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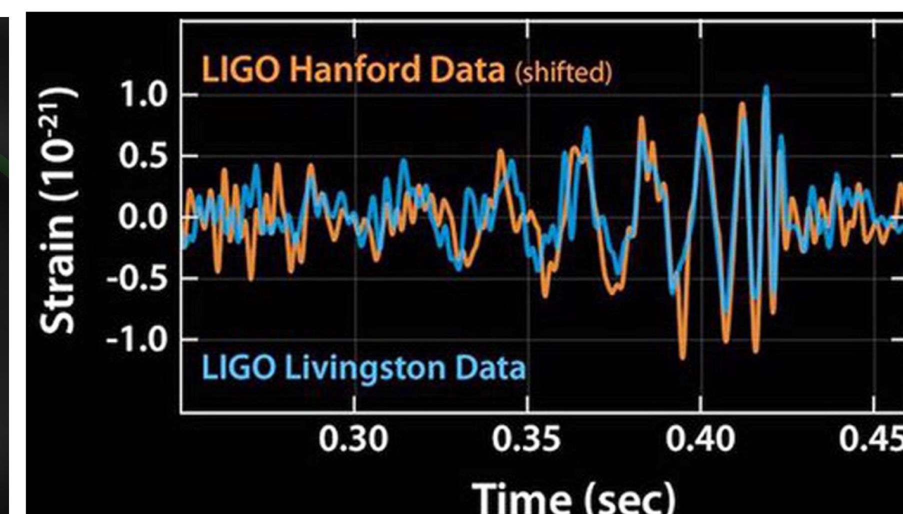
General Relativity
and
Sources of Gravitational Waves

Sascha Husa, UIB

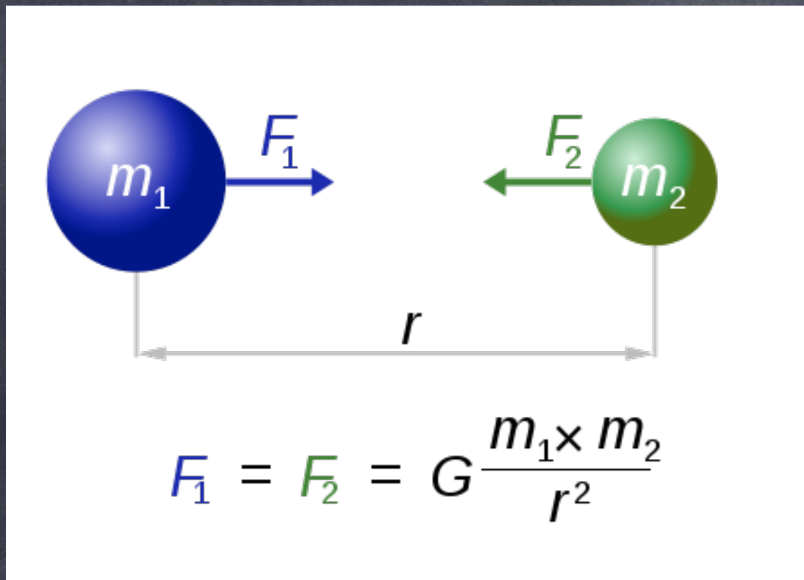
Santander 3/07/2019

Govern de les Illes Balears | Gobierno de España Ministerio de Economía y Competitividad | IEEC | UIB GRG

Selected References

- Textbooks on GR:
  Wald, Hartle, Carroll, …
- Textbooks on GWs:
  - Michele Maggiore
  - Creighton & Anderson
- Living Reviews in Relativity
  e.g. Sathyaprakash & Schutz,
  Physics, Astrophysics and Cosmology with Gravitational Waves
- arXiv.org
- *Wikipedia!*

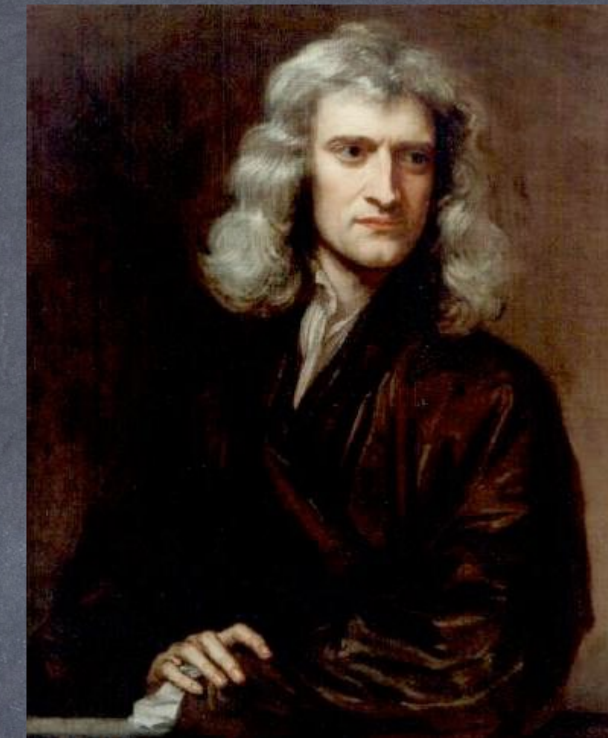
La Gravedad de Newton



$$m\ddot{\vec{x}} = -m\vec{\nabla}\phi$$

$$\vec{F} = -m\vec{\nabla}\phi$$

$$\Delta\phi = 4\pi G\rho(\vec{x}, t)$$



La gravedad funciona instantáneamente.

Las leyes de la gravedad de Newton son completamente análogas a las de la electrostática.

Einstein 1905: La velocidad de la luz es universal, información no puede propagar mas rápido que la luz
=> en realidad cambios en el campo gravitatorio no no pueden ser instantáneos! Necesitamos un teoría **relativista** de la gravedad!

¿Podemos generalizar de una manera similar el electromagnetismo? Gravitomagnetismo?

La Gravedad de Newton - 2

La fuerza electromagnética es mucho más fuerte que la fuerza gravitacional.

Para 2 protones con G = 6.67 x 10^{-11} m^3 kg^{-1} s^{-2}:

$$\frac{F_{\rm grav}}{F_{\rm elec}} = \frac{G m_{\rm proton}^2 / r^2}{e^2/(4\pi\epsilon_0 r^2)} \approx 10^{-36}$$

Objetos grandes (e.g. astrofísicos) tienden a ser eléctricamente neutro.

El universo parece ser eléctricamente neutro, y cualquier cargo neto a gran escala es neutralizado rápidamente, debido a que las fuerzas electromagnéticas son tan fuertes.

La gravedad es la interacción más débil conocida, per es **universalmente atractiva**, y es la **fuerza dominante** en el universo.

la fuerza que gobierna el universo

Cuatro fuerzas fundamentales:
- Fuerzas nucleares:
  fuerte (QCD), débil (QFD & unificación electrodébil),
- Electromagnetismo,
- La gravedad

Las fuerzas nucleares pueden considerarse una generalización no lineal del electromagnetismo con rango finito. EM y la gravedad tienen un alcance infinito, la fuerza disminuye con $1/r^2$.

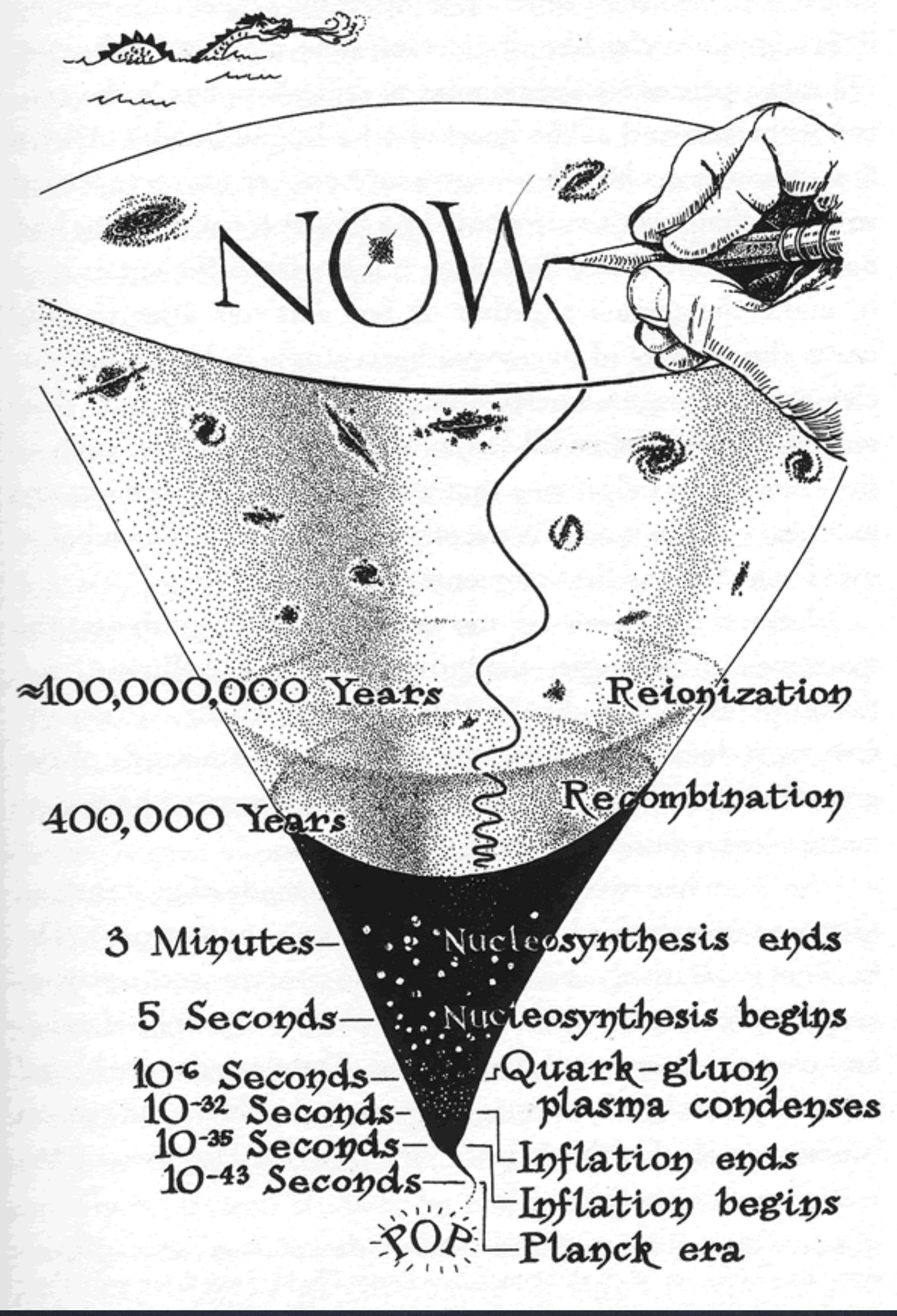
No hay cargos gravitacionales negativos - masa es siempre positivo, por lo tanto, no es posible blindar la interacción gravitatoria. La gravedad es siempre atractiva.

Vamos a aprender de la relatividad especial que la masa es una forma de energía:

$$E = mc^2$$

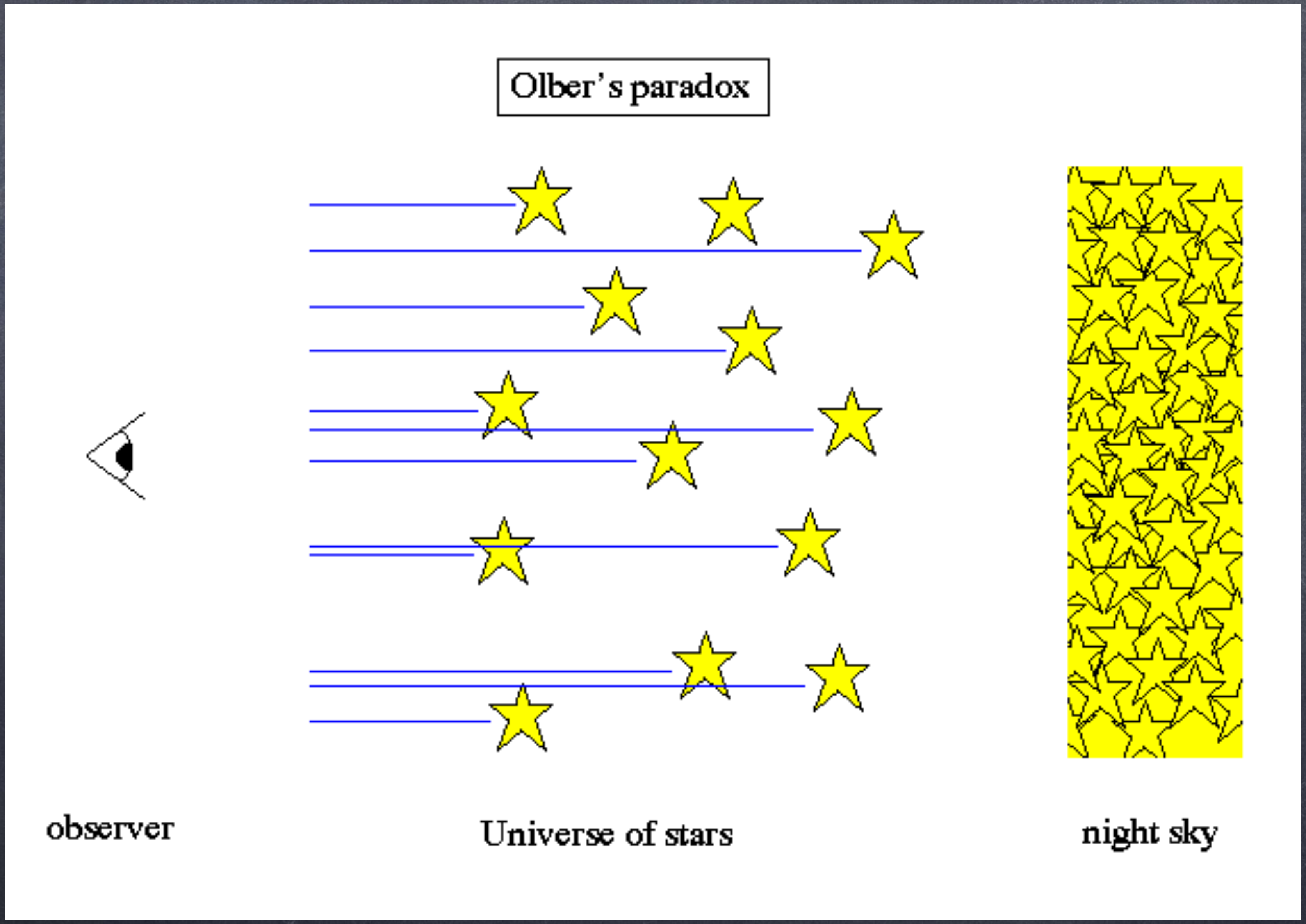
La gravedad es una interacción universal, entre todos los medios y formas de energía.

THE UNIVERSE SO FAR

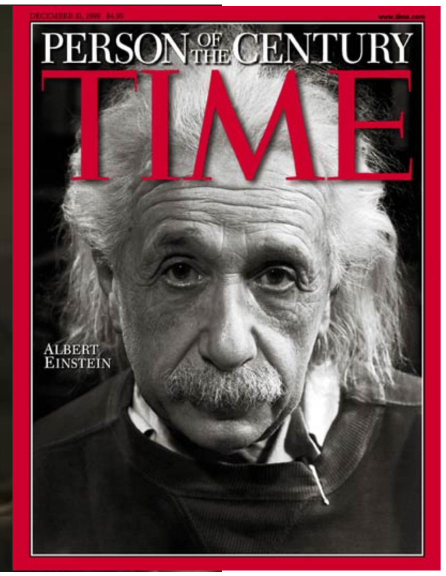


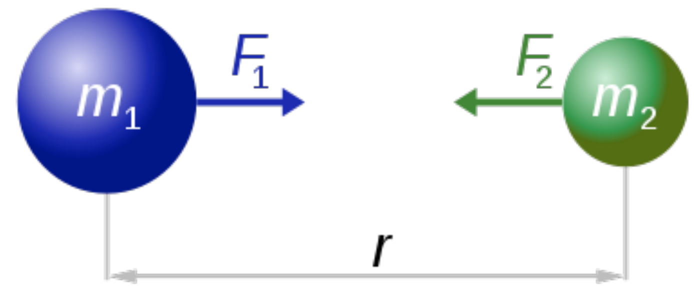
Paradoja de Olbers

Si el universo es infinito, ¿por qué el cielo es oscuro por la noche?

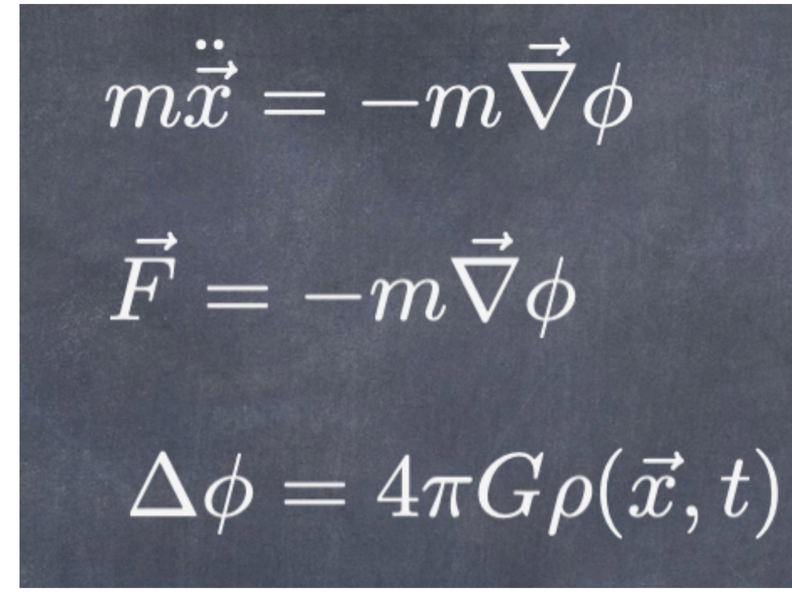


From Newton to Einstein

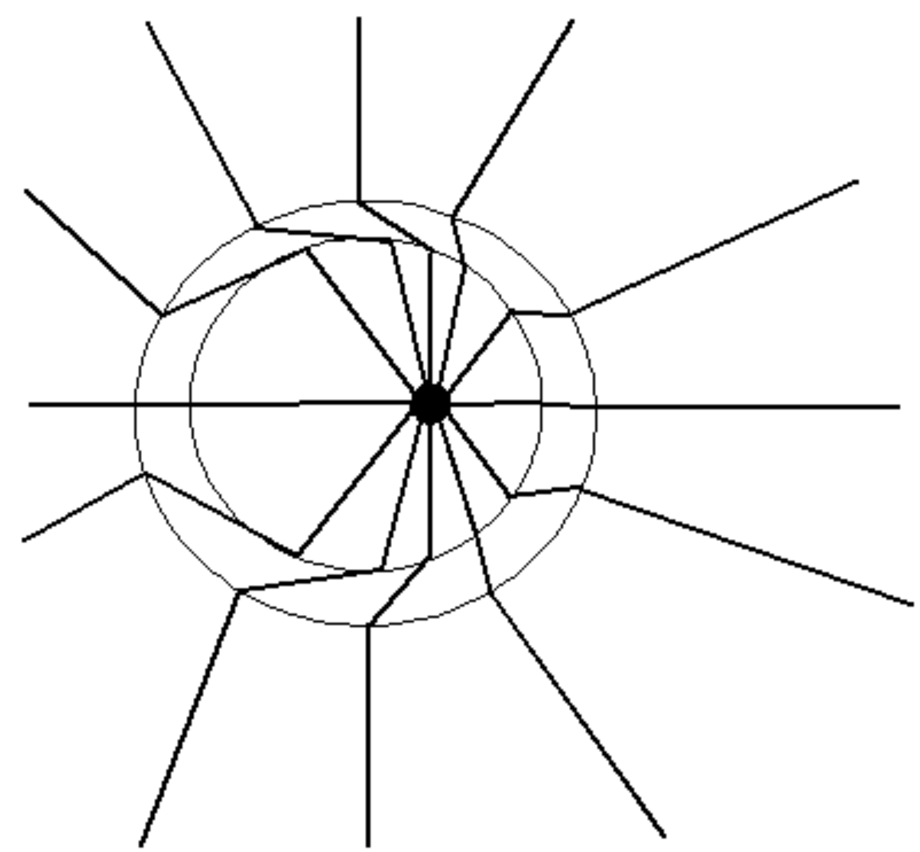




$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$



- 1905: Speed of light is the same for all inertial observers, information travels at finite speed.
- Need to change our theory of gravity!
    - 1915: theory of general relativity.
- Accelerated electric charges: at larger distance from the charge the field lines get updated later - outward propagating disturbance = EM wave.
- Finite propagation speed in GR -> expect GW.



RG es la teoría clásica de la gravedad

- No vamos a hablar de los efectos cuánticos. Por el momento no tenemos una teoría cuántica de la gravitación convincente. La gravitación es la única interacción fundamental para que no tenemos una teoría cuántica.

- RG es fundamental para la comprensión de los fenómenos fronterizos astrofísicos como los agujeros negros, pulsares, cuásares, el destino final de las estrellas, el Big Bang, y la historia del universo.

- RG describe las desviaciones pequeñas del movimiento de los planetas y los satélites de las leyes de Newton, y es necesaria para el funcionamiento del GPS.

- La gravedad es fundamental para la búsqueda de una teoría unificada de todas las interacciones.

gravedad cuántica

- G, c, $\hbar$! - No existe una teoría aceptada de la gravedad cuántica.
- Esperamos que los efectos cuánticos son importantes para la geometría del espacio-tiempo en la escala de Planck:
- $l_{Pl} = (G\,\hbar/c^3)^{1/2} = 1.62 \times 10^{-33}$ cm
- $t_{Pl} = (G\,\hbar/c^5)^{1/2} = 5.39 \times 10^{-44}$ s
- $E_{Pl} = (\hbar\, c^5/G)^{1/2} = 1.22 \times 10^{19}$ GeV (~ 4×10^7 de la energía de los rayos cósmicos con la energía más alta detectados hasta ahora)

Las dos fronteras de la gravedad

- Importante tanto para las escalas mayores y menores considerados en la física contemporánea.
- Escalas más grandes: la astrofísica y la cosmología.
- Escalas más pequeñas: la física de partículas cuánticas y elemental.
- Ambas escalas convertido en uno en el Big Bang.
- http://scaleofuniverse.com/
- En las fronteras de la física teórica y tambien experimental (la detección de ondas gravitacionales, así como la física del LHC, y experimentos de astropartículas).
- Observamos los rayos cósmicos con energías cinéticas de más de 10^{20} eV (~ béisbol). ¿Cuál es el origen de estas partículas?

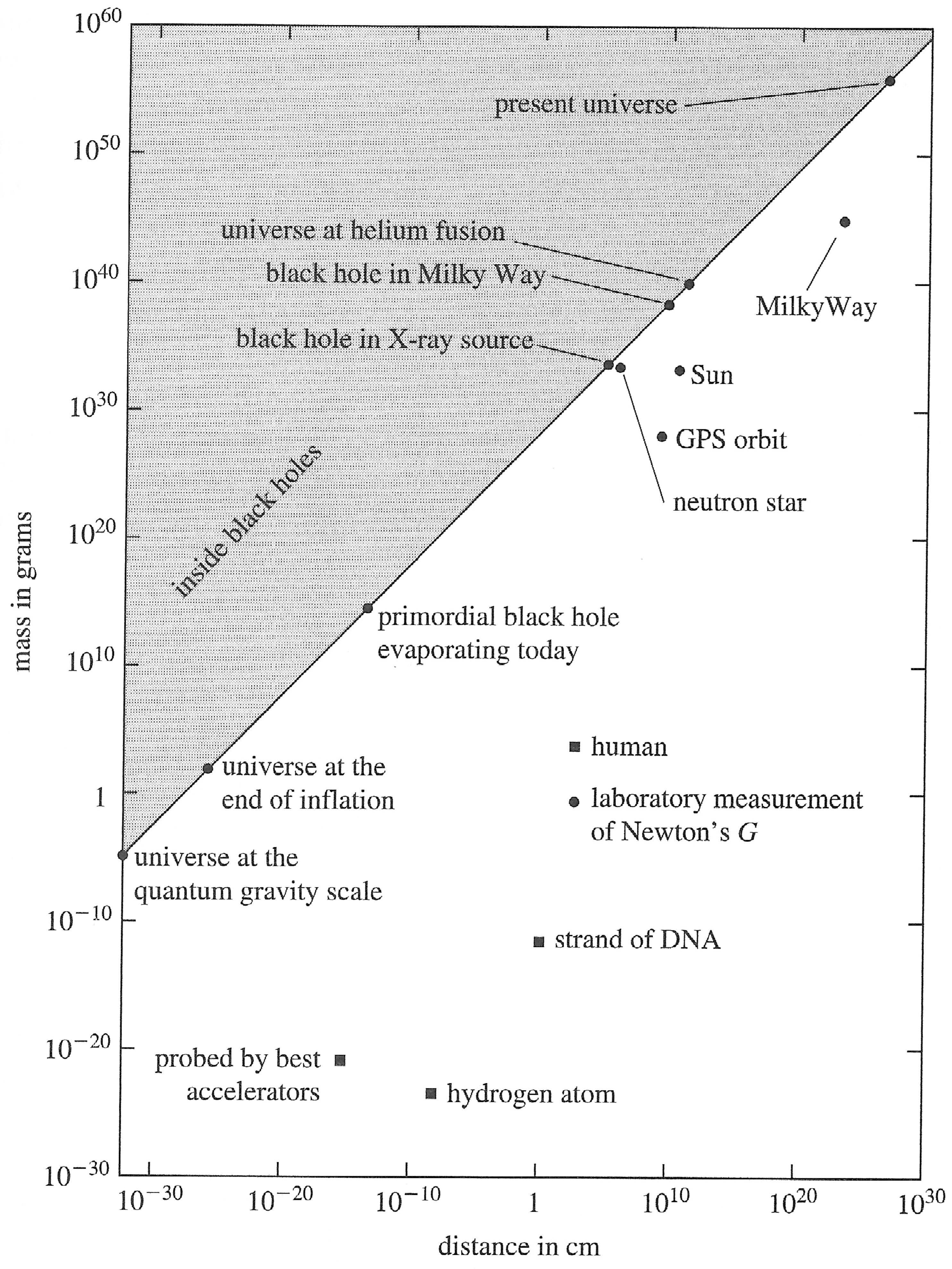
Gravedad relativista

Podemos usar la cantidad

$$\frac{2GM}{Rc^2}$$

como medida de la importancia de los efectos relativistas para el campo gravitacional.

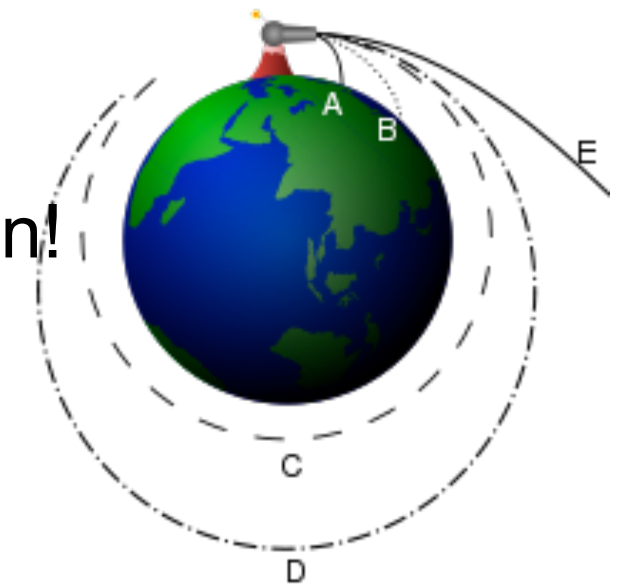
La gravedad Newtoniana se vuelve insuficiente, cuando se convierte en una fracción significativa de la unidad.



Gravitación en el sistema solar

- Tierra: $G M/R c^2 \approx 10^{-9}$ - no relativista en absoluto!
  - Pero: esta es la precisión requerida por el sistema de posicionamiento global (GPS)
- Sol: $G M/R c^2 \approx 10^{-6}$ - los efectos relativistas sobre las órbitas de los planetas son pequeños, pero detectable.
  - Mayor efecto: precesión del perihelio de Mercurio. Aproximación más cercana de Mercurio al Sol se desplaza en el tiempo - una de las primeras pruebas de RG!
  - RG predice que las trayectorias de los rayos de luz se curvan cerca del sol (recuerde $E = M c^2$). También el tiempo de los aumentos de pasaje - el tiempo se ralentiza en un campo gravitatorio. Necesidad de tener en cuenta tanto los efectos de la astronomía de precisión!

Desafiando la gravedad



Can we escape the earth's gravitational pull ? Newton's cannon!

Need to move at 40 000 km/h, in general

$$\frac{mv^2}{2} - \frac{GmM}{r} = 0 \longrightarrow v_{\rm esc} = \sqrt{\frac{2GM}{r}}$$

But can anything move faster than the speed of light?

Mitchell (1784) & Laplace (1796):
Light must fall back to the surface, if

$$R \approx 3 \frac{Mass}{Mass_{\rm sol}} km$$

The critical radius for the earth is 9 mm.

If the sun would stretch out to 40% of Jupiter's orbit at the same density, it would form such a “dark body”.

When nothing can stop gravity's pull

… and consequently, largest bodies in the universe could remain invisible to us … there exist, in the immensity of space, opaque bodies as considerable in magnitude, and perhaps equally as numerous as stars.

Pierre Simon Laplace, Exposition du système du Monde (1798/1799)

Stars are balls of gas – heated by nuclear burning of “lighter” elements.

If the fuel runs out – could a star shrink so much that it absorbs all light?

What stops the gravitational collapse of clouds of gas, or clusters of stars?

The universe could be filled with invisible dead stars!

Are stars dying quietly?

We need relativity to understand what happens when objects collapse under their own weight!

Teorema Virial

En un sistema estable, con fuerzas derivadas de una energía potencial entre las partículas (<,> promedio de tiempo):

$$V(r) = ar^n \qquad 2\langle T \rangle = n \langle V \rangle$$

Gravedad: n = -1:

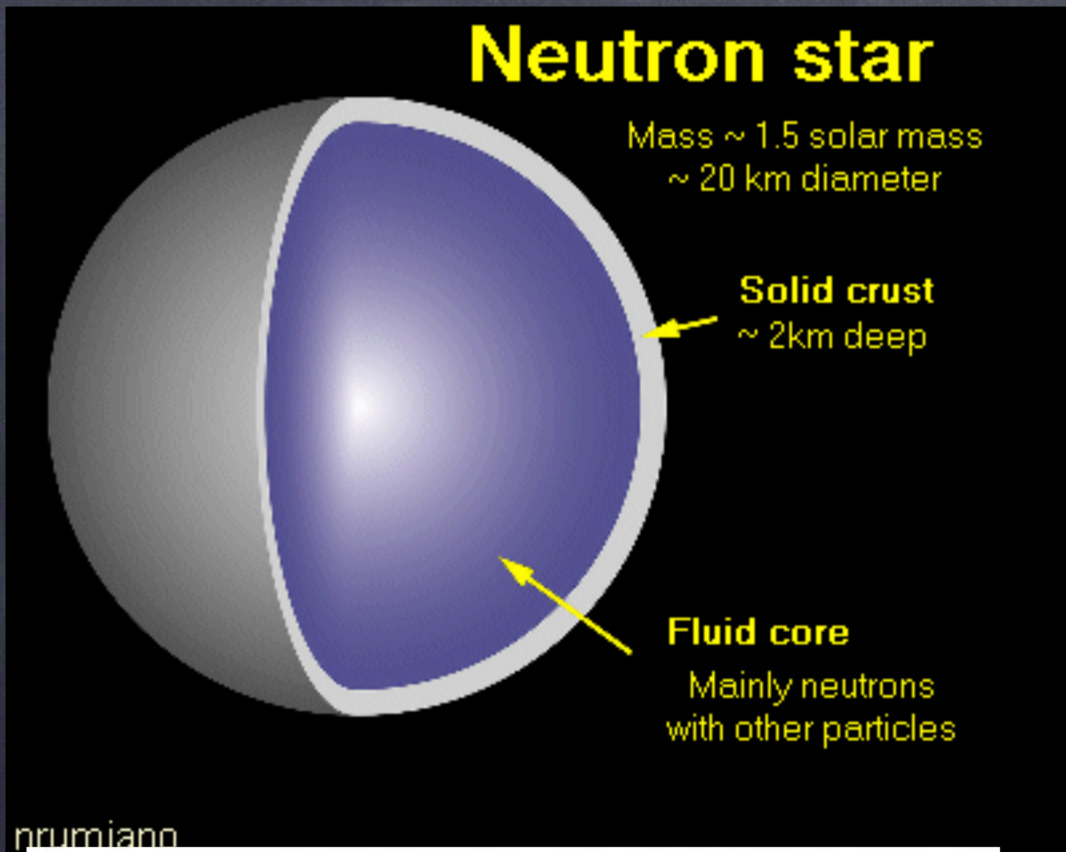
$$2\langle T \rangle = -\langle V \rangle = \frac{GM}{r}$$

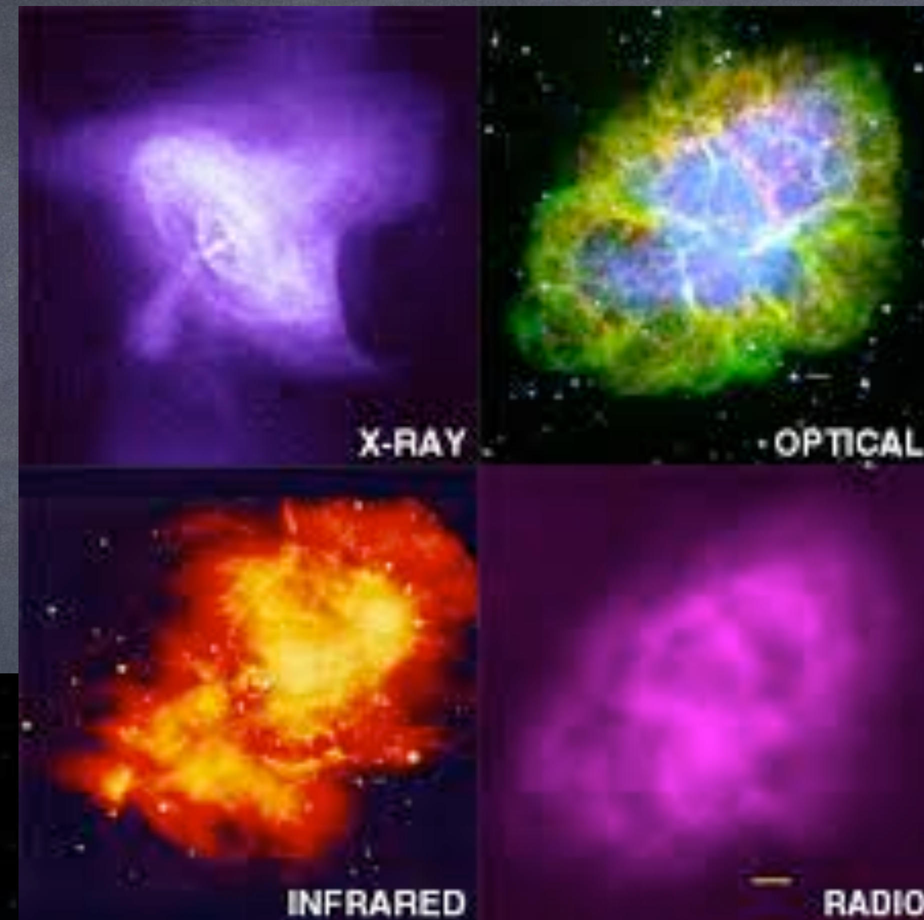
Cuando un "cuerpo" se comprime, la gravedad se hace más fuerte! ¿Qué impide que las estrellas y las galaxias colapsen bajo su propio peso?

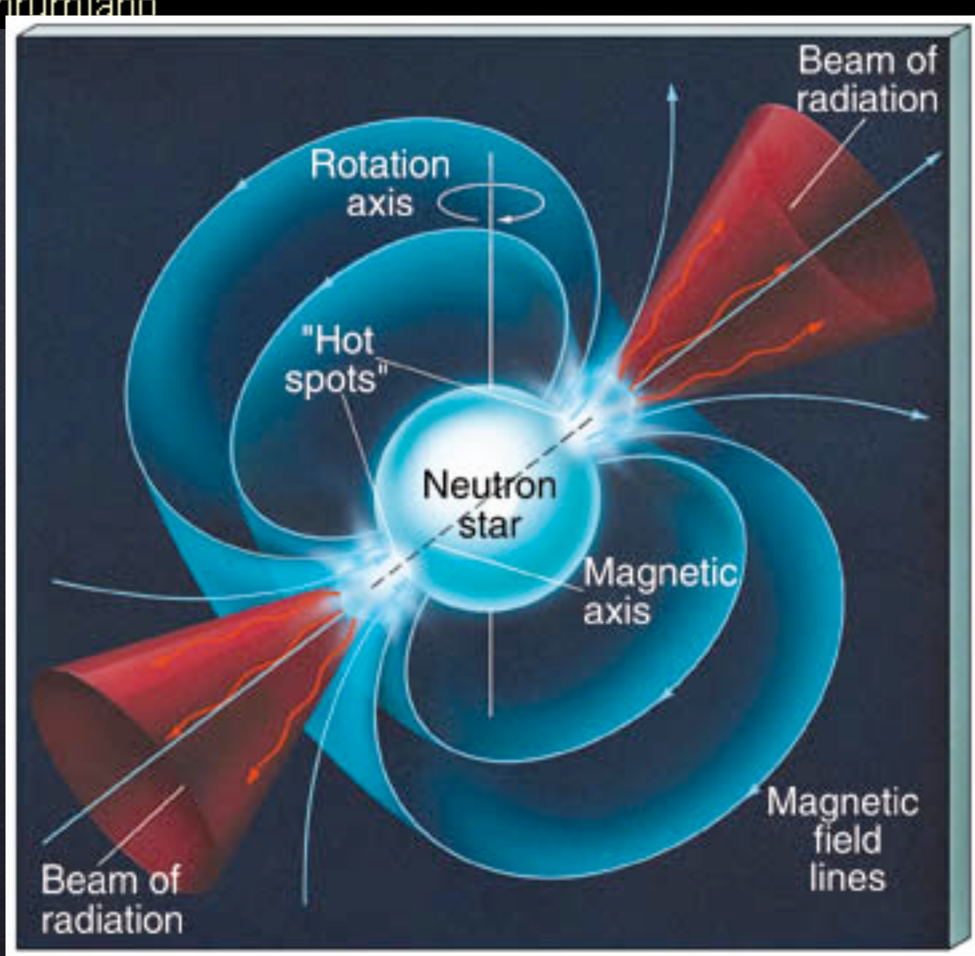
La compresión también aumenta la temperatura - este calor enciende las reacciones nucleares en las estrellas!

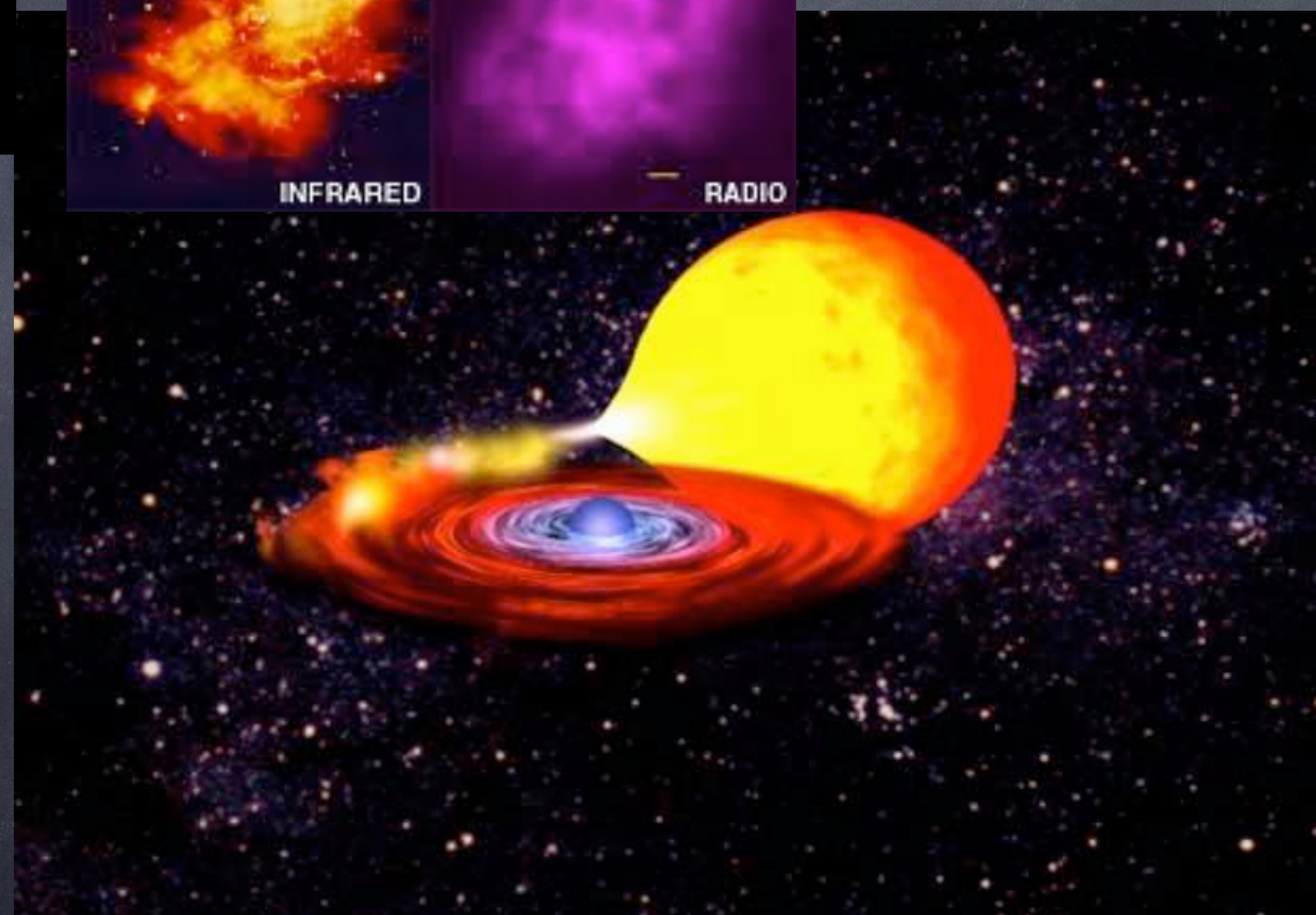
Estrellas relativistas

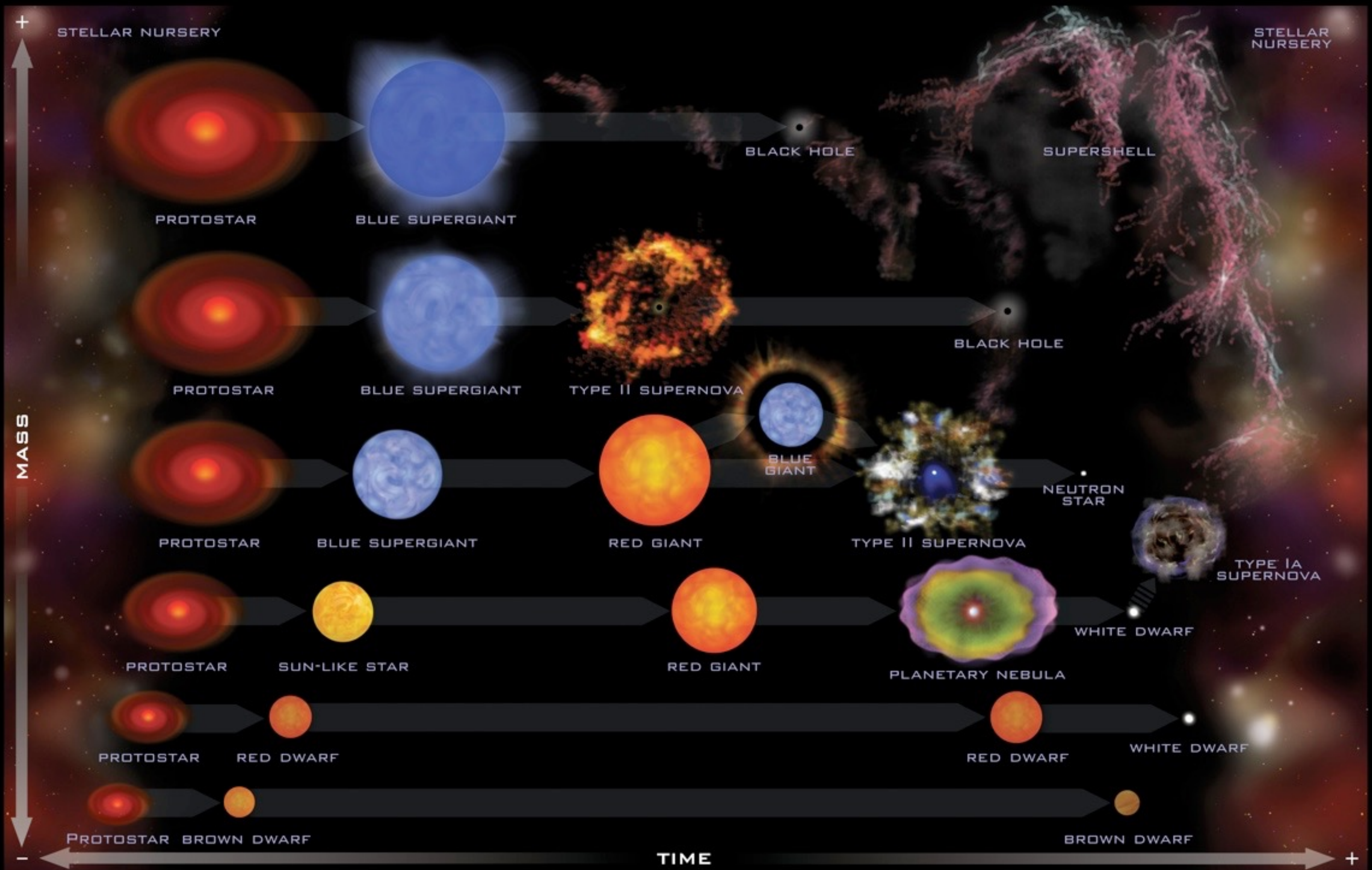
- Estrellas de neutrones: Cuando una estrella agota su combustible termonuclear, la consecuencia es el colapso gravitacional.
- Materia formada por fermiones (electrones, neutrones) a densidades extremas: la presión del principio de exclusión de Pauli, se detiene el colapso (gas de Fermi).
- Enanas blancas: ~ gas de Fermi de electrones
- Estrellas de neutrones: ~ gas de Fermi de neutrones, las masas de 1.5 - 3?? masas solares, radio de ~10 km. $GM/RC^2 = 0.1$
- Masa máxima de las enanas blancas y estrellas de neutrones: $< 3\ M_{sol}$
- Nuestro Sol se convertirá en una enana blanca, y luego enfriará a una enana marrón. Púlsares: Estrellas de neutrones que emiten ondas de radio periódicas.


Neutron star
Mass ~ 1.5 solar mass
~ 20 km diameter
Solid crust
~ 2km deep
Fluid core
Mainly neutrons
with other particles


X-RAY
OPTICAL
INFRARED
RADIO


Beam of
radiation
Rotation
axis
"Hot
spots"
Neutron
star
Magnetic
axis
Magnetic
field
lines
Beam of
radiation





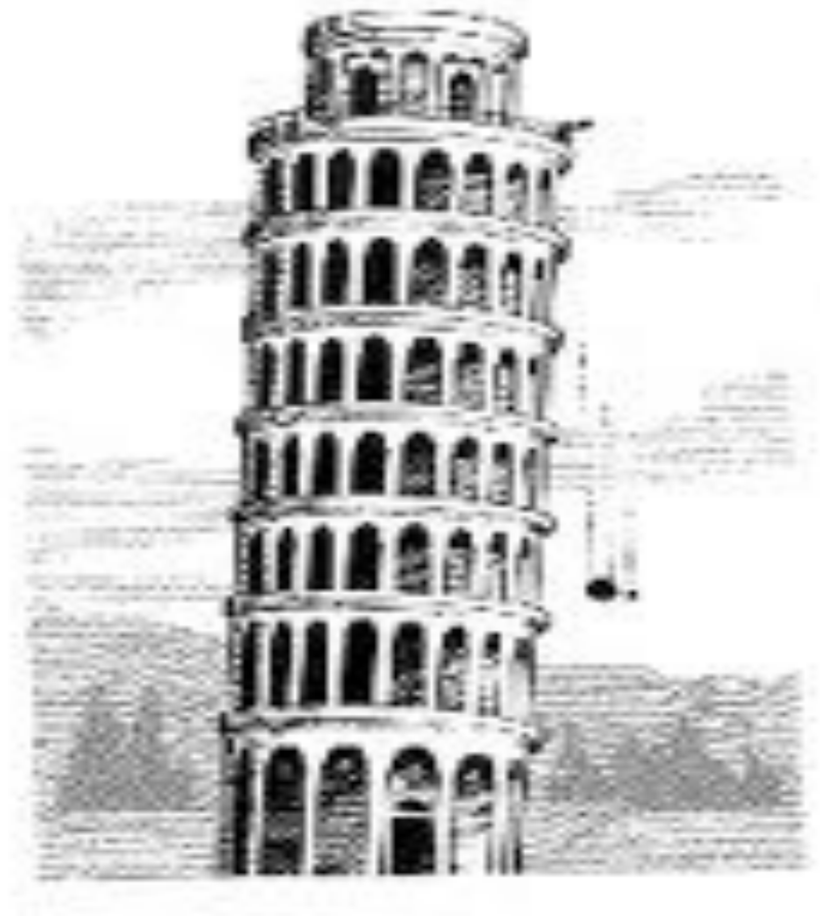
STELLAR NURSERY
PROTOSTAR
BLUE SUPERGIANT
BLACK HOLE
SUPERSHELL
STELLAR NURSERY
PROTOSTAR
BLUE SUPERGIANT
TYPE II SUPERNOVA
BLACK HOLE
PROTOSTAR
BLUE SUPERGIANT
RED GIANT
BLUE GIANT
TYPE II SUPERNOVA
NEUTRON STAR
TYPE IA SUPERNOVA
PROTOSTAR
SUN-LIKE STAR
RED GIANT
PLANETARY NEBULA
WHITE DWARF
PROTOSTAR
RED DWARF
RED DWARF
WHITE DWARF
PROTOSTAR
BROWN DWARF
BROWN DWARF
MASS
TIME

The equivalence principle

Galilei's experiments:

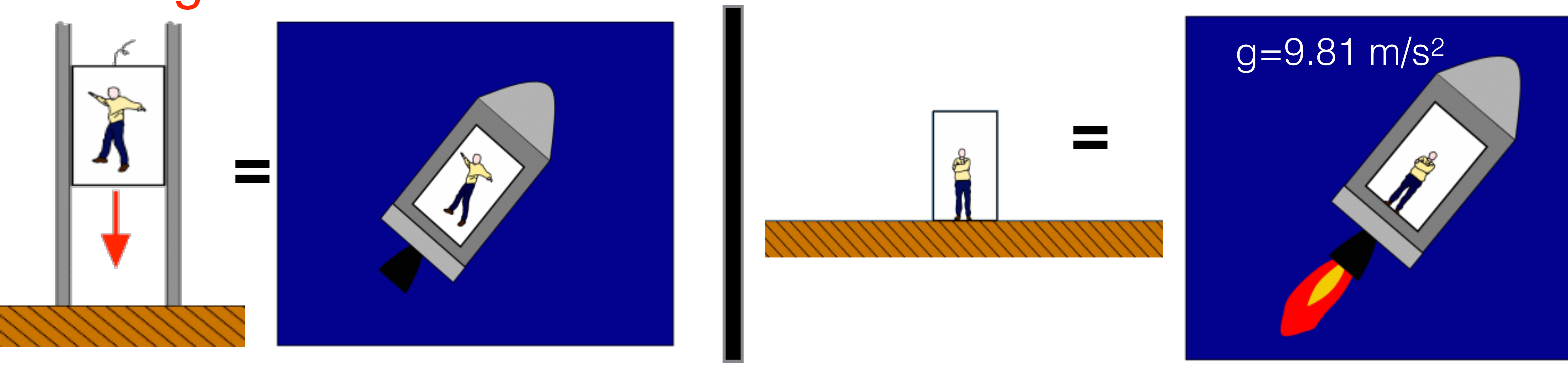
gravitational acceleration is the same for all bodies.

Gravitational mass=inertial mass



Einstein's version:

A homogeneous gravitational field can't be distinguished from uniform acceleration.



Consequences:
- geometric interpretation of gravitation
- no local energy density (without symm.)
- (radiated) E, GWs, … only defined at ∞

El principio de la equivalencia - II

$$\Delta\phi = 4\pi G\rho(\vec{x},t) \quad \vec{F} = -m\vec{\nabla}\phi \qquad m\ddot{\vec{x}} = -m\vec{\nabla}\phi \quad \ddot{\vec{x}} = -\vec{\nabla}\phi$$

Si la trayectoria de un cuerpo bajo la influencia de la gravedad es independiente de su masa y composición, podemos interpretar la gravitación como una propiedad del propio espacio-tiempo.

Cuerpos en caída libre se mueven a lo largo de "líneas de universo de caída libre" - existe una clase de curvas temporales "especiales" en la relatividad general (RG) en el espacio-tiempo. En la ausencia de masas, estas curvas son líneas rectas.

Idea clave de Einstein: la estricta igualdad de la masa gravitacional e inercial sugiere una descripción geométrica de la gravitación: donde, debido a la influencia gravitatoria, masas no se mueven a lo largo de "líneas rectas", como en el espacio plano.

Las trayectorias dependen de la curvatura del espacio-tiempo.
La curvatura del espacio-tiempo está determinado por su contenido de materia.

El principio de la equivalencia - III

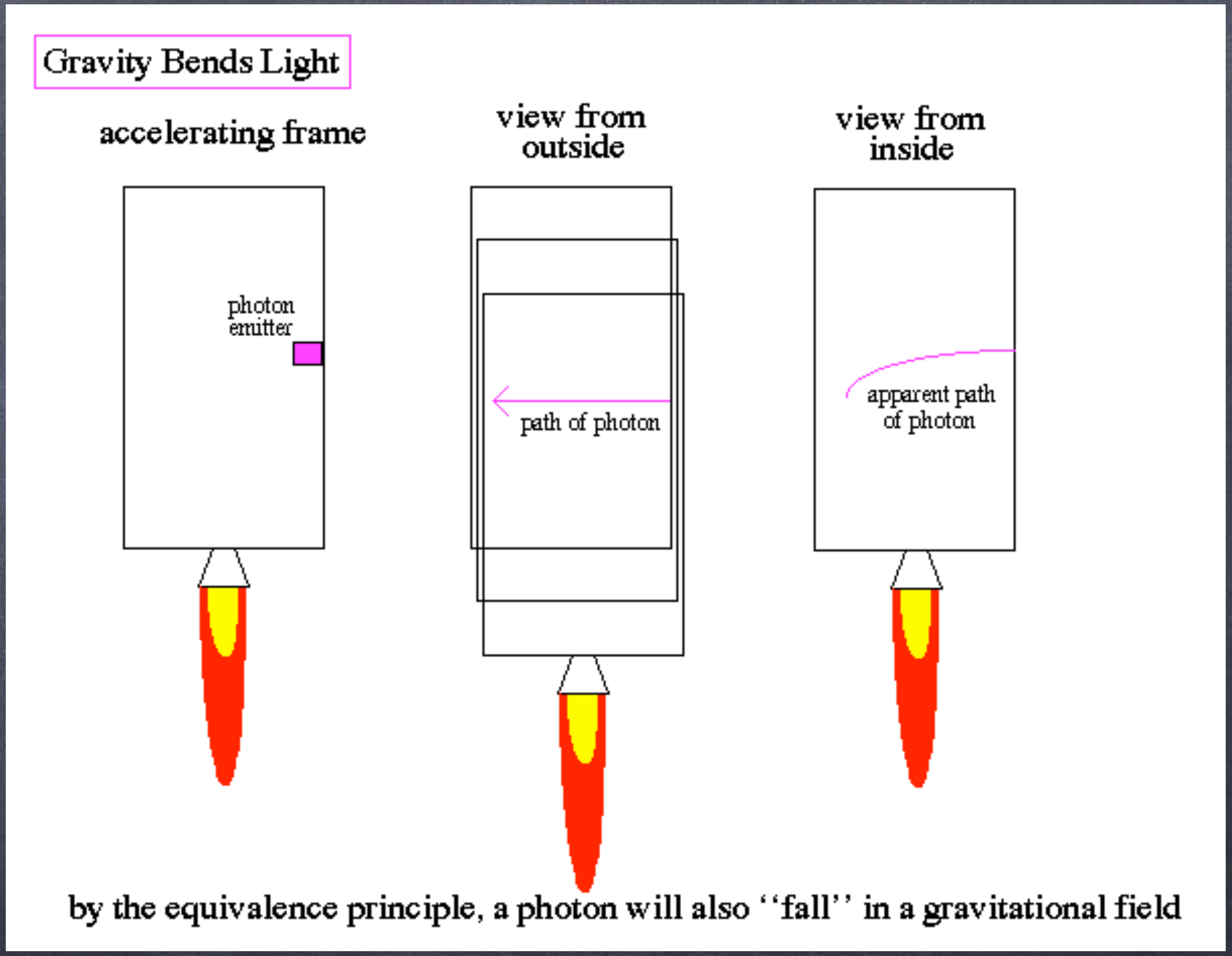
El campo gravitatorio tiene solo una existencia relativa! Equivalencia de Einstein afirma que no hay ningun experimento que pueda distinguir la aceleración uniforme de un campo gravitacional.

Ejemplo: un rayo de luz se mueve en una línea recta en un sistema inercial.

Supongamos que un rayo de luz se observa desde un laboratorio en movimiento transversal a la dirección de propagación.

En el marco del laboratorio, el rayo de luz saldrá en una posición inferior al que entró debido a la aceleración hacia arriba del laboratorio.

En el marco del laboratorio el rayo de luz se acelerará hacia abajo. Principio de equivalencia: el rayo de luz se acelerará hacia abajo en un campo gravitatorio!



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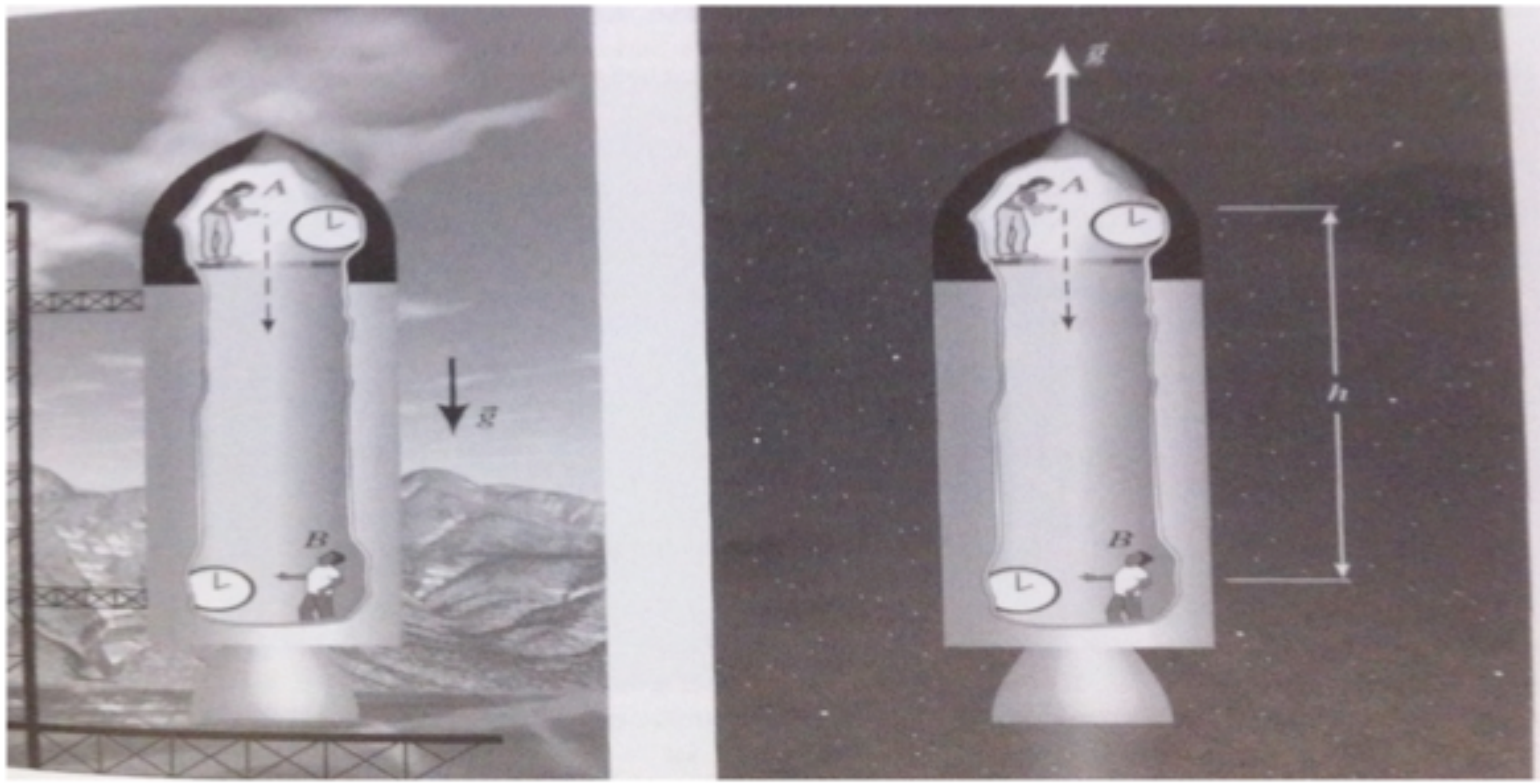
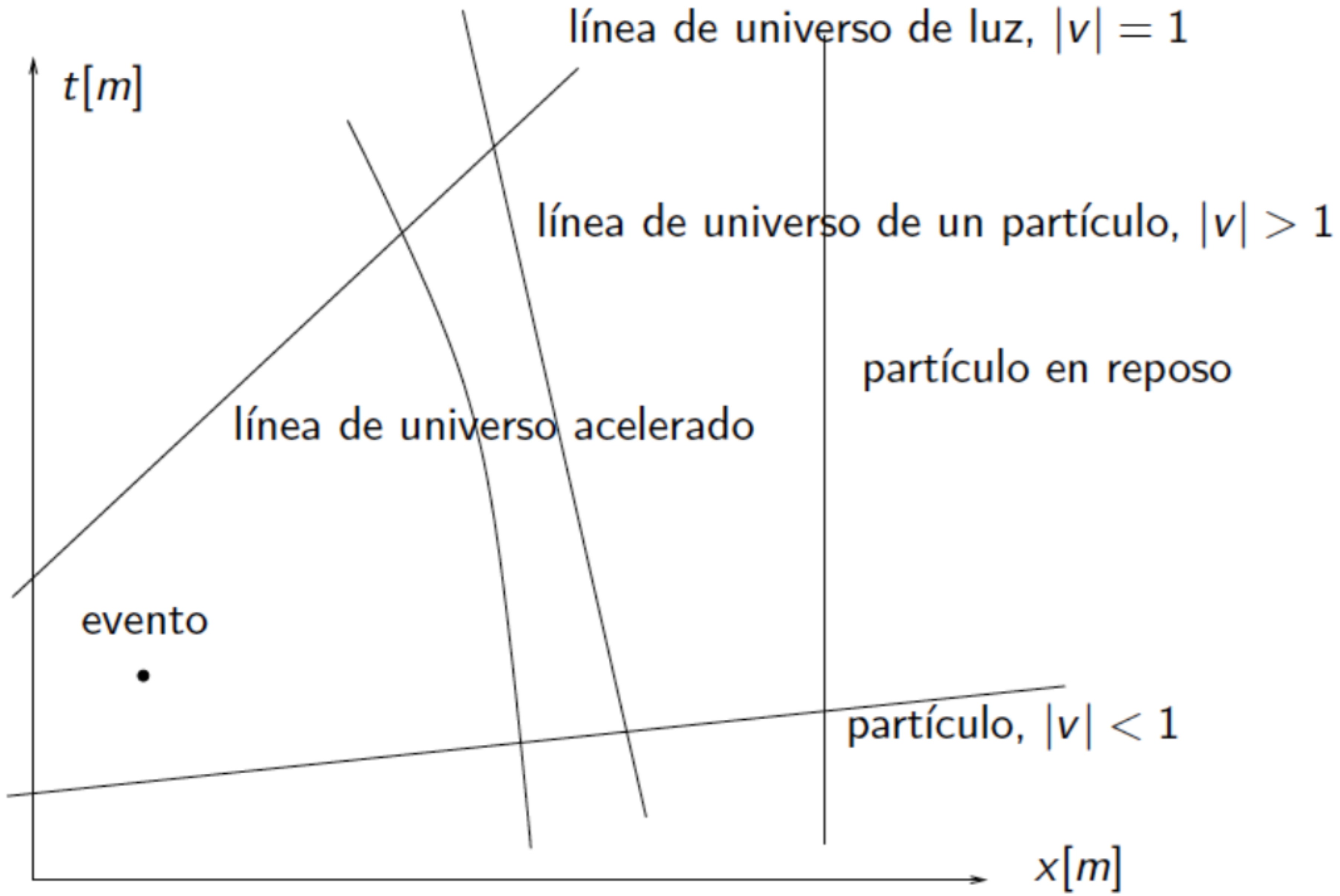
Observadora Alice emite señales de luz desde la parte superior de un cohete acelerado hacia Bob en la parte inferior.



Debido a la aceleración, Bob recibe las señales a un ritmo más rápido de lo que s emiten.

Según el PE, lo mismo va a suceder en un campo gravitatorio! Los relojes marcar más rápido cuando más lejos de la superficie de la tierra se encuentran!

Diagramas del espacio-tiempo



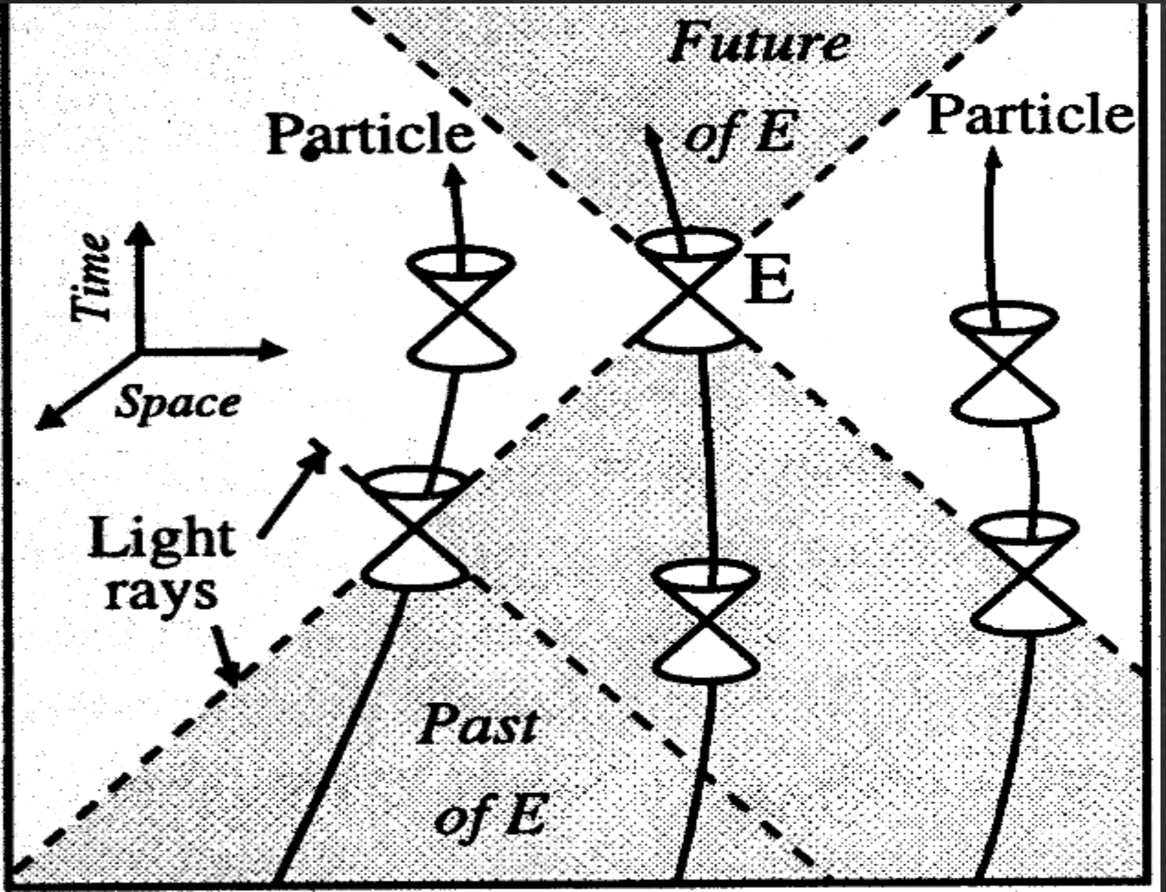
Pendiente de una línea de universo: $dt/dx = 1/v$.

Relatividad especial y el espacio-tiempo

Señales viajan dentro del "cono de luz".

En un diagrama de espacio-tiempo, el cono de luz nos dice cuáles son los posibles caminos de partículas y los rayos de luz

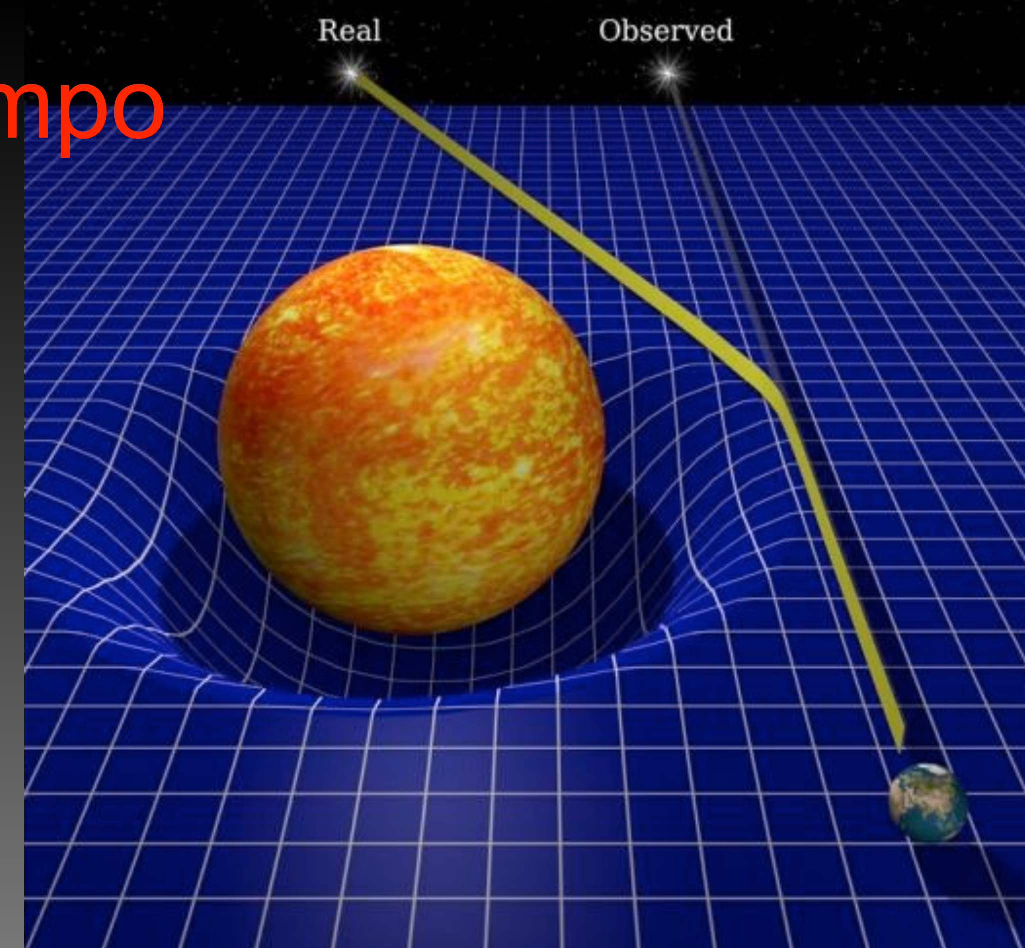
- y cuando los observadores no pueden envíar señales entre sí.



Vivimos en un universo de 4 dimensiones, con el tiempo como la cuarta dimensión:

el espacio-tiempo.

La curvatura del espacio-tiempo



La siguiente idea de Einstein:
El espacio-tiempo es suave y maleable!

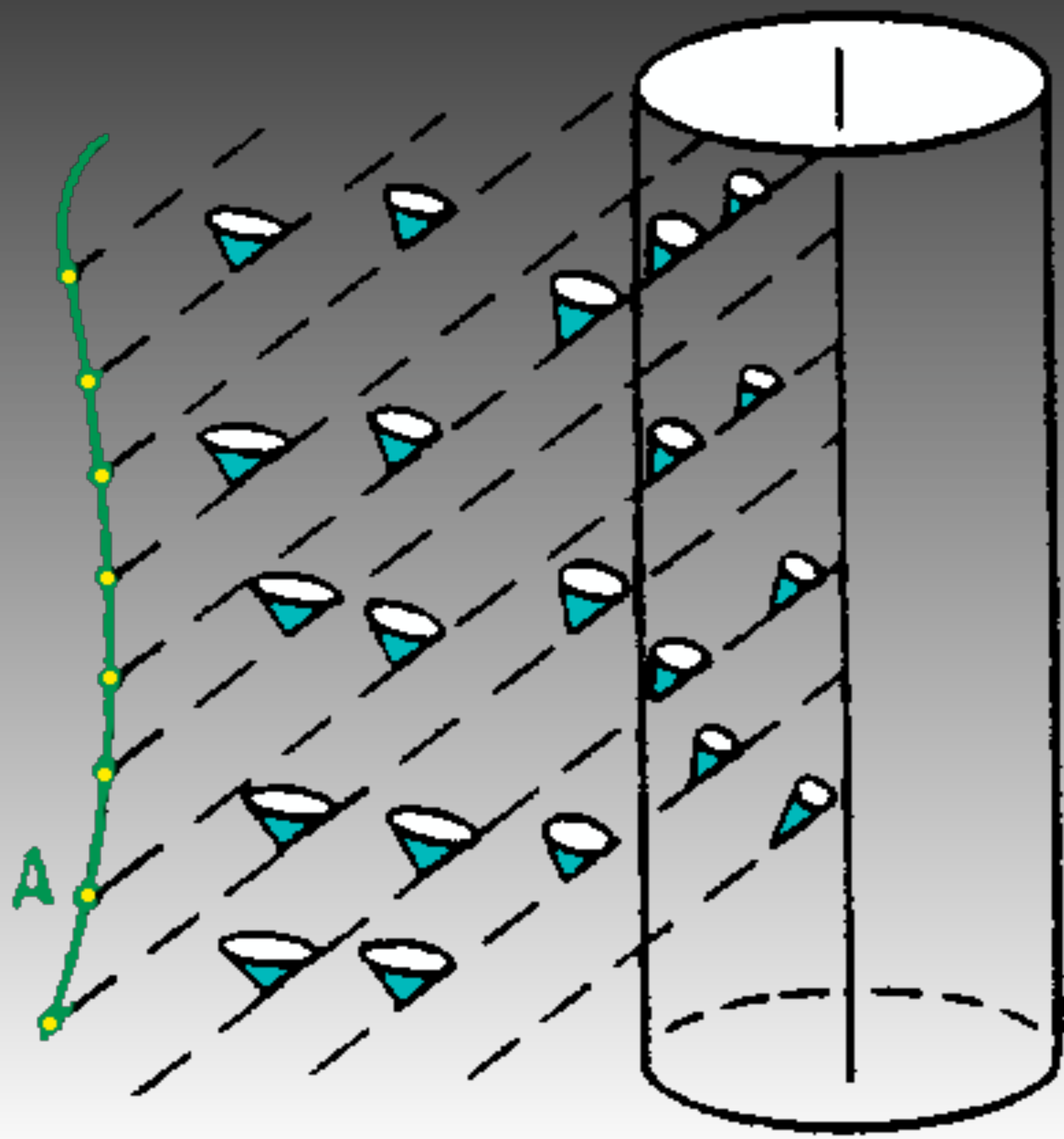
El 4-dimensional espacio-tiempo tiene una geometría curva, la curvatura es causada por la materia y corresponde a la gravedad!

$$
\begin{aligned}
R_{\eta\eta} = & -\frac{2a^2\frac{\partial\psi}{\partial\theta}\cot\theta}{\delta\psi} + \frac{2ac\frac{\partial\psi}{\partial\eta}\cot\theta}{\delta\psi} + \frac{a\frac{\partial c}{\partial\eta}\cot\theta}{\delta} - \frac{\frac{\partial a}{\partial\eta}c\cot\theta}{2\delta} - \frac{a\frac{\partial a}{\partial\theta}\cot\theta}{2\delta} - \frac{2a^2\frac{\partial^2\psi}{\partial\theta^2}}{\delta\psi} \\
& -\frac{2a^2(\frac{\partial\psi}{\partial\theta})^2}{\delta\psi^2} + \frac{4ac\frac{\partial\psi}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\psi^2} - \frac{a^2\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta d\psi} + \frac{ac\frac{\partial d}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta d\psi} + \frac{2a\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\psi} - \frac{\frac{\partial a}{\partial\eta}c\frac{\partial\psi}{\partial\theta}}{\delta\psi} \\
& -\frac{3a\frac{\partial a}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta\psi} - \frac{2a^2c\frac{\partial c}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} + \frac{2a^2b\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{a^2\frac{\partial b}{\partial\eta}c\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{a\frac{\partial a}{\partial\eta}bc\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} + \frac{a^3\frac{\partial b}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} \\
& +\frac{a^2\frac{\partial a}{\partial\theta}b\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{2ab\frac{\partial^2\psi}{\partial\eta^2}}{\delta\psi} - \frac{2\frac{\partial^2\psi}{\partial\eta^2}}{\psi} + \frac{4ac\frac{\partial^2\psi}{\partial\eta\partial\theta}}{\delta\psi} - \frac{2ab(\frac{\partial\psi}{\partial\eta})^2}{\delta\psi^2} + \frac{6(\frac{\partial\psi}{\partial\eta})^2}{\psi^2} \\
& +\frac{ac\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta d\psi} - \frac{ab\frac{\partial d}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta d\psi} - \frac{2c\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta\psi} + \frac{\frac{\partial a}{\partial\theta}c\frac{\partial\psi}{\partial\eta}}{\delta\psi} - \frac{2a\frac{\partial b}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta\psi} + \frac{\frac{\partial a}{\partial\eta}b\frac{\partial\psi}{\partial\eta}}{\delta\psi} \\
& +\frac{2a^2b\frac{\partial c}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{2abc\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{a^2\frac{\partial b}{\partial\theta}c\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{a\frac{\partial a}{\partial\theta}bc\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} + \frac{a^2b\frac{\partial b}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} + \frac{a\frac{\partial a}{\partial\eta}b^2\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} \\
& +\frac{a\frac{\partial c}{\partial\eta}\frac{\partial d}{\partial\theta}}{2\delta d} - \frac{\frac{\partial a}{\partial\eta}c\frac{\partial d}{\partial\theta}}{4\delta d} - \frac{a\frac{\partial a}{\partial\theta}\frac{\partial d}{\partial\theta}}{4\delta d} - \frac{\frac{\partial^2 d}{\partial\eta^2}}{2d} + \frac{(\frac{\partial d}{\partial\eta})^2}{4d^2} - \frac{c\frac{\partial c}{\partial\eta}\frac{\partial d}{\partial\eta}}{2\delta d} \\
& +\frac{\frac{\partial a}{\partial\theta}c\frac{\partial d}{\partial\eta}}{4\delta d} + \frac{\frac{\partial a}{\partial\eta}b\frac{\partial d}{\partial\eta}}{4\delta d} + \frac{a\frac{\partial^2 c}{\partial\eta\partial\theta}}{\delta} - \frac{a\frac{\partial^2 b}{\partial\eta^2}}{2\delta} - \frac{a\frac{\partial^2 a}{\partial\theta^2}}{2\delta} + \frac{ac\frac{\partial c}{\partial\eta}\frac{\partial c}{\partial\theta}}{\delta^2}
\end{aligned}
$$

Ecuaciones de Einstein:
La materia le dice al espacio-tiempo cómo debe curvarse, y la curvatura Cómo debe moverse la materia.

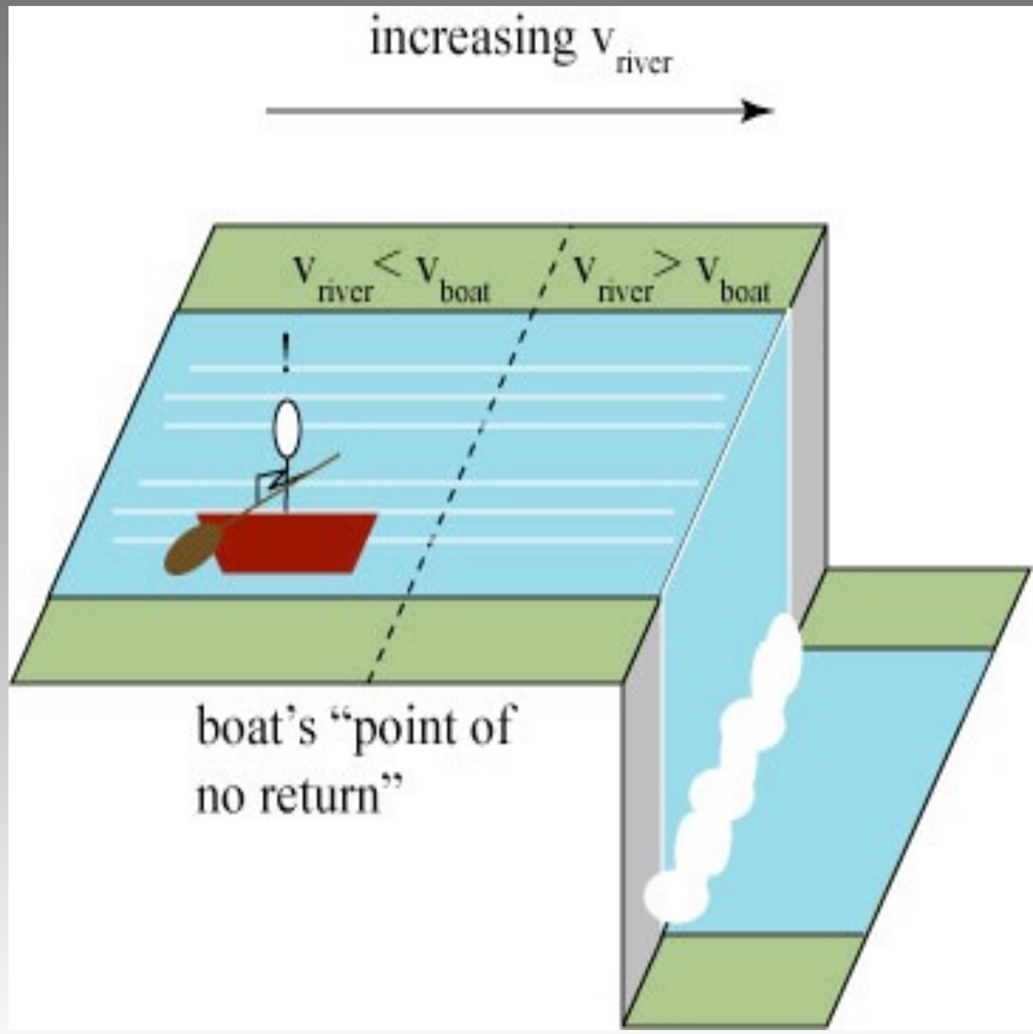
Conos de luz y agujeros negros

El espacio-tiempo puede deformarse tanto, que nada puede escaparse de los conos de luz.

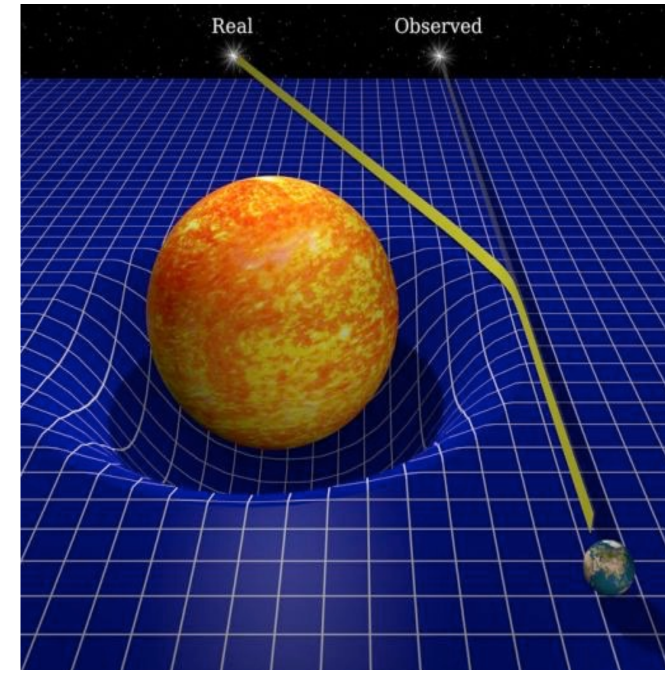


El **horizonte de eventos** es la superficie de no retorno, la region dentro del horizonte se llama

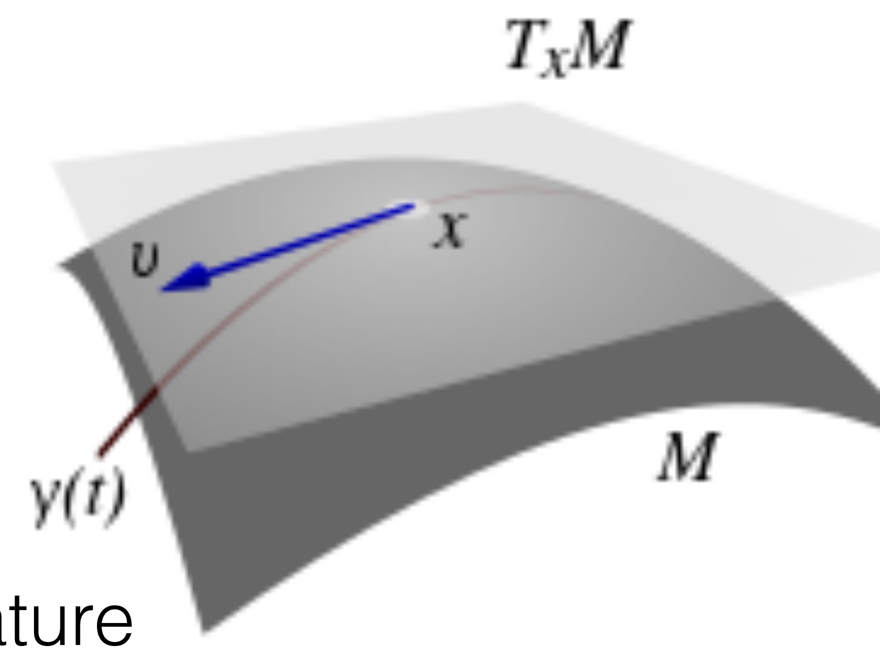
agujero negro.



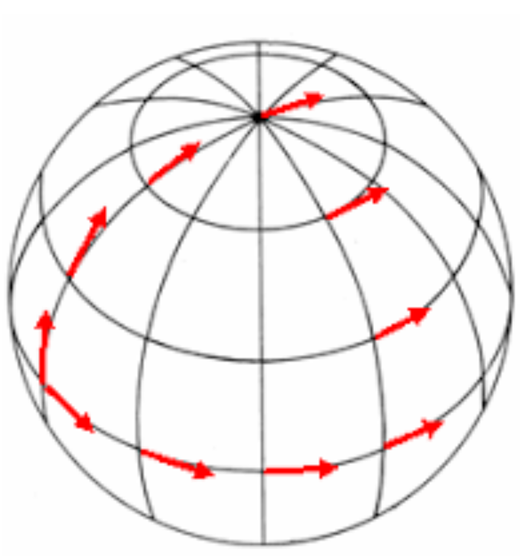
Toward Einstein's equations



- Gravity is geometry: freely falling objects correspond to a special class of curves:
- Geodesics = straightest possible
  = extremal length
- The mathematics of GR: differential geometry
  tensor fields on manifolds
- The fundamental object: the metric tensor



$$g_{ab} : \quad ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$



The Riemann tensor measures curvature
By telling us how a vector rotates
under parallel transport
along a closed curve.

e of general covariance: physics
n space-time structure only through
ic, no other special tensor fields.

$$R_{abcd}, \quad R_{bd} = R_{abcd} g^{ac}, \quad R = R_{ab} g^{ab}$$

Einstein's equations

$$m\ddot{\vec{x}} = -m\vec{\nabla}\phi$$

$$\vec{F} = -m\vec{\nabla}\phi$$

$$\Delta\phi = 4\pi G\rho(\vec{x},t)$$

- Generalize velocity to 4-velocity (tangent vector to world lines of observers):
  $$u^a u_a = g_{ab}u^a u^b = -1$$
- equation of motion
  -> geodesic equation: $u^a \nabla_a u^b = f^b = 0$
- matter density $\boldsymbol{\rho}$
  -> energy-momentum tensor
  $$T_{ab} = T_{ba} : \quad \nabla^a T_{ab} = 0$$
- Potential ϕ -> g_{ab}
- Δ -> $G_{ab} = R_{ab} - R\, g_{ab}/2$

$$G_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

Ideal fluid:

$$T_{ab} = \rho\, u_a u_b + P\,(g_{ab} + u_a u_b)$$

Solve consistently with
and possibly other matter equations.
Not: write down metric, compute T.

Geometric units

- $G = c = 1$
- Can measure time, length in units of mass
- Examples:
  - Schwarzschild radius $R_S = 2\ GM/c^2$
- Characteristic gravitational time scale for an object

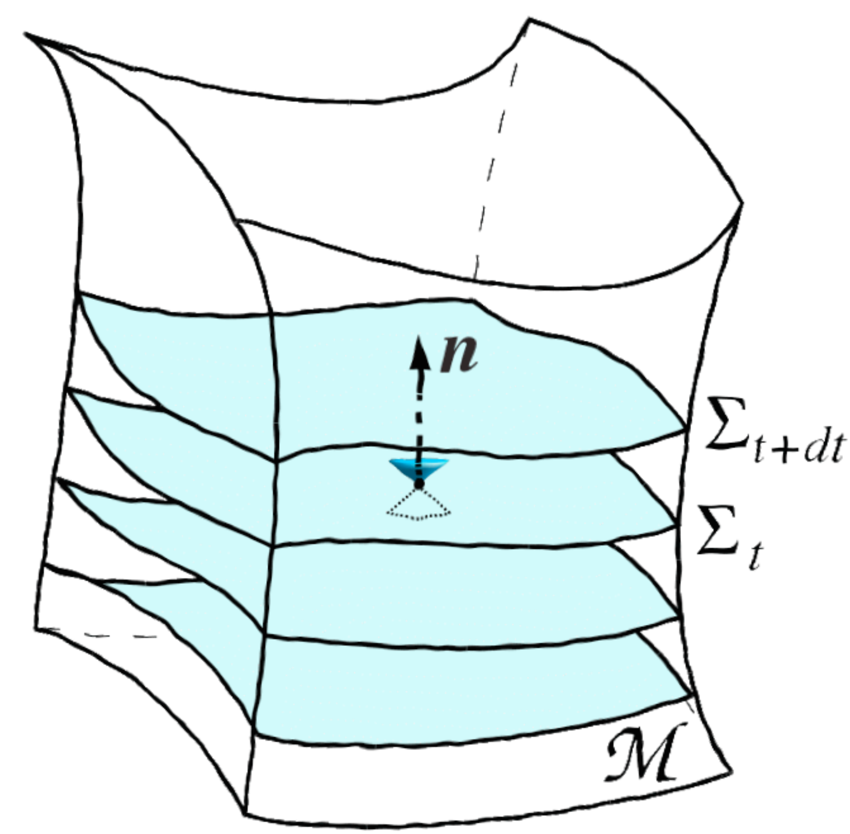
  GM/c^3
- Maximal radiated power scale 1M/1M.

Solving Einstein's equations

$$G_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

Interpretation as partial differential equations: 10 coupled nonlinear PDEs for the metric tensor.

$$\begin{aligned} G_{ab}[g_{cd}] &= R_{ab} - \frac{1}{2} R_c{}^c g_{ab} = 8\pi\kappa T_{ab}[g_{cd}, \phi^A], \qquad R_{bd} = R^a{}_{bad}. \\ R^a_{bcd} &= \Gamma^a_{bd,c} - \Gamma^a_{bc,d} + \Gamma^m_{bd}\Gamma^a_{mc} - \Gamma^m_{bc}\Gamma^a_{md}, \quad [\nabla_a, \nabla_b] v^c = R^c_{dab} v^d, \\ \Gamma^i{}_{k\ell} &= \frac{1}{2} g^{im}(g_{mk,\ell} + g_{m\ell,k} - g_{k\ell,m}). \end{aligned}$$



- algorithmic approach to solve 4-dimensional EEs: initial value formulation - many since 50's!
- EEs usually written in the form of elliptic constraints + hyperbolic evolution equations ~ nonlinear wave equations.

$$R_{\mu\nu} = -\frac{1}{2} g^{\lambda\rho} g_{\mu\nu,\lambda\rho} + \nabla_{(\mu}\Gamma_{\nu)} + \Gamma_\lambda{}^\eta{}_\mu\, g_{\eta\delta}\, g^{\lambda\rho}\, \Gamma_\rho{}^\delta{}_\nu + 2\,\Gamma_\delta{}^\lambda{}_\eta\, g^{\delta\rho}\, g_{\lambda(\mu}\, \Gamma_{\nu)}{}^\eta{}_\rho$$

Techniques to solve Einstein's equations

- **Exact solutions**: in the presence of high degree of symmetry. May not be stable!

- **Perturbative approaches**:

  - Post-Newtonian perturbation theory: Series in v/c

  - Post-Minkowski perturbation theory: Series in G

  - Self-force perturbation theory: Series in mass ratio.

- **Numerical Relativity**: discrete and solve numerically, small parameter is grid spacing, not physical. Error bars!

Which ones to use: combination of all of the above.

Linearised Einstein's equations & GWs

$$g_{ab} = \eta_{ab} + h_{ab} \qquad \bar{h}_{ab} = h_{ab} - \frac{1}{2}\eta_{ab}\, h^c{}_c \qquad \text{gauge: } \partial^a \bar{h}_{ab} = 0$$

=> $$\Box \bar{h}_{ab} = \eta^{cd}\partial_c\partial_d h_{ab} = -16\pi T_{ab}$$ wave eq. with T_{ab} as source.

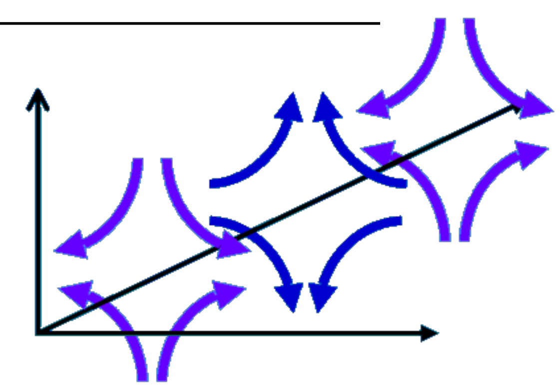
=> GWs travel at speed of light.

Propagation (no source):
Further gauge choices display 2 polarisation states.

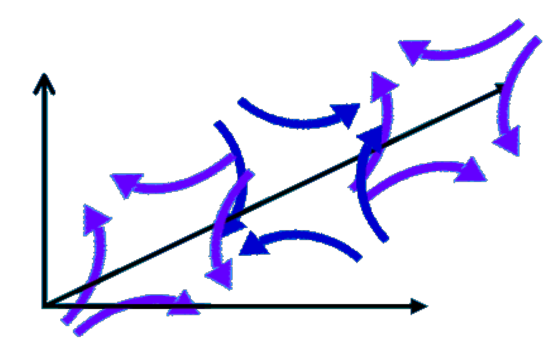
$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Slowly moving sources -> quadrupole formula:

$$\bar{h}_{ij}(t,r) = \frac{2G}{c^4 r}\ddot{I}_{ij}(t - r/c)$$



Energy flux for a binary:

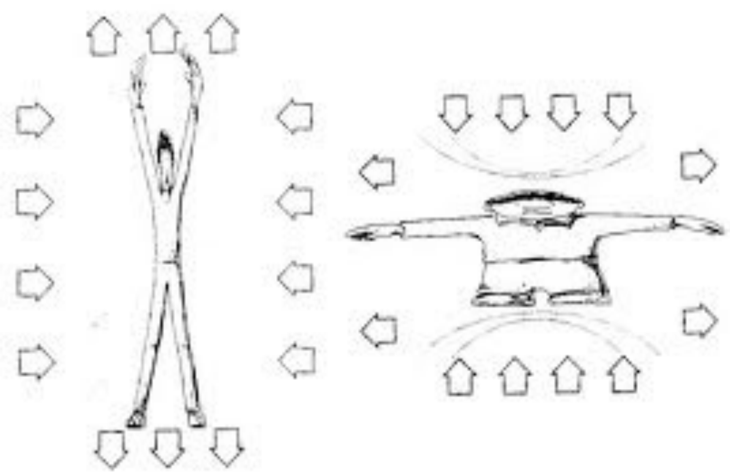


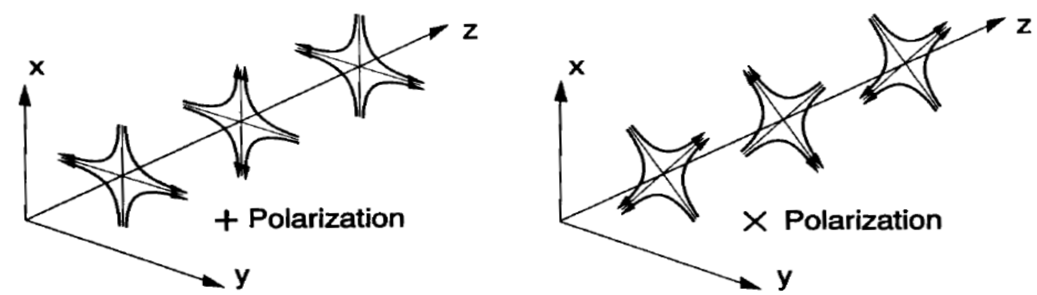
$$P = \dot{E} = \frac{32}{5}\frac{c^5}{G}\left(\frac{\pi G M_c f_{GW}}{c^3}\right)^{10/3} \qquad M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

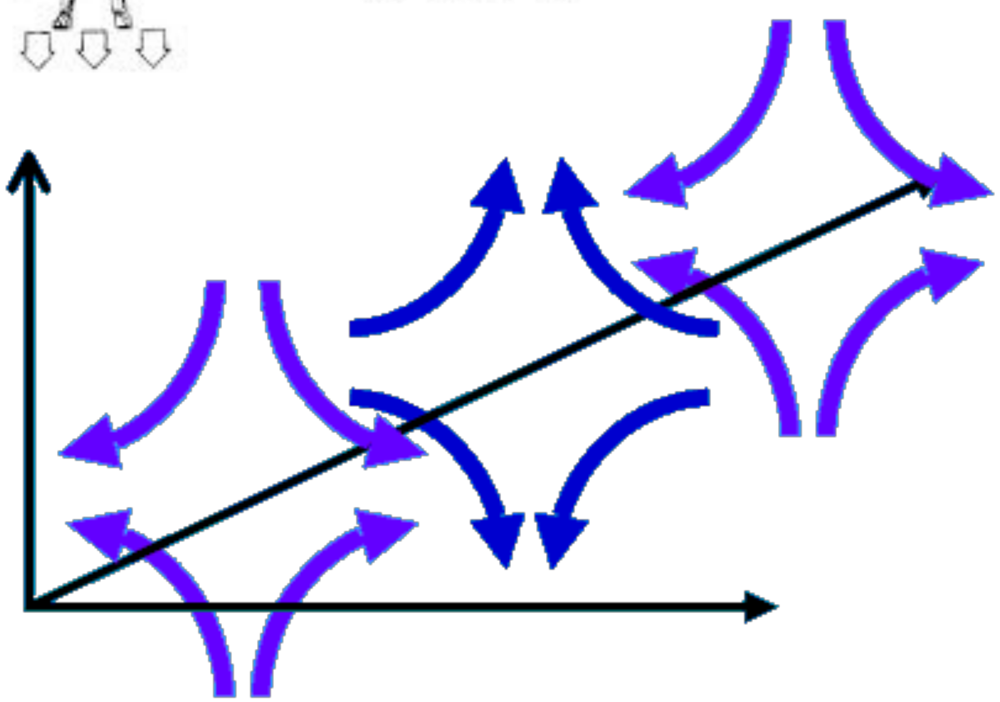
Gravitational wav effects on matter

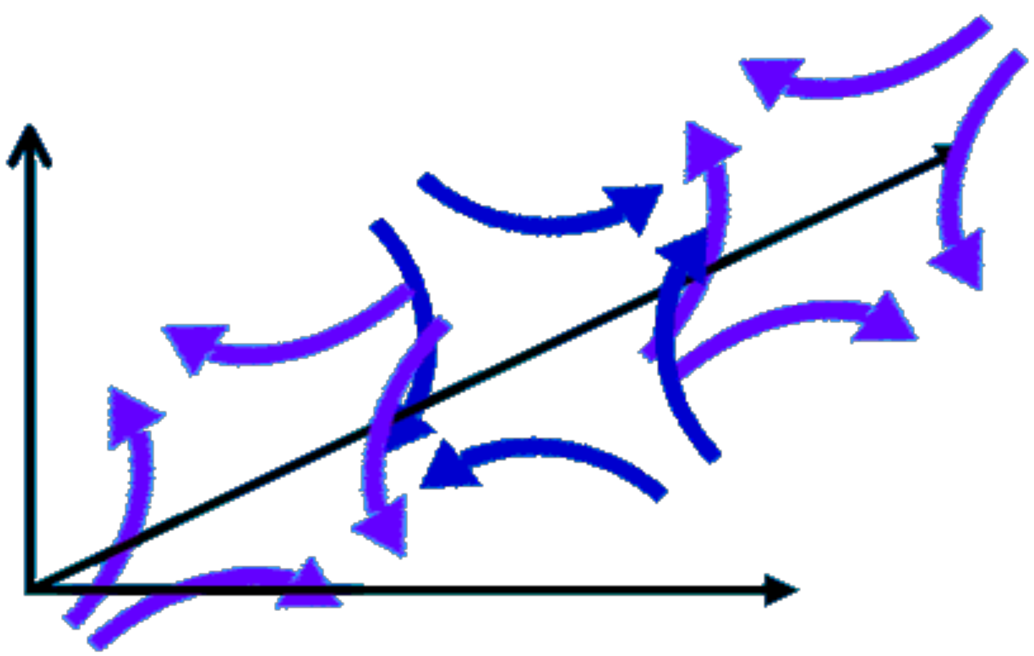
Gravitational waves produce tidal forces on matter:

- 2 polarisation states, rotated by 45°
- Due to the stiffness of spacetime, very large energies are required for very small deformations of spacetime.

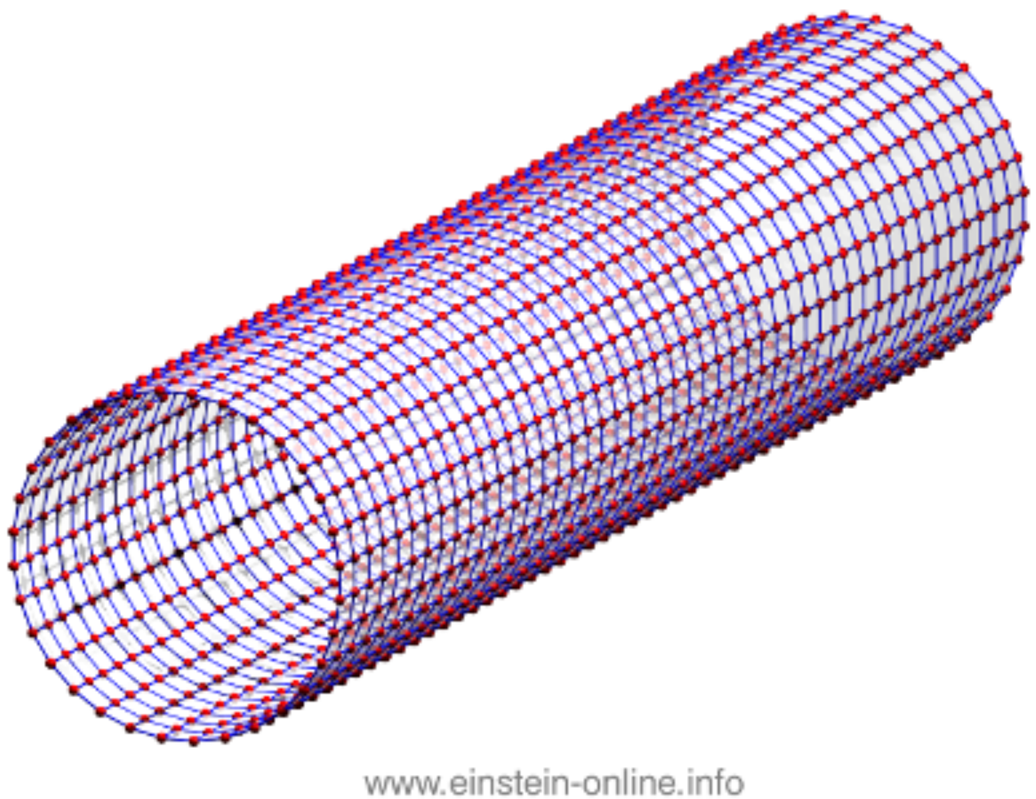


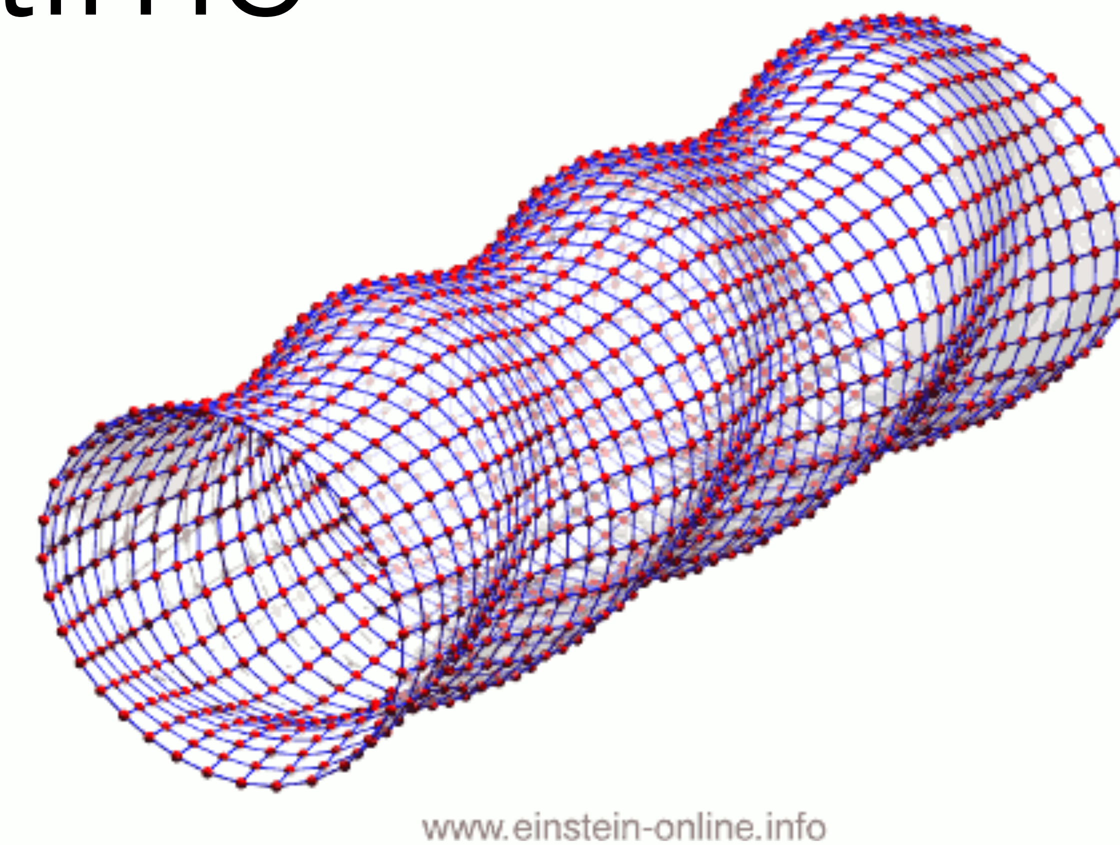


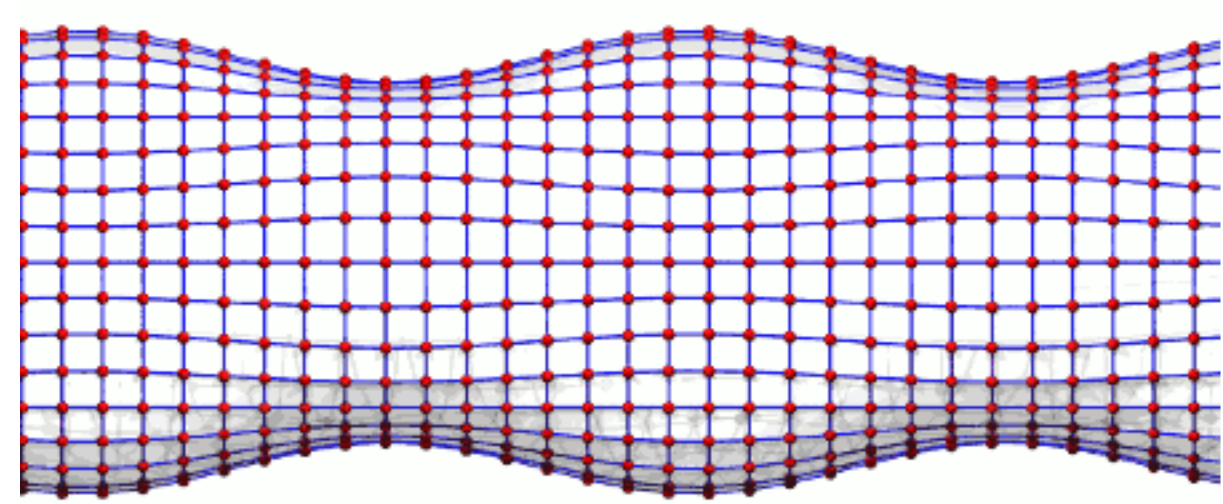




Gravitational waves: vibrations in spacetime



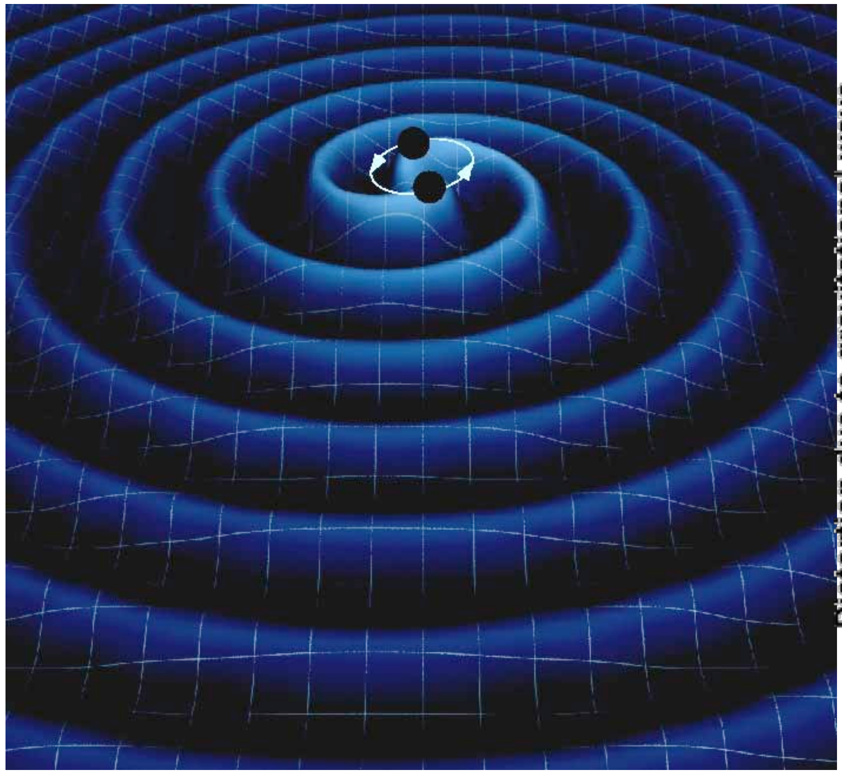


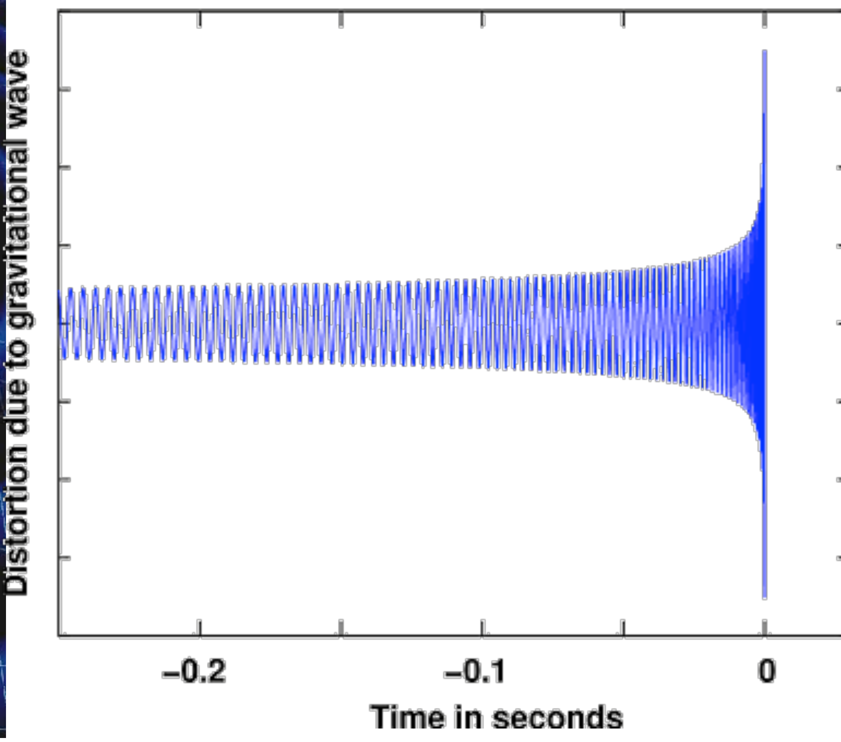


Animationen: Markus Pössel

Can we hear gravitational waves?

- Astronomical observation of electromagnetic waves:
  - Incoherent superposition of waves from many particles -> Loss of phase information
  - observe angular dependent intensity & spectrum -> source image





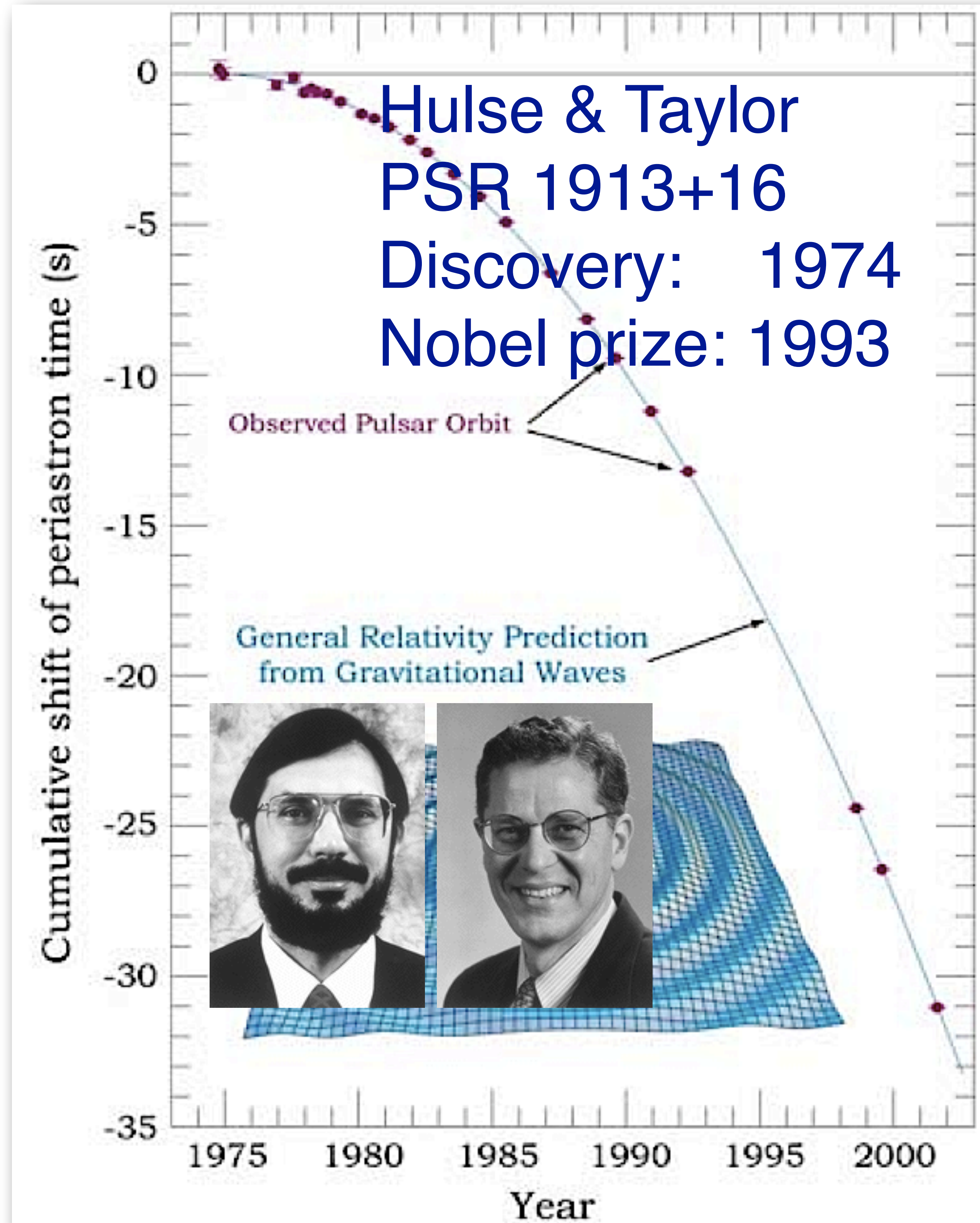
  - GW-signal carries information about the bulk motion of objects: ~ analogous to hearing.

- Amplitude falloff 1/distance (intensity: 1/distance^2) - event rate increases with sensitivity^3 (10-fold sensitivity increase -> 1000x events, ~31 for intensity).

Indirect observation of gravitational waves: Pulsars in binary systems

Pulsars discovered in July 1967

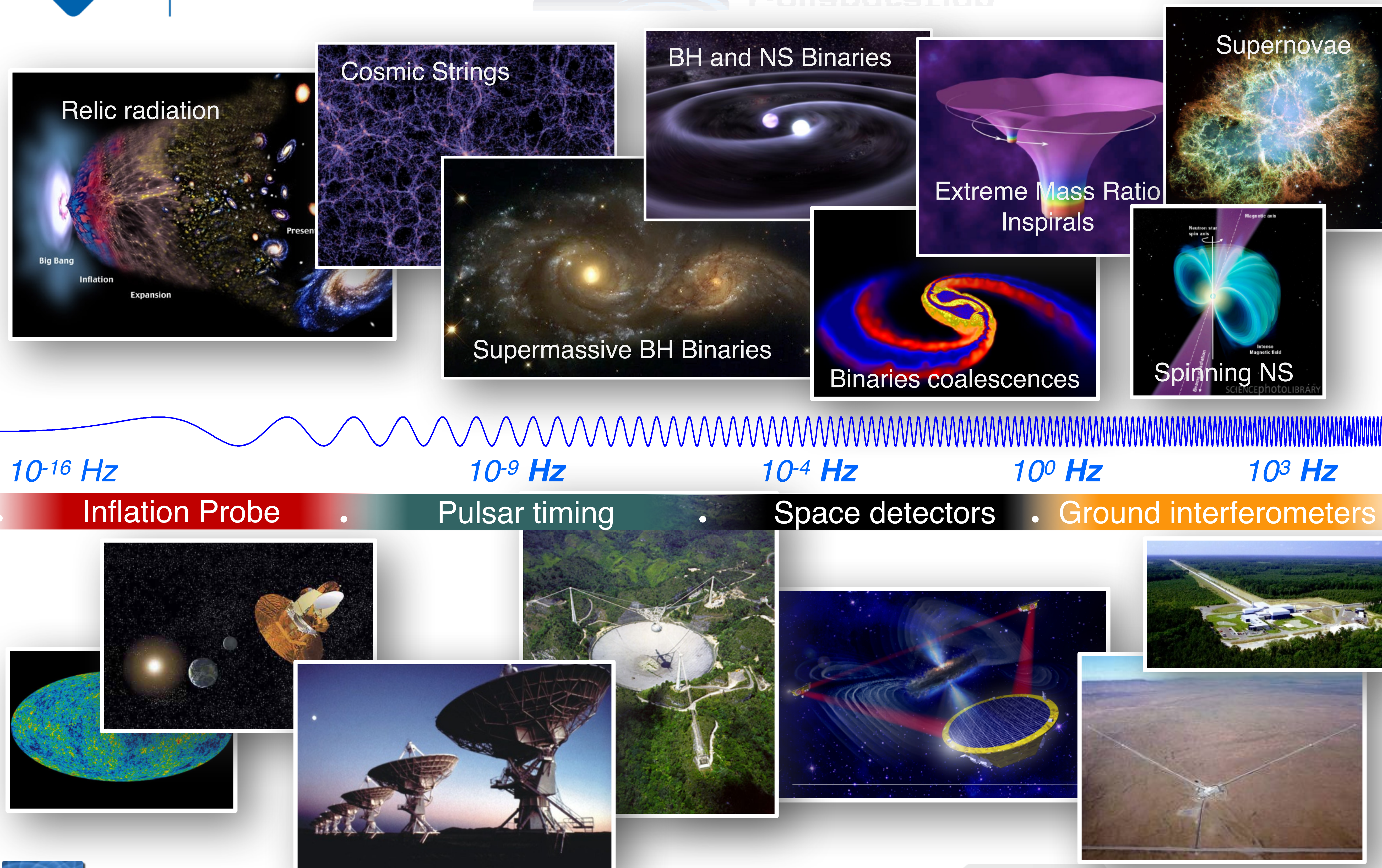
Pulsars:
neutron stars
emitting radio pulses
- act as precise clock.



Universitat de les Illes Balears

LSC LIGO Scientific Collaboration

LIGO@UIB



UIB GRG Grupo de Relatividad y Gravitación http://grg.uib.es/ligo/

IAC3 IEEC

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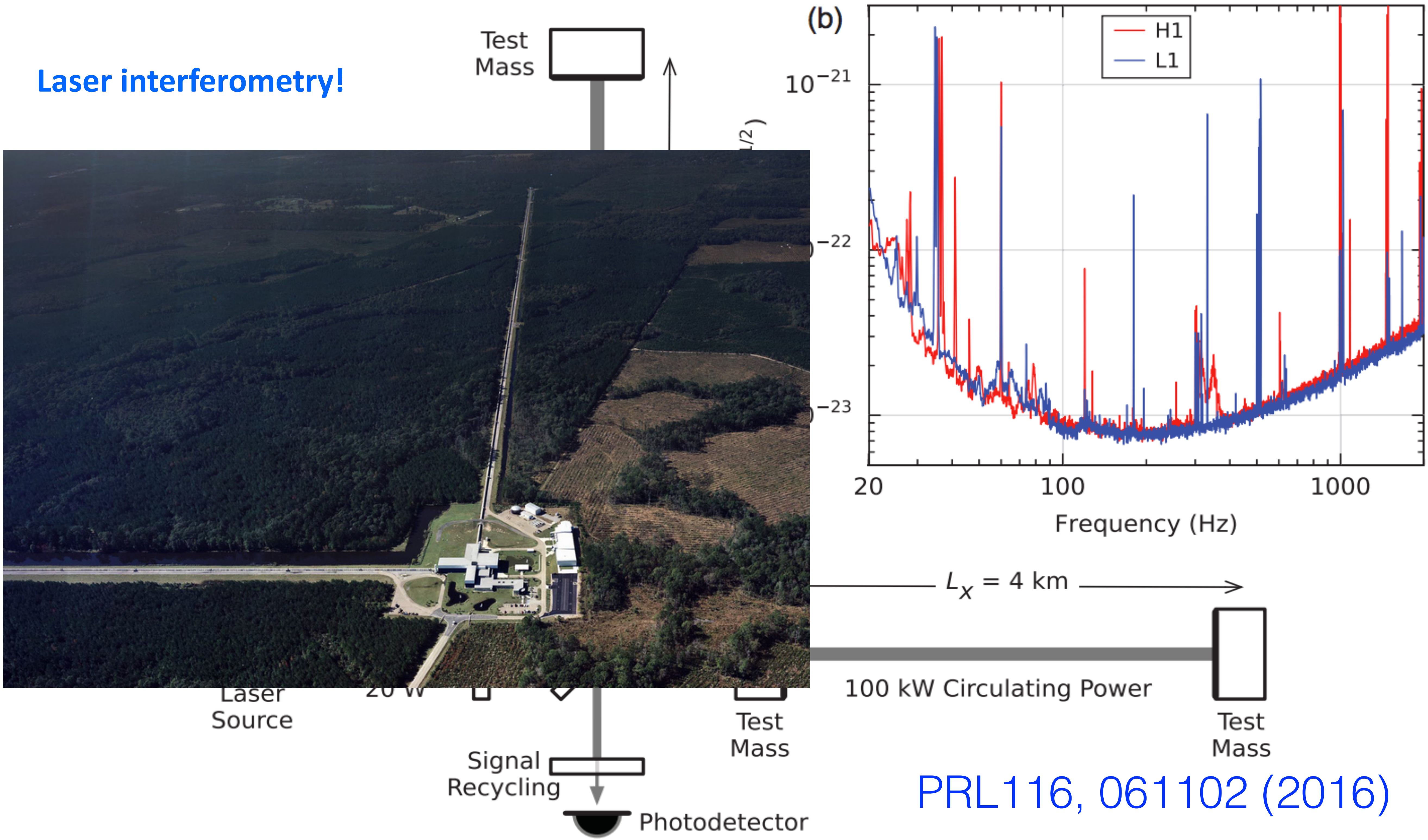
LIGO Scientific Collaboration

LIGO@UIB

Detect GWs: Need to measure strain ($\Delta L/L$) < 10^{-21}:

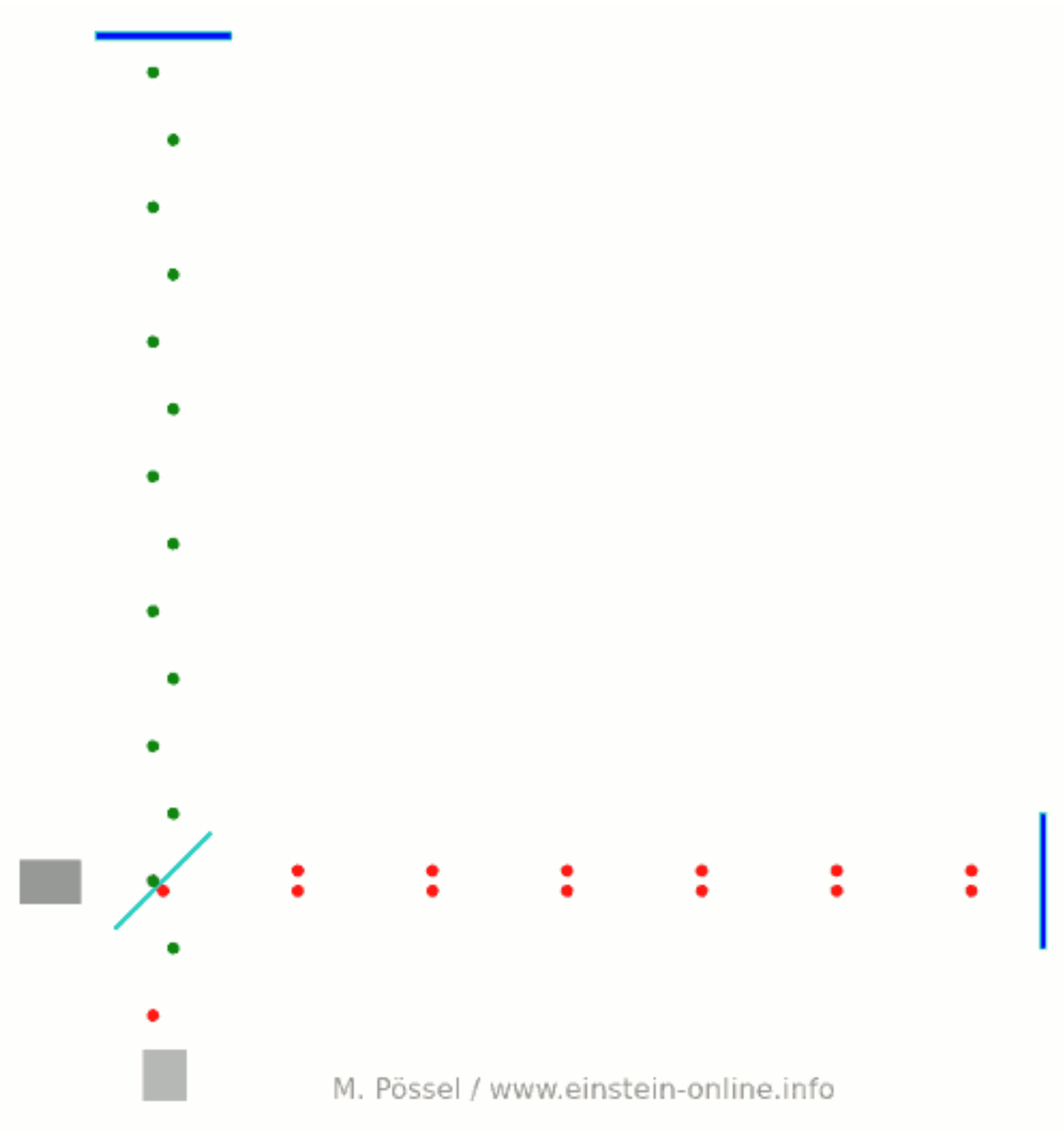
change in distance from mercury to sun by size of hydrogen atom

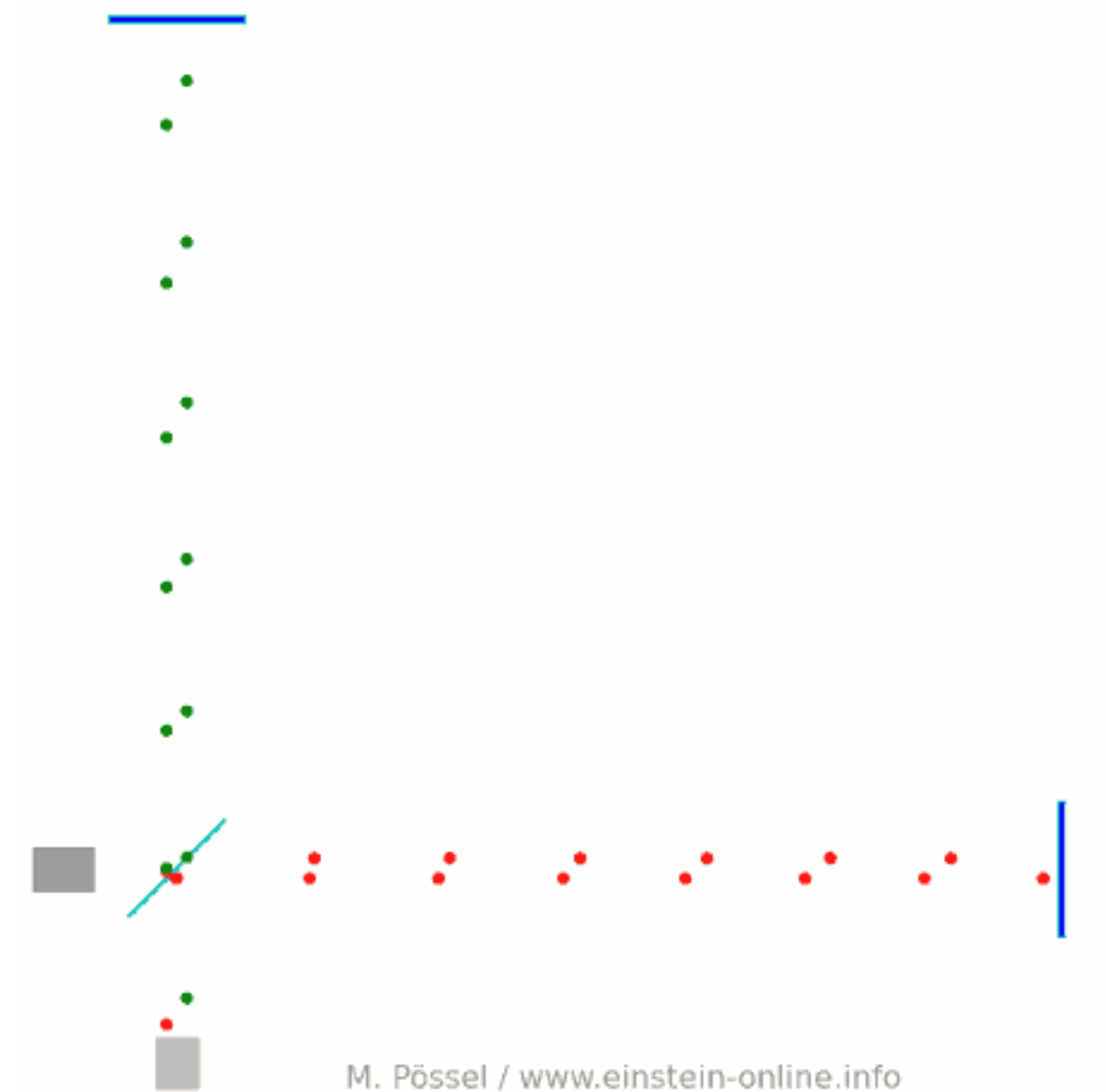
Laser interferometry!



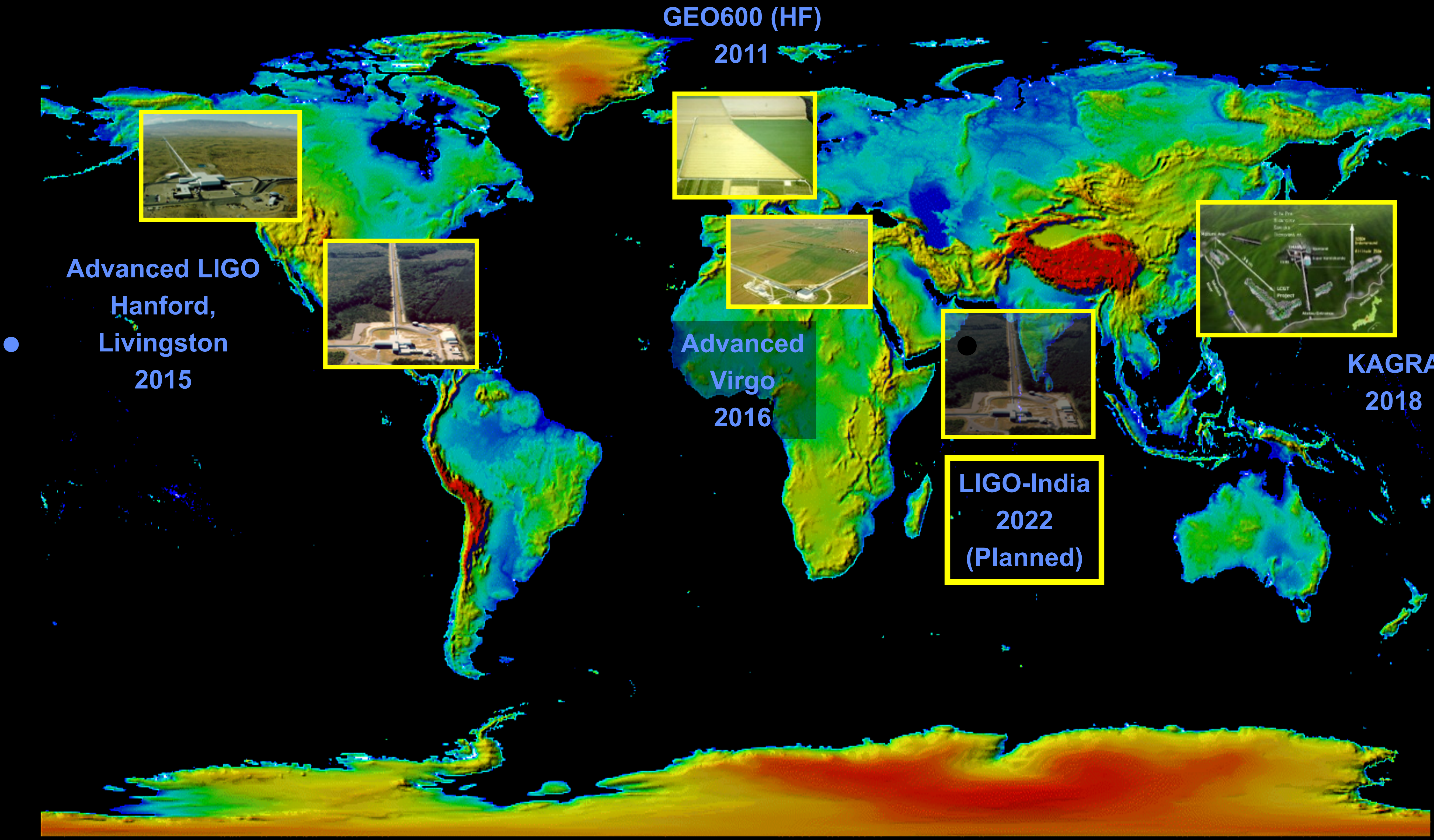
PRL116, 061102 (2016)

How does LIGO measure gravitational waves?





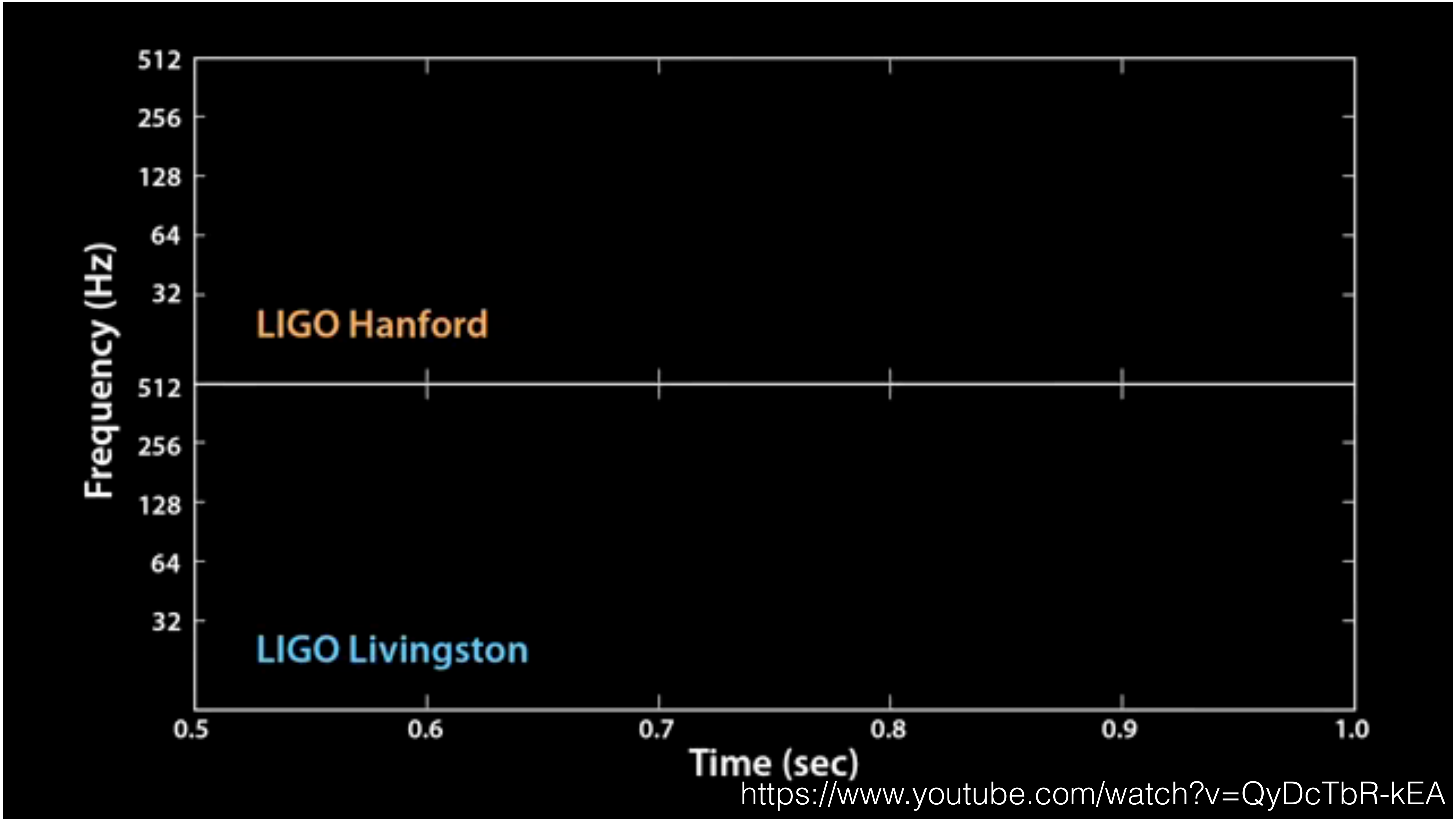
The advanced GW detector network



The LIGO Scientific Collaboration was established in 1997 to carry out LIGO science - 1200+ members, 108 Institutions, 18 countries. LSC + Virgo = LVC

www.ligo.org

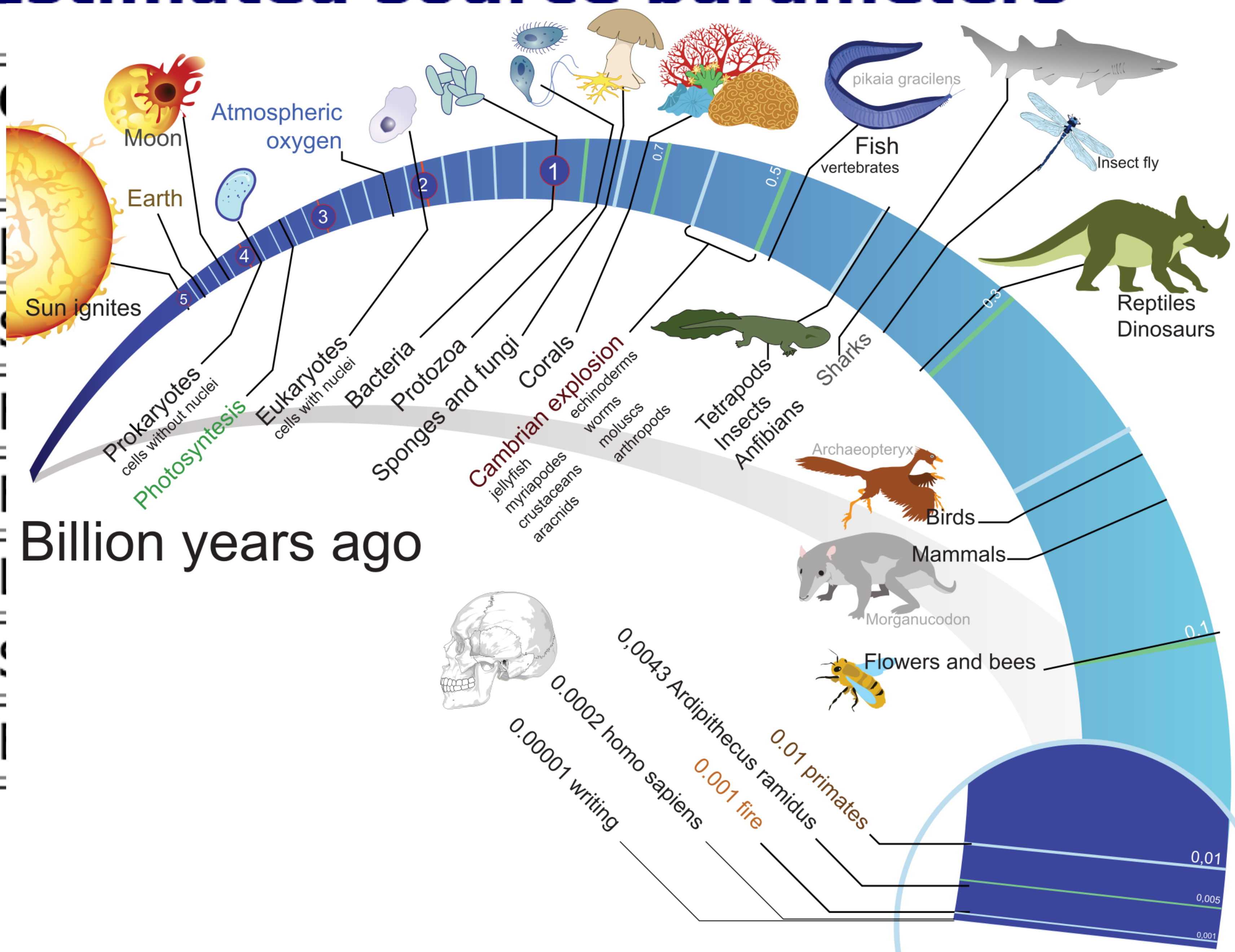
Grupo de Relatividad y Gravitación   http://grg.uib.es/ligo/



- GW150914: announced 11/02/2016.
- Consistent with the merger of 2 ~ 35 solar mass non-spinning black holes.
- SNR 21, statistical significance > 5 σ.

PRL116, 061102 (2016)

Estimated source parameters



Lower error ...e	Unit
	M sun
	M sun
	M sun
).07	
80	Mpc
).04	
.5	M sun

ver:

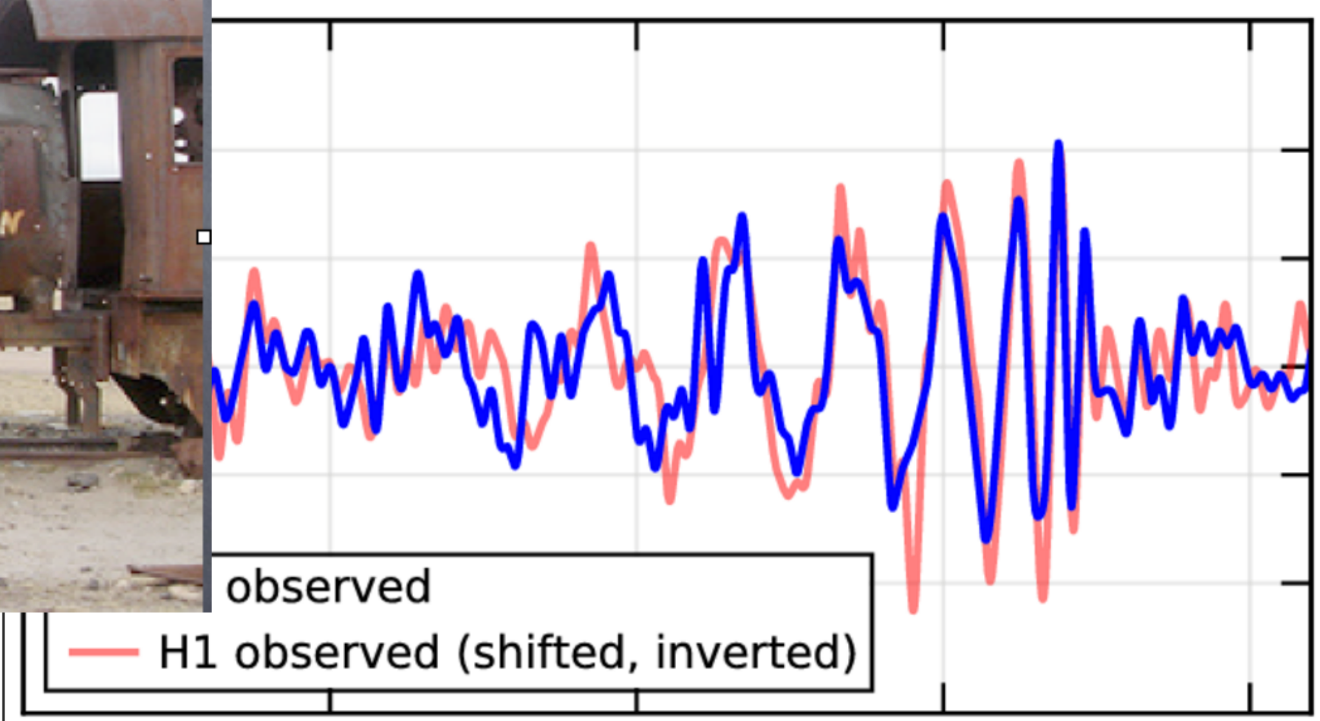
le universe

Masses are given in the source frame.

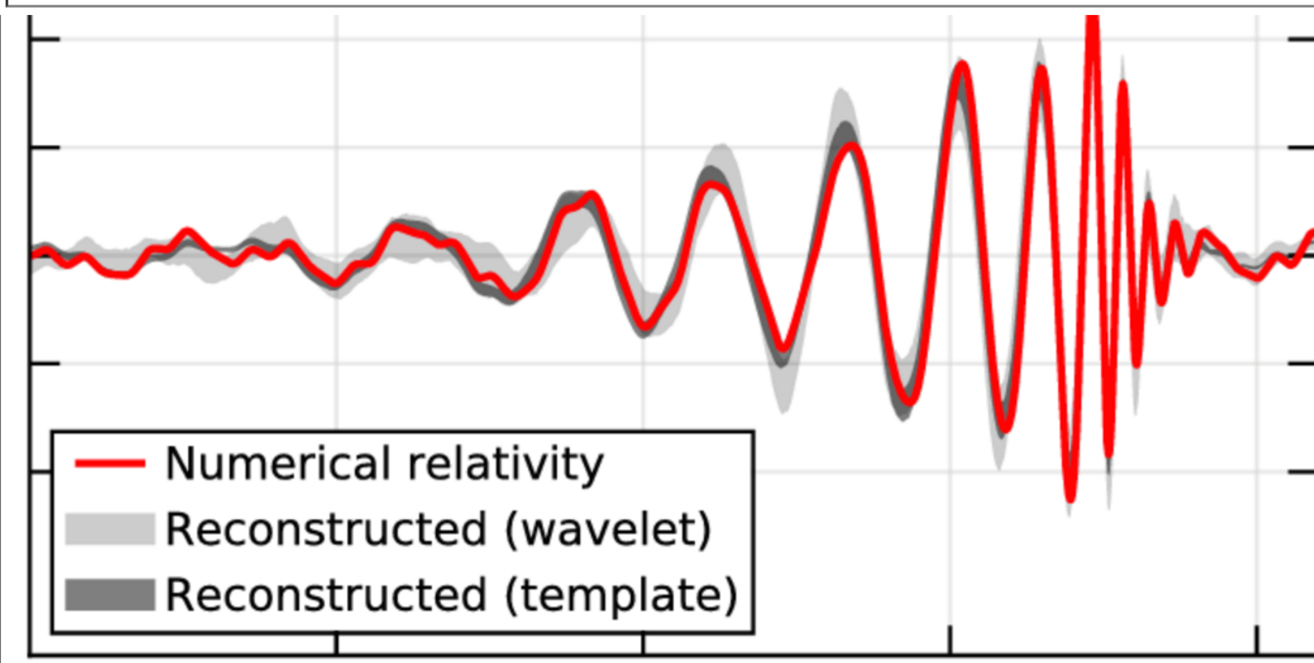
For detection we need to include cosmological redshift:
mass x (1 + z).

click for DATA

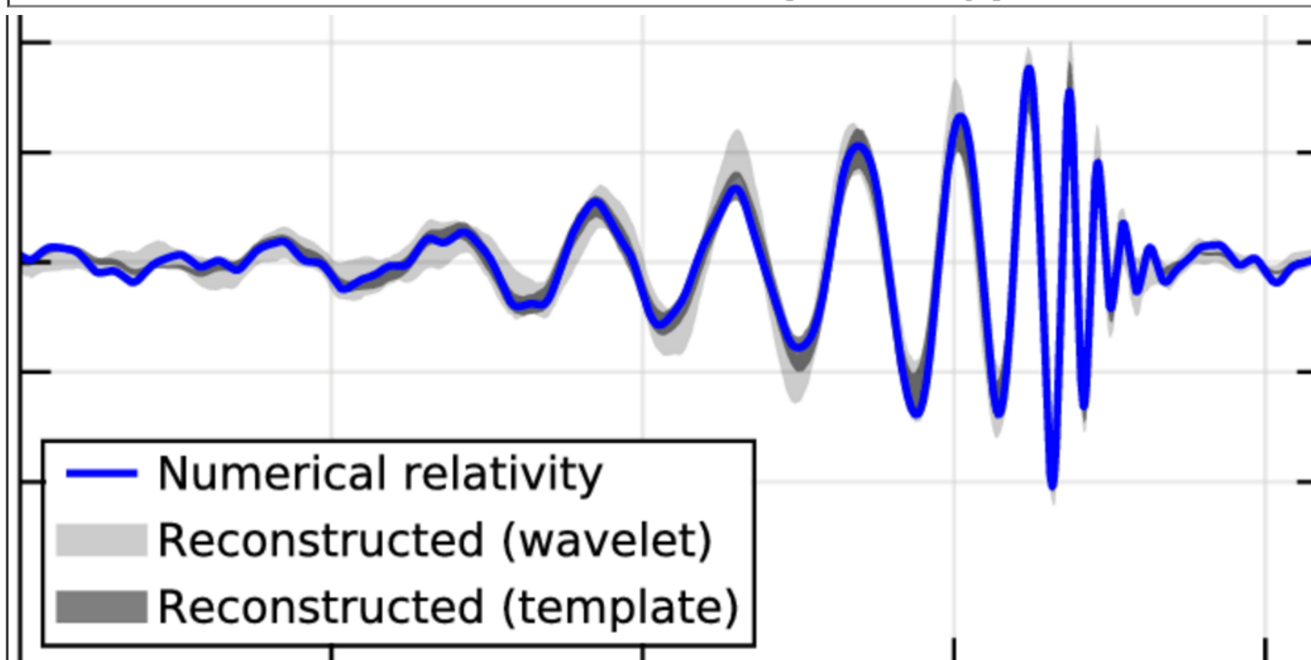
Livingston, Louisiana (L1)



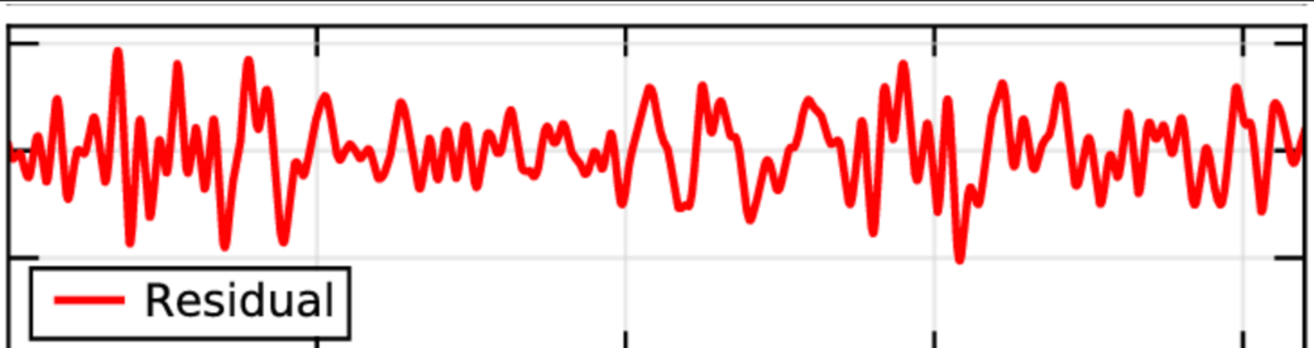
click for DATA (L1 only)

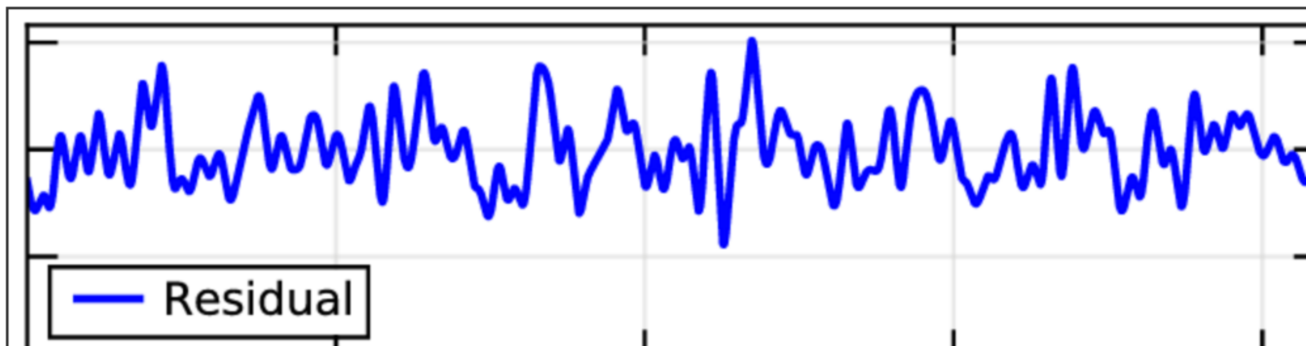


click for DATA (Numerical relativity)

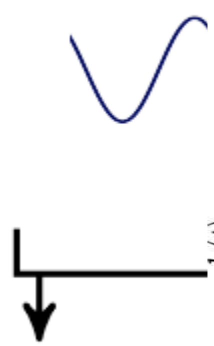


click for DATA (Numerical relativity)

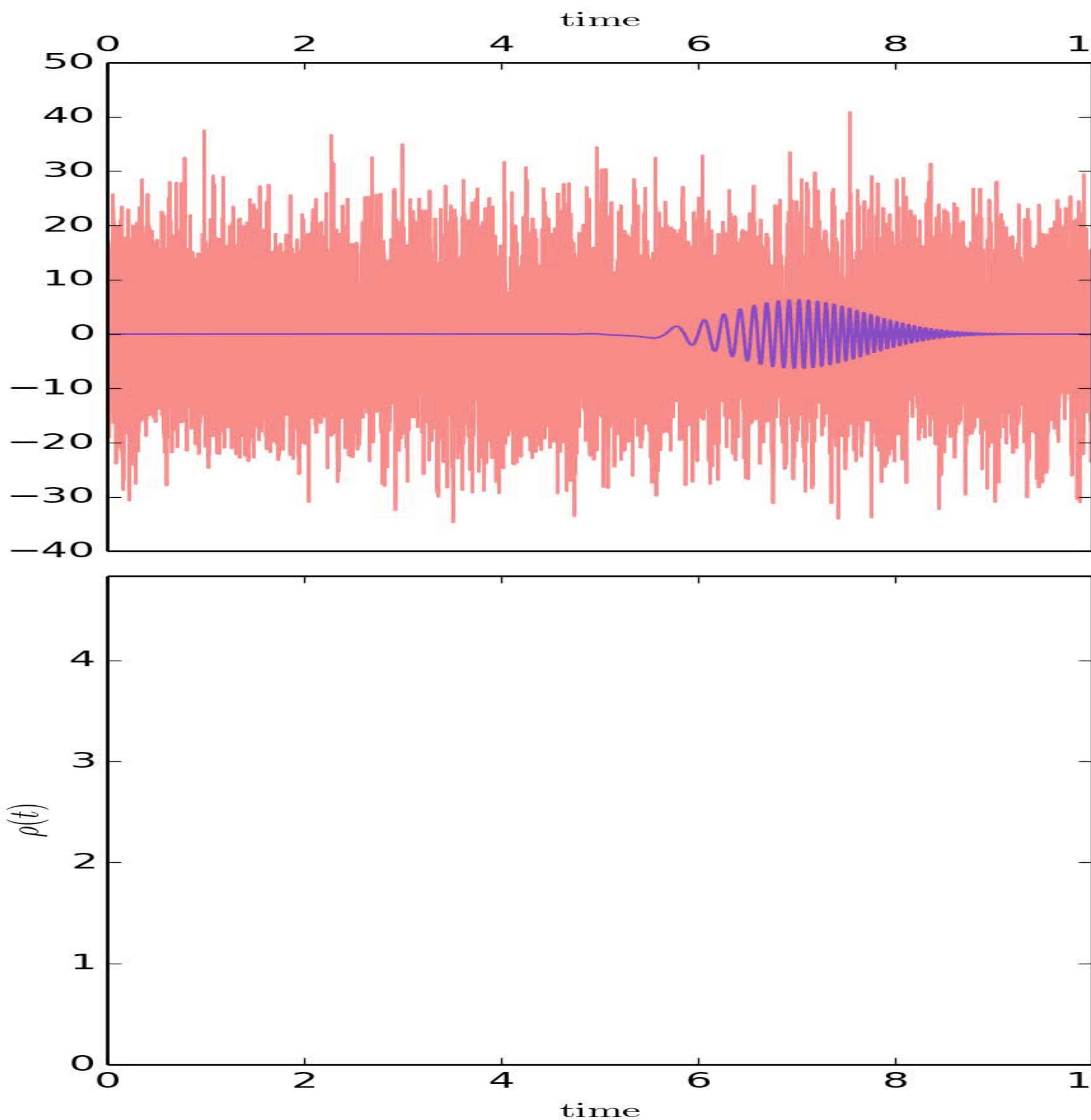


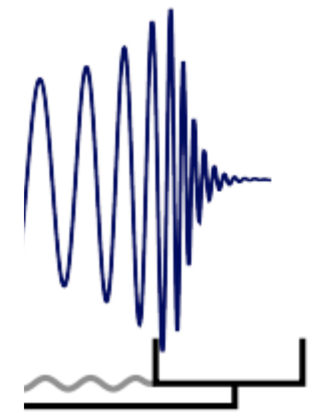


GW



Inspiral
post-Ne
Effective

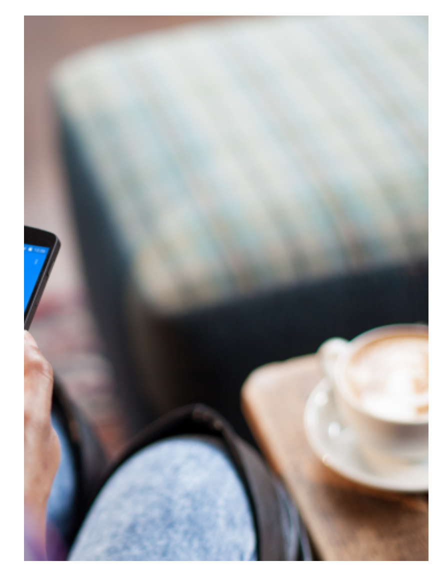




n theory

R)

The o … on the
match

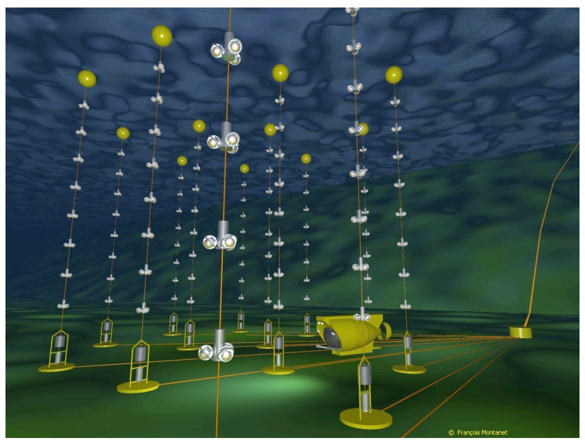


$\langle h_1, h_2 \rangle$

SNR: … $h_2||)$

Multi-Messenger Astronomy

gravitational waves



cosmic rays
neutrinos

gamma rays
X rays

ultraviolet
visible / infrared

radio

LIGO/Virgo has signed **MoU** with 92 partners from 19 countries, 70 observatories with 200 instruments covering the EM spectrum

Are short gamma ray bursts created by the collision of two neutron stars?

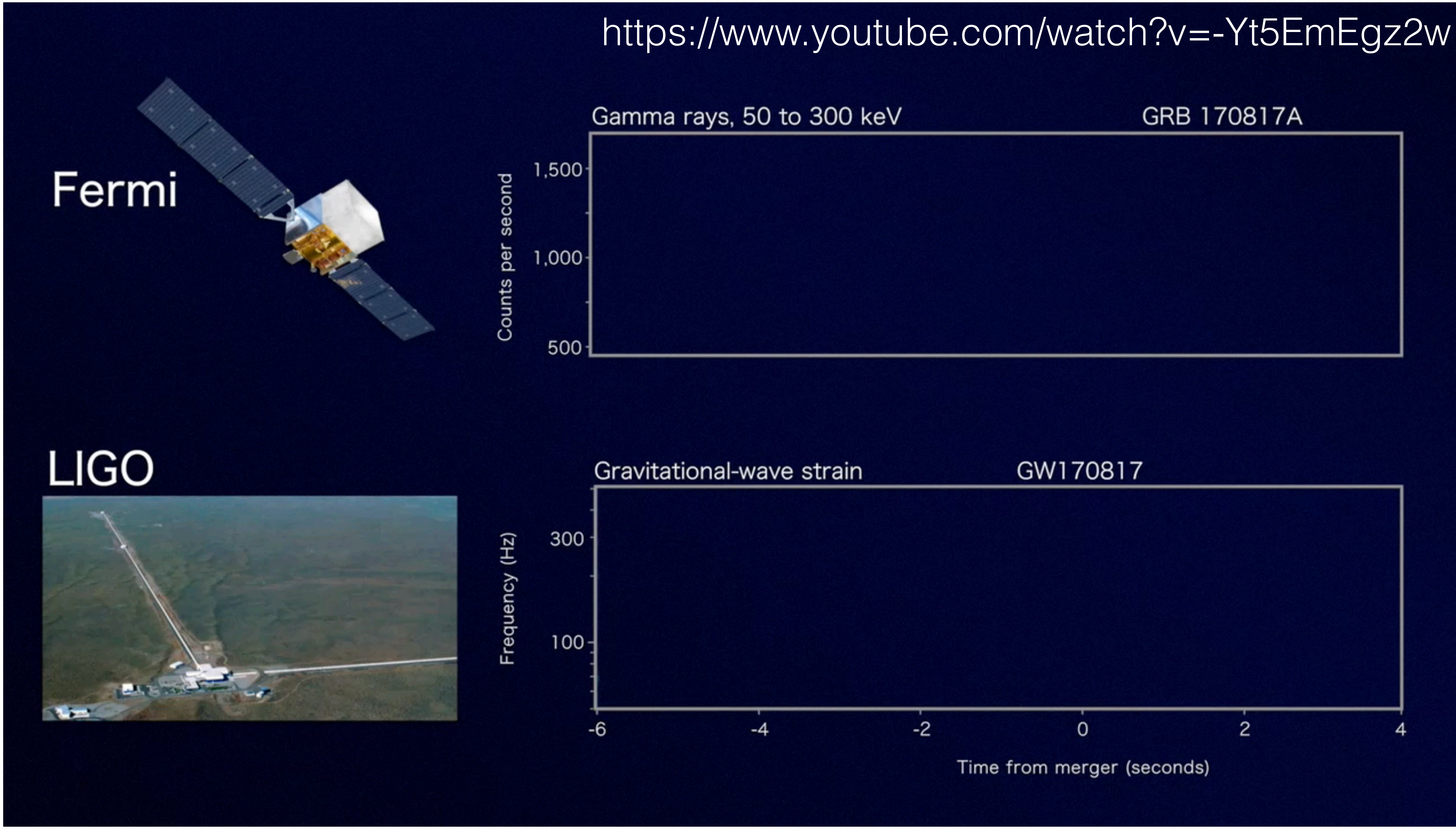
Gravitational wave signals from supernova explosions are hoped to tell us what is going on inside the explosion!

Only few neutron star collisions would be visible as gamma ray bursts due to small opening angle.

GRB 170817A y GW170817:

The beginning of the era of multi-messenger astronomy.

Coincident observation of GWs from a merger of two neutron stars with a short gamma ray burst.

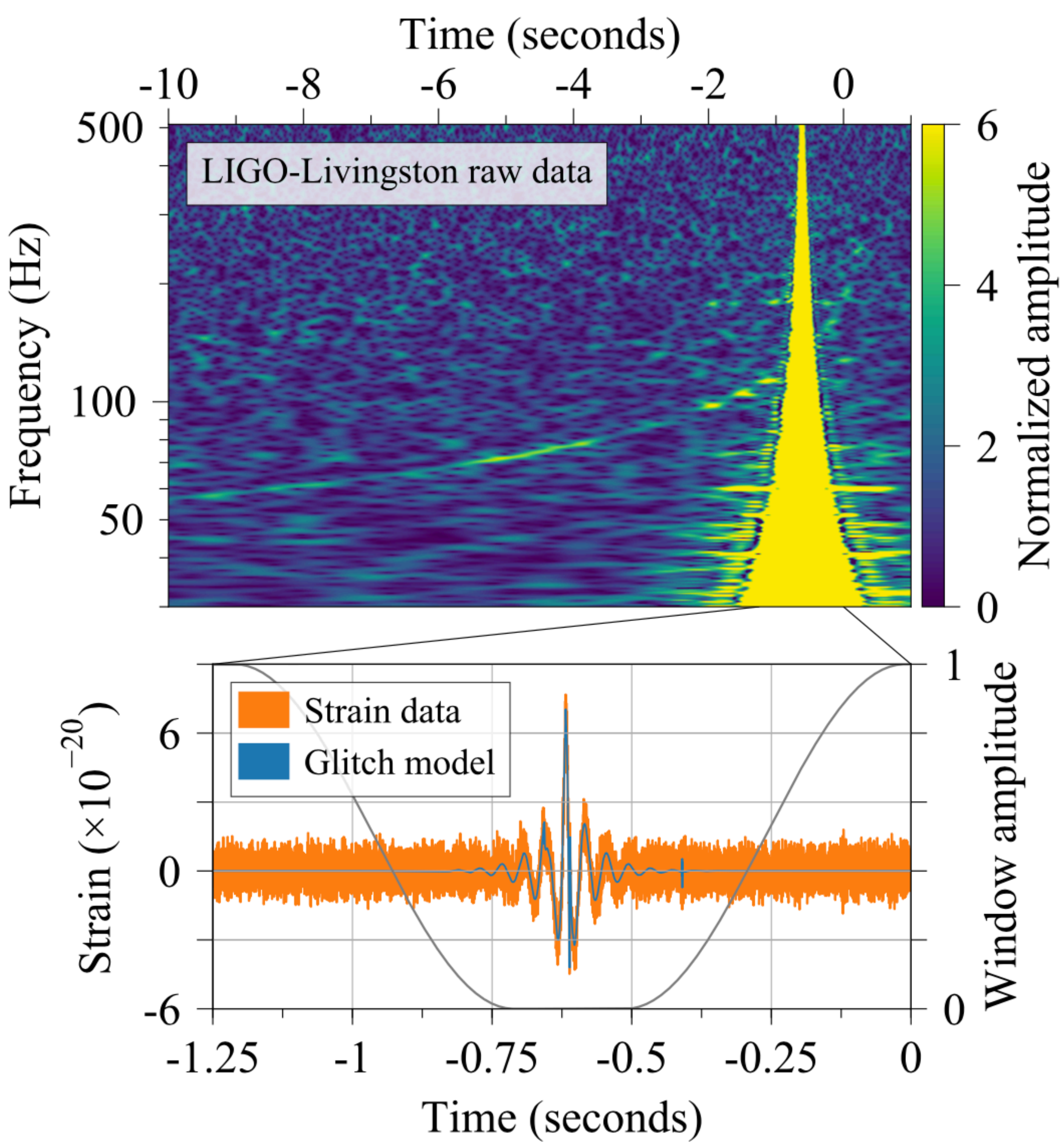


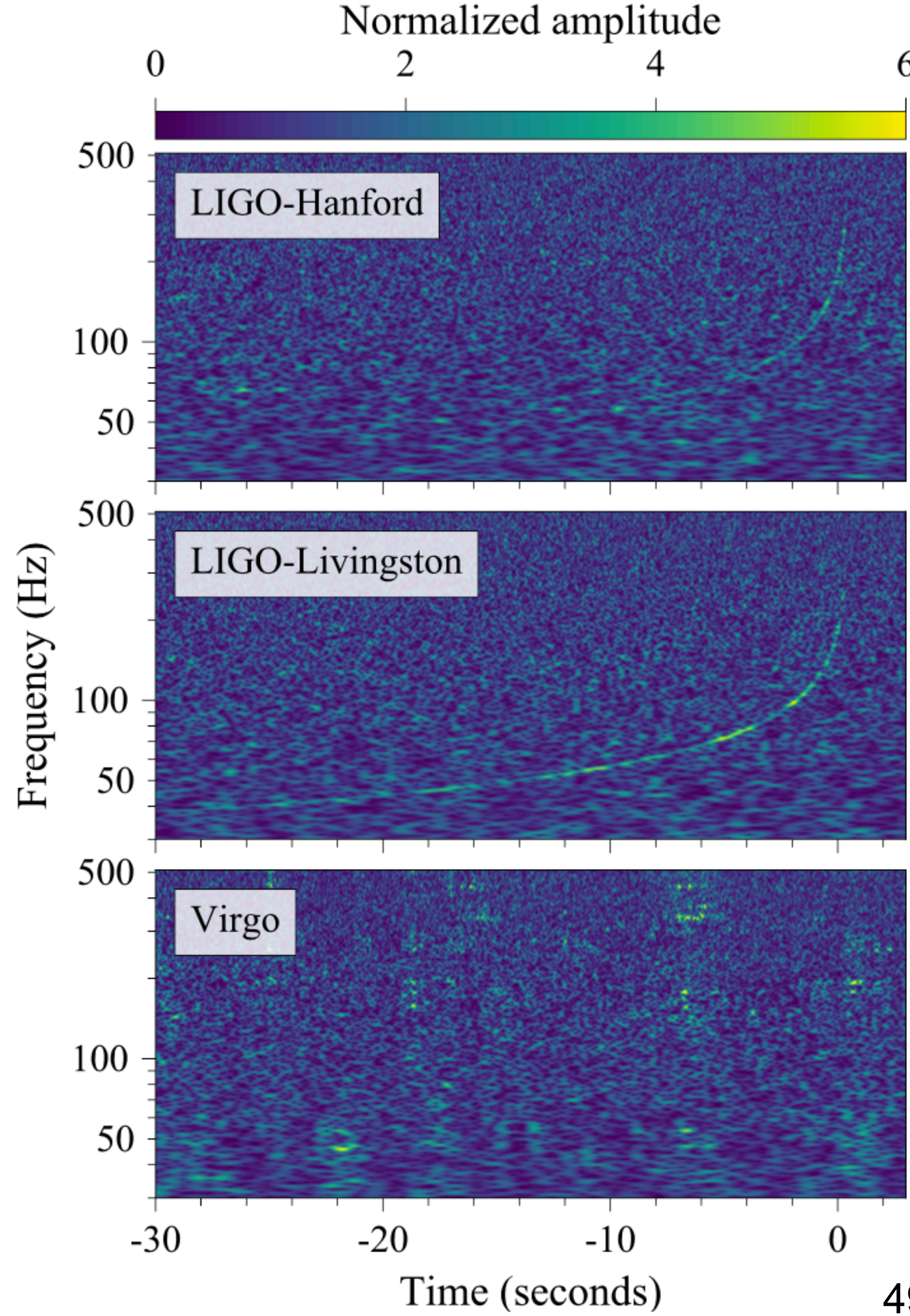
- Fermi-GBM: alert GRB170817A @ 14:41:20 CEST
- LIGO-Virgo signal @ 14:41:04 CEST, 2s before GRB170817A, detected after 6 minutes.
- LIGO-Virgo alert sent at 15:21:42 CEST, an extensive follow-up campaign starts within minutes.

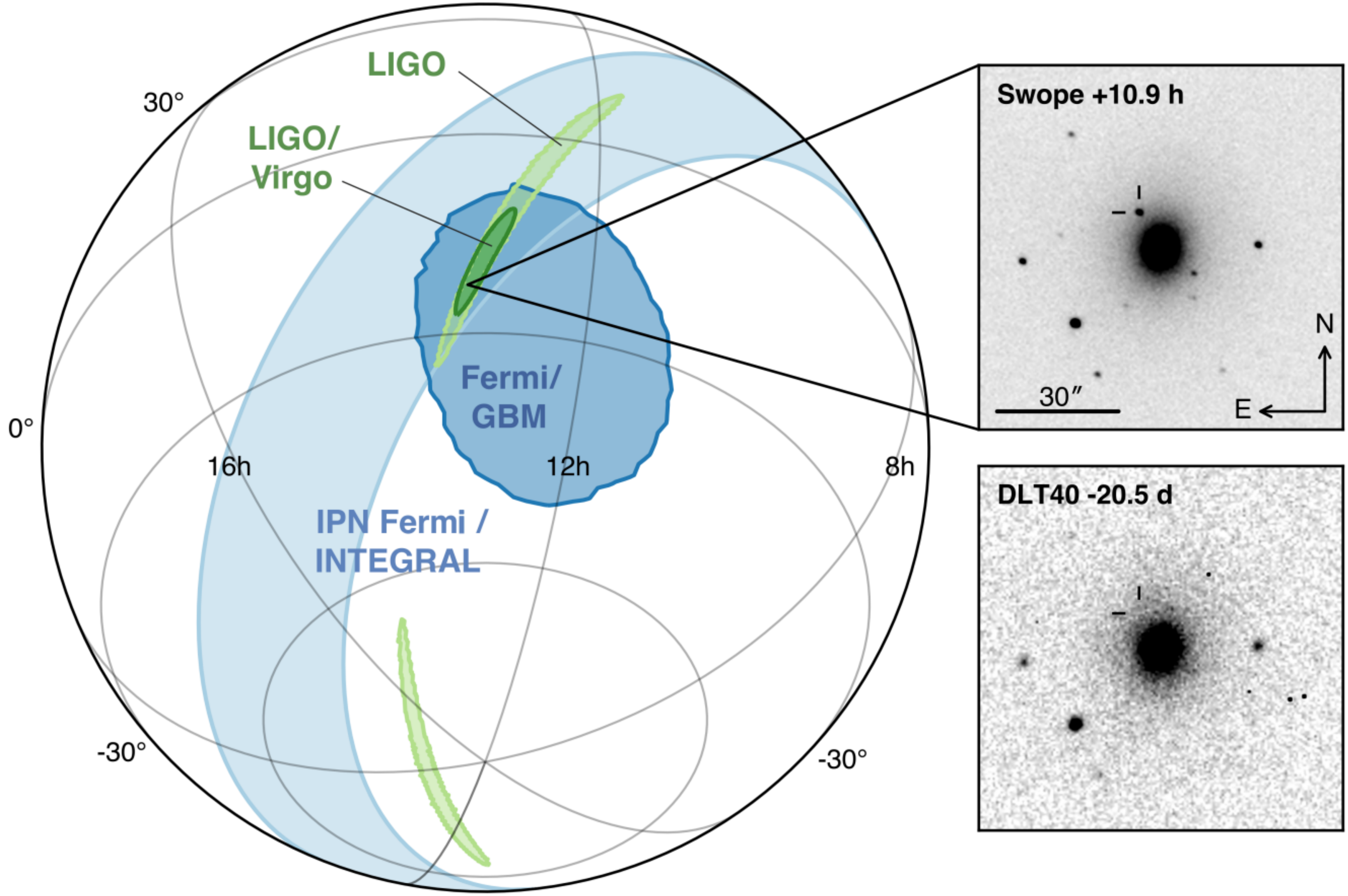
announced Oct. 16

GW170817:

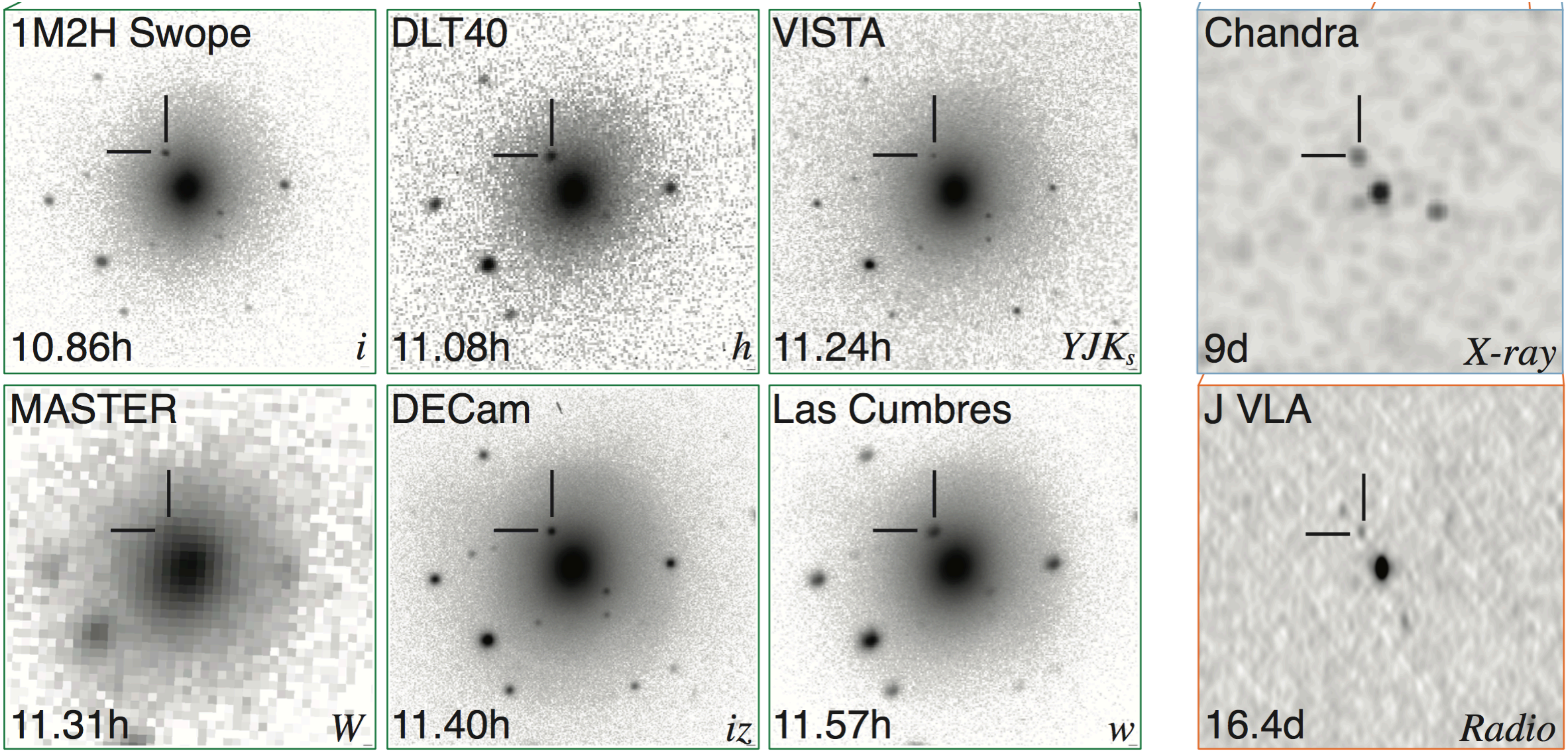
PRL 119, 161101 (2017)



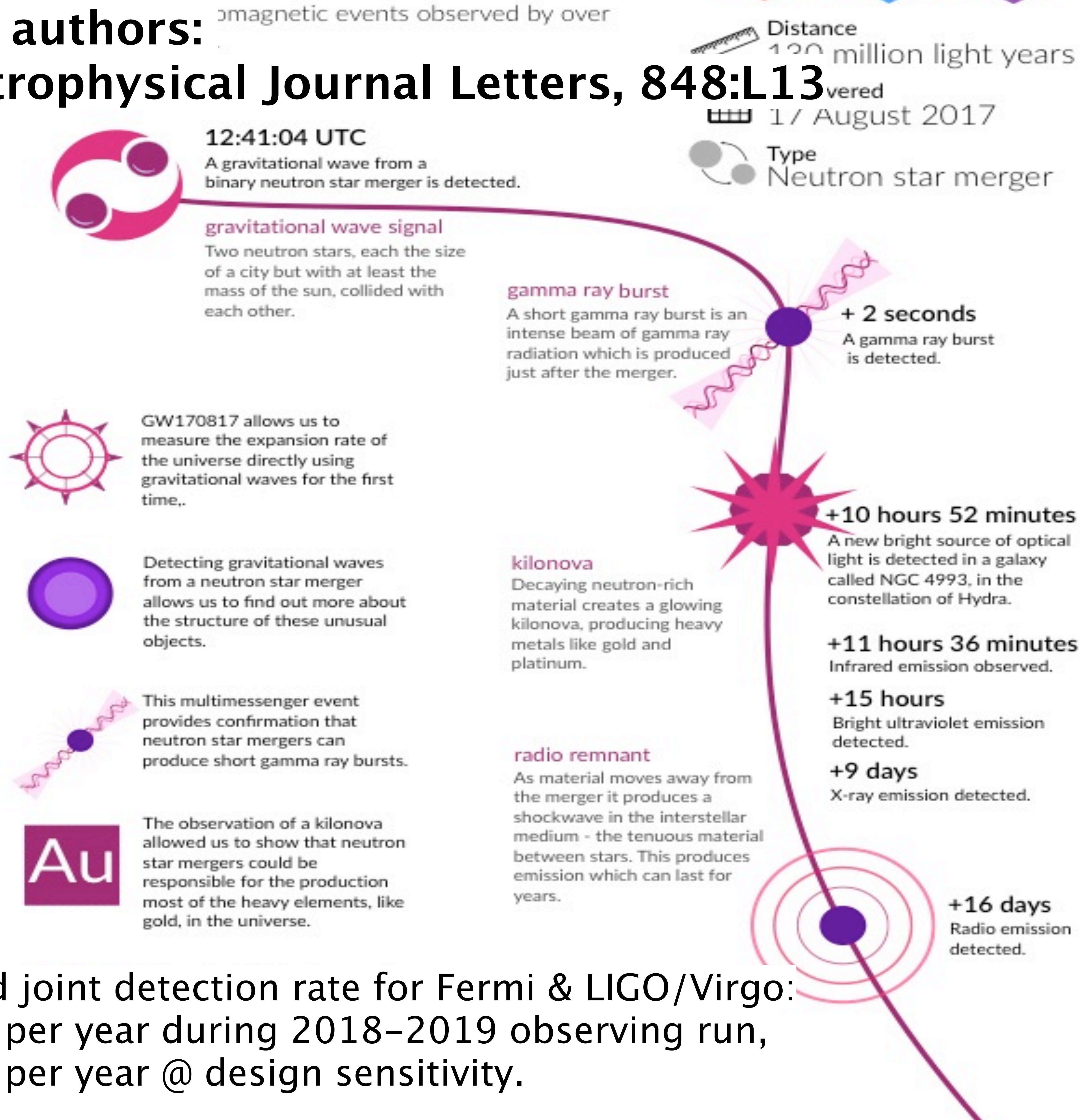



LIGO
LIGO/
Virgo
30°
0°
16h
12h
8h
-30°
-30°
Fermi/
GBM
IPN Fermi /
INTEGRAL
Swope +10.9 h
30″
N
E
DLT40 -20.5 d

In the evening in Chile, the telescope Swope identified an optical transient (SSS17a) in the galaxy NGC 4993, followed soon by many more telescopes.

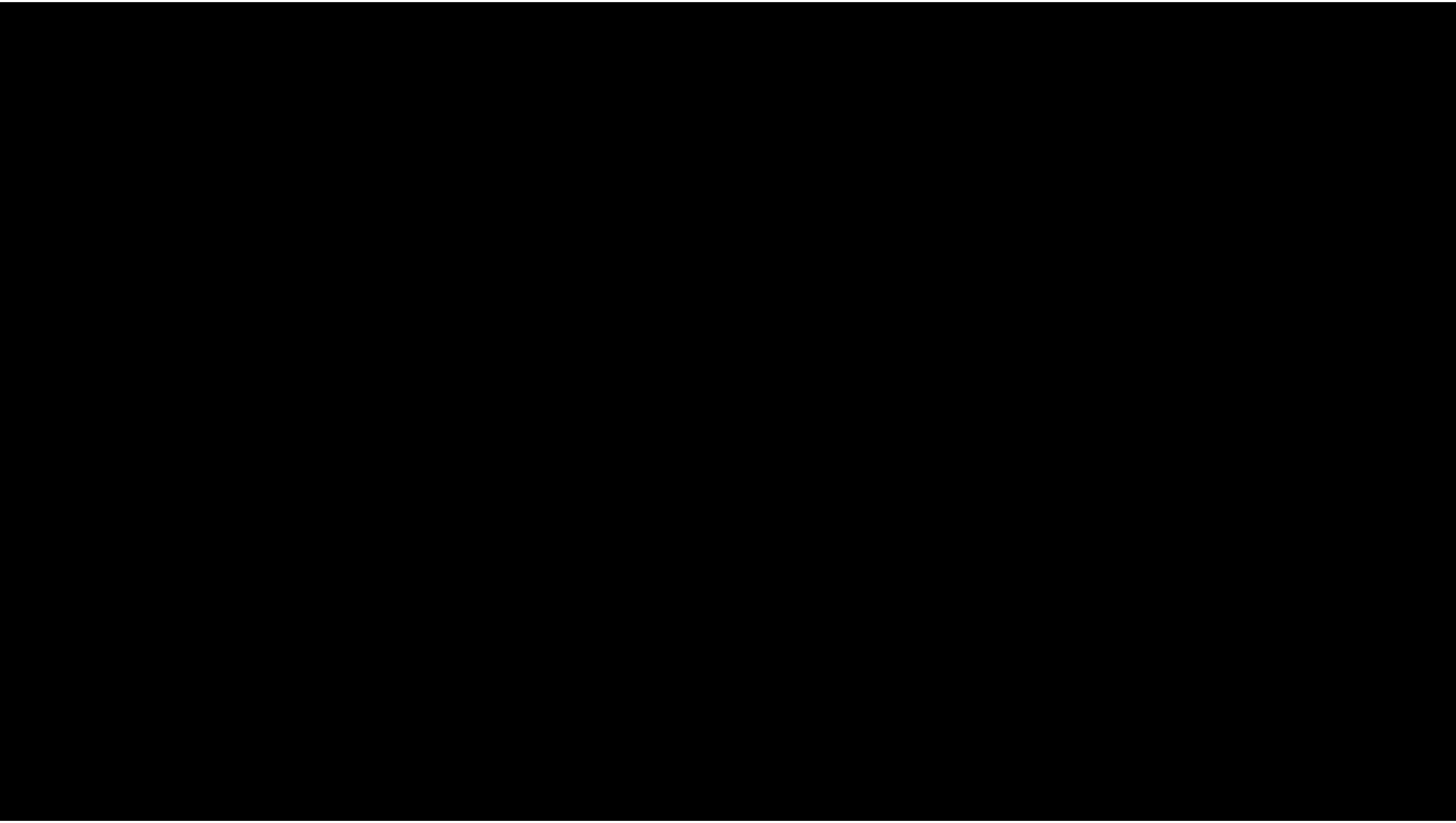


> 4000 authors:
The Astrophysical Journal Letters, 848:L13



Predicted joint detection rate for Fermi & LIGO/Virgo:
0.1-1.4 per year during 2018-2019 observing run,
0.3-1.7 per year @ design sensitivity.

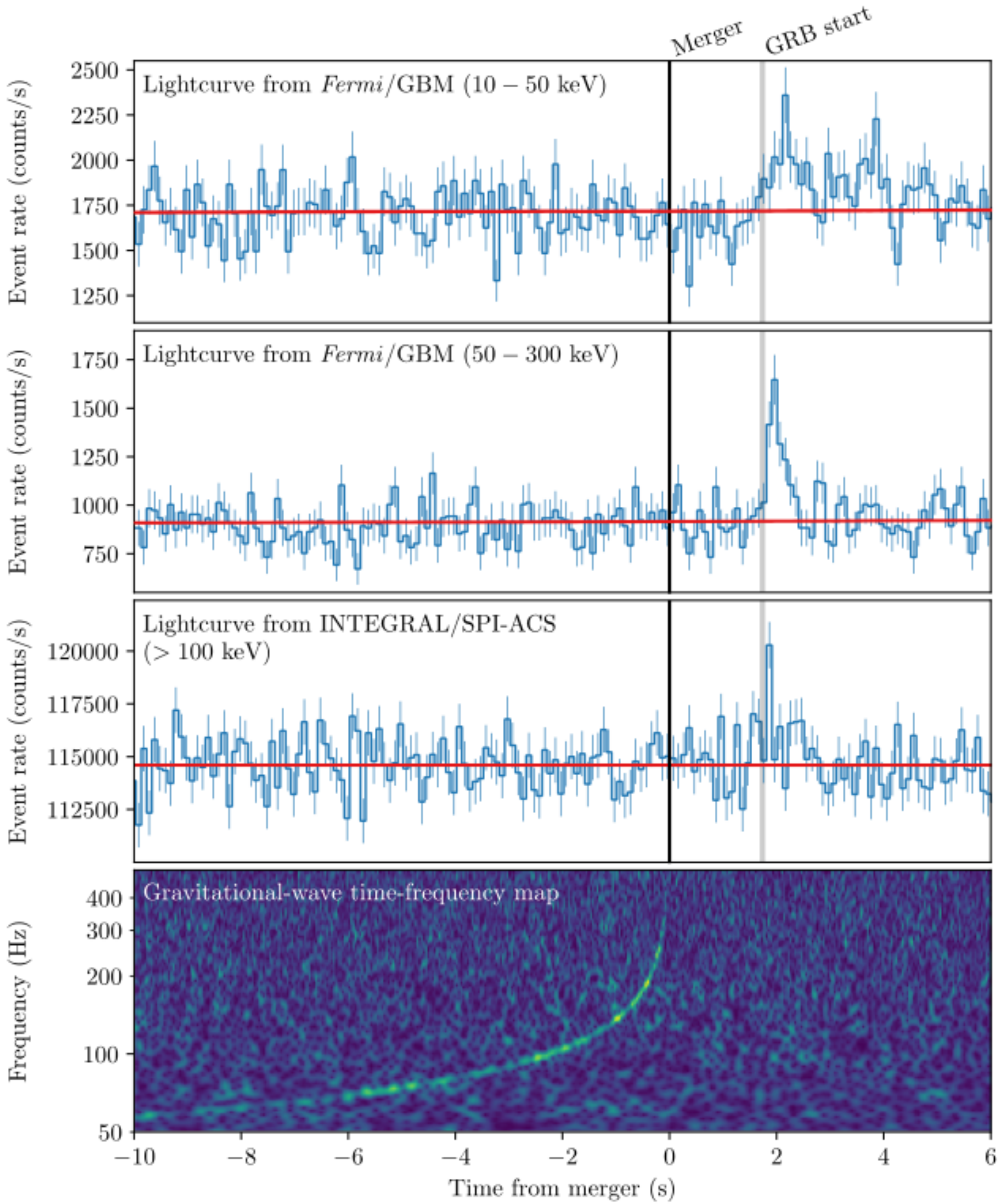
https://www.nasa.gov/press-release/nasa-missions-catch-first-light-from-a-gravitational-wave-event

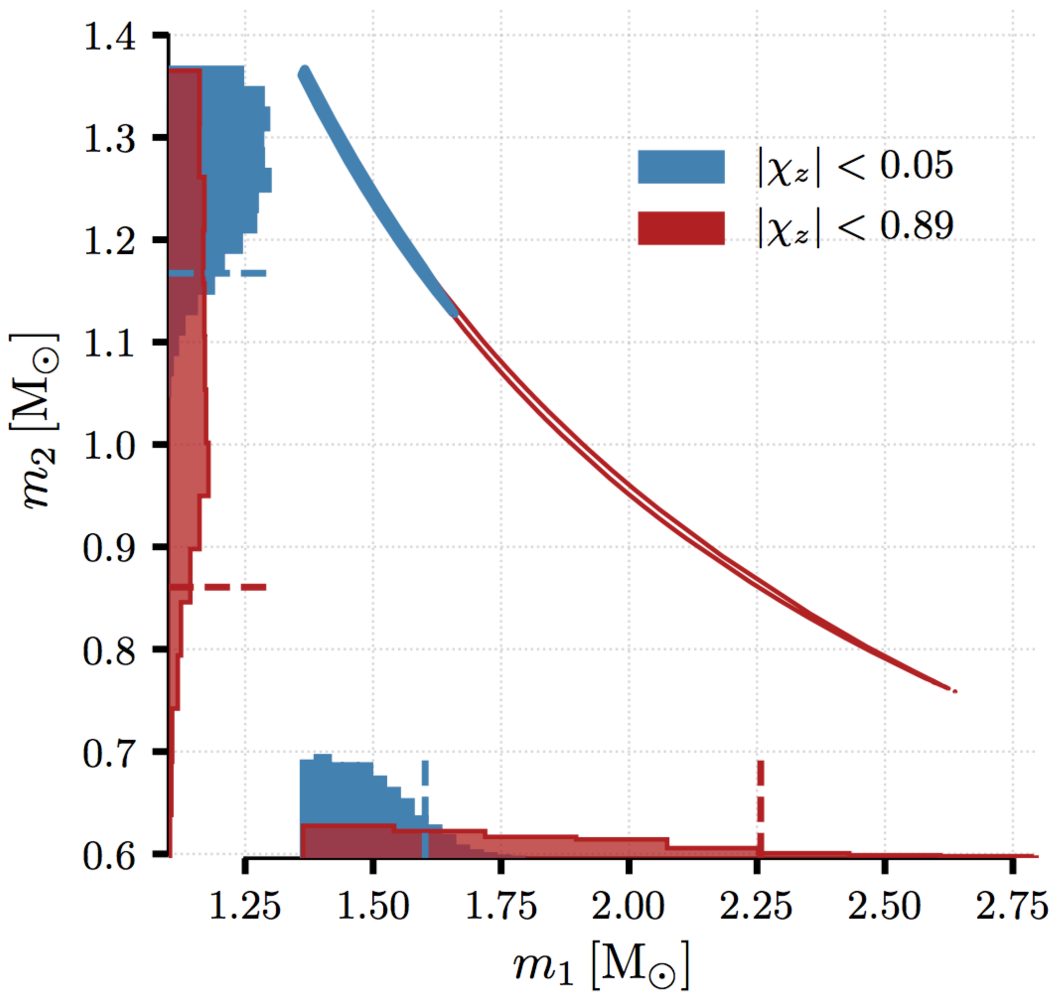


NASA's Goddard Space Flight Center/CI Lab

Component masses can be measured from GW signal:

- Consistent with typical masses of ~1.5 solar masses
- More accurate after assuming low NS spins.





Phys. Rev. Lett. 119 161101 (2017)

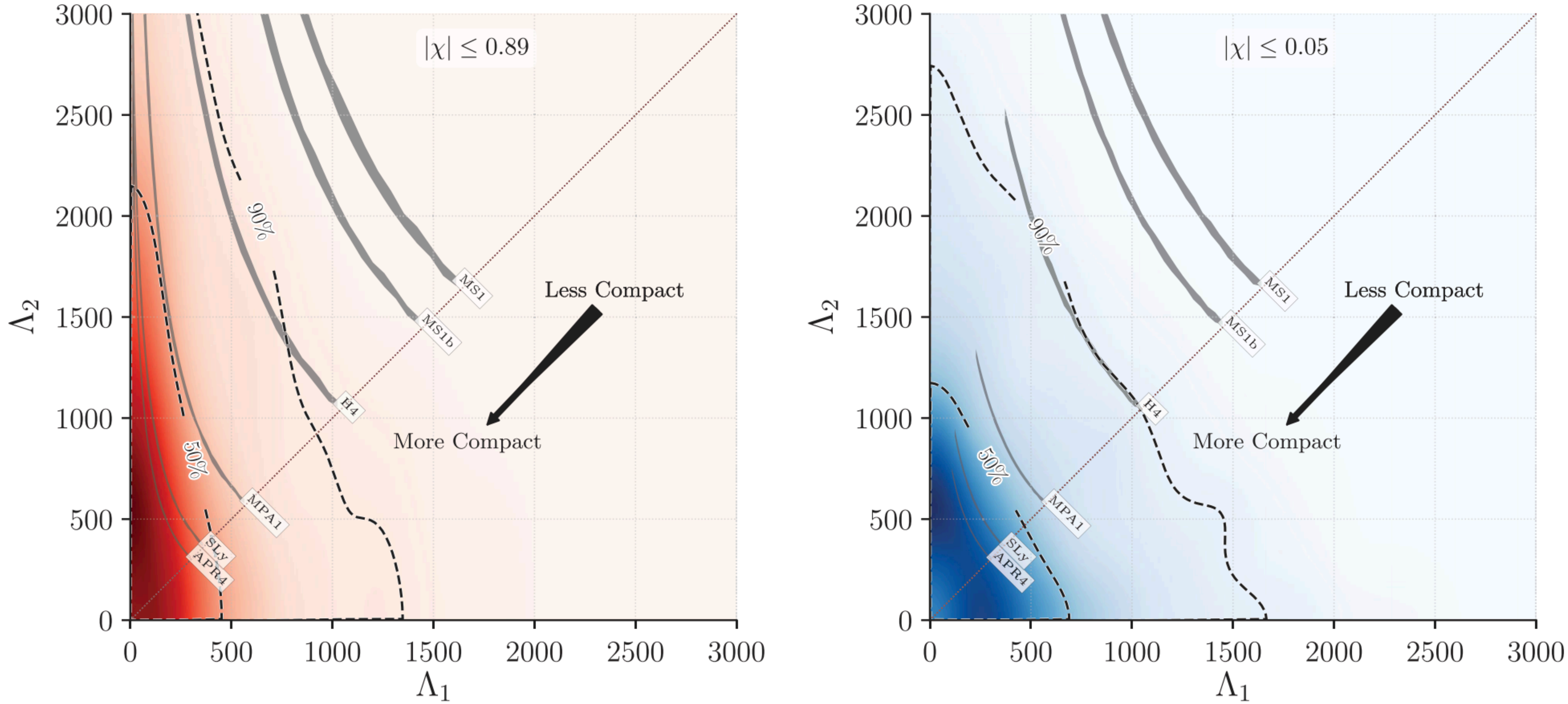
Love numbers: 5th order Post-Newtonian effect phase due to quadrupole deformation of neutron stars, depends on EOS:

$$\delta\Psi = -\frac{9}{16}\frac{v^5}{\mu M^4}\left[\left(11\frac{m_2}{m_1}+\frac{M}{m_1}\right)\lambda_1 + 1 \leftrightarrow 2\right]$$

Preliminary results exclude large neutron stars.

high spin

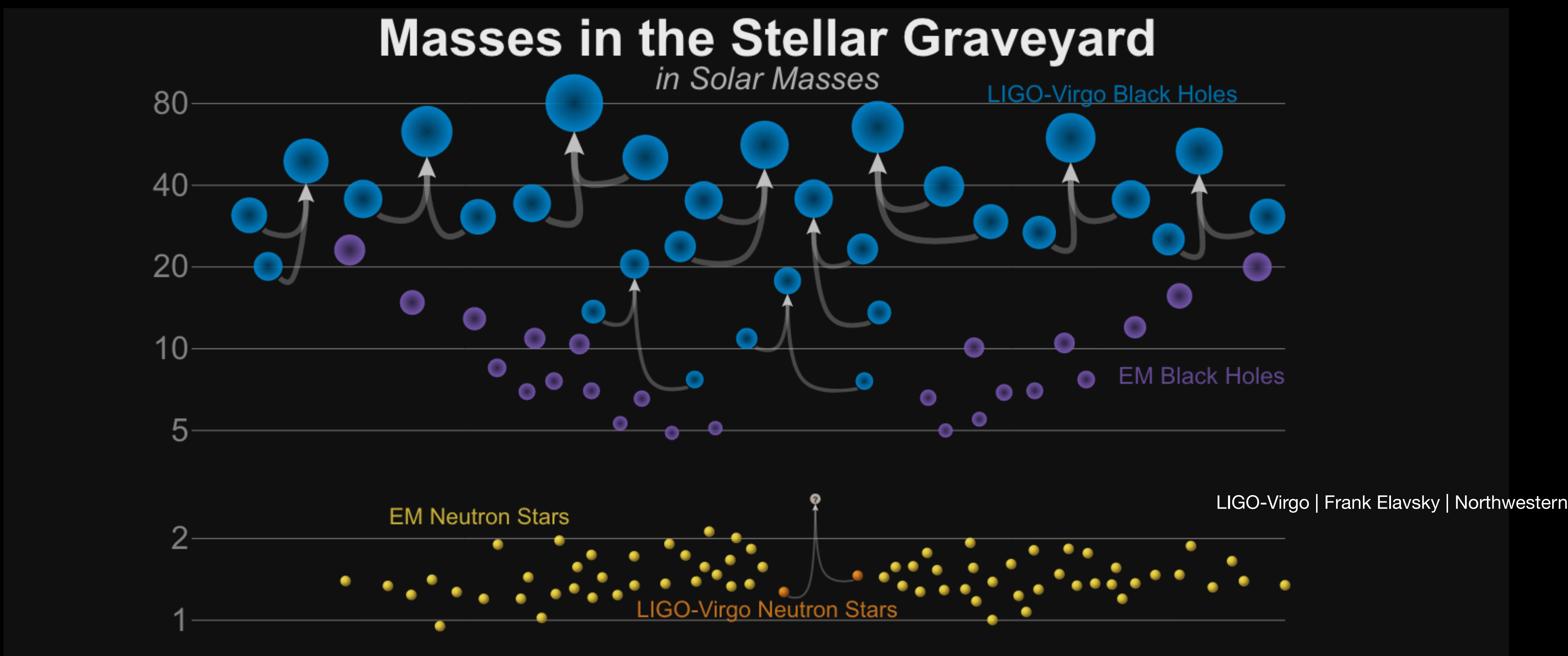
low spin



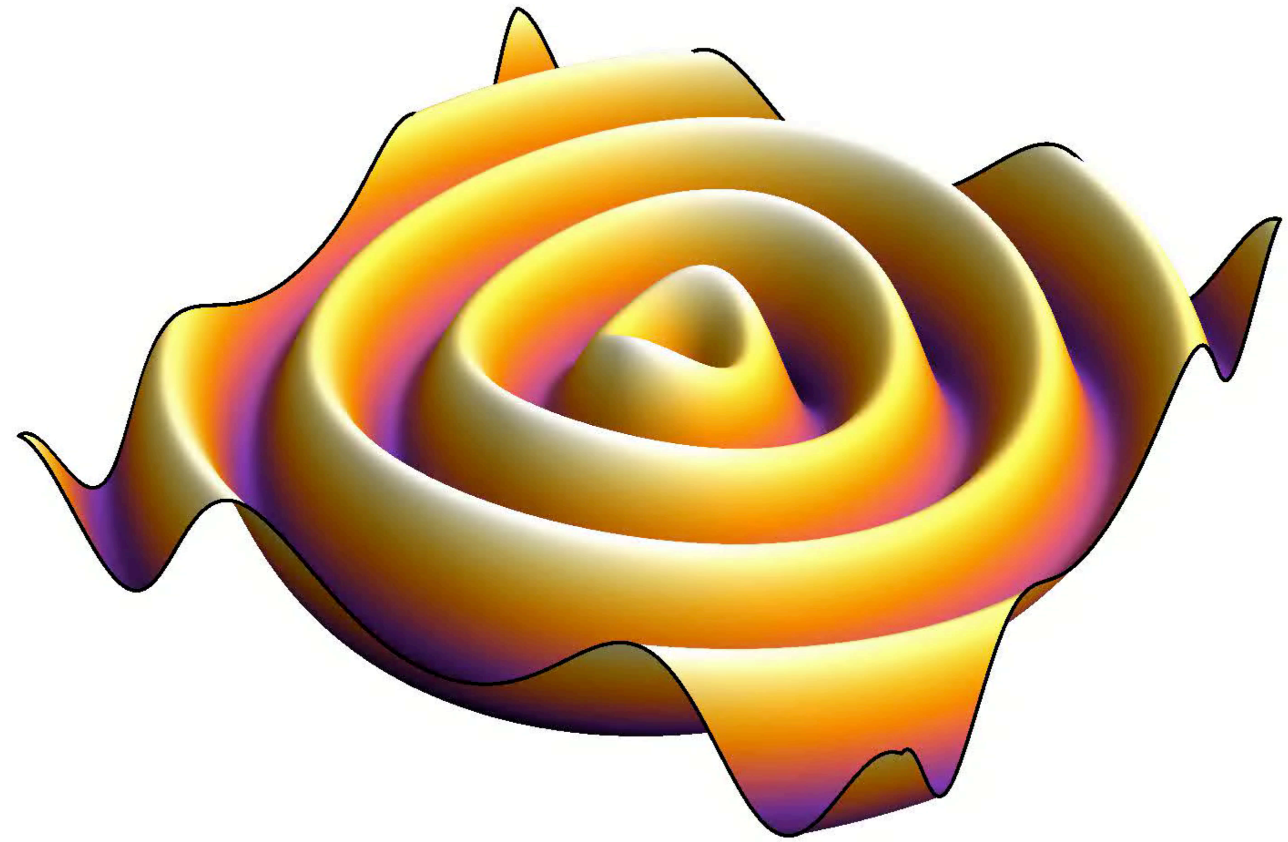
What have we seen so far: binary systems

- LIGO + Virgo have released their compact binary coalescences catalogue for the O1 + O2 observation runs.

- O1+O2 go to 11: 10 BBH mergers, 1 BNS.
- O3 since April 1 2019: https://gracedb.ligo.org



Gravitational waves from orbital motion



Radiation power of the sun (electromagnetic).		4×10^{26} W
Iron bar (1000 t, 100 m, 3 Hz)		10^{-26} W
Earth in orbit around sun		200 W
Stellar binary system		10^{15} - 10^{30} W
BNS system (100 km, 100 Hz)		10^{45} W
GW150914 maximal	How does this scale with mass?	3.6×10^{49} W

post-Newtonian 2-body problem

- Start with energy as function of separation R or orbital frequency ω:

$$E(R) = m_1 + m_2 - M\frac{\eta}{2}\frac{M}{R} \qquad E(\omega) = m_1 + m_2 - M\frac{\eta}{2}\left(\frac{(M\omega)^2}{G}\right)^{\frac{1}{3}}$$

  - Convert between ω and R with Kepler: $\omega^2 R^3 = G M$.
  - Or PN expansion: $$\omega^2(R) = \frac{GM}{R^3}\left(1 + f_1(R)\left(\frac{v}{c}\right)^2 + f_2(R)\left(\frac{v}{c}\right)^4 + \dots\right)$$

- Compute energy loss P=-dE/dt to some order in v/c, e.g. at leading order with the quadrupole formula.

$$\frac{dE}{dt} = -\frac{32}{5}\frac{G^4}{c^5}\eta^2\left(\frac{v}{c}\right)^{10}\left(1 + O(v^2) + \dots\right) \qquad v = (GM\omega)^{1/3} \qquad \eta = \frac{m_1 m_2}{(m_1+m_2)^2}$$

To compute the rate of change of any quantity X (e.g. X=ω, R) we write

$$\frac{dX}{dt} = \frac{\frac{dE}{dt}}{\frac{dE}{dX}} \qquad \frac{d\phi}{dt} = \omega$$ <- For GWs we also need phase.

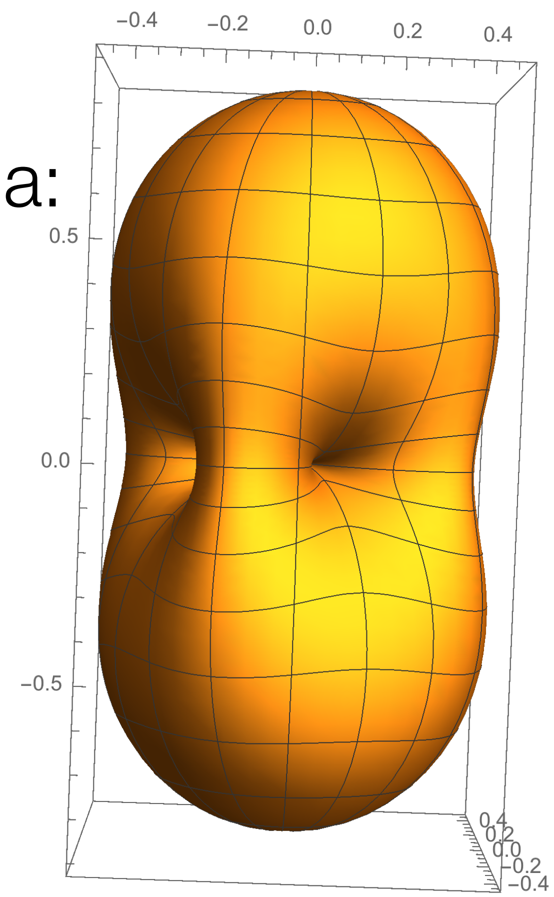
Exact solution for R: $$R(t) = \left(\frac{256}{5}\eta M^3\right)^{\frac{1}{4}}(t_c - t)^{\frac{1}{4}}$$

The Chirp - first step beyond Newton

Simplest approximation of the signal:
Newtonian point particles + energy loss from quadrupole formula:

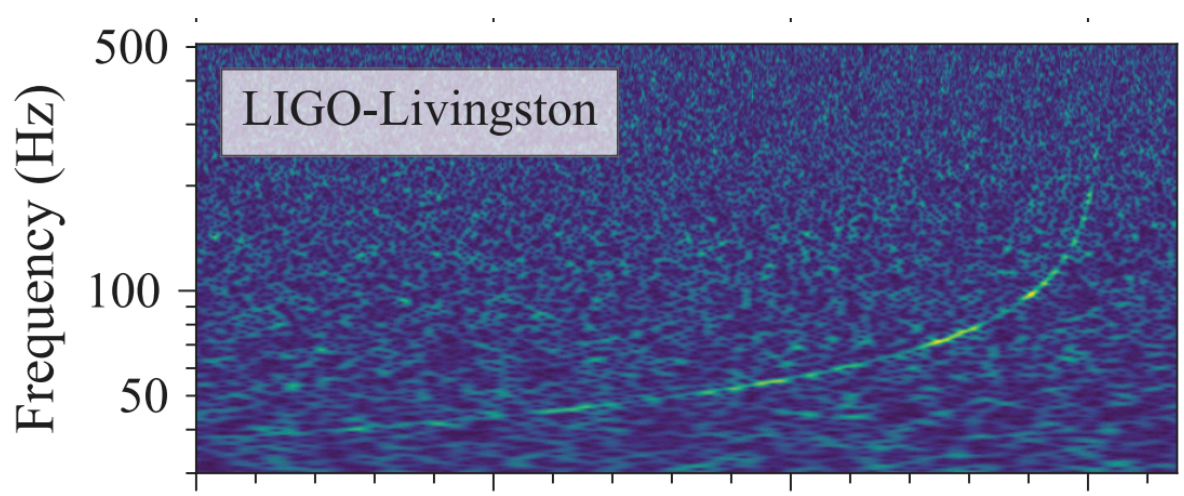
$$h_+(t) = \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\rm gw}}{c}\right)^{2/3}\frac{1+\cos^2\theta}{2}\cos(2\pi f_{\rm gw} t_{\rm ret} + 2\phi)\,,$$

$$h_\times(t) = \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\rm gw}}{c}\right)^{2/3}\cos\theta\,\sin(2\pi f_{\rm gw} t_{\rm ret} + 2\phi)\,,$$

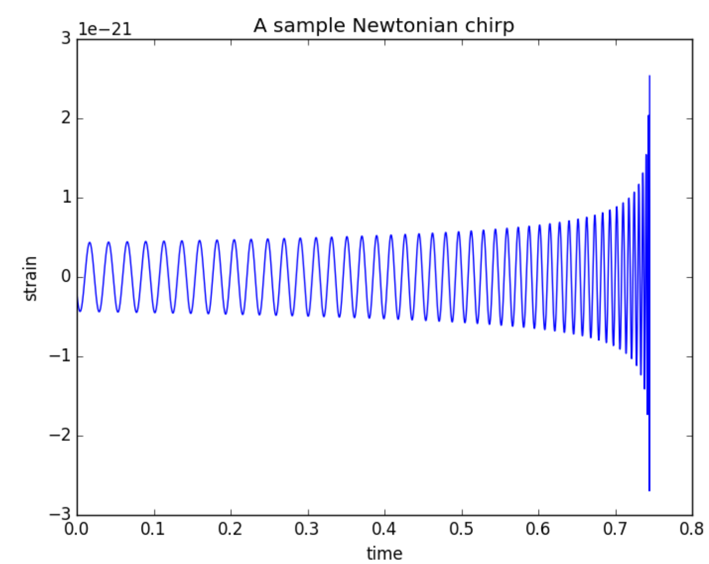


Quadrupole wave pattern

Only intrinsic source parameter: chirp mass Mc.



LSC+Virgo, PRL 119 161101 (2017)

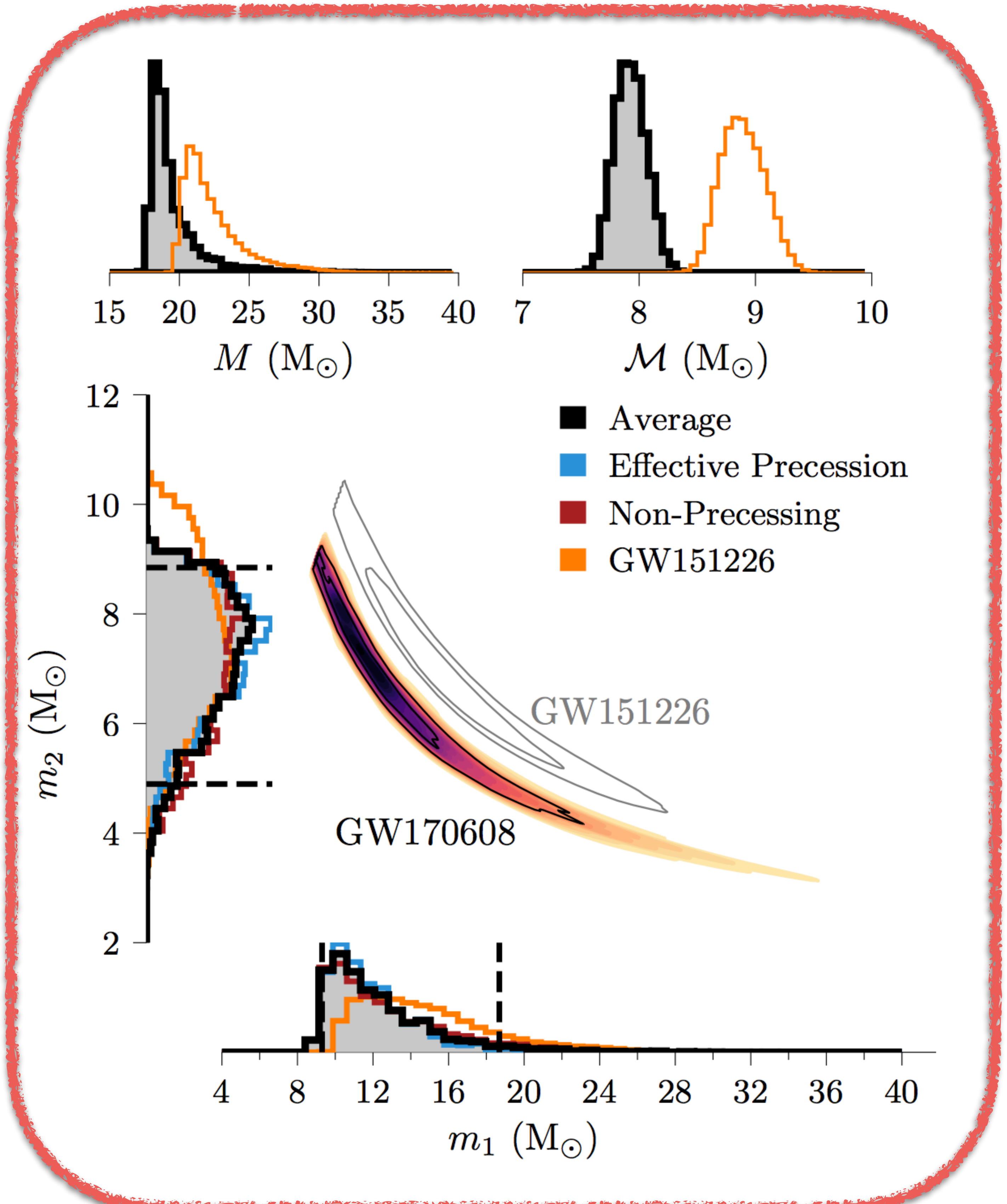


maximise emission:
- Face on/off
- Large mass
- High freq./small separation

Degeneracy:
only measure M_c, not both component masses.
Good for searches - bad for parameter estimation!

GW170608: lightest binary so far

The Astrophysical Journal Letters 851 (2017)



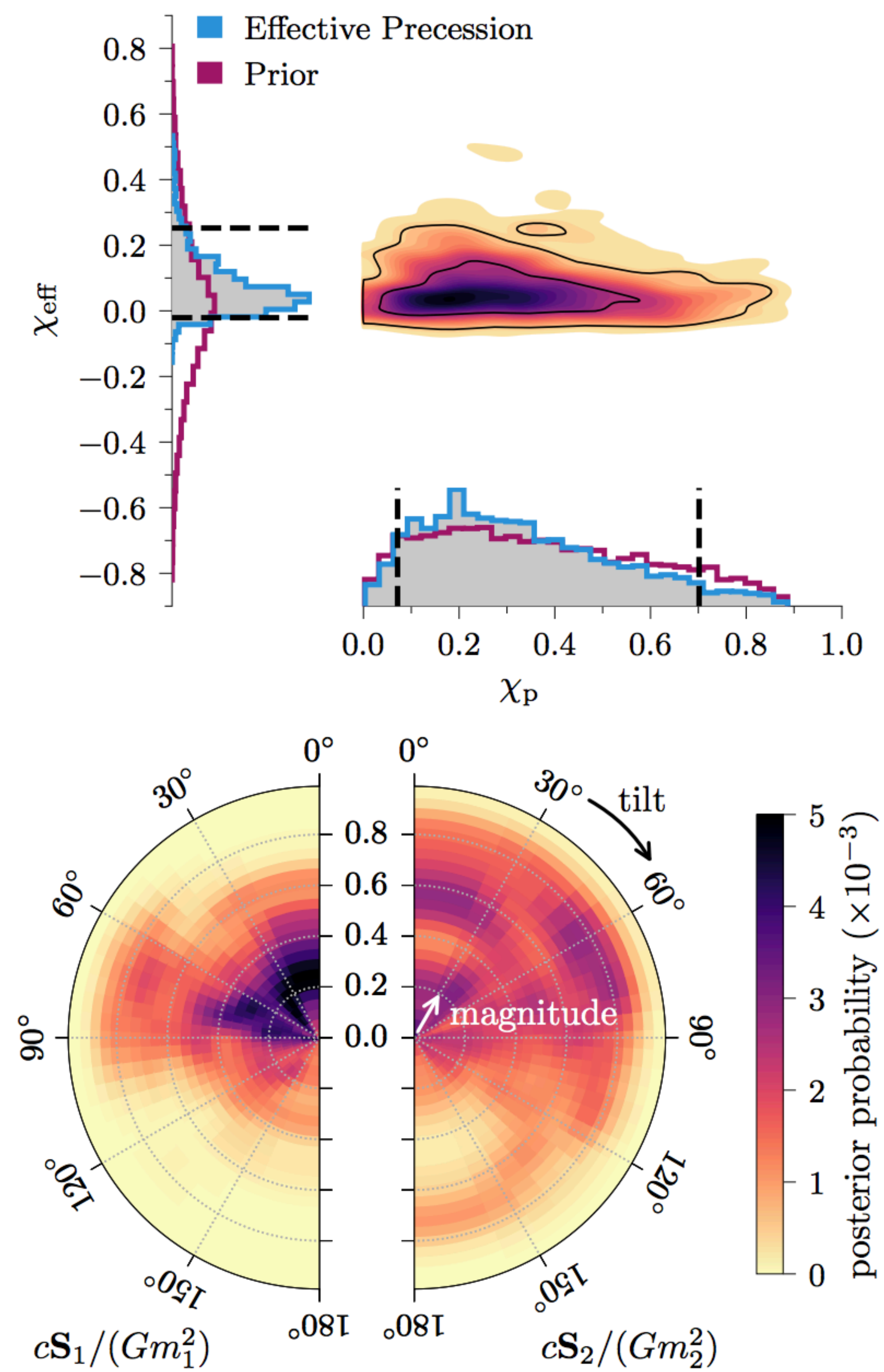
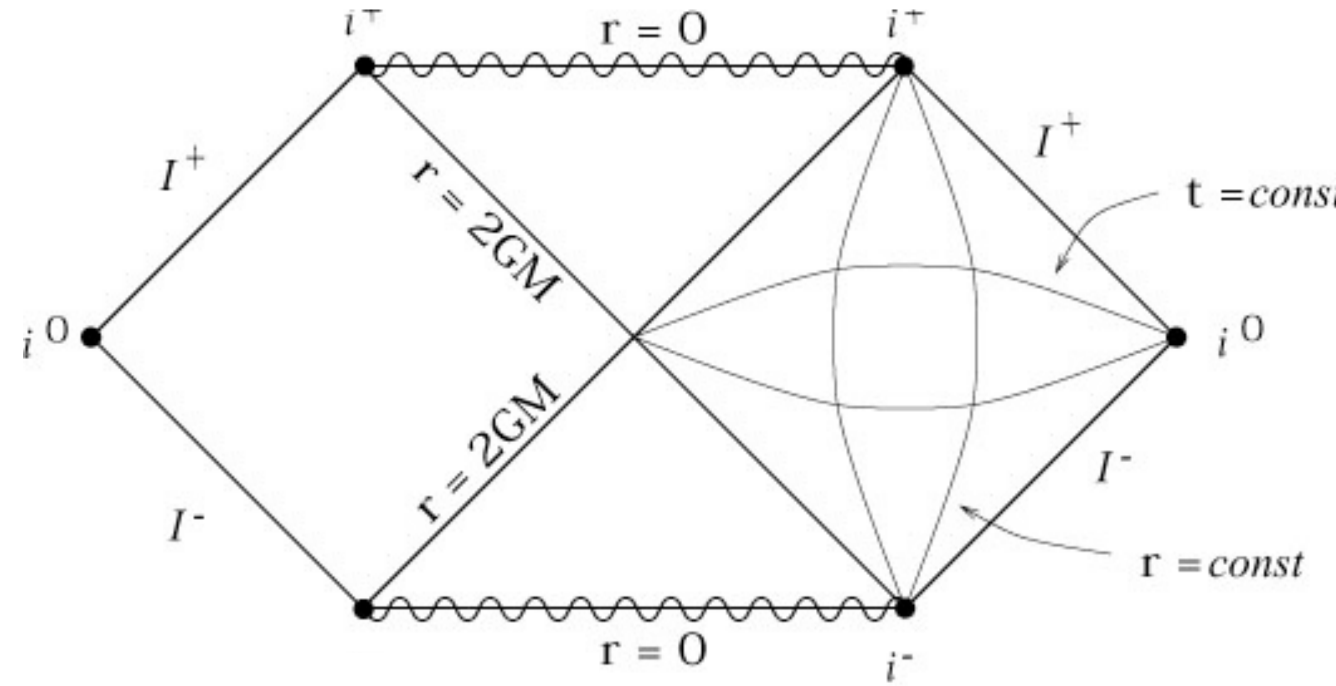


Figure 2. Posterior probability densities for binary component masses (m_1, m_2), total mass (M), and chirp mass

Vacuum (Exterior) Schwarzschild solution



- spherical symmetry + time independent
- r-coordinate: luminosity distance

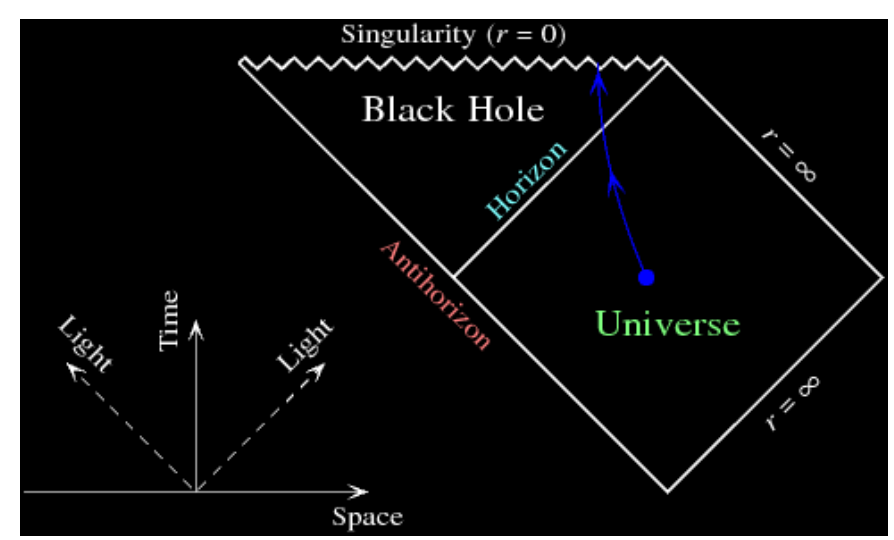
$$A_r = 4\pi r^2 \quad \Rightarrow \quad r = \sqrt{\frac{A_r}{4\pi}}$$

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2\, dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta\, d\phi^2)$$

- Curvature singularity at r=0
- Coordinate singularity at r = 2 GM/c^2

$$g_{ab}\left(\frac{\partial}{\partial t}\right)^a \left(\frac{\partial}{\partial t}\right)^b = (1 - \frac{2GM}{rc^2})$$ Move @ speed of light to stay @ r=2M.

- Can choose units G=c=M=1
  => all SS metrics with M>0 equivalent!



Event horizon: null surface traced backwards in time from i+, BH is the region inside the EH.

Interior Schwarzschild solution

- spherical symmetry + time independent + matter, e.g. EOS P(ρ)
- Interior solution with ideal fluid in equilibrium: TOV equation

$$ds^2 = e^\nu c^2\, dt^2 - \left(1 - \frac{2Gm}{rc^2}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

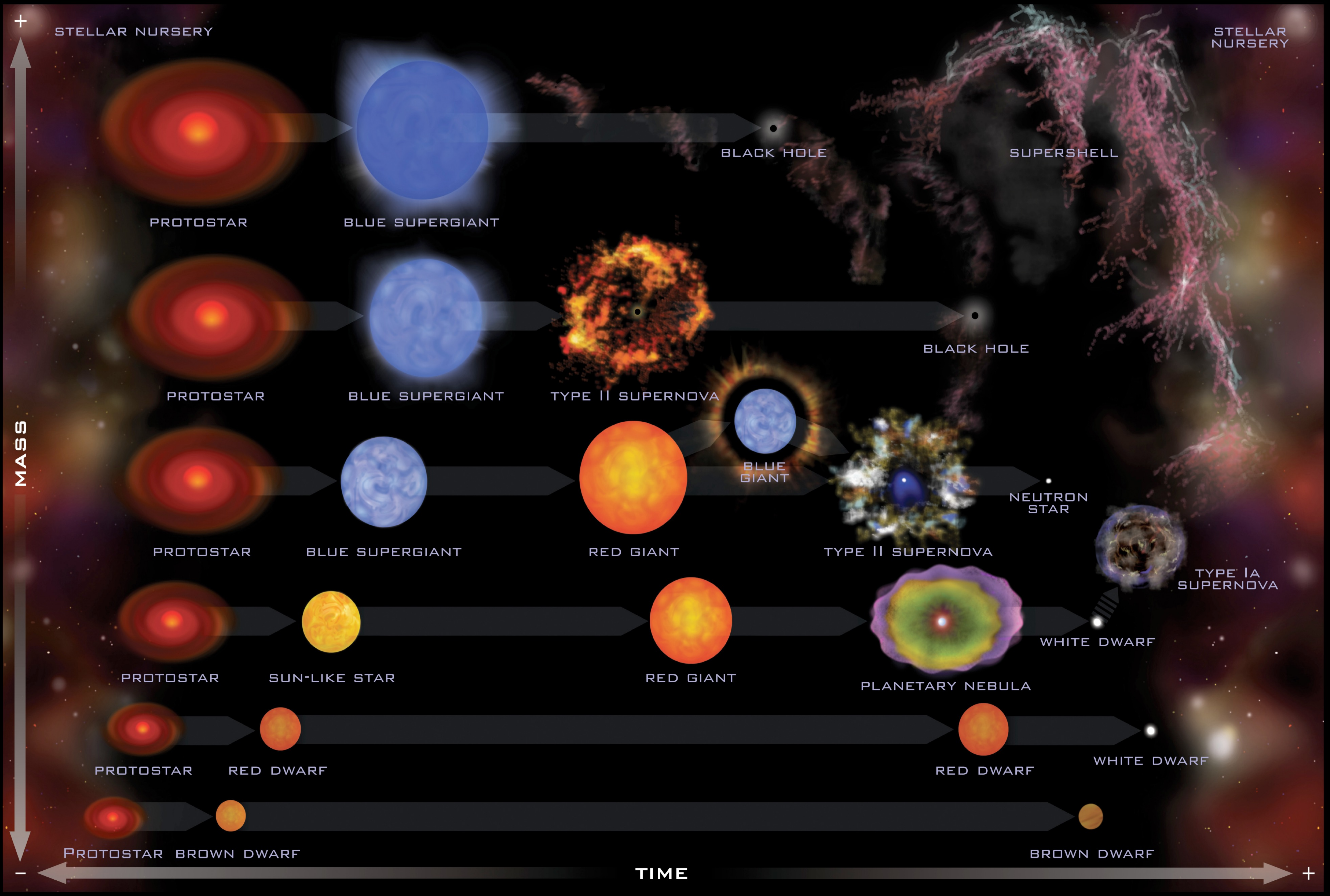
$$\frac{d\nu}{dr} = -\left(\frac{2}{P + \rho c^2}\right)\frac{dP}{dr} \qquad m(r) = 4\pi \int_0^r \rho(r') r'^2\, dr'$$

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi r^3 P}{mc^2}\right)\left(1 - \frac{2Gm}{rc^2}\right)^{-1}$$

GR vs. Newton: GR needs stronger pressure gradient for equilibrium.

- Qualitative behaviour captured by star with: ρ**=const.**, P_0 = central pressure.
  - Newton: solution always exists, $P(r) = \frac{2}{3}\pi\rho_0^2\left(R^2 - r^2\right)$
  - GR: the central pressure can diverge for finite ρ
    => there is a maximal mass for stability: $M_{max} = \frac{4}{9\sqrt{3\pi\rho_0}}$

The life cycle of stars



The simplicity of black holes in GR

- **How many equilibrium black hole solutions exist?**
  Consider fundamental long range interactions - GR + EM field.

- **Black hole uniqueness theorem / no hair theorem:**

  - All time independent solutions belong to Kerr-Newman family:
    parameterised by mass M, angular momentum J, charge Q.

  - Astrophysical bodies are ~ electrically neutral => **M, J**. (Kerr)

  - Cosmic censorship conjecture: dimensionless $\mathbf{J/M^2 < 1}$.
    (equality: extreme Kerr)

- **Parameter space of binary black holes:**

  - **Quasi-circular** (circular at infinite separation):

    - 2 masses + 2 spin vectors = 8 parameters (total mass = scale parameter)

  - **Eccentric:** 2 more. (but eccentricity is shed rapidly during inspiral)

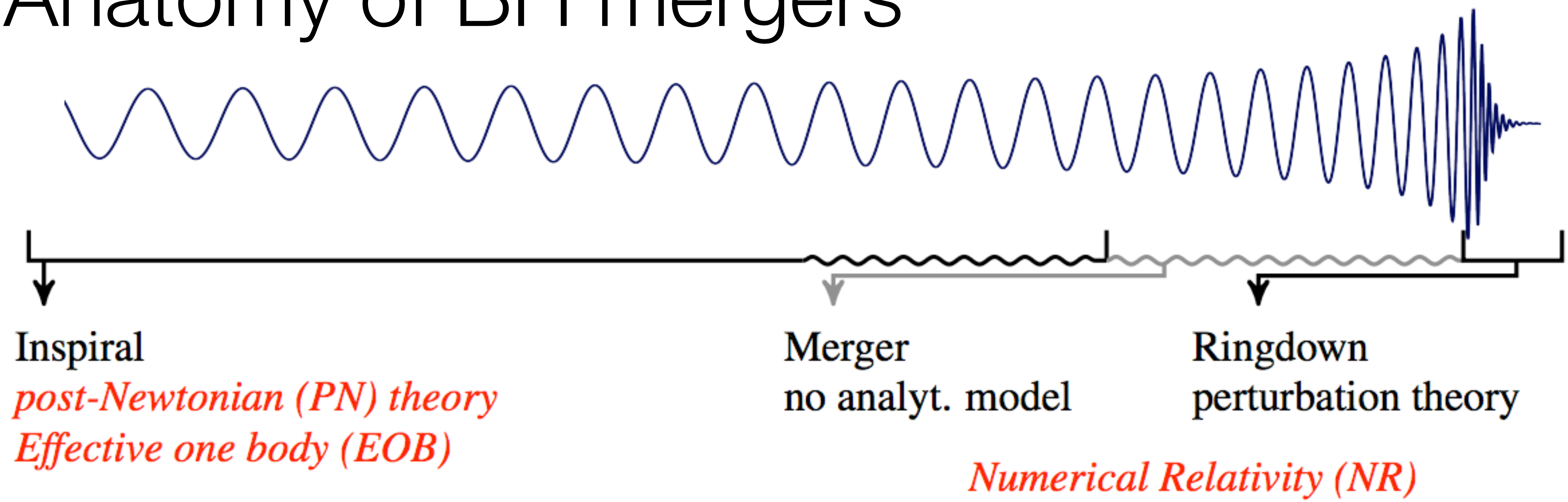
Quasi-normal modes

- [Kokkotas, K.D. & Schmidt, B.G. Living Rev. Relativ. (1999)]
- Normal modes: classical linear oscillating systems (strings, membranes, ...) will have preferred sinusoidal oscillations if the energy is conserved.
- If energy is lost, the systems will show damped oscillations (i.e. the frequency will become complex):

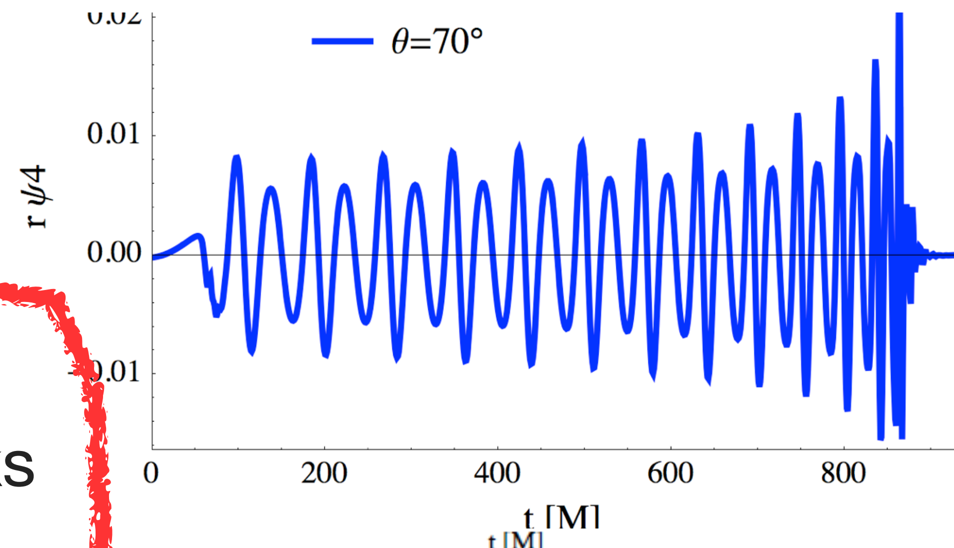
$$h(t) = \sum_{lmk} h_0 e^{i\omega_{lmk} t}$$

- Perturbed black holes loose energy to infinity, and energy is absorbed by the black hole -> quasi-normal modes, frequencies depend on the spin of the black hole (known e.g. from fit to NR data for final Kerr parameter and mass).
- Code to compute QNMs and data tables -> https://pages.jh.edu/~eberti2/ringdown.
- Ringdown period for l=m=2 quadrupole fundamental mode ≃ 17M.

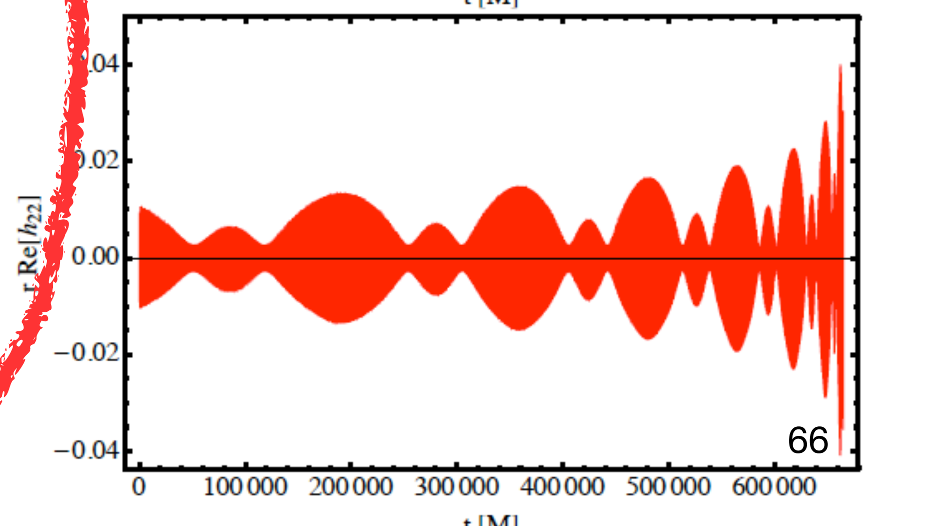
Anatomy of BH mergers



- Inspiral: energy loss to GWs leads to adiabatic inspiral, well described by post-Newtonian perturbation theory.



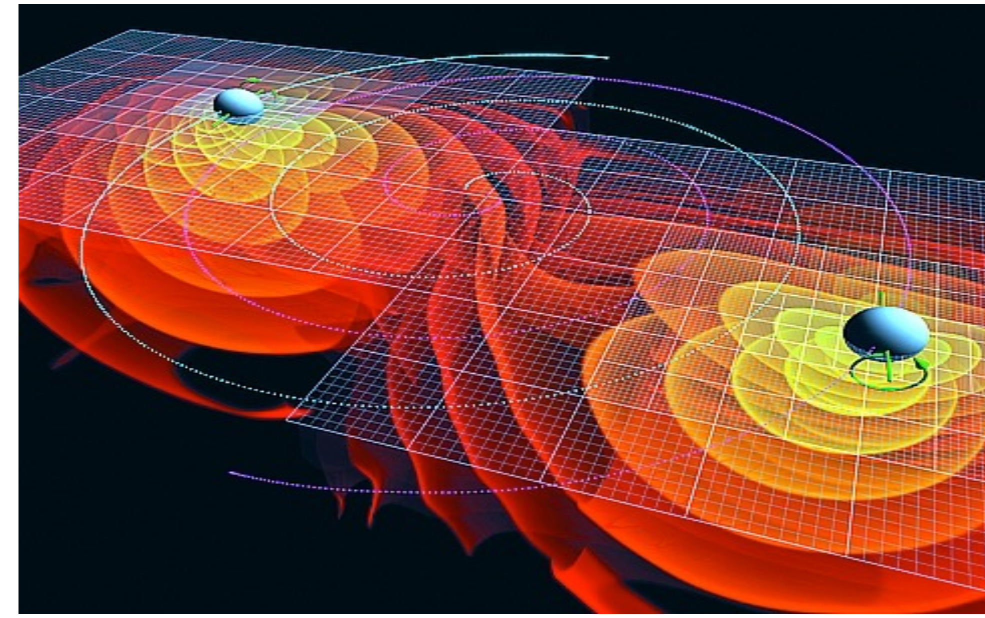
- Late inspiral & merger: post-Newtonian expansion breaks
  - solve full Einstein equations numerically as PDEs, "match" to post-Newtonian inspiral.
  - Most of the energy released (< 12 % of the mass).
- Ringdown: superposition of damped harmonics.



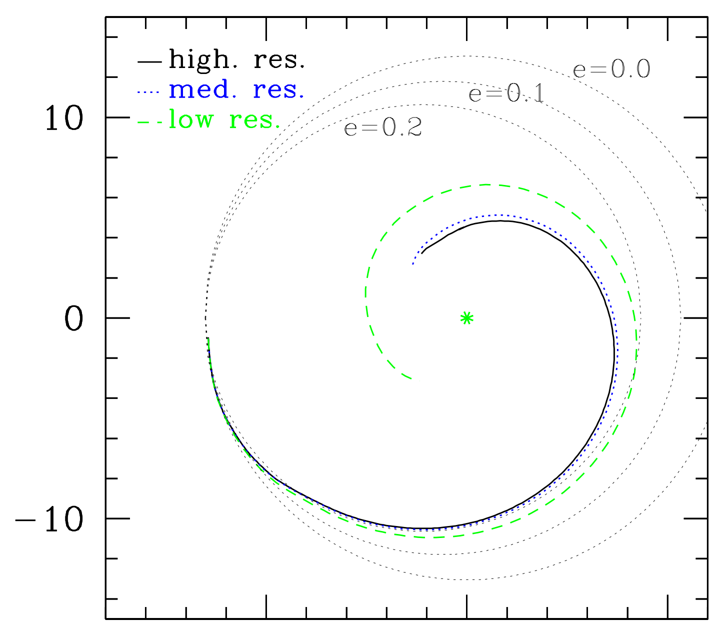
Numerical Relativity

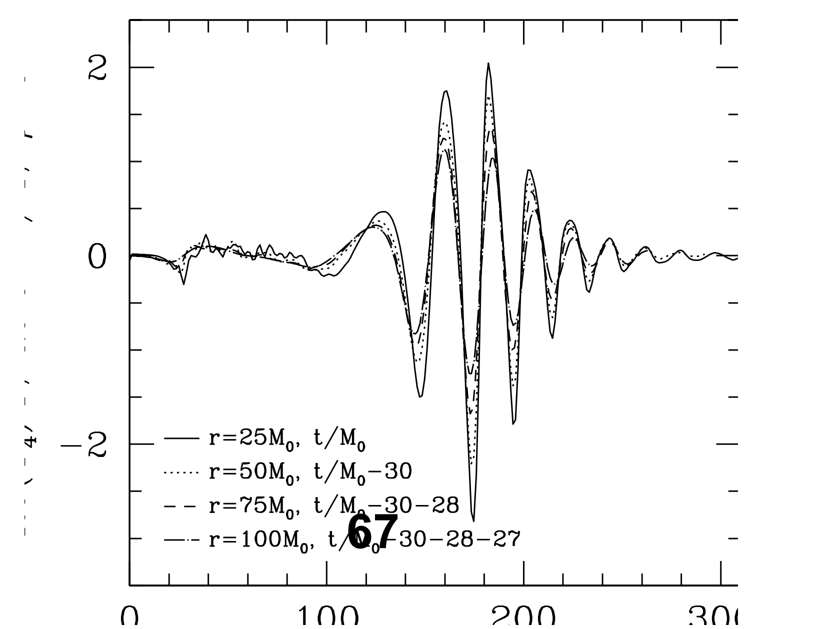
Starting with an initial configuration of the gravitational field (solving constraints), we can use evolution equations to predict the future dynamics.

Computing a few milliseconds of a collision of neutron stars or black holes, or a supernova explosion, can take many weeks on hundreds of processors.



- **First orbit + GWs: Frans Pretorius 2005**
- Surprise breakthrough after 4 decades of unstable formulations.
- Only 10 years to sufficiently accurate waveforms for first detection.





BSSN: the workhorse formulation for puncture evolutions

$\varphi = (1/12)\log(\det\gamma_{ij})$
$\tilde{\gamma}_{ij} = e^{-4\varphi}\gamma_{ij}$
$K = \gamma^{ij}K_{ij},$

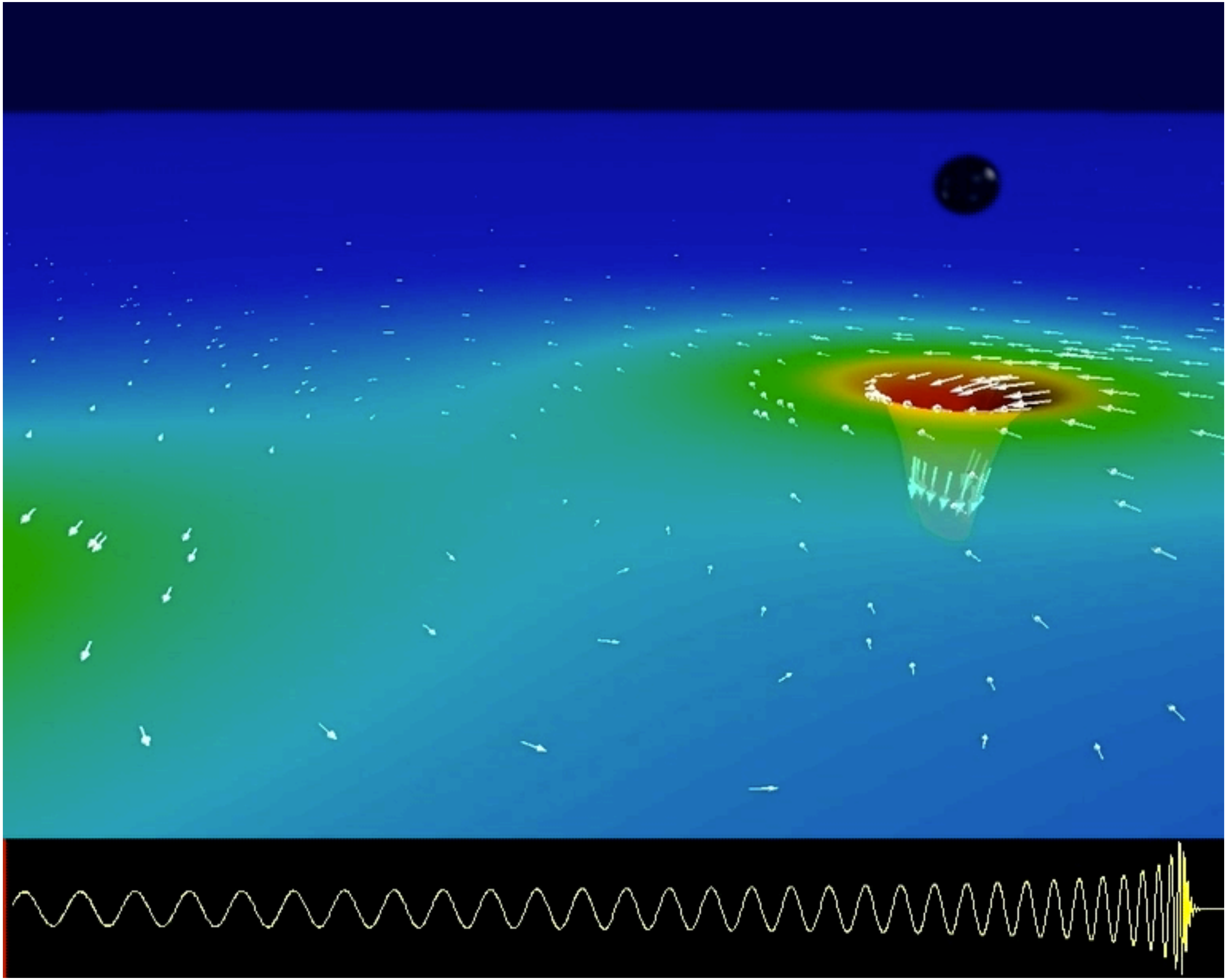
$\tilde{A}_{ij} = e^{-4\varphi}(K_{ij} - (1/3)\gamma_{ij}K),$
$\tilde{\Gamma}^i = \tilde{\Gamma}^i_{jk}\tilde{\gamma}^{jk}$

Making a standard choice for adding constraints (several "natural" ambiguities in this system – not yet completely analyzed!) we get

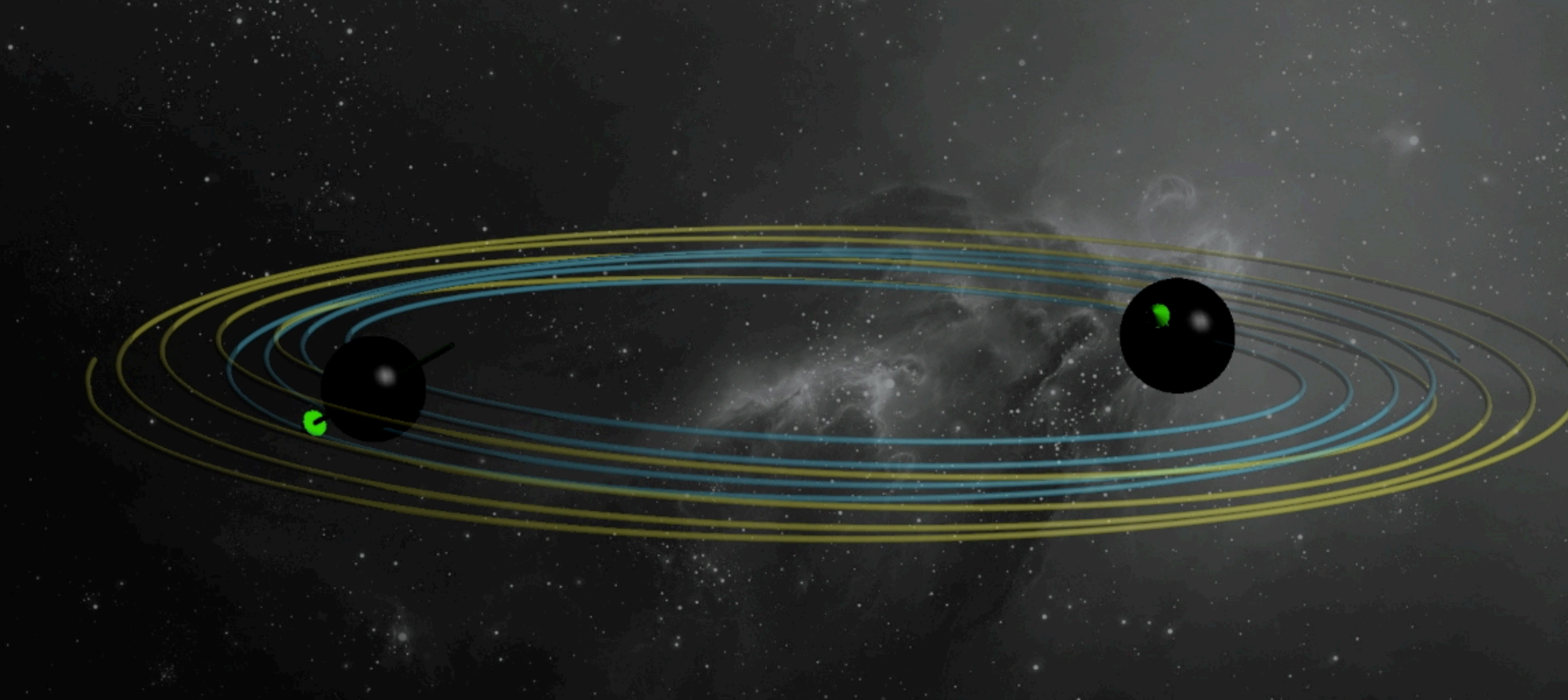
+4 Coordinate conditions

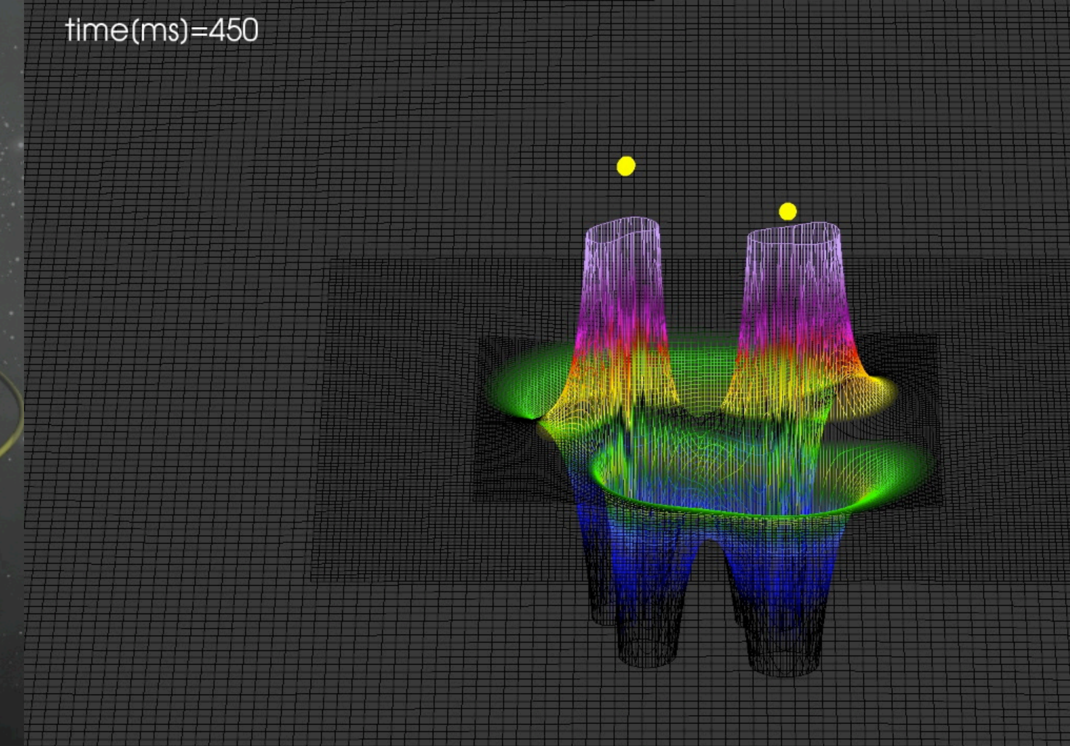
$$
\begin{aligned}
\mathcal{L}_n\varphi &= -(1/6)\alpha K,\\
\mathcal{L}_n\tilde{\gamma}_{ij} &= -2\alpha\tilde{A}_{ij},\\
\mathcal{L}_nK &= -D^iD_i\alpha + \alpha\tilde{A}_{ij}\tilde{A}^{ij} + (1/3)\alpha K^2,\\
\mathcal{L}_n\tilde{A}_{ij} &= -e^{-4\varphi}(D_iD_j\alpha)^{TF} + e^{-4\varphi}\alpha(R_{ij})^{TF} + \alpha K\tilde{A}_{ij} - 2\alpha\tilde{A}_{ik}\tilde{A}^k{}_j,\\
\mathcal{L}_n\tilde{\Gamma}^i &= -2(\partial_j\alpha)\tilde{A}^{ij} + 2\alpha\left(\tilde{\Gamma}^i_{jk}\tilde{A}^{kj} - (2/3)\tilde{\gamma}^{ij}(\partial_jK) + 6\tilde{A}^{ij}(\partial_j\varphi)\right).
\end{aligned}
$$

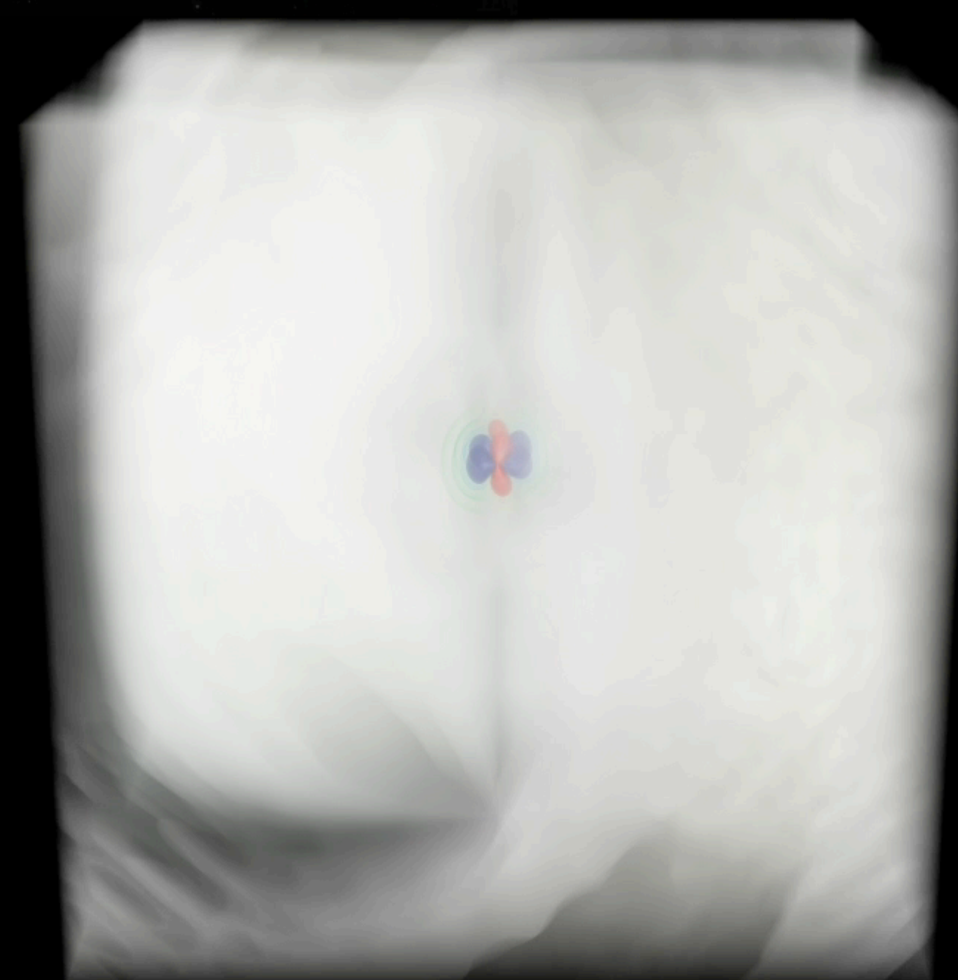
Symmetric hyperbolic with standard gauge conditions - but easy to trigger wild instabilities in this system!



Visualisation: SXS colaboration







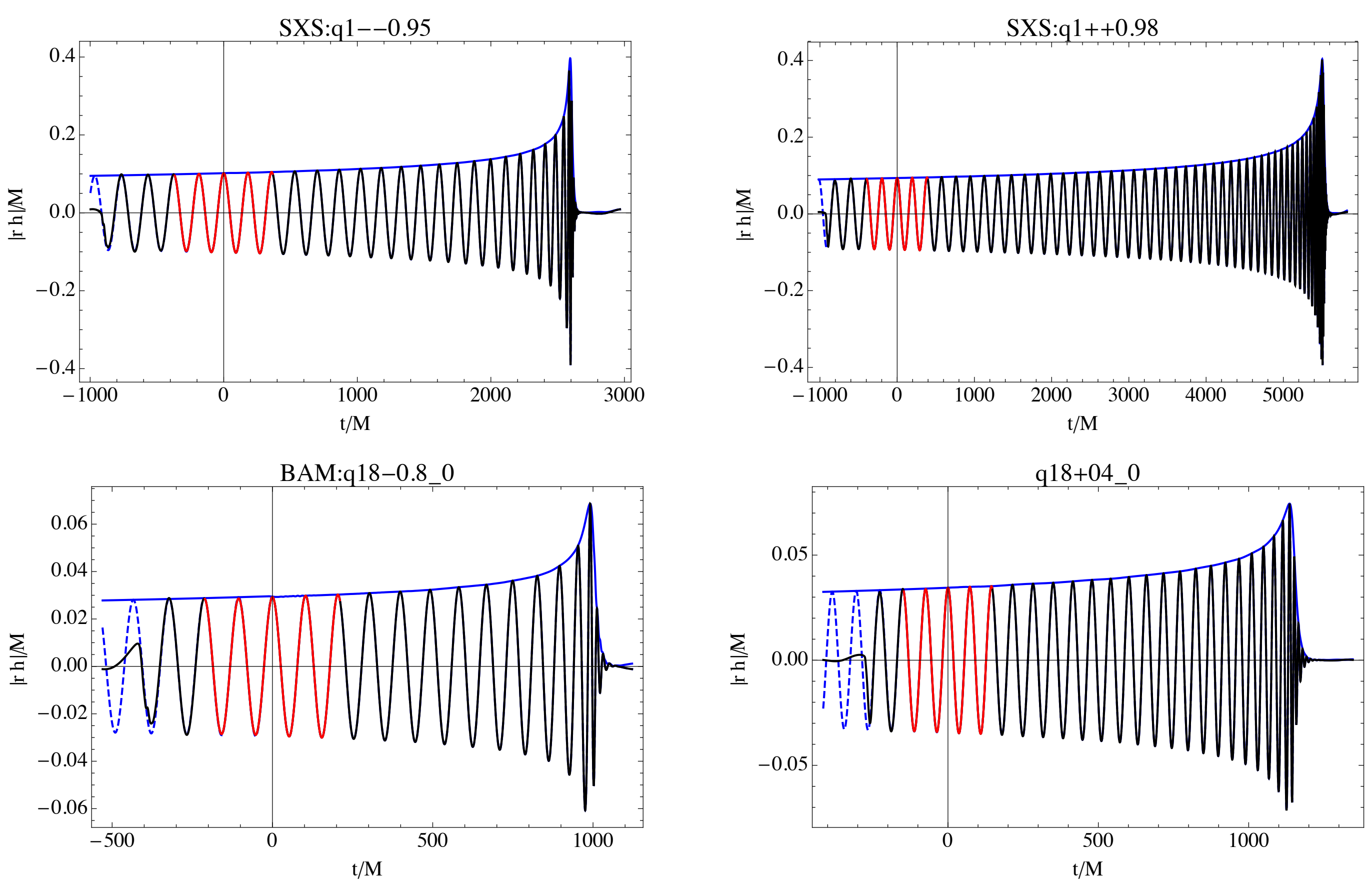
Simulaciones: Sascha Husa
Visualización: Rafel Jaume, UIB

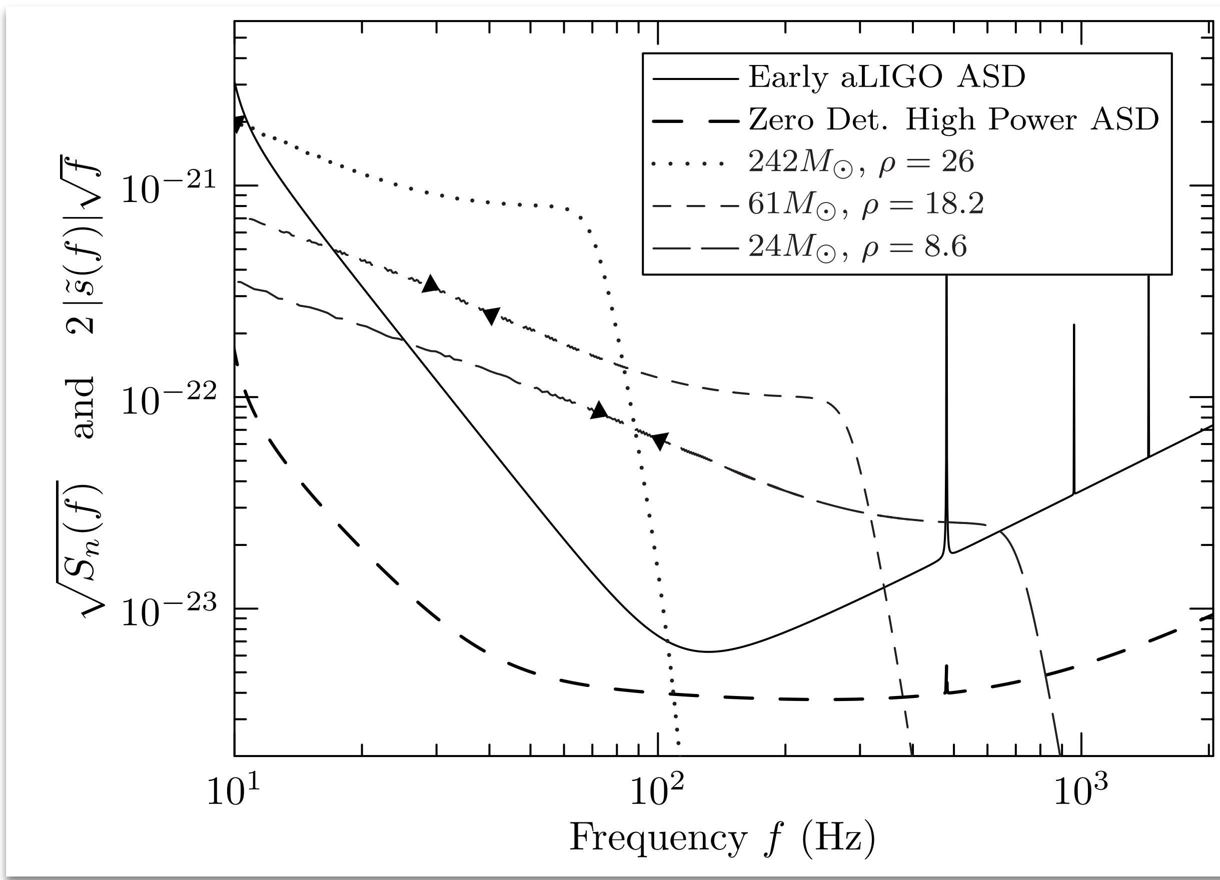
Gluing PN + NR: Hybrid waveforms

- Extending NR to low frequencies is very expensive!

$$T_{coalescence} \approx \eta^{-1} f_{initial}^{-8/3} \qquad \eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

- -> hybridise (glue) with post-Newtonian waveforms: input date to calibrate waveform models.





Wave extraction via Ψ_4

Wave-zone: adopt transverse-traceless (TT) gauge, all the information about the radiative degrees of freedom contained in h_{ij}:

$$h_{ij} = h_+(\mathbf{e}_+)_{ij} + h_\times(\mathbf{e}_\times)_{ij}, \tag{5}$$

$$(\mathbf{e}_+)_{ij} = \hat{\imath}_i\hat{\imath}_j - \hat{\phi}_i\hat{\phi}_j\,, \qquad \text{and} \qquad (\mathbf{e}_\times)_{ij} = \hat{\imath}_i\hat{\phi}_j + \hat{\imath}_j\hat{\phi}_i\,. \tag{6}$$

Newman-Penrose scalar method, in wave zone: $\mathrm{h} = h_+ - \mathrm{i}h_\times$ as

$$\mathrm{h} = \lim_{r\to\infty}\int_0^t \mathrm{d}t' \int_0^{t'} \mathrm{d}t''\Psi_4, \qquad \Psi_4 = -R_{\alpha\beta\gamma\delta}n^\alpha\bar{m}^\beta n^\gamma\bar{m}^\delta, \tag{7}$$

Null-tetrad ℓ (in), n (out), m, $\bar{m}$,

$$-\ell\cdot n = 1 = m\cdot\bar{m}, \tag{8}$$

Spin-weight -2 fields represent symmetric trace-free tensor fields on a sphere (in our case $R_{\alpha\beta\gamma\delta}n^\alpha n^\gamma$) in terms of a complex scalar field. Freedom in the choice of tetrad used in defining Ψ_4!

Radiated energy, linear and angular momentum

Radiated energy and linear & angular momentum:

$$\frac{dE}{dt} = \lim_{r\to\infty}\left[\frac{r^2}{16\pi}\int_\Omega \left|\int_{-\infty}^{t}\Psi_4 d\tilde{t}\right|^2 d\Omega\right], \tag{13}$$

$$\frac{dP_i}{dt} = -\lim_{r\to\infty}\left[\frac{r^2}{16\pi}\int_\Omega \ell_i \left|\int_{-\infty}^{t}\Psi_4 d\tilde{t}\right|^2 d\Omega\right], \tag{14}$$

$$\frac{dJ_z}{dt} = -\lim_{r\to\infty}\left\{\frac{r^2}{16\pi}\mathrm{Re}\left[\int_\Omega \left(\partial_\phi \int_{-\infty}^{t}\Psi_4 d\tilde{t}\right)\left(\int_{-\infty}^{t}\int_{-\infty}^{\hat{t}}\overline{\Psi_4}\, d\tilde{t}\, d\hat{t}\right) d\Omega\right]\right\}, \tag{15}$$

At finite extraction radius we need to perform "double Richardson extrapolation" – in extraction radius and grid spacing.

Spherical

Project onto

$$Y_{2-2}^{-2} \; \ldots \; s\,\iota)^2 \, e^{2i\phi}. \quad (9)$$

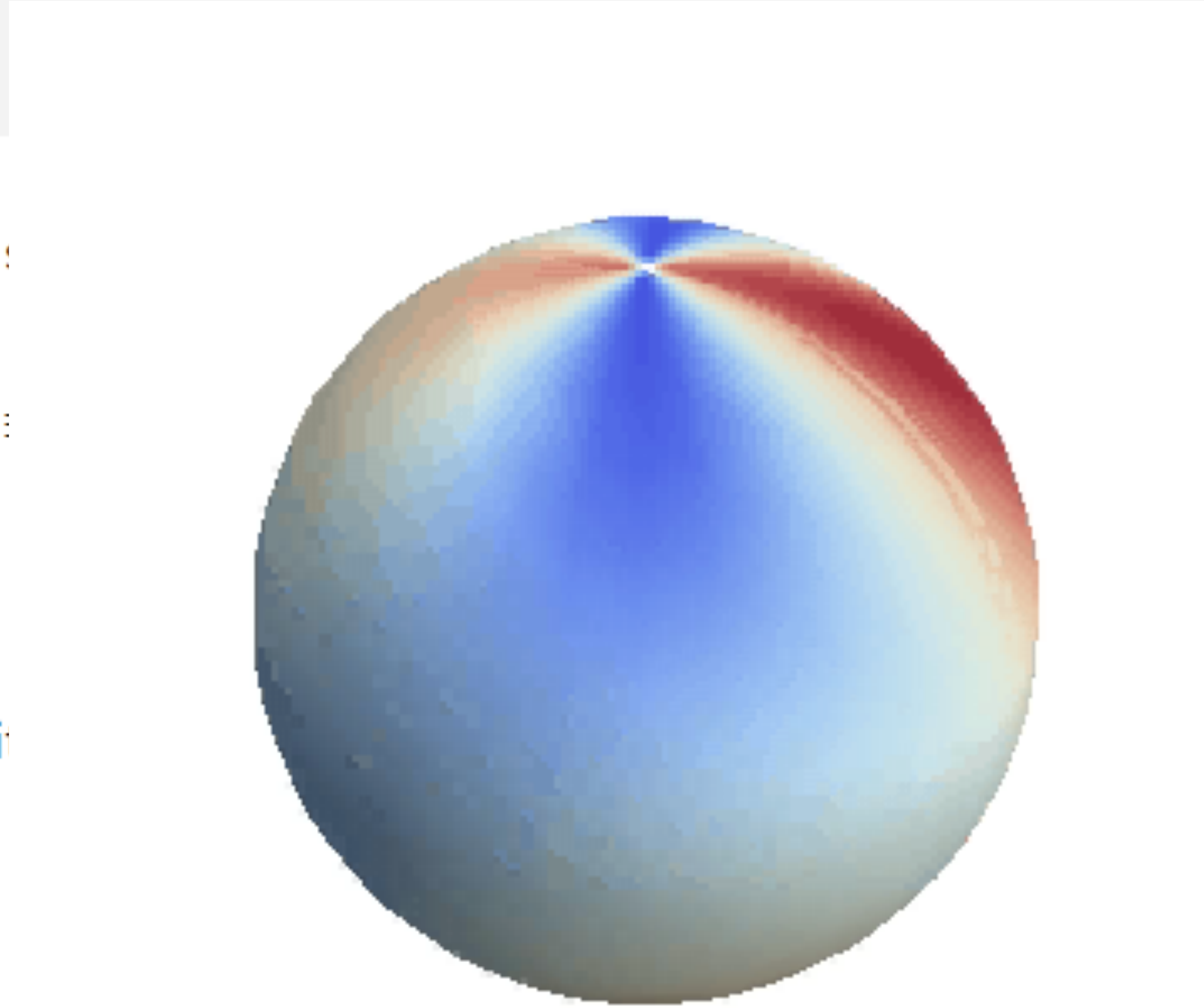
(10)

Orthonormali

(11)

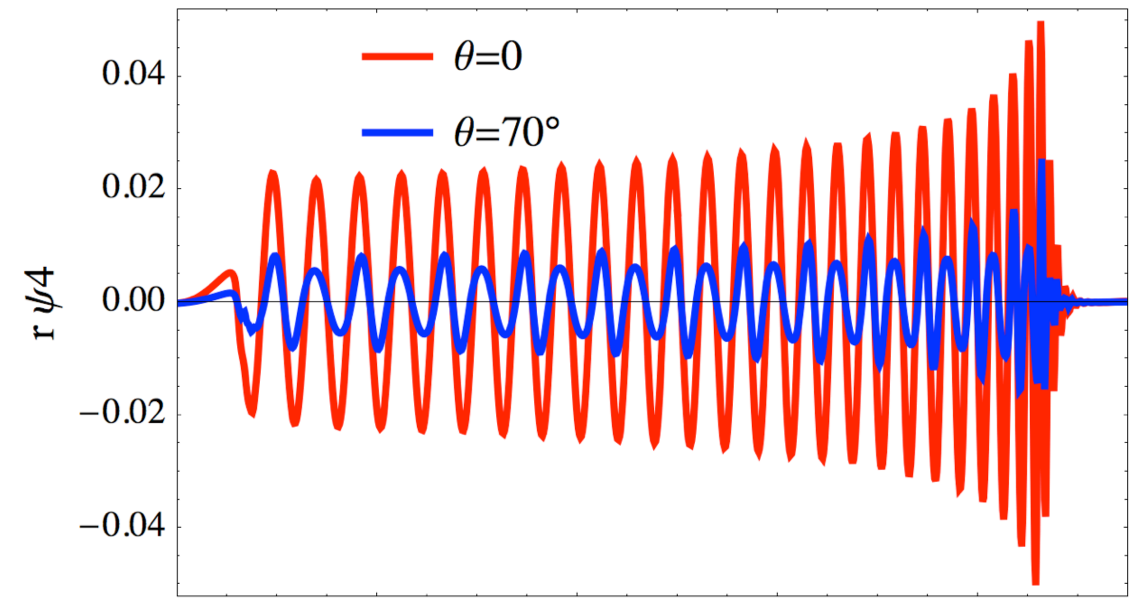
Amplitude-ph

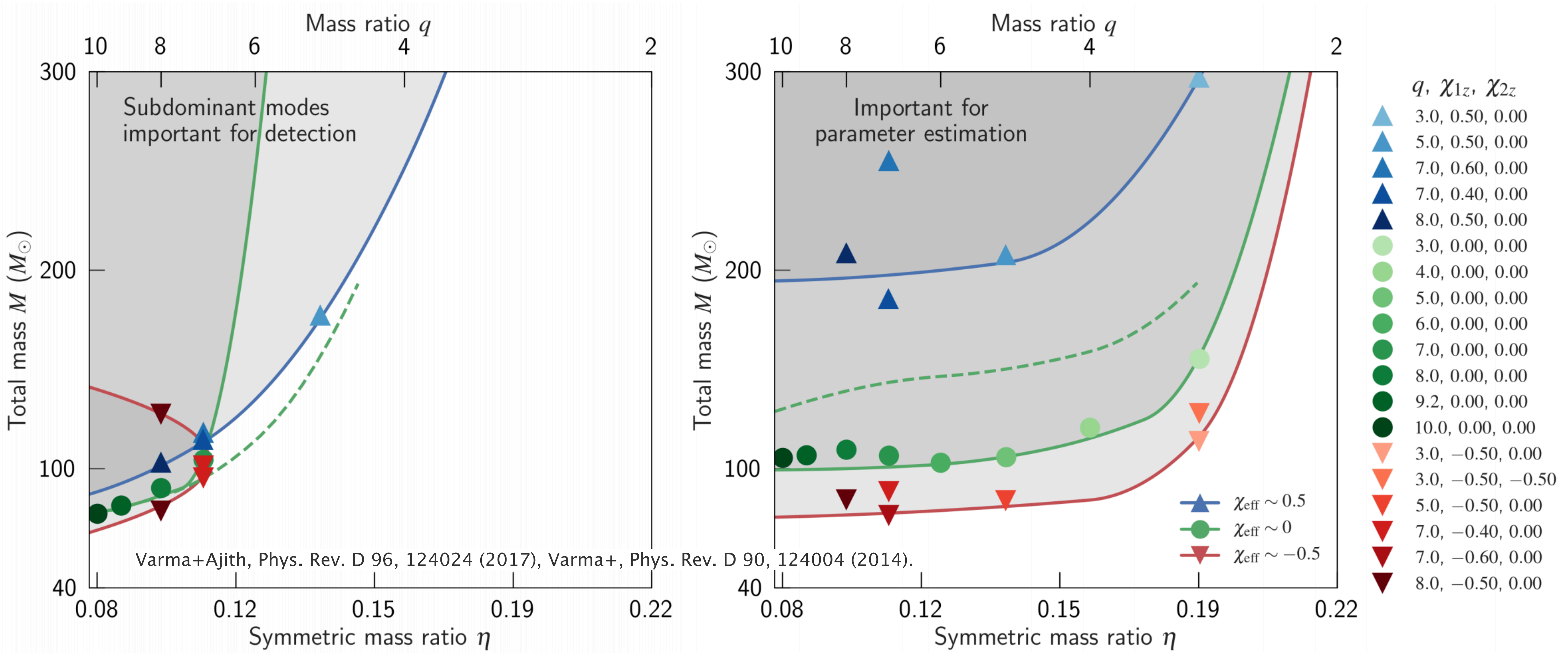
(12)



Subdominant spherical harmonics

- Data analysis used to only use l=|m|=2.
- Several efforts to incorporate harmonics into data analysis for O3
- [London+, PRL 2018, Cotesta+, PRD 2018]



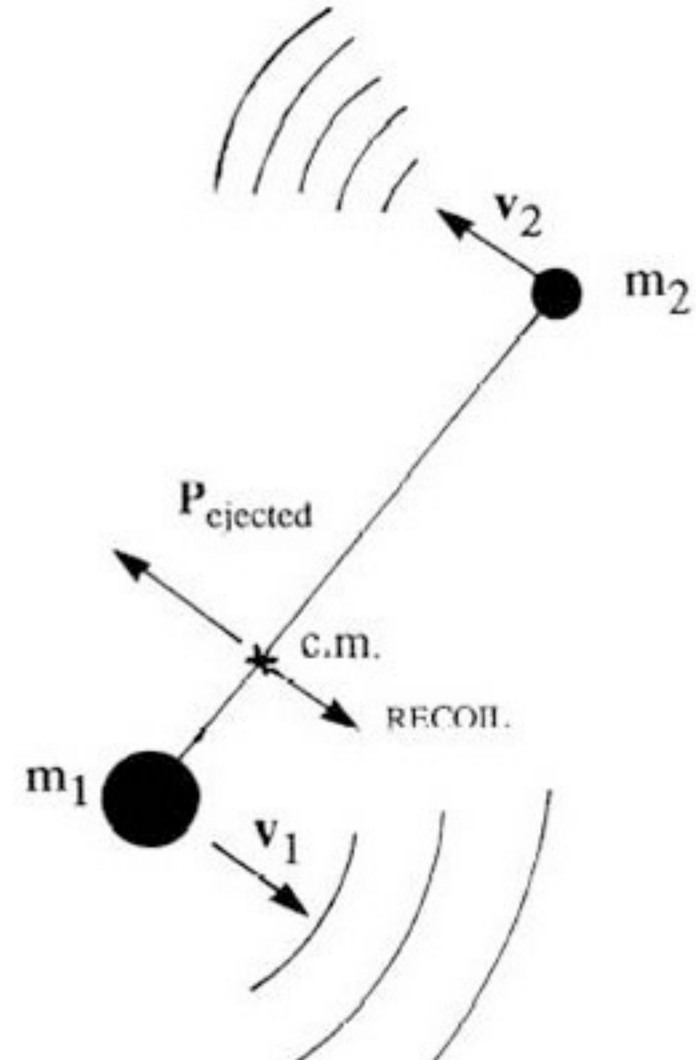


Varma+Ajith, Phys. Rev. D 96, 124024 (2017), Varma+, Phys. Rev. D 90, 124004 (2014).

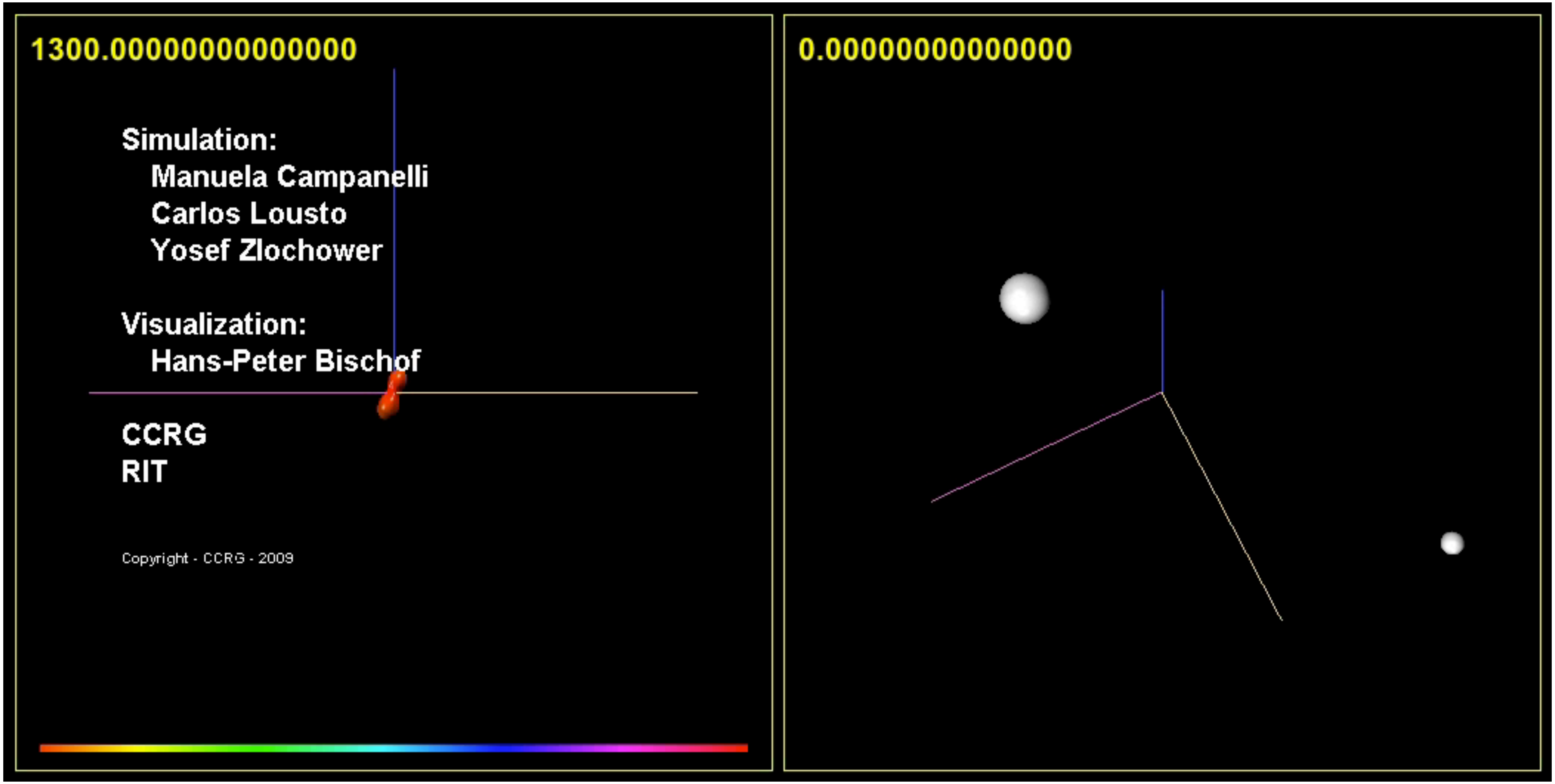
(a) For detection (b) For parameter estimation

Recoil from merging binaries

- For unequal BHs in a binary, GW emission is asymmetrical and produces a recoil of the final black hole - like a lawn sprinkler would move if the water intensity is asymmetric.



Anisotropy of GW emission and recoil: movie (RIT)



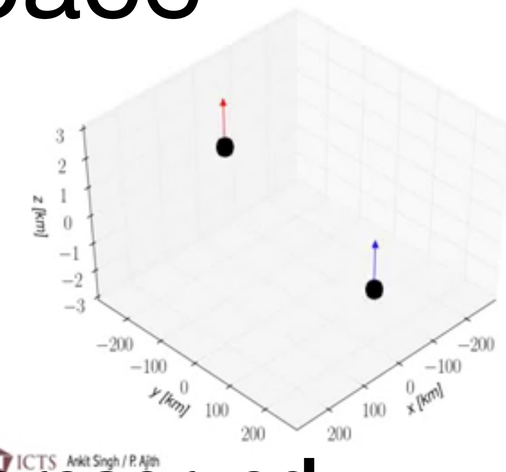
Maximal recoil velocities: > thousands of km/s

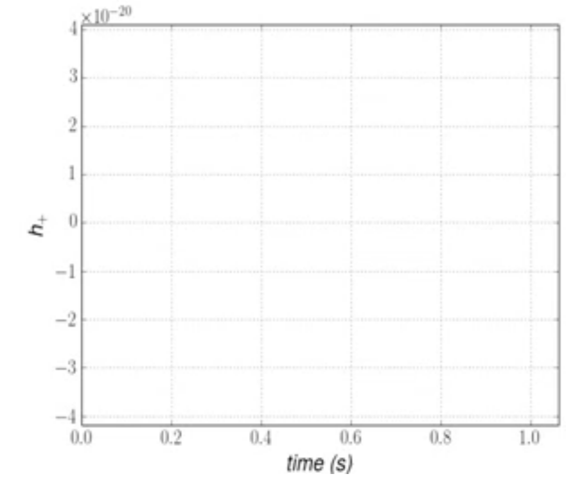
How does this scale with mass?

Spins & the BBH parameter space

- **Leading order PN spin effect: spin-orbit**

$$H_{SO} = 2\frac{\vec{S}_{\rm eff}\cdot\vec{L}}{r^3} \qquad \dot{S} = -2\frac{\vec{S}_{\rm eff}\times\vec{L}}{r^3}$$





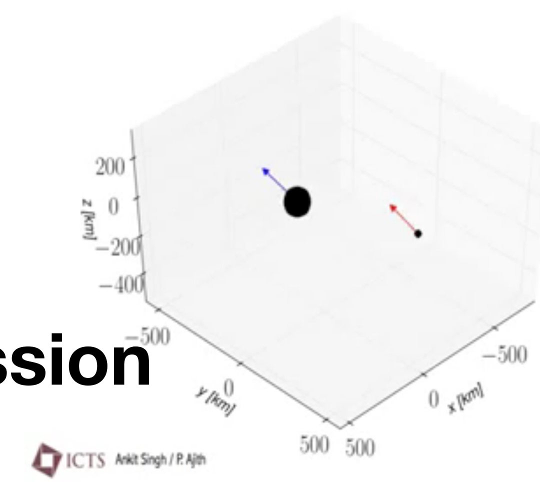
- **Spins orthogonal to orbital plane: plane preserved.**
  - 3-dimensional parameter space: dominant "average" spin

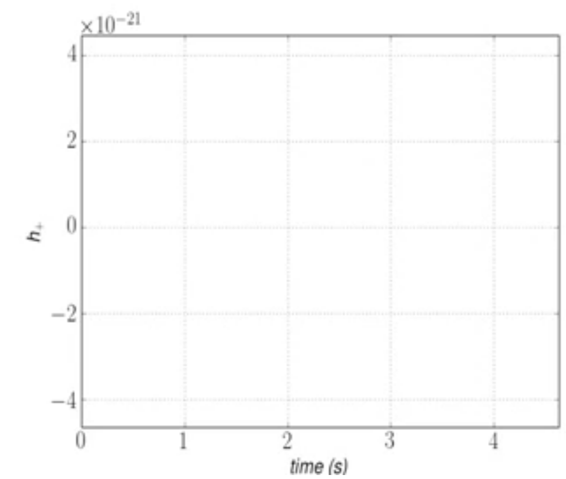
$$\chi_{eff} = \frac{\vec{S}_1\cdot\hat{\vec{L}}/m_2 + \vec{S}_1\cdot\hat{\vec{L}}/m_1}{m_1+m_2}$$

measure ok

  - subdominant: spin difference

not yet





- **Spin components in orbital plane: precession**
  7 dimensions (9 with eccentricity)

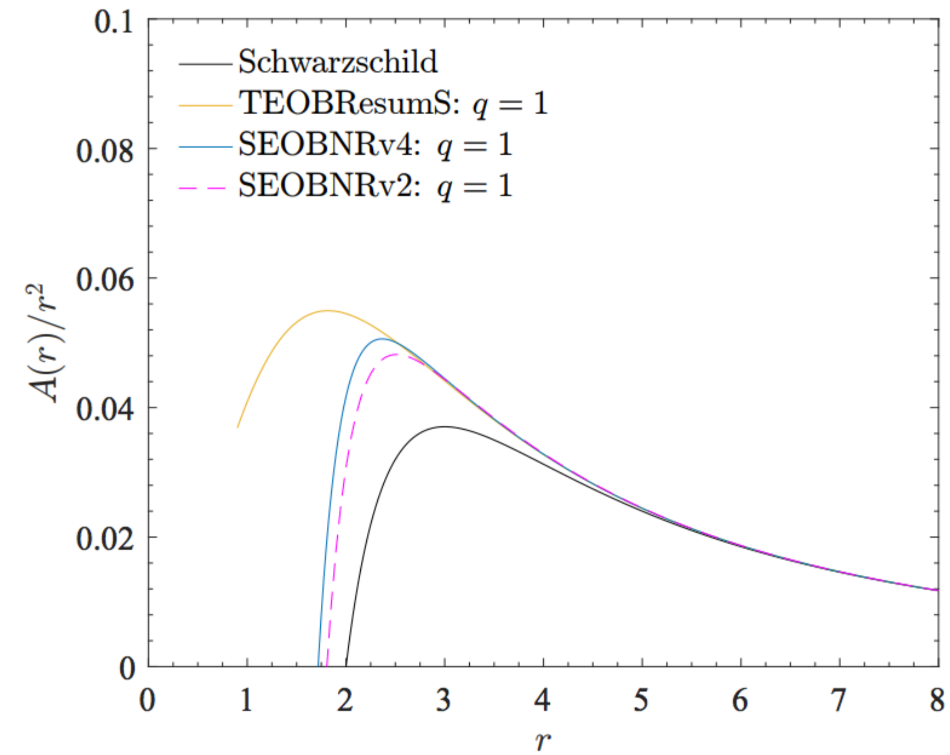
- Dominant precession effective spin [Hannam+ PRL 2013, Schmidt+ PRD2014]

$$A_1 = 2 + \frac{3m_2}{2m_1} \qquad \chi_p = \frac{\max(A_1 S_{1\perp}, A_2 S_{2\perp})}{A_2 m_>^2}$$

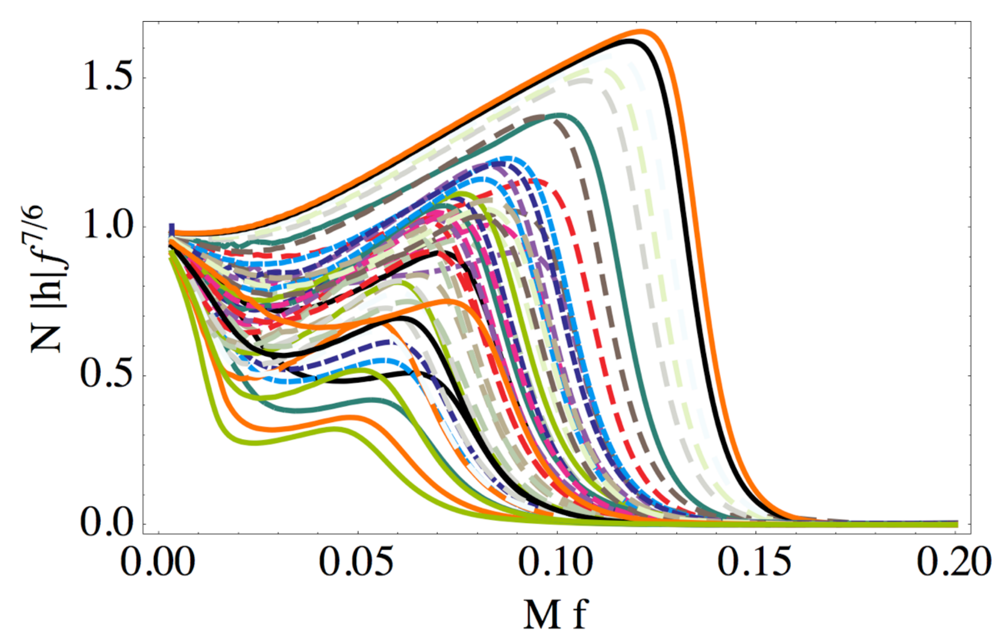
measure poorly

Waveform Modelling strategies I

- **Model simple functions:**
  e.g. split waveform into amplitude & phase.
- Frequency or time domain:
  - TD naturally suited for modelling dynamics
  - FD often more efficient for data analysis.



- **Discretize functions:**
  reduce to coefficients in some phenomenological ansatz, grid up, construct basis functions from waveforms.
- **Example:**
  - ~ 30 frequency points grid for amplitude and phase
  - polynomial interpolation in parameter space
  - reconstruct WF as spline



- **Avoid underfitting + overfitting to noise & systematic errors.**

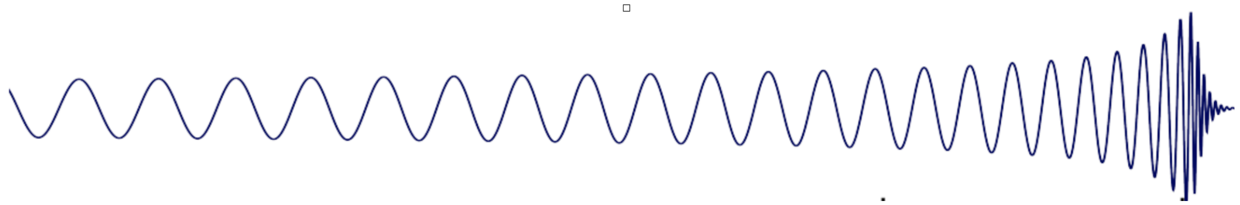
Waveform Modelling strategies II

- Currently 3 main strategies with different emphasis:
- effective one body (EOB) - push perturbative methods as far as possible
  - model the energy and flux of a particle inspiral in an effective metric, then integrate ODEs numerically.
  - Slow - need a fast model of the phenomenological EOB model.
- phenomenological (frequency domain) models - phenomenological understanding
  - piecewise closed form expressions - fast - IMRPhenom*
- "Surrogate models:" ROM for numerical data - algorithms to interpolate large parameter spaces
  - No intermediate phenomenological model, can use the same methods as for fast evaluation of EOB.
- Phenomenological + EOB: Make a physically motivated ansatz in terms of suitable parameters, fit to each waveform, then fit coefficients across parameter space.

Hierarchical Strategy to conquer parameter space

Model directions in parameter space in order of importance.

- Start with l=|m|=2 spherical harmonic mode (1 harmonic), no spins



  - 1D physical parameter space

- Non-precessing spins: single effective spin - 1 harmonic, 2D
- Leading precession effects via PN - 5 harmonics, ≧ 3D
- Non-precessing spins - 1 harmonic, 3D
- Non-precessing spins higher modes: handful of harmonics, 3D
- Extend to matter (incorporate some neutron star tidal effects)
- Eccentricity: non-spinning 22 mode - 2D
- NR calibration of leading precession effects - 7 D
- Generic black holes - 9 D

beyond GR

Some Open Challenges

- Observe/accurately model:
  - Highly spinning black holes
  - (Strongly) precessing black holes
  - BH-NS systems
  - Close to face-on with strong higher mode content
  - IMBHs
  - EMRIs - probably need to wait for LISA
  - Violations of GR/exotic compact objects - how to actually do that?
  - Continuous waves from spinning NS
  - Waves from the early universe

Until now we have seen black hole mergers up to about redshift z~0.5.

LISA and ET will allow to see black hole mergers back in the dark ages of the universe, and hopefully gravitational waves created in the first moments of the universe.