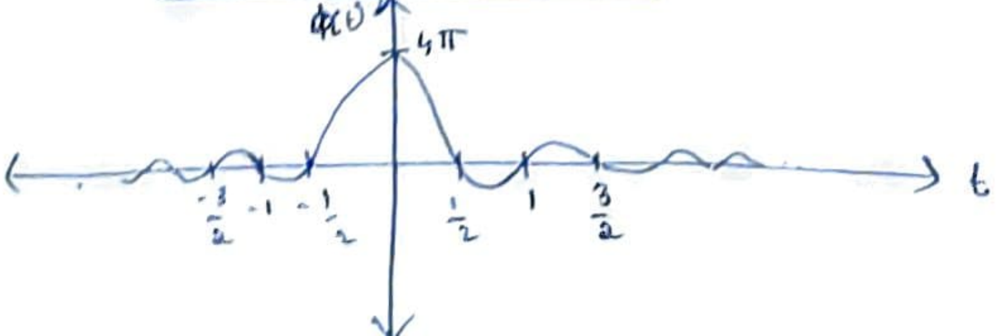
$$\Rightarrow \phi(f) = 2\pi\, I_{[-1,1]}(f)$$

We know that $I_{[-\frac{W}{2},\frac{W}{2}]}(f) \leftrightarrow W\,\mathrm{sinc}(Wt)$

$$\Rightarrow \boxed{\phi(t) = 2\pi \times 2\,\mathrm{sinc}(2t)}$$

$$= 4\pi\,\frac{\sin(2\pi t)}{2\pi t}$$

$$\boxed{\phi(t) = \frac{2\sin(2\pi t)}{t}}$$



b) $f(t)\Big|_{t=\frac{1}{4}} = ?$

$$f(t)\Big|_{t=\frac{1}{4}} = \frac{1}{2\pi}\,\frac{d\phi(t)}{dt}\Bigg|_{t=\frac{1}{4}}$$

$$= \frac{1}{2\pi} \times 2 \times \frac{d}{dt}\left(\frac{\sin(2\pi t)}{t}\right)\Bigg|_{t=\frac{1}{4}}$$

$$= \frac{1}{\pi}\left(\frac{2\pi\cos(2\pi t)\,t - \sin(2\pi t)}{t^2}\right)\Bigg|_{t=\frac{1}{4}}$$

$$= \frac{1}{\pi} \times \frac{-1}{\left(\frac{1}{4}\right)^2}$$

$$\boxed{f(t)\Big|_{t=\frac{1}{4}} = -\frac{16}{\pi}}$$

c) $\Delta f_{max} = \left|-\frac{16}{\pi}\right| = \frac{16}{\pi}$

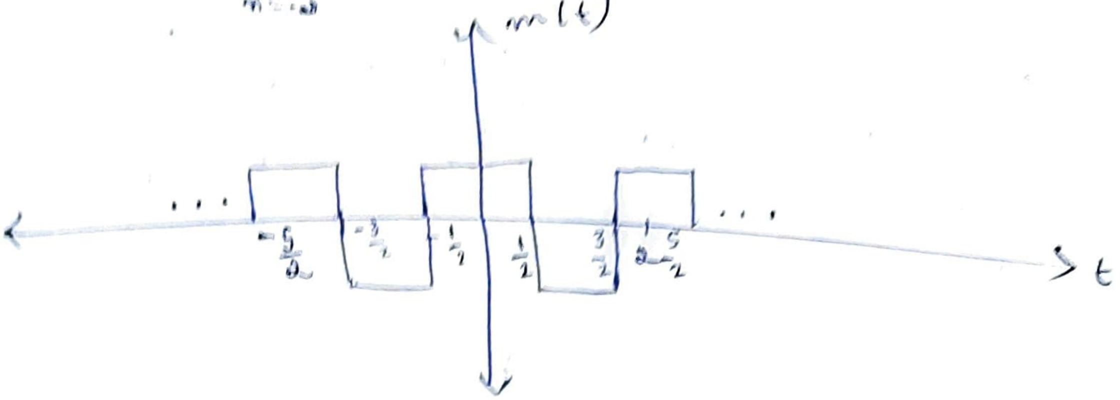
$$B_{FM} = 2B + 2\Delta f_{max}$$

$$= 2 + \frac{32}{\pi}$$

$$\boxed{B_{FM} = 12.186}$$

3a) $p(t) = I_{[-\frac{1}{2}, \frac{1}{2}]}(t)$

$$m(t) = \sum_{n=-\infty}^{\infty} (-1)^n p(t-n)$$



$$\phi(t) = \left(20\pi \int_{-\infty}^{t} m(\tau)\,d\tau\right) + a$$

Clearly from the plot, when $t > 0$,

$$\int_{-\infty}^{0} m(\tau)\,d\tau = 0$$

$\cancel{\phi(t) = 20\pi \int_0^t m(\tau)\,d\tau}$ $\int_0^t m(\tau)\,d\tau$ for $0 < t \le \frac{1}{2}$

$$= t$$

In the interval $\frac{1}{2} < t \le \frac{3}{2}$

$$\int_0^t m(\tau)\,d\tau = \frac{1}{2} + \int_{\frac{1}{2}}^{t} -1\,dt$$

$$= \frac{1}{2} - \left(t - \frac{1}{2}\right)$$

$$= 1 - t$$

Ily in $\frac{3}{2} \le t \le 2$ $\int_0^t m(\tau)\,d\tau = t - 2$

Since $m(t)$ is periodic we can simply extend this for other intervals.

We know that $\phi(t) = \phi(0) + 2\pi k_f \int_0^t m(\tau)\,d\tau$

By comparing we can get $\phi(0) = a$; $\boxed{k_f = 10}$

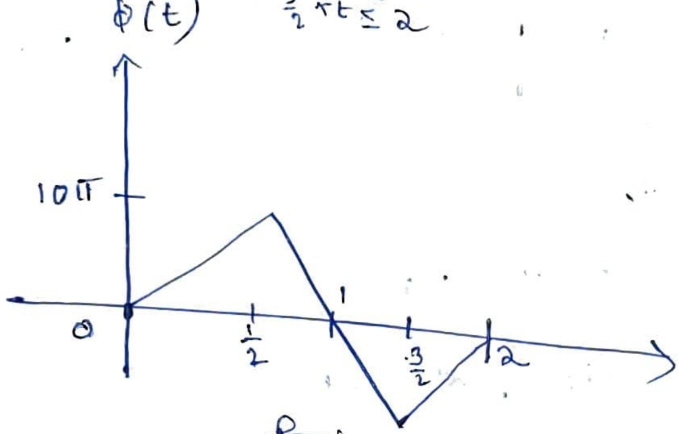
But a is chosen such that $\phi(0) = 0$

$$\Rightarrow \boxed{a = 0}$$

$\phi(t) = 2\pi \cdot 10 \int_0^t m(t)\,dt$ where $t > 0$,

we will again write the $t < 0$ part based on the fact that we integrate an even signal and thus get odd signal

$$\therefore \phi(t) = \begin{cases} 20\pi t & 0 < t \leq \frac{1}{2} \\ 20\pi(1-t) & \frac{1}{2} < t \leq \frac{3}{2} \\ 20\pi(2-t) & \frac{3}{2} < t \leq 2 \end{cases}$$



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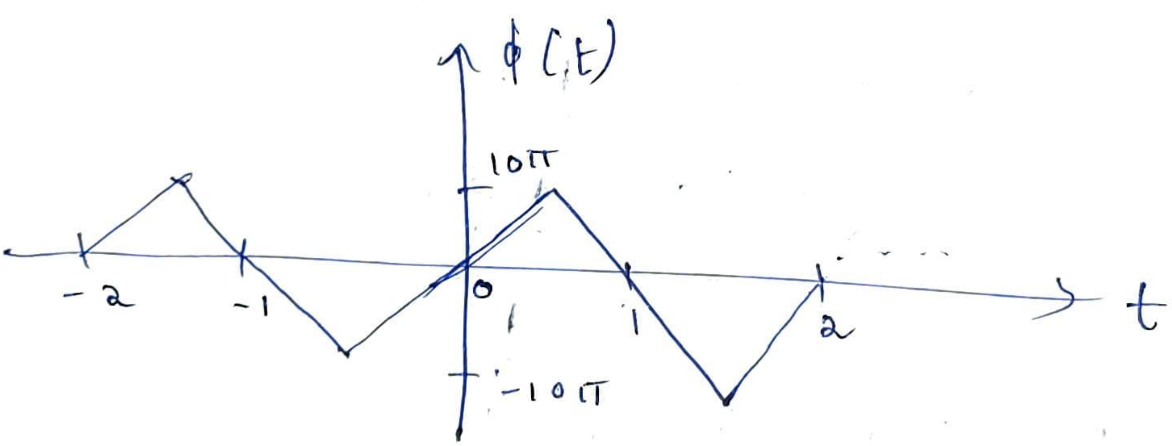
~~But~~ From odd signal idea, and periodicity property,



b) $W \approx 2$

using Carsons Formula,

$B_{FM} = 2W + 2\Delta f_{max}$

$\Delta f_{max} = k_f \, \max_t |m(t)|$

$= 10$

$= 2 \times 12$

$= 24\,Hz$

c) We know that $m(t)$ is periodic with period 2.

Also $\phi(t)$ and $e^{j\phi(t)}$ have period 2.

We need to find the Fourier series components for complex envelope at $n f_m$. This is because the output of the narrow ideal BPF gives the above mentioned components.

$$\boxed{U(f) = \sum_{n=-\infty}^{\infty} J_n(\beta)\, \delta(f - n f_m)}$$ This is the FM spectrum and J is Bessel function.

So, Spectrum of passband signal has components at $f_c + n f_m$ $(= f_c + \alpha)$

We know that $f_m = \frac{1}{T} = \frac{1}{2}$

From this we can conclude that for $\boxed{\alpha = 0.75}$ we get non zero power whereas for the rest α we get 0 power