Assignment 5

Communication Theory - 1 (EC5.203 - Spring 2021)

March 15, 2021

Deadline: 20/03/2021 - 11:00 PM for Questions 1-4 and 29/03/2021 - 11:00 PM for Matlab Question - 5

Please do not copy from your peers or online. Copied assignments will fetch 0 marks.

Submission Format:

- For analytical problems, write on A4 sheets and scan them in pdf format. For simulation code (if any) create simulation as folder name and add .m files. Submit .zip file (Rollnumber_Assignment5) containing pdf file and simulation folder.
- For simulation part, along with the codes, submit a report (pdf format) clearly depicting the generated plots (if any) with answers to questions asked as part of simulation exercise. State the parameter values used for simulation in the report clearly. Marks obtained will depend upon clarity in report writing.

Questions

1. Consider the pulse
$$p(t) = \begin{cases} t/a, & 0 \leq t \leq a \\ 1, & a \leq t \leq 1-a \\ (1-t)/a, & 1-a \leq t \leq 1 \\ 0, & \text{else} \end{cases}$$
where $0 \leq a \leq \frac{1}{2}$.
   (a) Sketch $p(t)$ and find its Fourier transform $P(f)$.
   (b) Consider the linearly modulated signal $u(t) = \sum_n b[n]p(t-n)$, where $b[n]$ take values independently and with equal probability in a 4 -PAM alphabet $\{\pm1,\pm3\}$. Find an expression for the PSD of u as a function of the pulse shape parameter a.
   (c) Numerically estimate the 95% fractional power containment bandwidth for u and plot it as a function of $0 \leq a \leq \frac{1}{2}$. For concreteness, assume the unit of time is 100 picoseconds and specify the units of bandwidth in your plot.

2. Consider linear modulation with a signaling pulse $p(t) = \text{sinc}(at)\,\text{sinc}(bt)$, where a and b are to be determined.
   (a) How should a and b be chosen so that $p(t)$ is Nyquist with 50% excess bandwidth for a data rate of 40 Mbps using 16QAM? Specify the occupied bandwidth.
   (b) How should a and b be chosen so that $p(t)$ can be used for Nyquist signaling both for a 16QAM system with 40 Mbps data rate, and for an 8PSK system with 18 Mbps data rate? Specify the occupied bandwidth.

3. In this problem, we derive the time domain response of the frequency domain raised cosine pulse. Let $R(f) = I_{[-\frac{1}{2},\frac{1}{2}]}(f)$ denote an ideal boxcar transfer function, and let $C(f) = \frac{\pi}{2a}\cos\left(\frac{\pi}{a}f\right) I_{[-\frac{a}{2},\frac{a}{2}]}$ denote a cosine transfer function.
   (a) Sketch $R(f)$ and $C(f)$, assuming that $0 < a < 1$.

(b) Show that the frequency domain raised cosine pulse can be written as

$$S(f) = (R * C)(f)$$

(c) Find the time domain pulse $s(t) = r(t)c(t)$. Where are the zeros of $s(t)$? Conclude that $s(t/T)$ is Nyquist at rate $1/T$

(d) Sketch an argument that shows that, if the pulse $s(t/T)$ is used for BPSK signaling at rate $1/T$, then the magnitude of the transmitted waveform is always finite.

4. Any pulse timelimited to duration T is square root Nyquist (up to scaling) at rate $1/T$. Find whether this statement is True or False with valid explanation.

5. [MATLAB Simulation - Nyquist and square root Nyquist pulses]

   (a) Run the code fragment [Present on Madhow Textbook, Code Fragment 4.B.1 (Sampled raised cosine pulse) on Page 193] for $25\%, 50\%$ and 100% excess bandwidths and plot the time domain waveforms versus normalized time t/T over the interval $[-5T, 5T]$, sampling fast enough (e.g. at rate $32/T$ or higher) to obtain smooth curves. Comment on the effect of varying the excess bandwidth on these waveforms.

   (b) For excess bandwidth of 50%, numerically explore the effect of time domain truncation on frequency domain spillage. Specifically, compute the Fourier transform for two cases: truncation to $[-2T, 2T]$ and truncation to $[-5T, 5T]$, using the DFT as described in code fragment [Refer Madhow TextBook, Code Fragment 2.5 .1, Page 59] to obtain a frequency resolution at least as good as $\frac{1}{64T}$. Plot these Fourier transforms against the normalized frequency fT, and comment on how much of increase in bandwidth, if any, you see due to truncation in the two cases.

   (c) Numerically compute the 95% bandwidth of the two pulses in (b), and compare it with the nominal bandwidth without truncation.