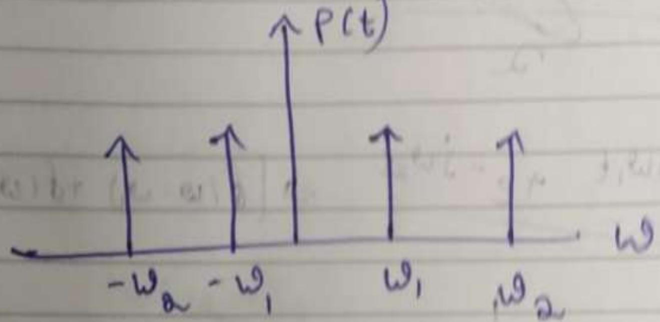
8.1 a)

$$p(t) = \cos(\omega_1 t) + \cos(\omega_2 t)$$

$$P(\omega) = \pi(\delta(\omega-\omega_1) + \delta(\omega+\omega_1) + \delta(\omega-\omega_2) + \delta(\omega+\omega_2)$$

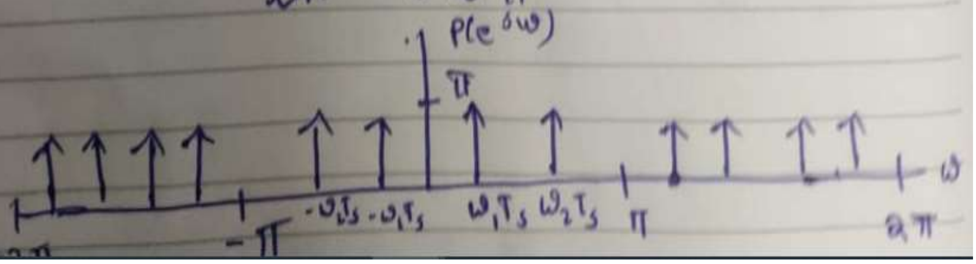


b) $P(e^{j\omega})$ of $p[n]$

$$p[n] = \cos(\omega_1 n T_s) + \cos(\omega_2 n T_s)$$

$$P(e^{j\omega}) = \pi(\delta(\omega-\omega_1 T_s) + \delta(\omega+\omega_1 T_s) + \delta(\omega-\omega_2 T_s) + \delta(\omega+\omega_2 T_s))$$

in $-\pi$ to π



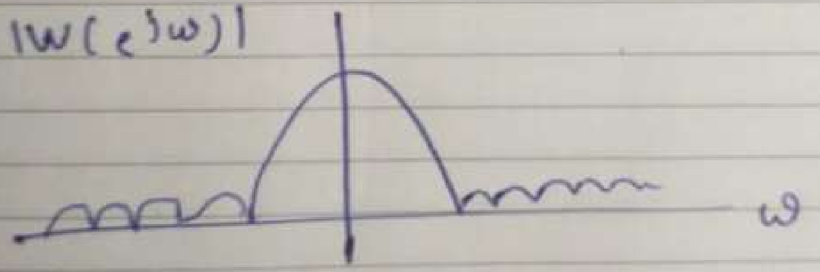
c) $x[n] = p[n] \cdot w[n]$

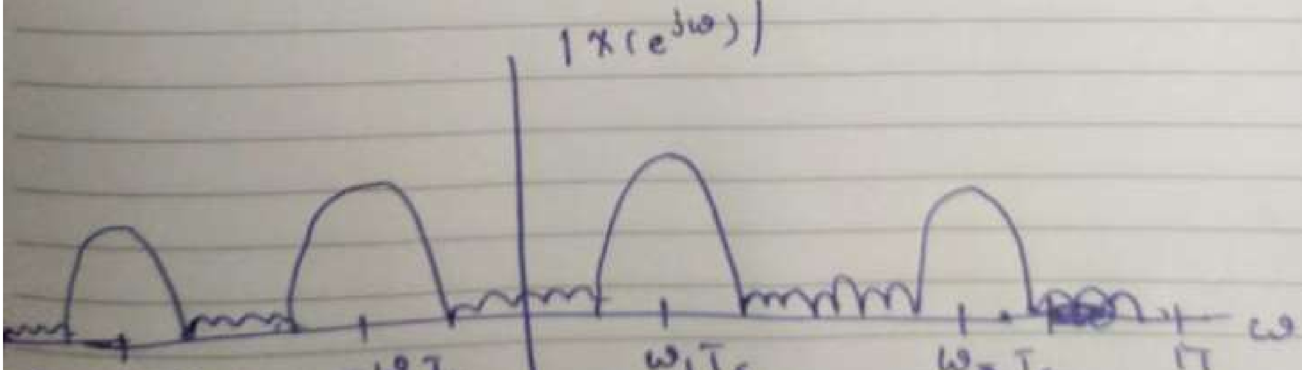
$$X(e^{j\omega}) = P(e^{j\omega}) * W(e^{j\omega})$$

$$w[n] = 1 \qquad 0 \le n \le L-1$$

$$W(e^{j\omega}) = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega\left(\frac{L}{2}-1\right)}$$

Now convolution gives $W(e^{j\omega})$ at those 4 points





8.1 d)

We notice that the graph compresses as we increase L value.

8.1 e)

We observe only two sinc waves as w1 and w2 are very close.

L affects the compression and frequency resolution won't change as N is not changing

8.1 g) We have the plot of DFT. We notice where the peaks are.

W1*Ts and W2*Ts are values at those peaks. As we are given ws, we can calculate Ts and

from that we can get w1 and w2 values.

For example N=1000, L=100 plot, one peak is ~130=w1*Ts

Given ws=8000 =>Ts=2*pi/8000

=>w1*Ts=130 in k scale.

=>w1*Ts=130*pi/500 in w scale

=> w1*2*pi/8000 =130*pi/500

=> **w1=130*8=940**

Actual w1 value is ws/8=1000

So our approximation is **correct**.

Similarly we calculate w2*2*pi/8000=370*pi/500 (RHS is value from graph)

=>**w2=370*8=3030**

Actual w2 value is 3ws/8=3000

So our approximation is **correct.**

8.1 f) The procedure is same as mentioned above. So we can easily make an approximation from the graph

8.2 c)

We need to find the time complexity of fft

Let us assume the method used is radix 2 fft.

We have logN(base2) stages and in each stage we have N/2 multiplications and N additions.

For that we have N/2 *logN(base 2) multiplications and N*logN(base 2) additions.

So the total complexity is O(NlogN(base2)).

8.2 d)

For direct convolution in time, we have N multiplications and N-1 additions.

So the complexity is N*(N-1)=N^2.