P-unit model by Henriette Walz & Alexander Ott

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1 The model

The input into the P-unit model, $x(t)$, is

- the fish's own EOD

$$x(t) = x_{EOD}(t) = \cos(2\pi f_{EOD} t) \tag{1}$$

  with EOD frequency f_{EOD} and amplitude normalized to one.

- the EOD multiplied with an amplitude modulation $AM(t)$:

$$x(t) = (1 + AM(t))\cos(2\pi f_{EOD} t) \tag{2}$$

  For a random amplitude modulaten ($AM(t) = RAM(t)$) random numbers are drawn for each frequency up to $f_{EOD}/2$ in Fourier space. After backtransformation the resulting signal is scaled to the desired standard deviation relative to the EOD carrier.

- a superposition of two EODs

$$x(t) = x_{EOD}(t) + x_{EOD1}(t) = \cos(2\pi f_{EOD} t) + \alpha_1 \cos(2\pi f_{EOD1} t) \tag{3}$$

  with the EOD of the second fish having frequency f_{EOD1} and amplitude α_1 relative to the amplitude of the receiving fish.

- a superposition of many EODs

$$x(t) = x_{EOD}(t) + \sum_{i=1}^{n} x_{EODi}(t) = \cos(2\pi f_{EOD} t) + \sum_{i=1}^{n} \alpha_i \cos(2\pi f_{EODi} t)\,. \tag{4}$$

  For $n = 2$ this is our cocktail-party problem.

The input $x(t)$ is a normalized EOD and thus is unitless.

First, the input $x(t)$, is thresholded potentially at the synapse between the receptor cells and the afferent, and then low-pass filtered with time constant τ_d by the afferent's dendrite:

$$\tau_d \frac{dV_d}{dt} = -V_d + \lfloor x(t) \rfloor_0^p \tag{5}$$

Because the input is unitless, the dendritic voltage is unitless, too. $\lfloor x(t) \rfloor_0$ denotes the threshold operation that sets negative values to zero:

$$\lfloor x(t) \rfloor_0 = \left\{ \begin{array}{ll} x(t) & ; \quad x(t) \geq 0 \\ 0 & ; \quad x(t) < 0 \end{array} \right. \tag{6}$$

Usually the exponent p is set to one (pure threshold). In our advanced models p is set to three in order to reproduce responses to beats with difference frequencies above half of the EOD frequency.

This thresholding and low-pass filtering extracts the amplitude modulation of the input $x(t)$. The dendritic voltage $V_d(t)$ is the input to a leaky integrate-and-fire (LIF) model

$$\tau_m \frac{dV_m}{dt} = -V_m + \mu + \alpha V_d - A + \sqrt{2D}\xi(t) \tag{7}$$

where τ_m the membrane time constant, μ a fixed bias current, α a scaling factor for V_d, and $\sqrt{2D}\xi(t)$ a white noise of strength D. All terms in the LIF are unitless.

The adaptation current A follows

$$\tau_A \frac{dA}{dt} = -A \tag{8}$$

with adaptation time constant τ_A.

Whenever the membrane voltage $V_m(t)$ crosses the threshold $\theta = 1$ a spike is generated, $V_m(t)$ is reset to 0, the adaptation current is incremented by ΔA, and integration of $V_m(t)$ is paused for the duration of a refractory period t_{ref}:

$$V_m(t) \geq \theta \;:\; \left\{ \begin{array}{rcl} V_m & \mapsto & 0 \\ A & \mapsto & A + \Delta A/\tau_A \end{array} \right. \tag{9}$$

2 Parameter values

[JB: Sascha, list all parameters (table or itemize) plus time step of the model with typical values and the right (time) units]

Table 1: Model parameters.

parameter	explanation	median parameter
α	stimulus scaling factor	$90.533695\,\mathrm{cm}$
τ_m	membrane time constant	$0.001847\,\mathrm{s}$
μ	bias current	$-17.1875\,\mathrm{mV}$
$\sqrt{2D}$	noise strength	$0.01848\,\mathrm{mV}\sqrt{\mathrm{s}}$
τ_A	adaption time constant	$0.111759\,\mathrm{s}$
ΔA	adaption strength	$0.122197\,\mathrm{mVs}$
τ_d	time constant of dendritic low-pass filter	$0.002463\,\mathrm{s}$
t_{ref}	absolute refractory period	$0.000965\,\mathrm{s}$
Δt	time step	$0.00005\,\mathrm{s}$

[SR: Die Tabelle ist sehr ähnlich der in der Masterarbeit von Alex (Table 3), die Einheiten in der Tabelle der Masterarbeit scheinen jedoch nicht zu stimmen (ich denke es müssten Sekunden und keine Millisekunden sein). Die Einheiten habe ich mir bei einem Abgleich der Werte der csv Tabelle und dem Parameter Wertebereich in den Bildern der Masterabeit abgeleitet (figure 19).]

3 Numerical implementation

[JB: Sascha: This is Alexander Ott's code from the git repository, right?] [SR: Yes that is correct.] The ODEs are integrated by the Euler forward method with time-step Δt.

For the intrinsic noise of the model $\xi(t)$ in each time step i a random number is drawn from a normal distribution $\mathcal{N}(0, 1)$ with zero mean and standard deviation of one. This number is multiplied with $\sqrt{2D}$ and divided by $\sqrt{\Delta t}$:

$$V_{m_{i+1}} = V_{m_i} + \left(-V_{m_i} + \mu + \alpha V_{d_i} - A_i + \sqrt{\frac{2D}{\Delta t}} \mathcal{N}(0, 1)_i \right) \frac{\Delta t}{\tau_m} \tag{10}$$

[JB: Benjamin: wir haben das Rauschen innerhalb der Klammer. Damit ist zwar das $\sqrt{\Delta t}$ richtig, aber wir teilen noch durch τ_m. Damit sind effektiv unsere D Werte andere als wenn wir den Rauschterm ausserhalb der Klammer haetten (so machst du das glaube ich immer).]

The noise strength values from the table in fact equal $\sqrt{2D}$ and not D.

[JB: we need to clean up that noise issue] [SR: Wie genau?]