

Modern Language Models

Language Models

Why are language models important?

Many tasks can be expressed as a sequence prediction problem

- Programming Languages: Generate code
- Language of Mathematics: Solving equations

Language of Mathematics

$$49 + 10 =$$

Language of Mathematics

$$49 + 10 = 59$$

Language of Mathematics

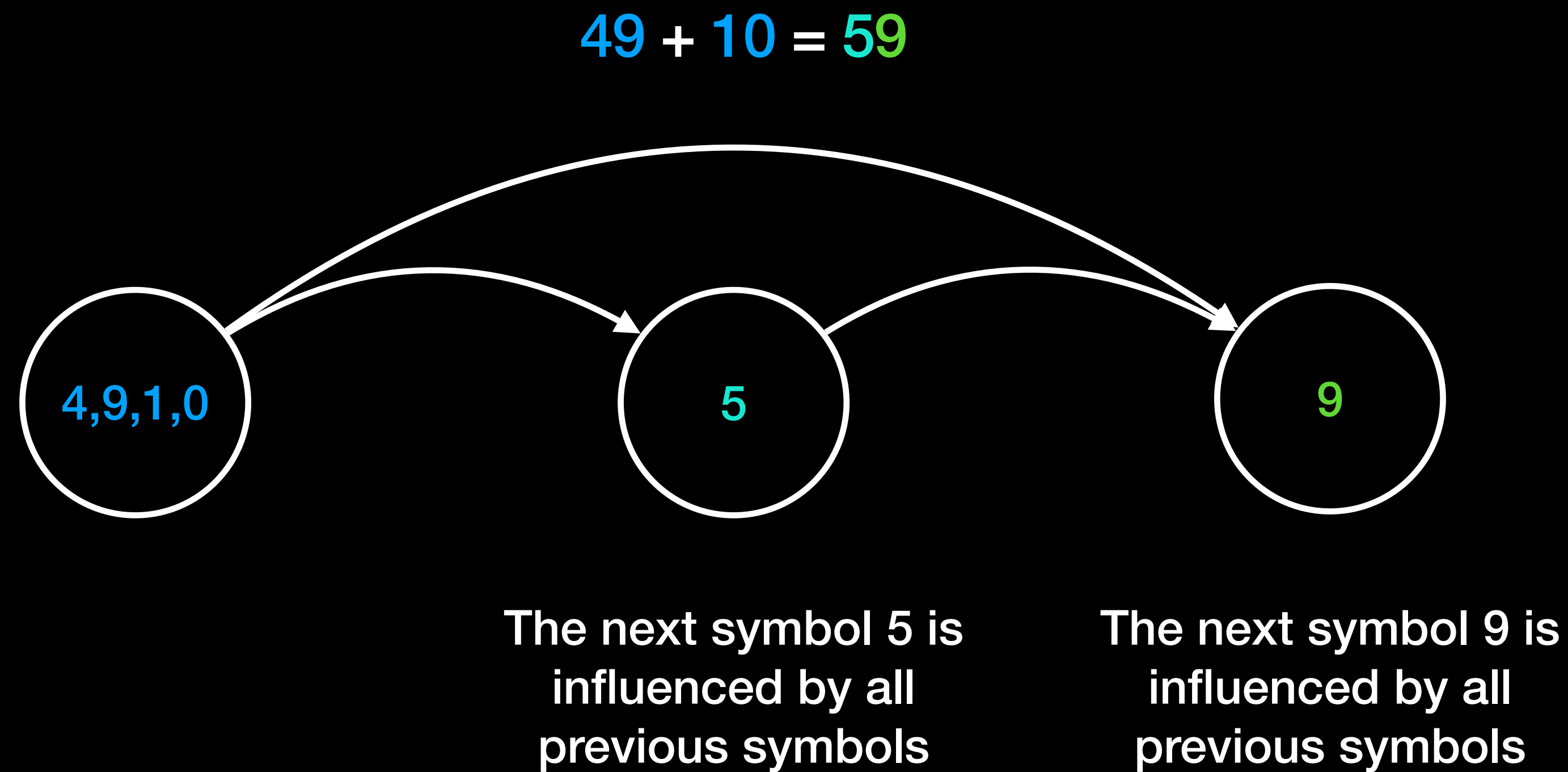
49 10 59

Language of Mathematics

4,9,1,0,5,9

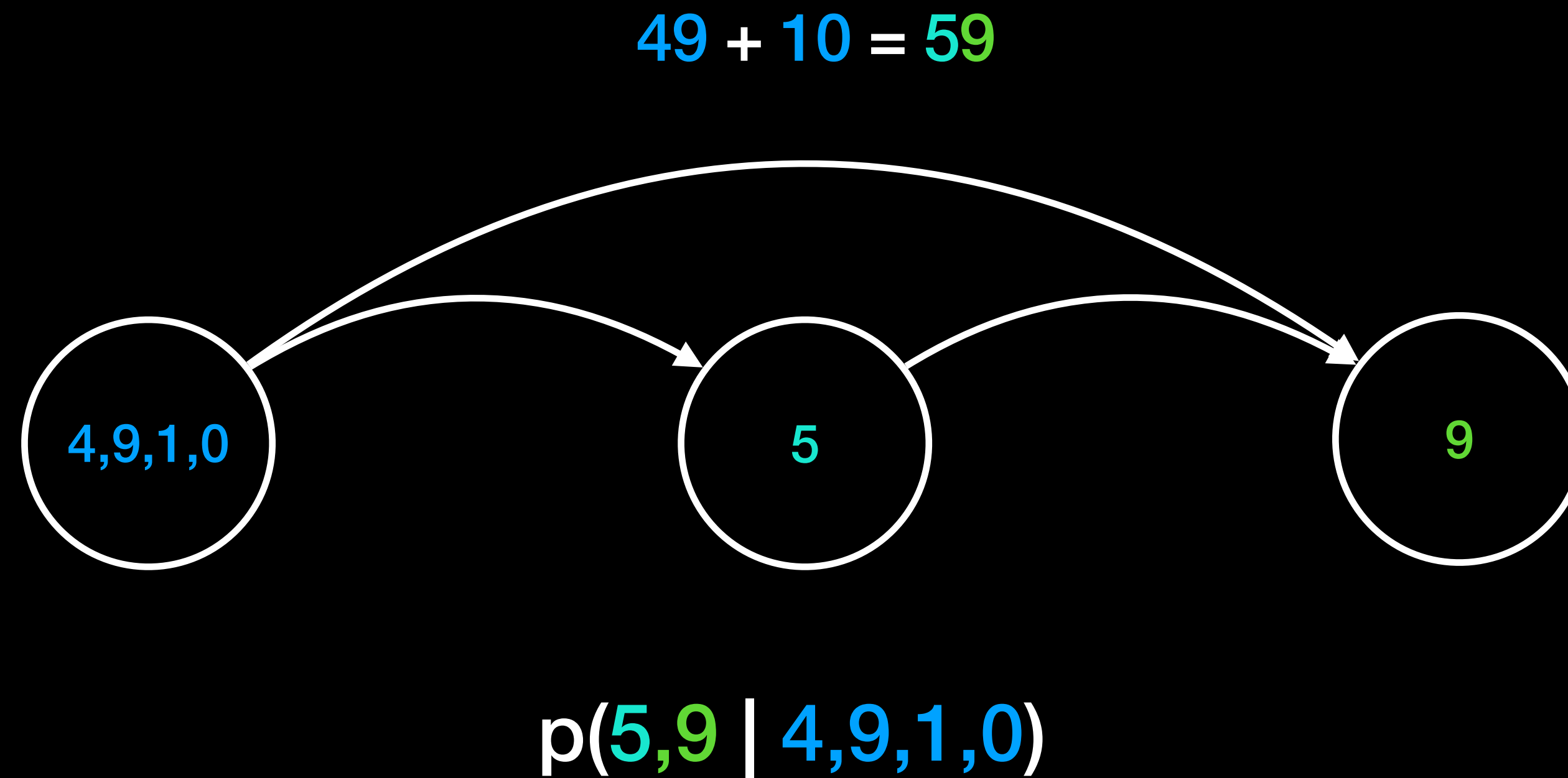
Language Model

n^{th} order Markov assumption



Language Model

Sequence Probability



Language Model

Predict a Sequence

$$p(5,8 \mid 4,9,1,0) = 0$$

$$p(5,9 \mid 4,9,1,0) = 1$$

$$p(6,0 \mid 4,9,1,0) = 0$$

Language Model

Predict a Sequence

$$p(5,8 \mid 4,9,1,0) = 0$$

$$p(5,9 \mid 4,9,1,0) = 1$$

$$p(6,0 \mid 4,9,1,0) = 0$$

Sequence Probability Combinations

$$49 + 10 = 59$$

$$27 + 30 = 57$$

$$00 + 26 = 26$$

$$40 + 47 = 87$$

$$03 + 32 = 35$$

...

Sequence Probability Combinations

$$49 + 10 = 59$$

$$27 + 30 = 57$$

$$00 + 26 = 26$$

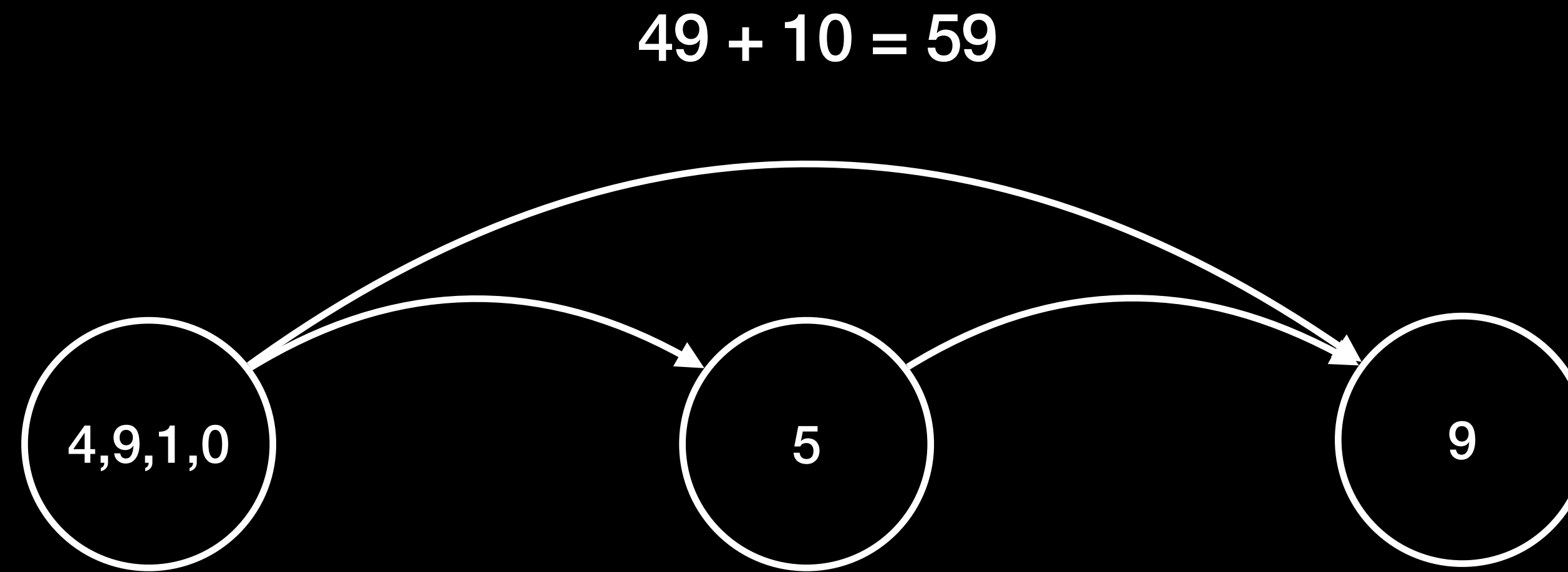
$$40 + 47 = 87$$

$$03 + 32 = 35$$

...

Language Model

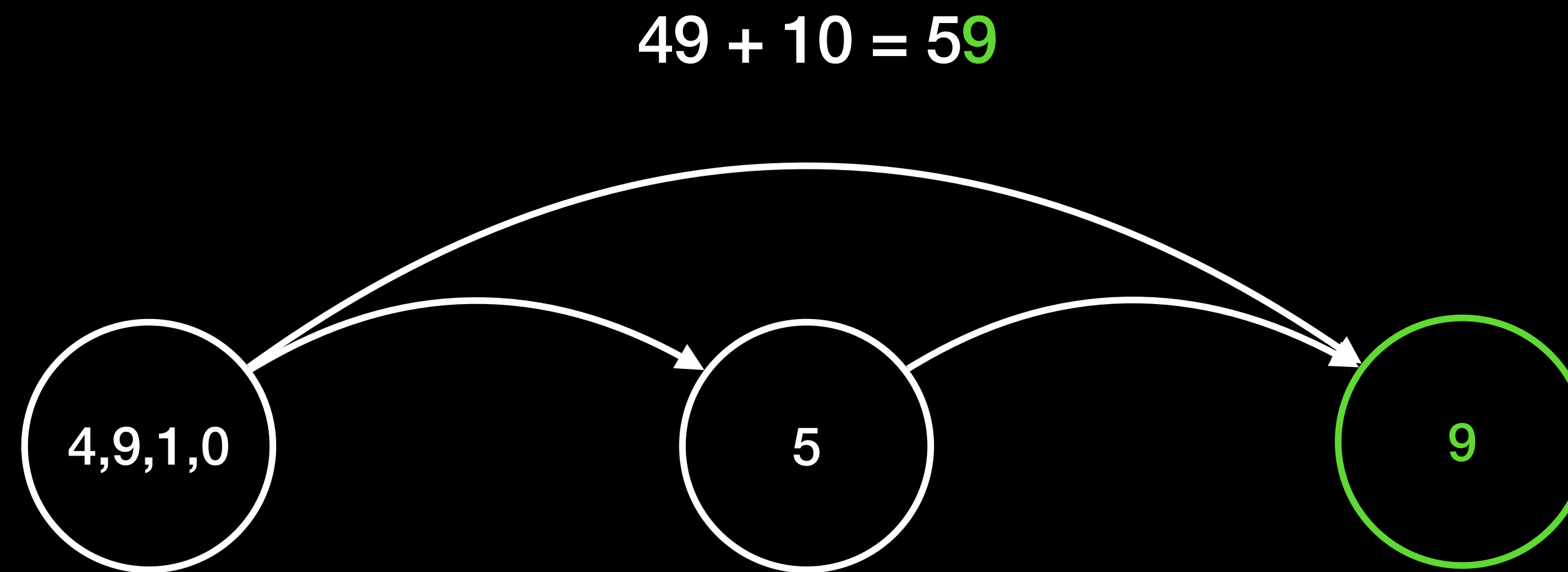
Sequence Probability



$$p(5 \mid 4,9,1,0) \cdot p(9 \mid 4,9,1,0,5) = p(5,9 \mid 4,9,1,0)$$

Language Model

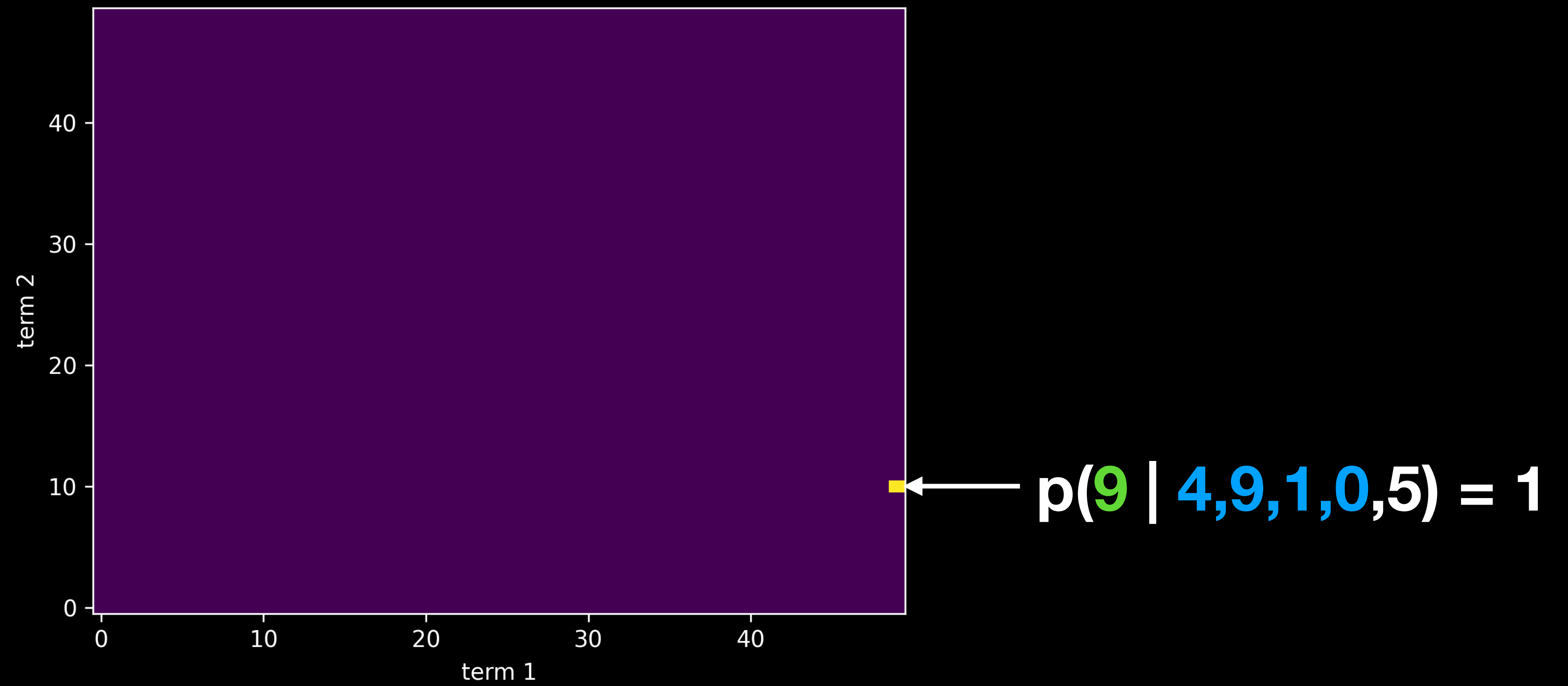
Sequence Probability



$$p(5 \mid 4,9,1,0) \cdot \underline{p(9 \mid 4,9,1,0,5)} = p(5,9 \mid 4,9,1,0)$$

Perfect Model

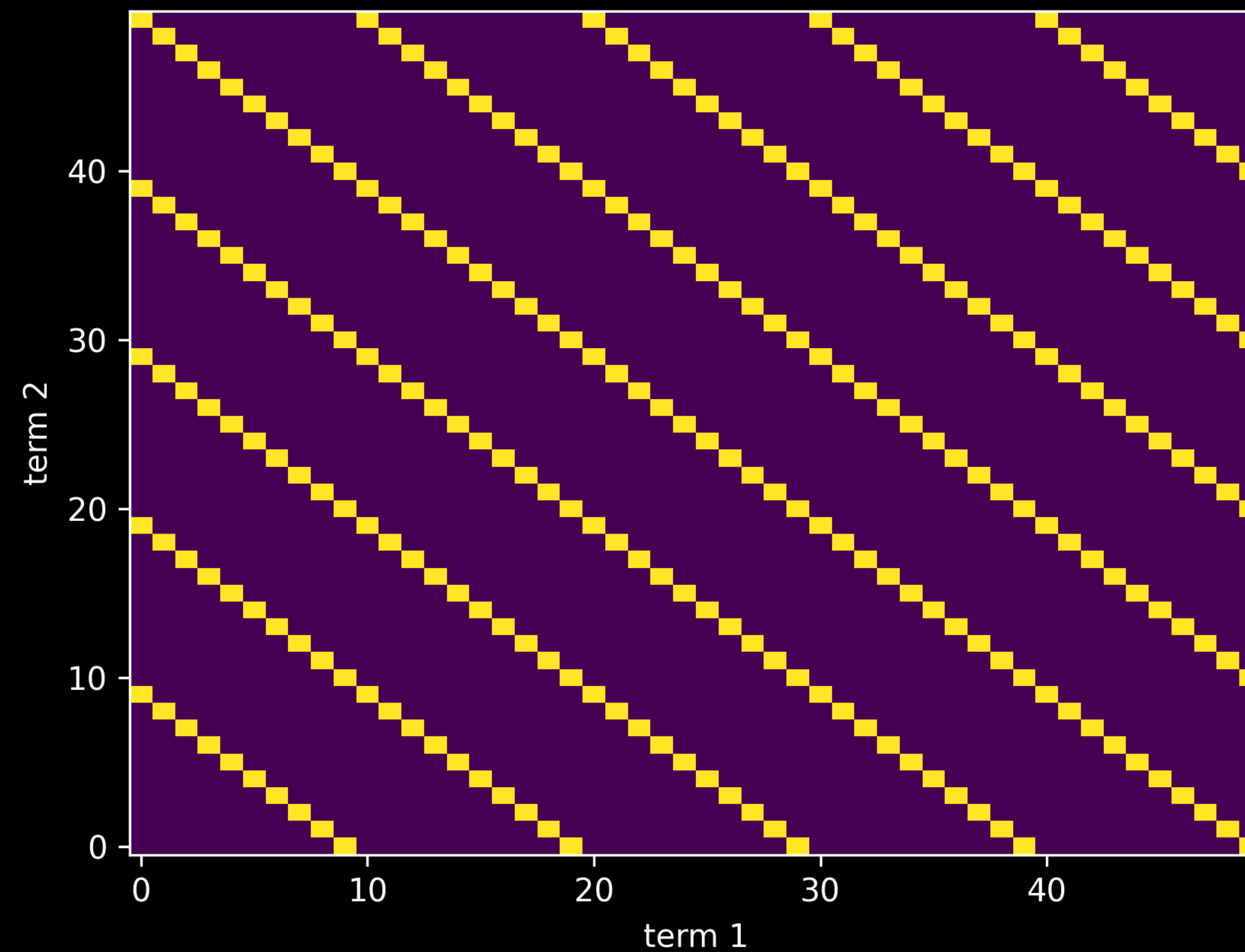
Training Data 100% – Accuracy 100%



Perfect Model

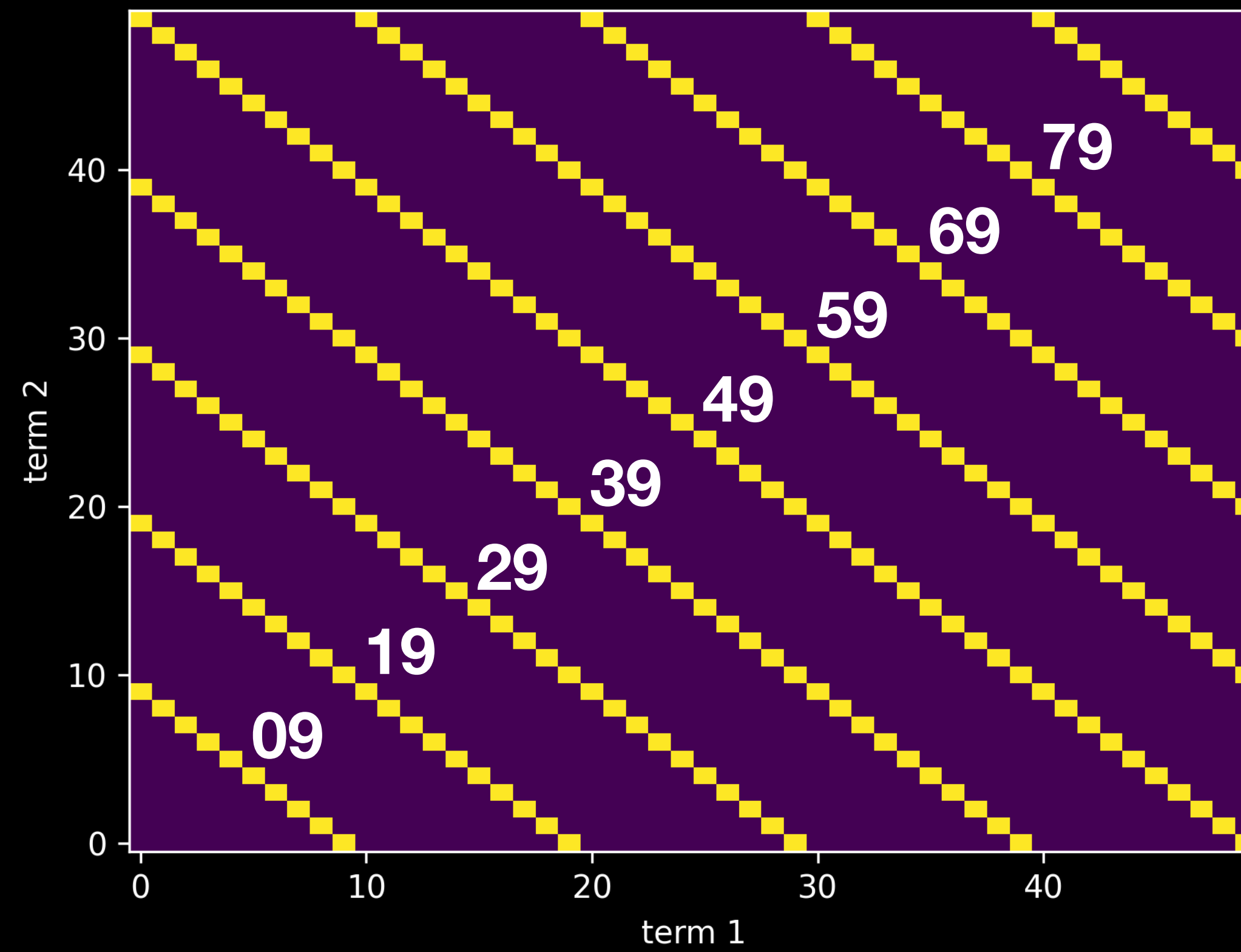
Training Data 100% – Accuracy 100%

$$p(\text{digit2}=\textcolor{red}{9} \mid \text{term1}, \text{term2}, \text{digit1})$$



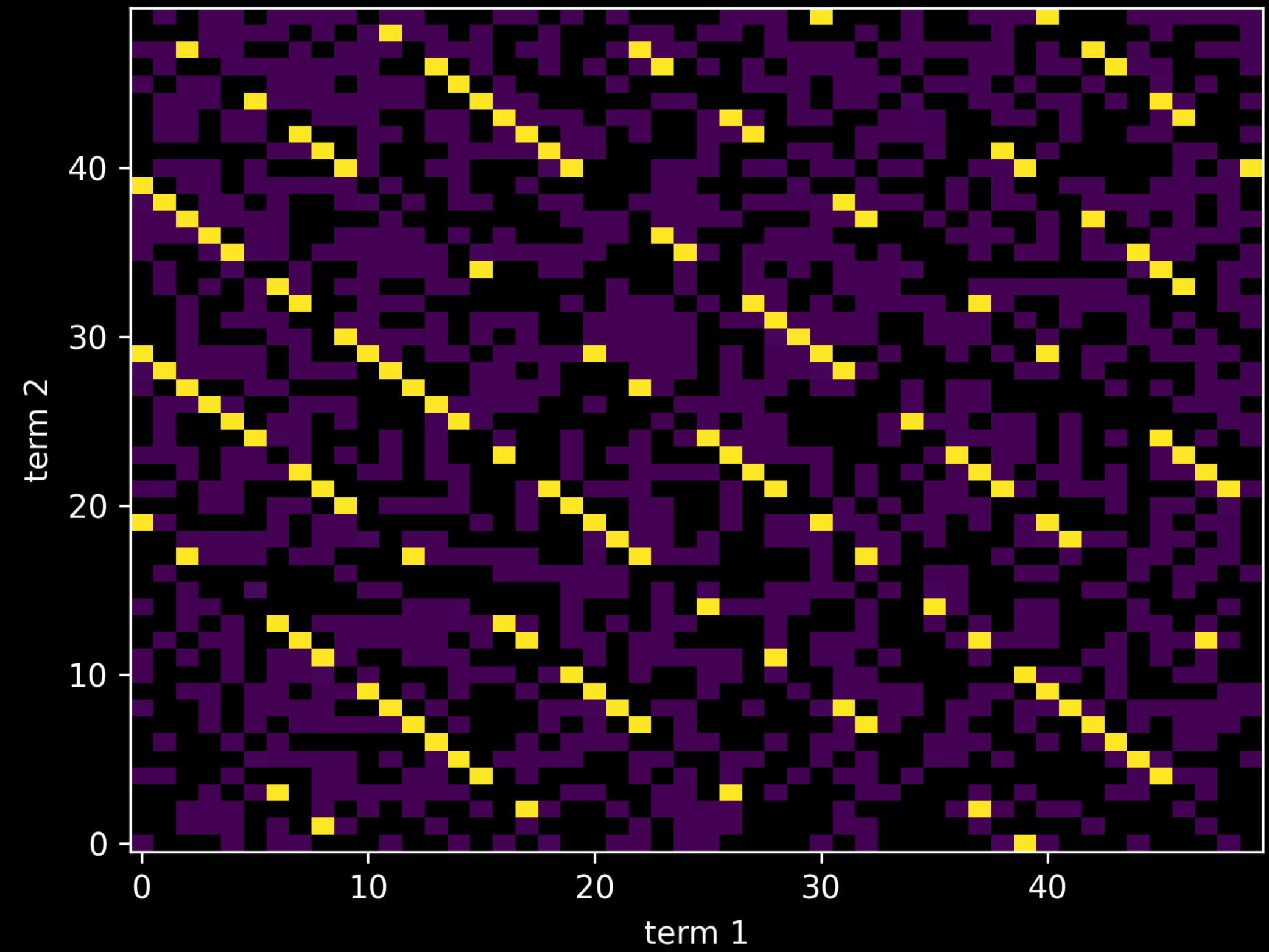
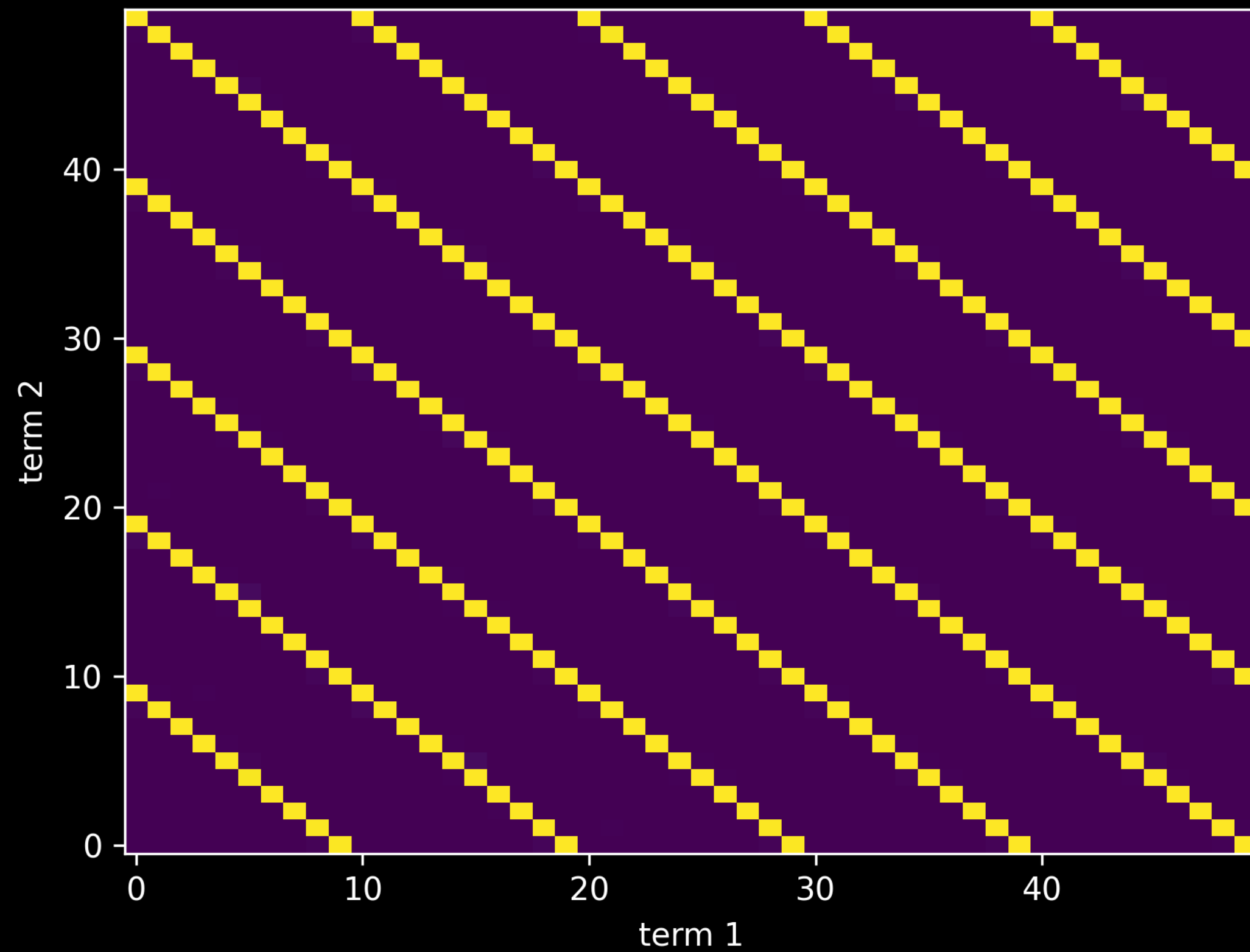
Perfect Model

Training Data 100% – Accuracy 100%



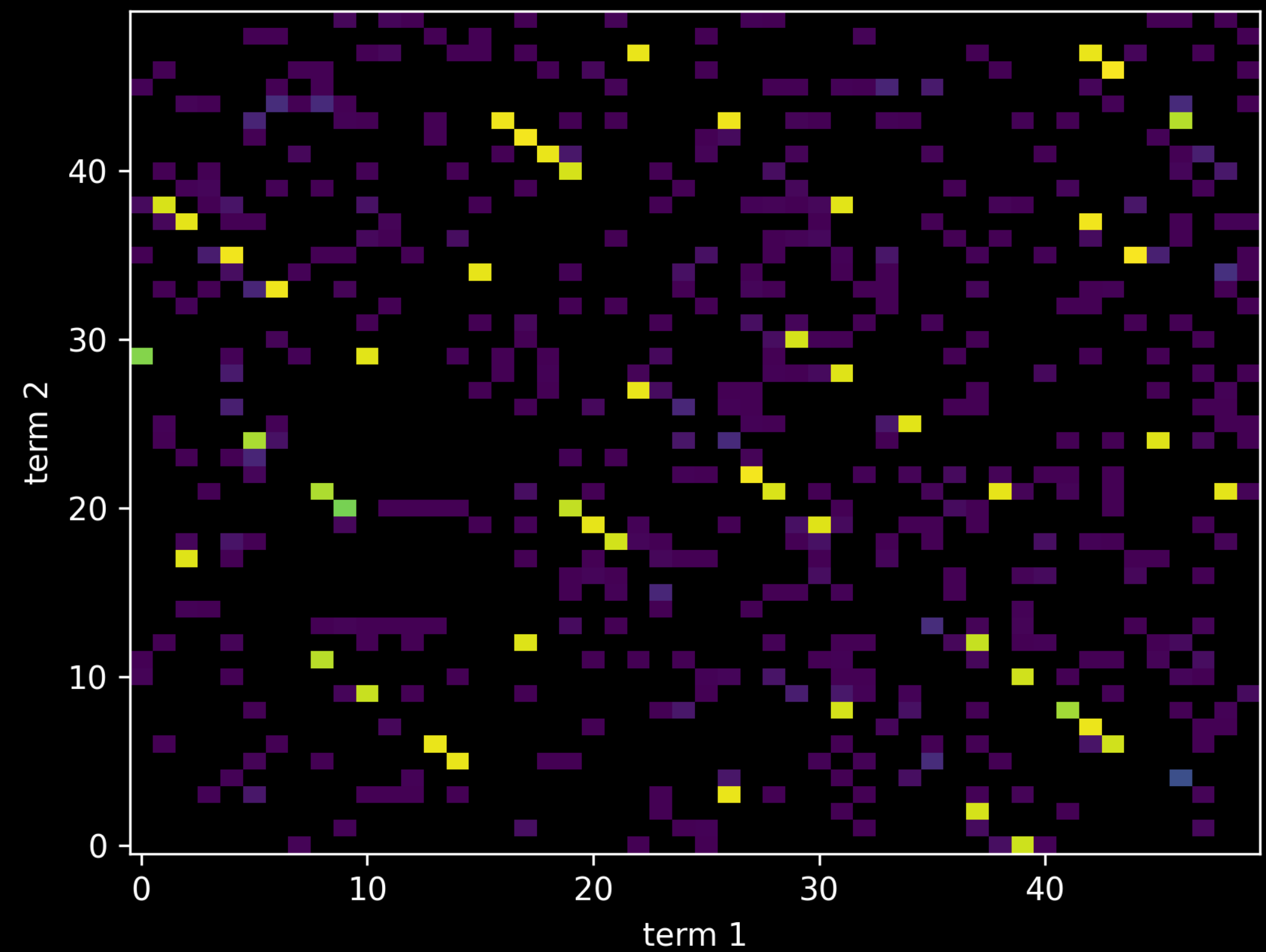
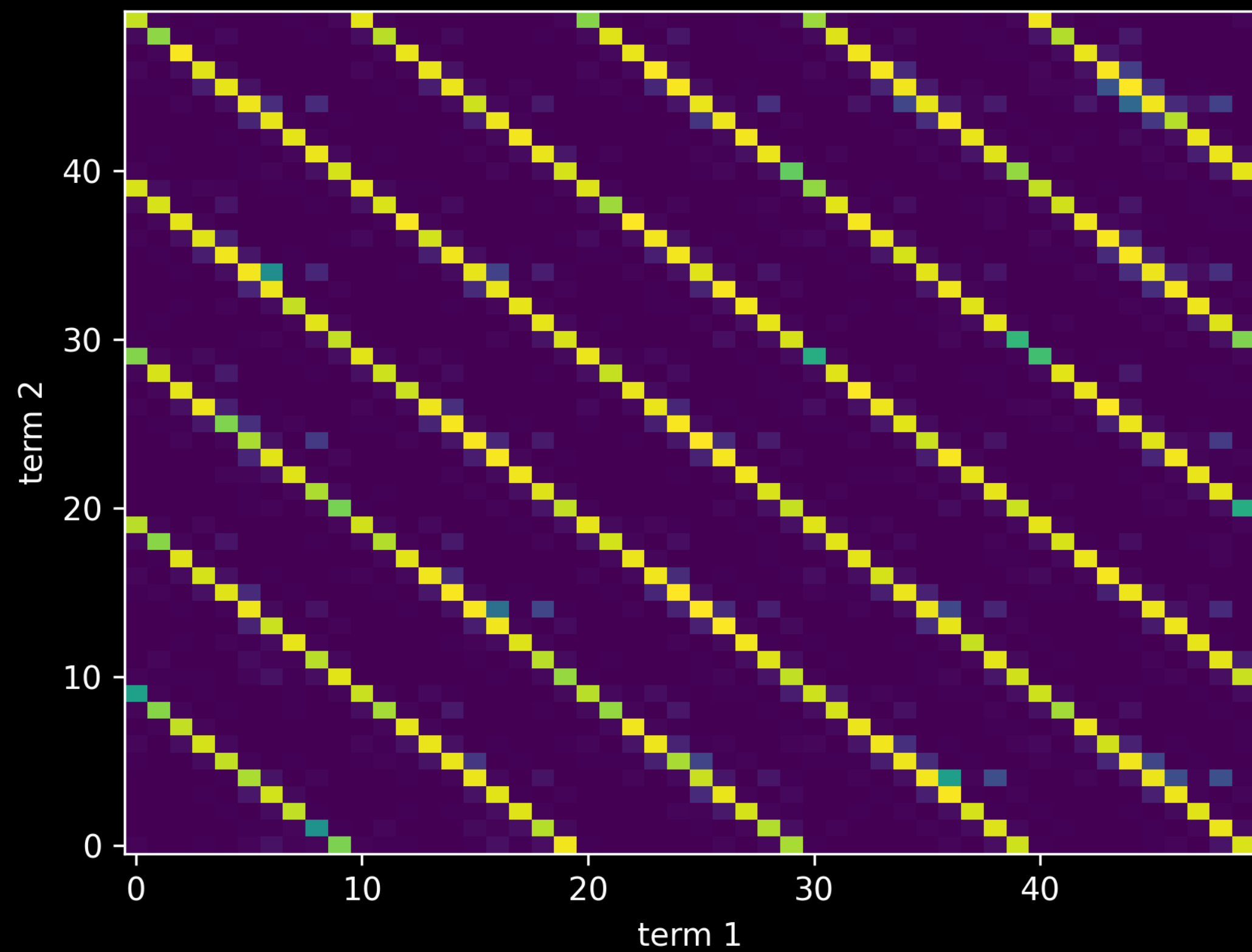
Autoregressive Decoder

Training Data 50% – Accuracy 100%



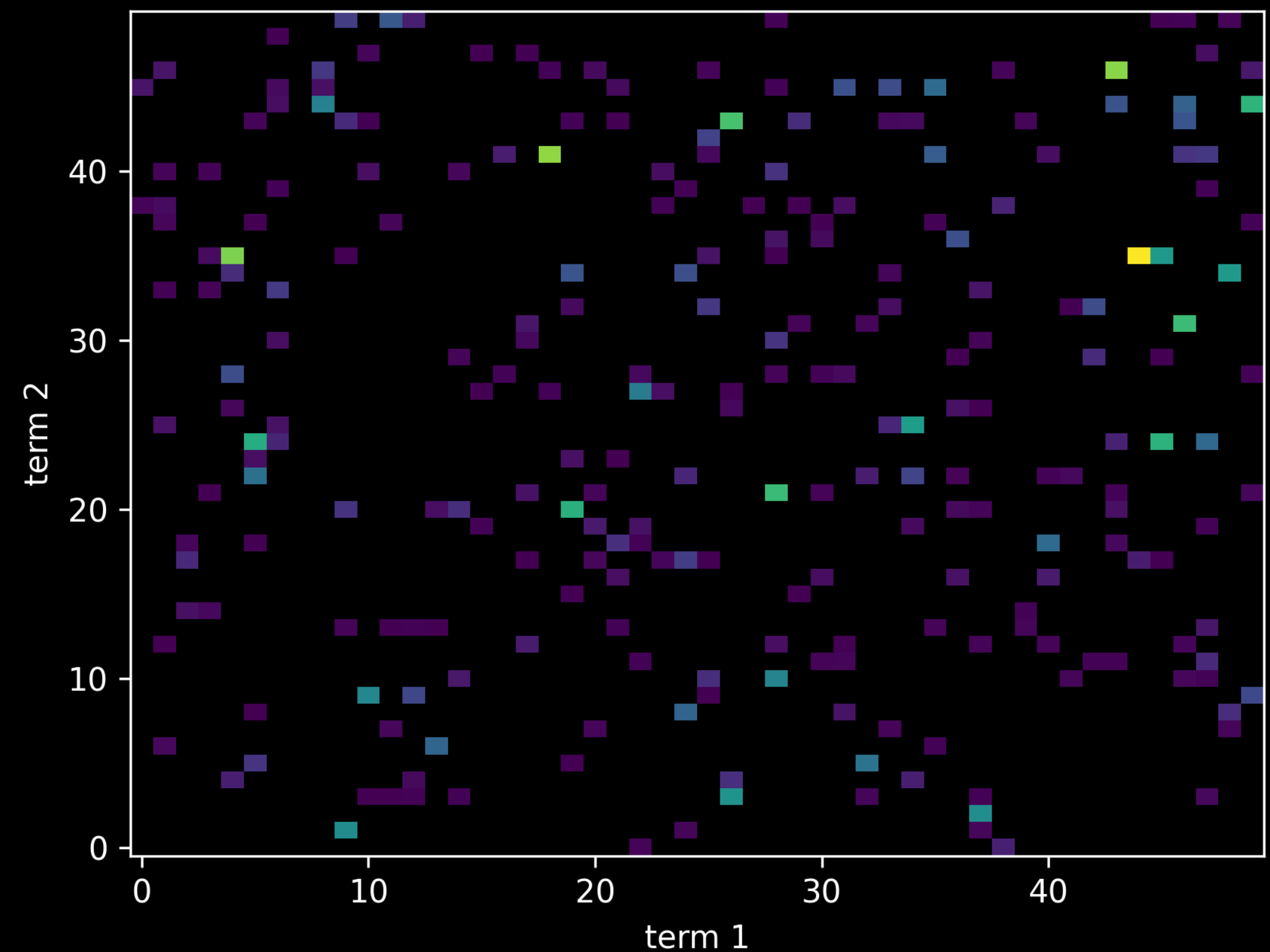
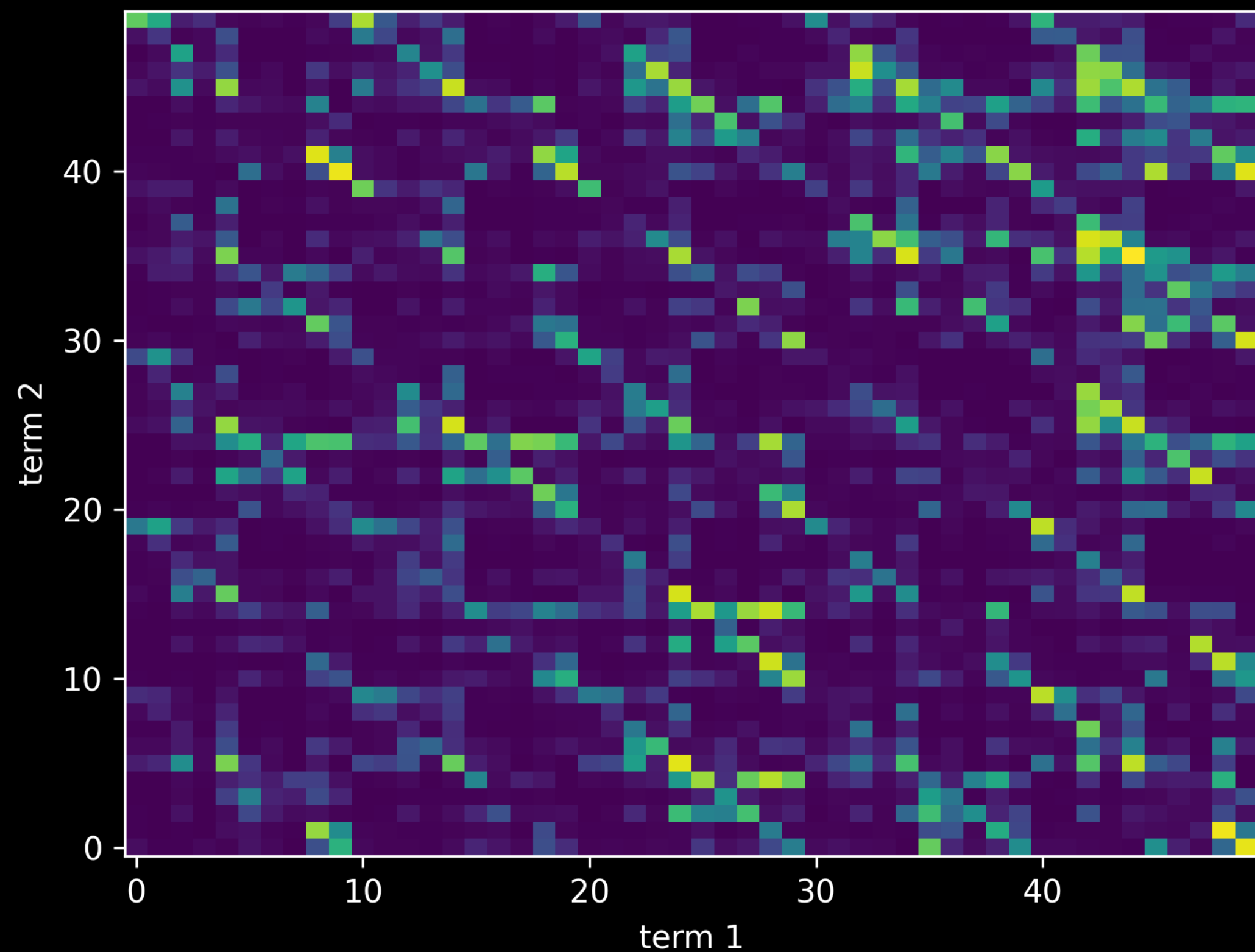
Autoregressive Decoder

Training Data 20% – Accuracy 82%



Autoregressive Decoder

Training Data 10% – Accuracy 36%



Recent Achievements of Language Models

- 2022-02: AlphaCode achieved on average a ranking of top 54.3% in programming competitions.
- 2022-02: GPT-f solves 2 problems from the Internationale Mathematik-Olympiade.
- 2022-03: Tesla FSD Beta 10.11 – Upgraded modeling of lane geometry from dense rasters (“bag of points”) to an autoregressive decoder that directly predicts and connects “vector space” lanes point by point using a transformer neural network. This enables us to predict crossing lanes, allows computationally cheaper and less error-prone post-processing, and paves the way for predicting many other signals and their relationships jointly and end-to-end.

Chain Rule of Probability

$$\begin{aligned} p(s_1, s_2, s_3) &= p(s_3 \mid s_1, s_2) \cdot p(s_1, s_2) \\ &= p(s_3 \mid s_1, s_2) \cdot p(s_2 \mid s_1) \cdot p(s_1) \end{aligned}$$

Predicting a sequence

$$p(s_1, s_2, s_3) = p(s_3 \mid s_1, s_2) \cdot p(s_2 \mid s_1) \cdot p(s_1)$$

$$p(s_2, s_3 \mid s_1) \cdot p(s_1) = p(s_3 \mid s_1, s_2) \cdot p(s_2 \mid s_1) \cdot p(s_1)$$

$$p(s_2, s_3 \mid s_1) = p(s_3 \mid s_1, s_2) \cdot p(s_2 \mid s_1)$$