

1) Small error PLL,

i) S.T. First-order loop cannot track incoming signal whose instantaneous frequency varying linearly with time.

$$\text{i.e. } \theta_i(t) = kt^2$$

For tracking incoming signal,

$$\lim_{s \rightarrow 0} s \theta_o(s) = 0$$

$$\theta_i(t) \longleftrightarrow \theta_i(s)$$

$$\text{We know that } t^n \longleftrightarrow \frac{n!}{s^{n+1}}$$

$$\Rightarrow \theta_i(s) = \frac{2k}{s^3}$$

For a first order system,

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{s}{s + kH(s)}$$

$$\begin{aligned} \text{For } H(s) &= 1 \\ \Rightarrow \theta_o(s) &= \frac{2k}{s^2(s+k)} \end{aligned}$$

— ①

$$\begin{aligned} \lim_{s \rightarrow 0} s \theta_o(s) &= \lim_{s \rightarrow 0} \frac{2k}{s(s+k)} \\ &= \infty (\neq 0) \end{aligned}$$

⇒ We cannot track the signal.

ii)

It can be tracked only when we have zero-phase error i.e. $H(s) = \frac{s^2 + as + b}{s^2}$

Substituting this in — (1),

$$\frac{\Theta_o(s)}{\frac{2k}{s^2}} = \frac{s}{s + k \left(\frac{s^2 + as + b}{s^2} \right)}$$

$$\Rightarrow \Theta_o(s) = \frac{2k}{s^2} \times \frac{s^2}{s^3 + k(s^2 + as + b)}$$

$$\Theta_o(s) = \frac{2k}{s^3 + k(s^2 + as + b)}$$

$$\lim_{s \rightarrow 0} s\Theta_o(s) = \lim_{s \rightarrow 0} \frac{2sk}{s^3 + ks^2 + kas + kb}$$

Numerator = 0 & denominator is kb

$$\Rightarrow \boxed{\lim_{s \rightarrow 0} s\Theta_o(s) = 0}$$

⇒ We can track the signal.

2) Given,
Dual band radio operates at 900 MHz and 1.8 GHz
channel spacing in each band is 1 MHz .

Superheterodyne receiver with IF 250 MHz .

LO uses synthesizer tunable from 1.9 GHz to 2.25 GHz .

a) Superhet receiver should receive passband
restricted to $1800 \text{ MHz} - 1801 \text{ MHz}$.

$$f_{RF} = 1.8 \text{ to } 1.801 \text{ GHz} \quad \text{Avg } f_{RF} = 1.8005 \text{ GHz}$$

We know that $f_{IF} = |f_{RF} - f_{LO}|$.

$$\Rightarrow 0.25 = |1.8 \text{ to } 1.801 - f_{LO}|$$

$$\Rightarrow f_{LO} = 2.05 \text{ to } 2.051 \quad \text{or} \quad f_{LO} = 1.55 \text{ to } 1.551$$

Given that we ~~have~~ use 1.9 GHz to 2.25 GHz

$$\therefore \boxed{f_{LO} = 2.05 \text{ to } 2.051 \text{ GHz}}$$

$$\text{Avg } f_{LO} = 2.0505 \text{ GHz}$$

~~We have channel spacing of f_{IM}~~

$f_{IM} = f_{LO} + f_{IF}$ where f_{IM} is frequency of image

$$\Rightarrow f_{IM} = 2.05 \text{ to } 2.051 + 0.25 \text{ GHz}$$

$$\Rightarrow \boxed{f_{IM} = 2.3 \text{ to } 2.301 \text{ GHz}}$$

On average, $f_{IM} = 2.3005 \text{ GHz}$

RF Filter: should pass 1800 - 1801 MHz

\Rightarrow centre is 1800.5 MHz and allows 1800 - 1801 MHz ~~this signal but~~
and rejects image frequency 2300 - 2301 MHz
so the ^{bandwidth can} ~~cut-off should~~ be ideally 2-5 MHz
from 1800.5 MHz

IF Filter: It is at 250 MHz & should pass the message
in 249.5 to 250.5 with a sharp cut-off
beyond.

b) $f_{RF} = 900 \text{ to } 901 \text{ MHz}$, $\text{avg } f_{RF} = 900.5 \text{ MHz}$

We know that $f_{IF} = |f_{RF} - f_{LO}|$

$$250 = |900 \text{ to } 901 \text{ MHz} - f_{LO}|$$

$$\Rightarrow f_{LO} = 1150 \text{ to } 1151 \text{ MHz} \text{ OR } f_{LO} = 650 \text{ to } 651 \text{ MHz}$$

Range 1.9 GHz to 2.25 GHz.

$$\Rightarrow f_{LO} = 0.65 - 0.651 \text{ GHz} \quad (\text{because it is a multiple})$$

Now image frequency is $f_{LO} - f_{IF}$ as RF is at $f_{LO} + f_{IF}$

$$\Rightarrow f_{IM} = (0.65 \text{ to } 0.651) - 0.250 \text{ GHz}$$
$$= 0.4 \text{ to } 0.401 \text{ GHz}$$

RF Filter: should pass 900 - 901 MHz and
reject f_{IM} which is around 400 MHz.
 \Rightarrow centre is at 900.5 MHz and bandwidth can be
~~cut-off is~~ ideally 2-5 MHz ~~bandwidth~~

IF Filter: It is at 250 MHz & should pass the
message in 249.5 to 250.5 with a
sharp cut-off beyond.