Assignment 1

Communication Theory - 1 (EC5.203 - Spring 2021)

January 16, 2021

Deadline: 25/01/2021 - 11:30 AM

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Submission Format:

- For analytical problems, write on A4 sheets and scan them in pdf format. For simulation code (if any) create simulation as folder name and add .m files. Submit .zip file (Rollnumber_Assignment1) containing pdf file and simulation folder.
- For simulation part, along with the codes, submit a report (pdf format) clearly depicting the generated plots (if any) with answers to questions asked as part of simulation exercise. State the parameter values used for simulation in the report clearly. Marks obtained will depend upon clarity in report writing.

Questions

1. Consider the signal $x(t)$ defined as $x(t) = e^{-at}$ for $t \geq 0$ and 0 otherwise. What is the bandwidth required to transmit 95% of the signal?
   Hint: It will be a function of 'a'.

2. Consider the tent signal $s(t) = (1 - |t|)I_{[-1,1]}(t)$.
   (a) Find and sketch the Fourier transform $S(f)$.
   (b) Compute the 99% energy containment bandwidth in KHz, assuming that the unit of time is milliseconds.

3. Let $x(t)$ and $y(t)$ be two periodic signals with period T_0 , and let x_n and y_n denote the Fourier series coefficients of these two signals.
   (a) Show that

$$\frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} x(t)\ y^*(t)\ dt = \sum_{n=-\infty}^{\infty} x_n\ y_n^*$$

   This relation is known as Parseval's relation for Fourier series. Show that Rayleigh's relation for periodic signals is a special case of this relation.
   Rayleigh's Relation:

$$\sum_{n=-\infty}^{\infty} \mid x_n \mid^2$$

   (b) Show that for all periodic physical signals that have finite power, the coefficients of the Fourier series expansion x_n tend to zero as $n \to \infty$.
   (c) Use Parseval's relation in part (a) to prove the following identity.

$$1 + \frac{1}{3^4} + \frac{1}{5^4} + ... + \frac{1}{(2n+1)^4} = \frac{\pi^4}{96}$$

   Hint: Find Fourier series expansion of $f(x) = x^2, x \in [-\pi, \pi]$ and then use Parseval's identity.

4. Determine the Fourier transform of each of the following signals.

   (a) $\text{sinc}^3\, t$

   (b) $t\,\text{sinc}\, t$

   (c) $t\ e^{-\alpha t}\cos(\beta t)$

5. Using the properties of the Fourier transform, evaluate the following integrals

   (a) $\int_0^\infty e^{-\alpha t}\,\text{sinc}^2(t)\ dt$

   (b) $\int_0^\infty e^{-\alpha t}\cos(\beta t)\ dt$

6. Consider the following two passband signals:

$$u_p(t) = \text{sinc}(2t)\cos(100\pi t)$$

   and

$$u_p(t) = \text{sinc}(t)\sin(101\pi t + \frac{\pi}{4})$$

   (a) Find the complex envelopes $u(t)$ and $v(t)$ for u_p and v_p respectively, with respect to the frequency reference $f_c = 50$ Hz.

   (b) What is the bandwidth of $u_p(t)$? What is the bandwidth if $v_p(t)$?

   (c) Find the inner product $\langle u_p\,, v_p\rangle$, using the result in (a).

   (d) Find the convolution $y_p(t) = (u_p * v_p)(t)$, using the result in (a).

7. The passband signal $u(t) = I_{[-1;1]}(t)\cos(100\pi t)$ is passed through the passband filter $h(t) = I_{[0;3]}(t)\sin(100\pi t)$. Find an explicit time-domain expression for the filter output.

8. Consider a passband signal of the form

$$u_p(t) = a(t)\cos(200\pi t)$$

   where $a(t) = \text{sinc}(2t)$, and where the unit of time is in microseconds.

   (a) What is the frequency band occupied by $u_p(t)$?

   (b) The signal $u_p(t)\cos(199\pi t)$ is passed through a lowpass filter to obtain an output $b(t)$. Give an explicit expression for $b(t)$, and sketch $B(f)$ (if $B(f)$ is complex-valued, sketch its real and imaginary parts separately).

   (c) The signal $u_p(t)\sin(199\pi)t$ is passed through a lowpass filter to obtain an output $c(t)$. Give an explicit expression for $c(t)$, and sketch $C(f)$ (if $C(f)$ is complex-valued, sketch its real and imaginary parts separately).

   (d) Can you reconstruct $a(t)$ from simple real-valued operations performed on $b(t)$ and $c(t)$? If so, sketch a block diagram for the operations required. If not, say why not.

MATLAB Coding Question

1. Consider a pair of independently modulated signals, $u_c(t) = \sum_{n=1}^{N} b_c[n]p(t-n)$ and $u_s(t) = \sum_{n=1}^{N} b_s[n]p(t-n)$, where the symbols $b_c[n], b_s[n]$ are chosen with equal probability to be $+1$ and -1, and $p(t) = I_{[0,1]}(t)$ is a rectangular pulse. Let $N = 100$.

   (a) Use Matlab to plot a typical realization of $u_c(t)$ and $u_s(t)$ over 10 symbols. Make sure you sample fast enough for the plot to look reasonably "nice."

   (b) Upconvert the baseband waveform $u_c(t)$ to get

$$u_{p,1}(t) = u_c(t)\cos 40\pi t$$

(c) This is a so-called binary phase shift keyed (BPSK) signal, since the changes in phase due to the changes in the signs of the transmitted symbols. Plot the passband signal $u_{p,1}(t)$ over four symbols (you will need to sample at a multiple of the carrier frequency for the plot to look nice, which means you might have to go back and increase the sampling rate beyond what was required for the baseband plots to look nice).

(d) Now, add in the Q component to obtain the passband signal

$$u_p(t) = u_c(t) \cos 40\pi t - u_s(t) \sin 40\pi t$$

Plot the resulting Quaternary Phase Shift Keyed (QPSK) signal $u_p(t)$ over four symbols.

(e) Downconvert $u_p(t)$ by passing $2u_p(t)\cos(40\pi t+\theta)$ and $2u_p(t)\sin(40\pi t+\theta)$ through crude lowpass filters with impulse response $h(t) = I_{[0,0.25]}(t)$. Denote the resulting I and Q components by $v_c(t)$ and $v_s(t)$, respectively. Plot v_c and v_s for $\theta = 0$ over 10 symbols. How do they compare to u_c and u_s? Can you read off the corresponding bits $b_c[n]$ and $b_s[n]$ from eyeballing the plots for v_c and v_s?

(f) Plot v_c and v_s for $\theta = \pi/4$. How do they compare to u_c and u_s ? Can you read off the corresponding bits $b_c[n]$ and $b_s[n]$ from eyeballing the plots for v_c and v_s?

(g) Figure out how to recover u_c and u_s from v_c and v_s if a genie tells you the value of θ (we are looking for an approximate reconstruction-the LPFs used in downconversion are non-ideal, and the original waveforms are not exactly bandlimited). Check whether your method for undoing the phase offset works for $\theta = \pi/4$, the scenario in $part(e)$. Plot the resulting reconstructions $\tilde{u}_c$ and $\tilde{u}_s$, and compare them with the original I and Q components. Can you read off the corresponding bits $b_c[n]$ and $b_s[n]$ from eyeballing the plots for $\tilde{u}_c$ and $\tilde{u}_s$?