CT Assignment - 2

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1) $u(t) = [20 + 2\cos 3000\pi t + 10\cos 6000\pi t] \cos 2\pi f_c t$

$f_c = 10^5$ Hz

a) (Voltage) spectrum of $u(t)$

$$1 \xleftrightarrow{FT} \delta(f)$$

$$\cos 2\pi f_0 t \xleftrightarrow{FT} \frac{\delta(f - f_0) + \delta(f + f_0)}{2}$$

$U(f) = FT\{20 + 2\cos 3000\pi t + 10\cos 6000\pi t\} * FT\{\cos 2\pi f_c t\}$

(Multiplication in time is convolution in frequency)

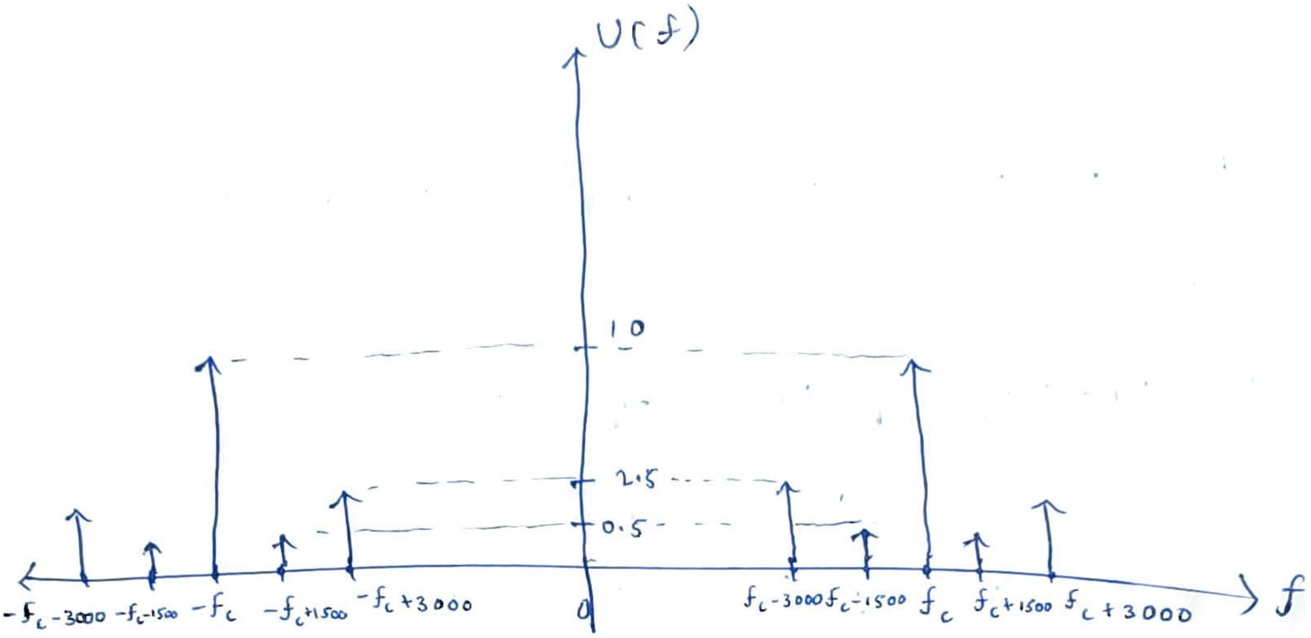
$$= \left(20\delta(f) + 2\left(\frac{\delta(f - 1500) + \delta(f + 1500)}{2}\right) + 10\left(\frac{\delta(f - 3000) + \delta(f + 3000)}{2}\right)\right)$$

$$* \left(\frac{\delta(f - f_c) + \delta(f + f_c)}{2}\right)$$

We know that $X(f) * \delta(f - f_0) = X(f - f_0)$

$$= \frac{1}{2}\Big(20\delta(f - f_c) + \delta(f - (1500 + f_c)) + \delta(f - (f_c - 1500))$$
$$+ 5\delta(f - (f_c + 3000)) + 5\delta(f - (f_c - 3000))$$

$$+ \frac{1}{2}\Big(20\delta(f + f_c) + \delta(f + (f_c - 1500)) + \delta(f + (f_c + 1500))$$
$$+ 5\delta(f + (f_c - 3000))$$
$$+ 5\delta(f + (f_c + 3000))\Big)\Big)$$



b) We know the basic idea that Power of $a\cos\theta$ is $\frac{a^2}{2}$

We have i) $20\cos 2\pi f_c t$ as one term

$$\frac{(20)^2}{2} = \underline{200W}$$

ii) $2\cos 3000\pi t \cos 2\pi f_c t = \cos\theta_1 + \cos\theta_2$

$$\Rightarrow \frac{1^2}{2} + \frac{1^2}{2} = \underline{1W}$$

iii) $10\cos 6000\pi t \cos 2\pi f_c t = 5\cos\theta_3 + 5\cos\theta_4$

$$\Rightarrow \frac{5^2}{2} + \frac{5^2}{2} = \underline{25W}$$

$P_{TOTAL} = \underline{226W}$

c)

$m(t) = 2\cos 3000\pi t + 10\cos 6000\pi t$

$\min_t m(t) = ?$

$m(t) = \cancel{\sqrt{2^2+10^2}}$ $\quad m(t) = 2\cos\theta + 10\cos 2\theta$

$= 2\cos\theta + 20\cos^2\theta - 1$

$= 20\cos^2\theta + 2\cos\theta - 1$

$$\min(m(t)) = \frac{-(4/20)(-10)-4)}{4 \cdot 20}$$

$$= -10.05$$

$$M.I = \frac{A \left| \min_t m(t) \right|}{A_c}$$

$A = 1 \quad A_c = 20$

$$\Rightarrow MI = \frac{10.05}{20}$$

$$\boxed{MI = 0.5025}$$

d)

$20 + 2\cos 3000\pi t + 10\cos 6000\pi t$

We need sidebands energy.

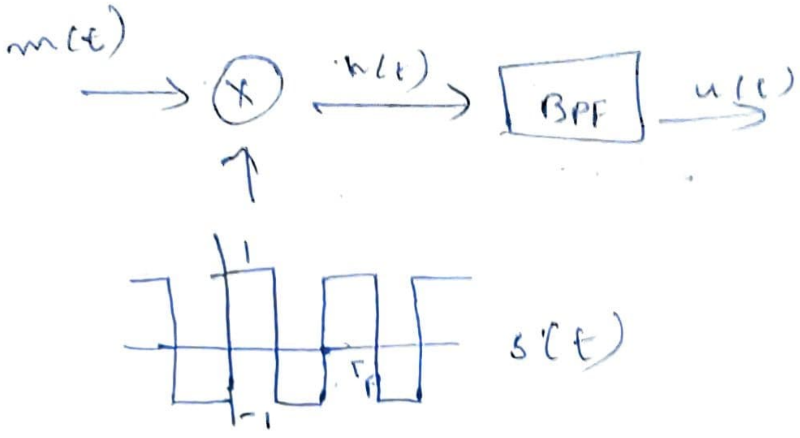
So we take energy corresponding to $2\cos 3000\pi t + 10\cos 6000\pi t$

so ii) + iii) in b)

$1 + 25 = 26W$ = Power of side bands

$$\Rightarrow \boxed{\text{Power ratio} = \frac{26}{226} = 0.115}$$

2) Given, DSB-SC signal is generated as



Let $h(t) = m(t)s(t)$

$s(t)$ is periodic with period T_p

$$\Rightarrow f_p = \frac{1}{T_p}$$

Since $s(t)$ is periodic, we can find the fourier series

since the signal is odd, we only have the sin terms

$$s(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$b_n = \frac{2}{T_0}\int_0^{T_0} s(t)\sin(n\omega_0 t)\,dt$$

$$= \frac{2}{T_0}\left(\int_0^{\frac{T_0}{2}} \sin(n\omega_0 t)\,dt - \int_{\frac{T_0}{2}}^{T_0} \sin(n\omega_0 t)\,dt\right)$$

$$= \frac{2}{T_0}\left[\frac{-\cos(n\omega_0 t)}{n\omega_0}\Bigg|_0^{\frac{T_0}{2}} + \frac{\cos(n\omega_0 t)}{n\omega_0}\Bigg|_{\frac{T_0}{2}}^{T_0}\right]$$

$$= \frac{2}{T_0}\left[\frac{2}{n\omega_0} - \frac{2\cos(n\omega_0 \frac{T}{2})}{n\omega_0}\right]$$

$$= \frac{2(1-\cos(n\pi))}{n\pi}$$

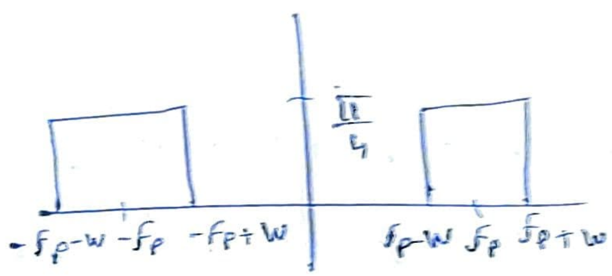
$$b_n = \frac{4\sin^2\left(\frac{n\pi}{2}\right)}{n\pi}$$

For $n = 2k$, $b_n = 0$

$n = 2k+1$, $b_n = \frac{4}{\pi n}$

$$h(t) = m(t) \cdot \left(\sum_{k=0}^{\infty} \frac{4}{\pi(2k+1)} \sin((2k+1)\omega_0 t) \right)$$

We pass this through bandpass filter.



After passing through BPF given only one frequency component passes i.e containing f_p.

$$\Rightarrow \quad u(t) = \frac{\pi}{4}\left(\frac{4}{\pi} \sin(2\pi f_p t) \right) m(t)$$

$$\Rightarrow \quad \boxed{u(t) = m(t)\sin(\omega_p t)}$$

Hence proved.

3)

$s(t)$ is any other signal but is periodic.

$s(t) = \sum_n x(t - nT_0)$ The period is T_0

$v(t) = m(t) s(t)$

$$s(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jn\omega_0 t} dt$$

$$\Rightarrow v(t) = m(t) \cdot \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} C_n \left(m(t) e^{jn\omega_0 t} \right)$$

The only frequency component that passes is $-f_p, f_p$ through the band pass filter.

$$y(t) = A\, m(t) \left(C_{-1} e^{-j\omega_0 t} + C_1 e^{j\omega_0 t} \right)$$

If $s(t)$ is even, $C_1 = C_{-1}$

$$y(t) = A\, m(t)\, C_1 \left(e^{j\omega_0 t} + e^{-j\omega_0 t} \right)$$

$$= B_p\, m(t) C_1 \cos(\omega_0 t) \quad \text{--- } (1)$$

If $s(t)$ is odd $C_1 = -C_{-1}$

$$y(t) = A\, m(t)\, C_1 \left(e^{j\omega_0 t} - e^{-j\omega_0 t} \right)$$

$$= B_0\, m(t) C_1 \sin(\omega_0 t) \quad \text{--- } (2)$$

So from (1) & (2), we get that $y(t)$ is of the form $k\, m(t) \sin(\omega_0 t)$

4) $M(f) = I_{[-2,2]}(f)$

a) $u_{DSB-SC} = 10m(t)\cos(300\pi t)$

Energy of $m(t) = \int_{-\infty}^{\infty} |m(t)|^2 dt$

$= \int_{-\infty}^{\infty} |M(f)|^2 df$ (Parseval's)

$= \int_{-2}^{2} 1\, df$

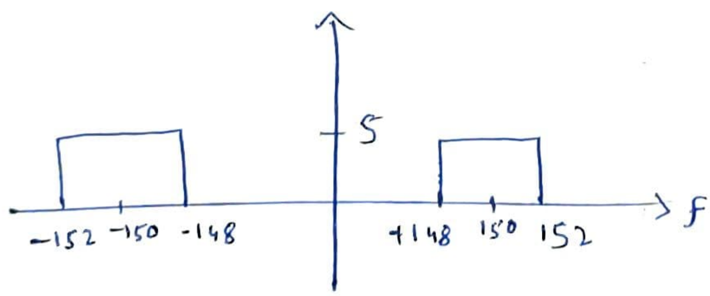
$= 4$

Since an energy signal (finite energy) cannot be a power signal, we can say that.

Power of the signal $= \underline{\underline{0}}$

$U(f) = 5M(f) * (\delta(f-150) + \delta(f+150))$

$= 5(M(f-150) + M(f+150))$



$\boxed{\text{Bandwidth} = 2 \times 2 = 4\,Hz}$

b) $u_{DSB-SC} = 10m(t)\cos(300\pi t)$

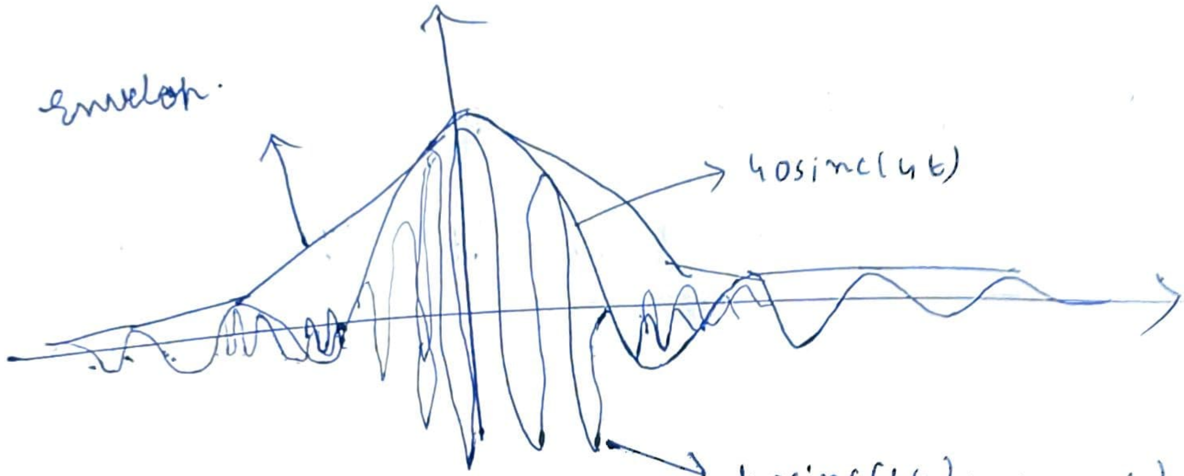
We know $\text{sinc}(Wt) \longleftrightarrow \frac{1}{W} I_{[-\frac{W}{2}, \frac{W}{2}]}(f)$

Here $W = 4$

$\Rightarrow m(t) = 4\,\text{sinc}(4t)$

$\therefore u(t) = 40 sinc(4t) \cos(300\pi t)$

We pass this signal through envelope detector.



The envelope detector is ~~take~~ RC circuit.

It just connects peaks.

$\therefore$ We do not find output to be exact message signal and info is lost.

However, we find that the peaks of ~~input~~ message signal is lost.

c) $u_{AM} = (A + m(t)) \cos(300\pi t)$

$\min_t m(t) = ?$

$m(t) = 40 sinc(4t)$

$= \dfrac{40 \sin(4\pi t)}{\pi t}$

$\min_t m(t) = -8.689$ (from internet)

We want $MI \leq 1$

$\Rightarrow \dfrac{A\,|-8.689|}{A_c} \leq 1$

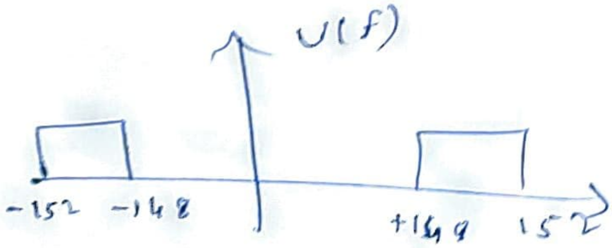
$\Rightarrow$ Here $A = 1$

$\Rightarrow A_c \geq 8.689$

$\Rightarrow \boxed{A \geq 8.689}$

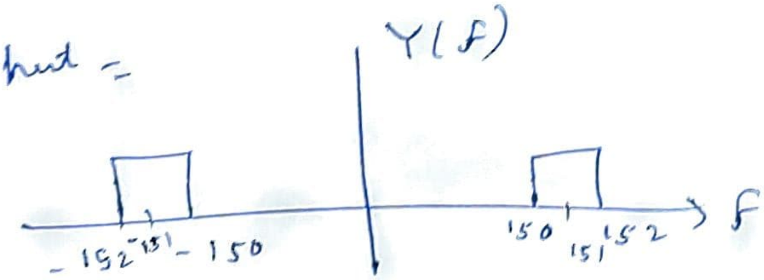
$A = 8.689$ is the smallest value that can be obtained without distortion.

d)



cutoff 150 Hz

$\Rightarrow$ output =



$$y(t) = I_{[-1,1]} \cos(2\pi \cdot 151 t)$$

$$= I_{[-1,1]} \cos(2\pi \cdot 150 t + 2\pi t)$$

$$= I_{[-1,1]} \left[\cos(2\pi t)\cos(2\pi\, 150 t) - \sin(2\pi t)\sin(2\pi \cdot 150 t)\right]$$

given $u_p(t) = u_c(t)\cos(300\pi t) - u_s(t)\sin(300\pi t)$

$$u_c(t) = I_{[-1,1]} \cos(2\pi t)$$

$$u_s(t) = I_{[-1,1]} \sin(2\pi t)$$