

Simulating Einstein's Special Relativity Using R Tilde Matrix and MATLAB

Lorentz Transformation

$$\begin{aligned} t' &= \gamma \left(t - \frac{ux}{c^2} \right) \\ x' &= \gamma (x - ut) \\ y' &= y \\ z' &= z \end{aligned}$$

$$L = \begin{pmatrix} \gamma & -u\gamma & 0 & 0 \\ -u\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Linear Transformation?

$$\bar{R}_\theta = \begin{pmatrix} \cosh \theta & \sinh \theta & 0 \\ -\sinh \theta & \cosh \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Using hyperbolic function?

Euler Rotation Mat.

$$R_\theta = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R Tilde Matrix

$$\tilde{R}_\theta = \begin{pmatrix} \cosh \theta & -\sinh \theta \\ -\sinh \theta & \cosh \theta \end{pmatrix}$$

$$\tilde{R}_{\theta'} = \begin{pmatrix} \cosh \theta' & -\sinh \theta' & 0 \\ -\sinh \theta' & \cosh \theta' & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where

$$\theta' = \cosh^{-1} \gamma$$

Analogies to
Rotation Mat.

$$\det(\tilde{R}_\theta) = 1$$

$$\tilde{R}_{\theta_1} \tilde{R}_{\theta_2} = \tilde{R}_{\theta_1 + \theta_2}$$

R Tilde Rotating Curves

Line

$$\begin{aligned} ax + by + c &= 0 \\ \text{if } a \neq \pm b, \text{ wlog,} \\ \sinh \theta_0 x + \cosh \theta_0 y + c &= 0 \\ \text{or} \\ \cosh \theta_0 x + \sinh \theta_0 y + c &= 0 \end{aligned}$$

Line to Line

$$\begin{aligned} \sinh(\theta_0 - \theta)x' + \cosh(\theta_0 - \theta)y' &= 0 \\ \cosh(\theta_0 - \theta)x' + \sinh(\theta_0 - \theta)y' &= 0 \end{aligned}$$

Circle

Unit Circle Equation :

$$x^2 + y^2 = 1$$

Circle to Ellipse

$$\begin{aligned} x'^2 (\cosh \theta + \sinh \theta)^2 \\ + y'^2 (\cosh \theta - \sinh \theta)^2 &= 1 \end{aligned}$$

Hyperbola

Unit Hyperbola Equation :

$$x^2 - y^2 = 1$$

Hyperbola to Hyperbola

$$x'^2 - y'^2 = 1$$

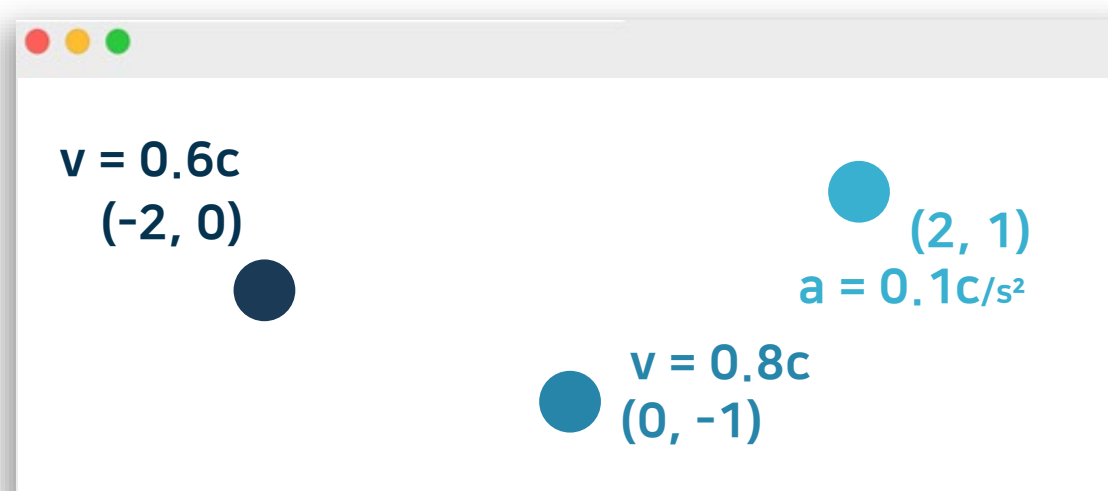
Linear Transformation Using R Tilde Matrix

$$O'_i = R_\theta \times \tilde{R}_{\theta'} \times R \times (O_i - S)$$

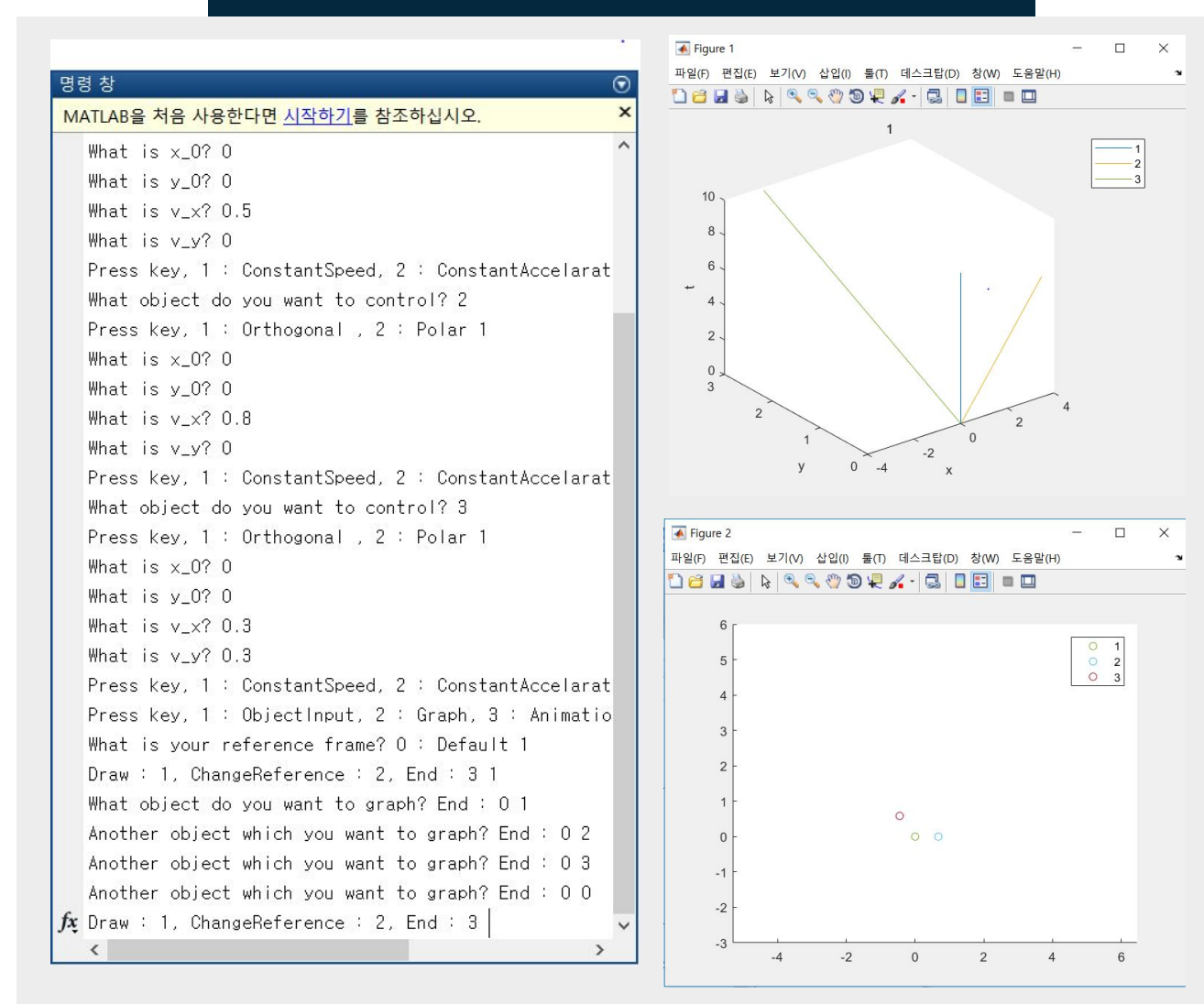
SRE Simulation

Objective

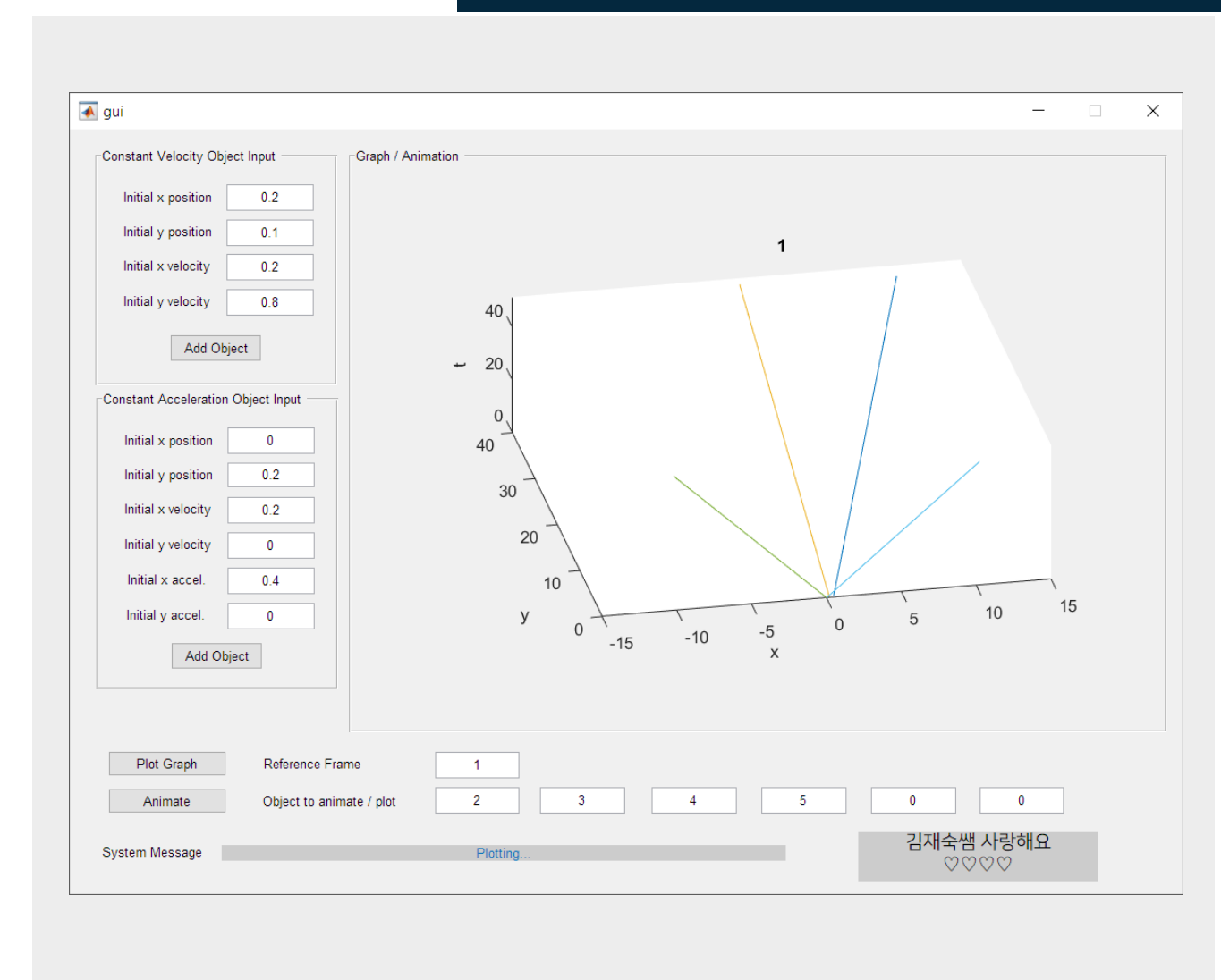
- Number of objects to simulate?
- (Relative) Position of each of the objects?
- Under what type of motion(const. acc / const. vel)?
- (Relative const.) Velocity of each of the objects?
- (Relative const.) Acceleration of each of the objects?
- Viewed from which Frame of Reference?



Text-Based



GUI



Conclusion of Research

- Finding Properties of R Tilde Matrix; especially similarities with Euler rotation matrix
- Make a program with MATLAB; Can draw Minkowski Diagram and Animate many objects
- Can help people understand many non-intuitive cases in SRE; especially with Minkowski Diagram
- Find mathematical properties of Lorentz transformation by R Tilde Matrix
- Get a generalized form of Lorentz transformation Matrix with R Tilde Matrix