

Conflict-Driven Clause Learning

Sasha Fedchin¹

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Tufts University

Motivation: why DPLL is not enough

Looking for satisfying interpretation of:

$$\wedge (a \vee \neg h)$$

$$\wedge (\neg b \vee c)$$

$$\wedge (\neg b \vee d)$$

$$\wedge (\neg c \vee e)$$

$$\wedge (\neg e \vee \neg d \vee \neg b)$$

$$\wedge (b \vee f)$$

$$\wedge (b \vee g)$$

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Implication Graph:



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Implication Graph:

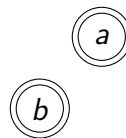


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$$\begin{aligned}
 &\wedge (a \vee \neg h) \\
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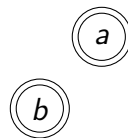
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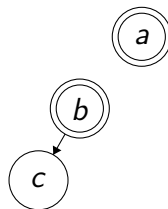
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$$\wedge (\neg b \vee c)$$

$$\wedge (\neg \textcolor{red}{b} \vee d)$$

$$\wedge (\neg \textcolor{red}{c} \vee e)$$

$$\wedge (\neg e \vee \neg d \vee \neg \textcolor{red}{b})$$

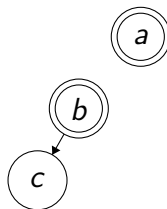
$$\wedge (b \vee f)$$

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$$\wedge (\neg h \vee \neg g \vee \neg \textcolor{red}{a})$$

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$$\wedge (\neg \textcolor{red}{b} \vee \textcolor{blue}{d})$$

$$\wedge (\neg \textcolor{red}{c} \vee e)$$

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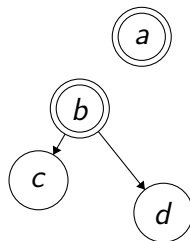
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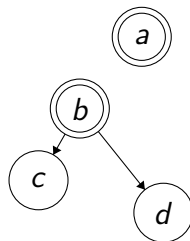
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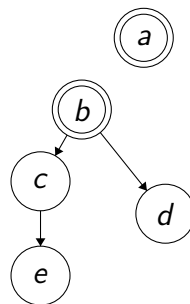
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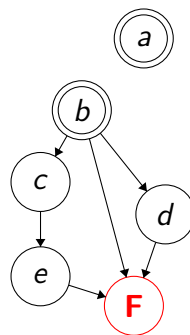
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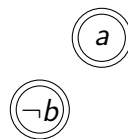
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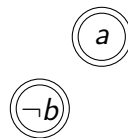
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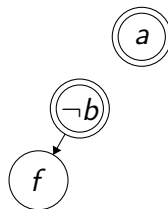
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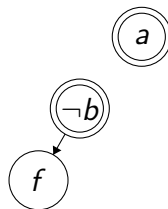
$$\wedge (b \vee f)$$

$$\wedge (\textcolor{red}{b} \vee g)$$

$$\wedge (\neg \textcolor{red}{f} \vee h \vee \textcolor{red}{b})$$

$$\wedge (\neg h \vee \neg g \vee \neg \textcolor{red}{a})$$

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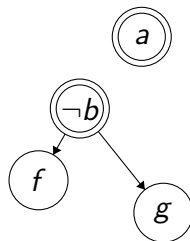
$$\wedge (b \vee f)$$

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$$\wedge (\neg \textcolor{red}{f} \vee h \vee \textcolor{red}{b})$$

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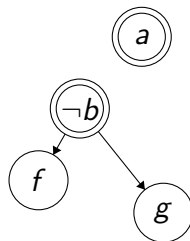
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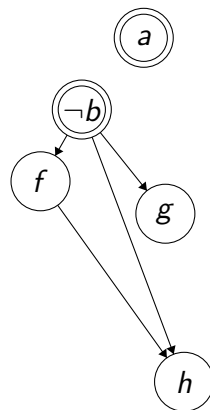
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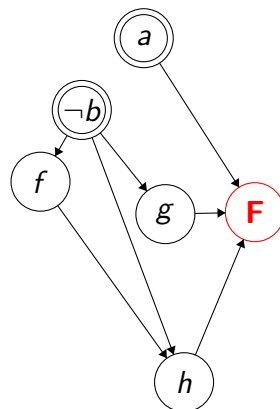


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$$\wedge (\neg c \vee e)$$

$$\wedge (\neg e \vee \neg d \vee \neg b)$$

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$$\wedge (b \vee g)$$

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$$\wedge (\neg h \vee \neg g \vee \textcolor{blue}{\neg a})$$

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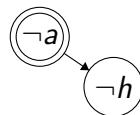
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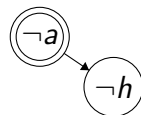
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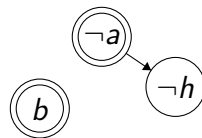
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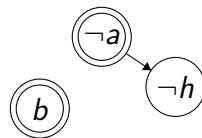
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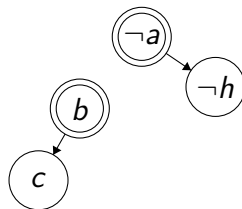
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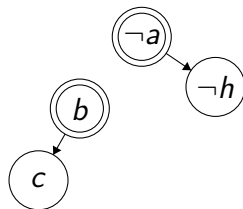
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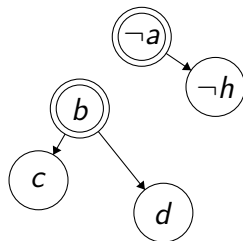
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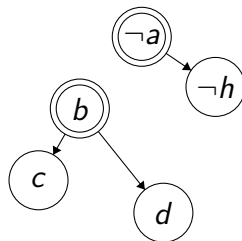
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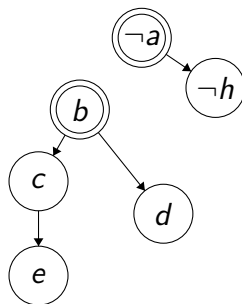
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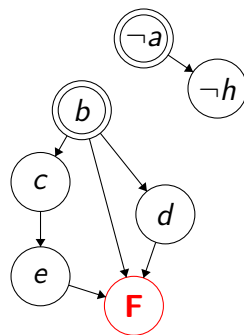
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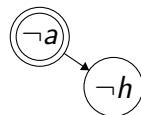
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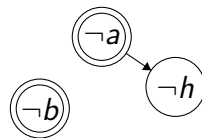
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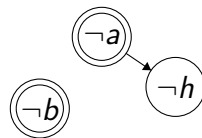
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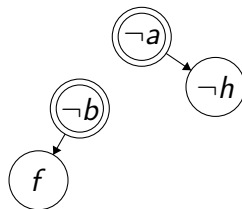
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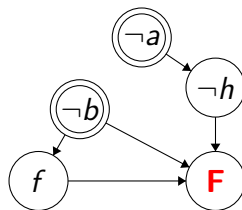


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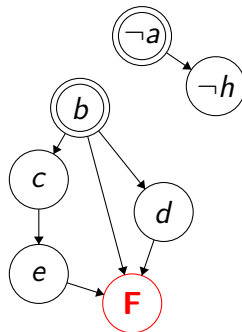
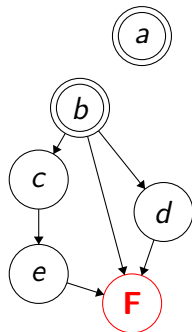
$$\begin{aligned}
 &\wedge (a \vee \neg h) \\
 &\wedge (\neg b \vee c) \\
 &\wedge (\neg b \vee d) \\
 &\wedge (\neg c \vee e) \\
 &\wedge (\neg e \vee \neg d \vee \neg b) \\
 &\wedge (\textcolor{red}{b} \vee \textcolor{blue}{f}) \\
 &\wedge (\textcolor{red}{b} \vee g) \\
 &\wedge (\neg \textcolor{red}{f} \vee h \vee \textcolor{red}{b}) \\
 &\wedge (\neg h \vee \neg g \vee \neg a)
 \end{aligned}$$

Implication Graph:



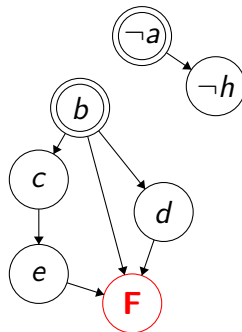
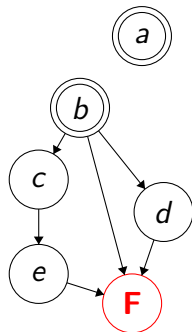
Conflict Driven Clause Learning - Motivation

- We have encountered two conflicts that had similar implication graphs:



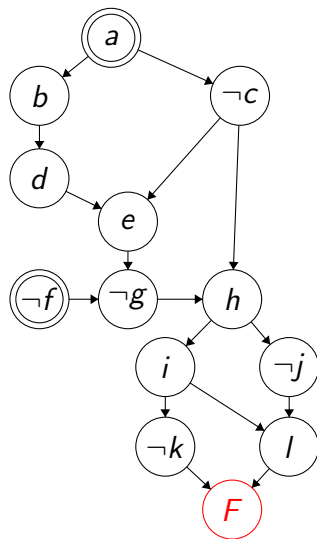
Conflict Driven Clause Learning - Motivation

- We have encountered two conflicts that had similar implication graphs:

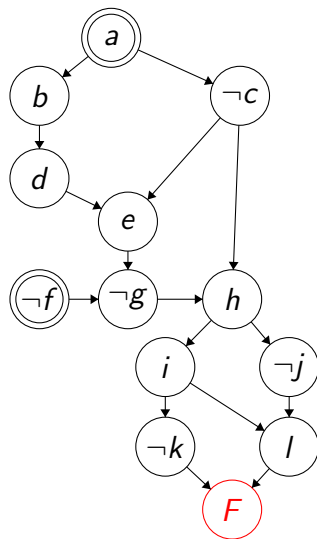


- We can learn the $\neg b$ clause from the conflict on the left and then avoid deriving it for the second time. This is **conflict-driven clause learning**.

Learning Clauses from Conflicts

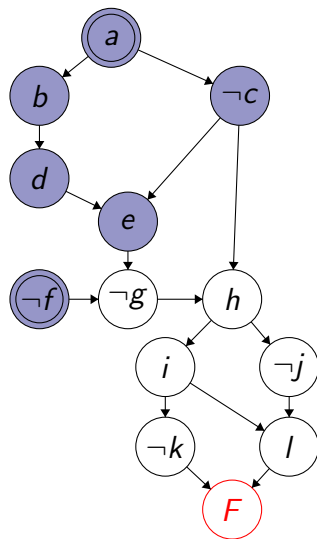


Learning Clauses from Conflicts



To learn a clause:

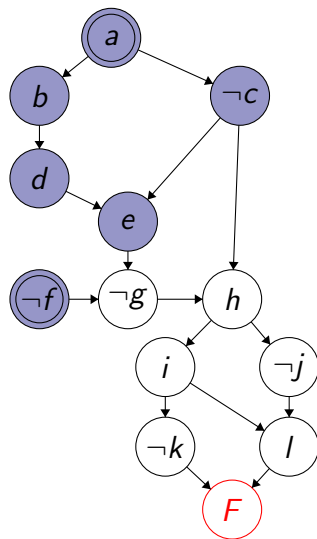
Learning Clauses from Conflicts



To learn a clause:

- Cut the graph in two sets A and B .

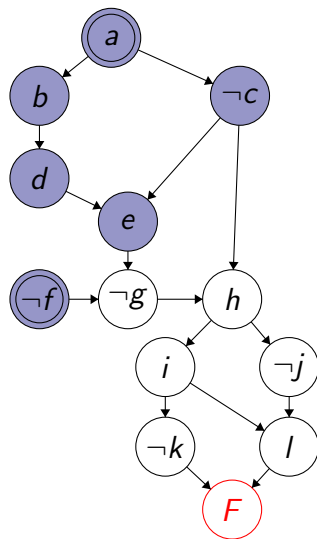
Learning Clauses from Conflicts



To learn a clause:

- ▶ Cut the graph in two sets A and B .
- ▶ Decision vertices a and $\neg f$ must belong to A .

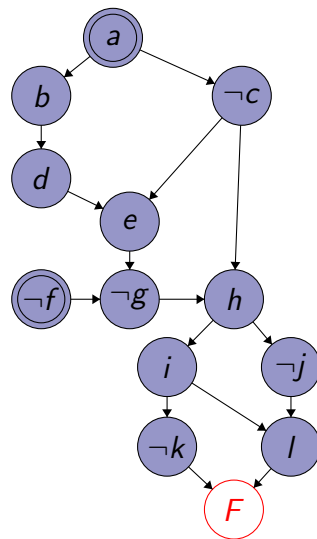
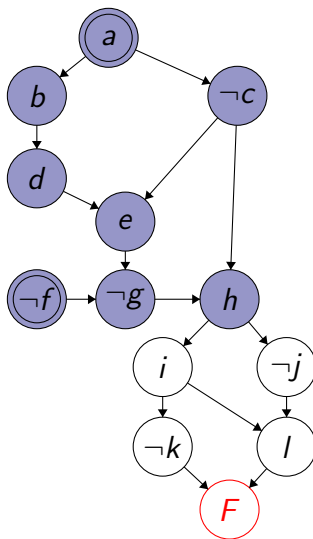
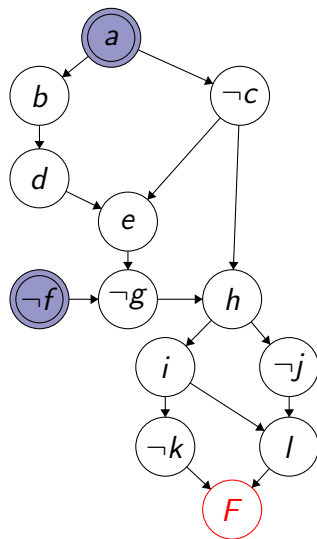
Learning Clauses from Conflicts



To learn a clause:

- ▶ Cut the graph in two sets A and B .
- ▶ Decision vertices a and $\neg f$ must belong to A .
- ▶ Vertices in A that have edges into B form the literals of a new clause ($f \vee \neg e \vee c$).

Multiple cuts are possible



CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\wedge (a \vee \neg h)$$

$$\wedge (\neg b \vee c)$$

$$\wedge (\neg b \vee d)$$

$$\wedge (\neg c \vee e)$$

$$\wedge (\neg e \vee \neg d \vee \neg b)$$

$$\wedge (b \vee f)$$

$$\wedge (b \vee g)$$

$$\wedge (\neg f \vee h \vee b)$$

$$\wedge (\neg h \vee \neg g \vee \neg a)$$

CDCL: Motivating Example Revisited

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Implication Graph:



CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\begin{aligned} &\wedge (a \vee \neg h) \\ &\wedge (\neg b \vee c) \\ &\wedge (\neg b \vee d) \\ &\wedge (\neg c \vee e) \\ &\wedge (\neg e \vee \neg d \vee \neg b) \\ &\wedge (b \vee f) \\ &\wedge (b \vee g) \\ &\wedge (\neg f \vee h \vee b) \\ &\wedge (\neg h \vee \neg g \vee \neg a) \end{aligned}$$

Implication Graph:

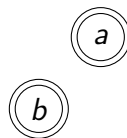


CDCL: Motivating Example Revisited

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Implication Graph:

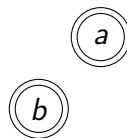


CDCL: Motivating Example Revisited

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Implication Graph:

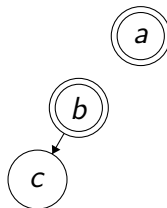


CDCL: Motivating Example Revisited

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Implication Graph:



CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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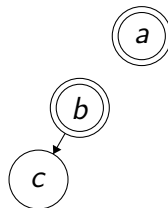
$$\wedge (b \vee f)$$

$$\wedge (b \vee g)$$

$$\wedge (\neg f \vee h \vee b)$$

$$\wedge (\neg h \vee \neg g \vee \neg a)$$

Implication Graph:



CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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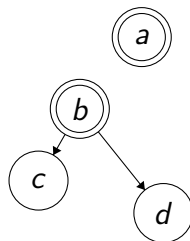
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$$\wedge (\neg f \vee h \vee b)$$

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Implication Graph:



CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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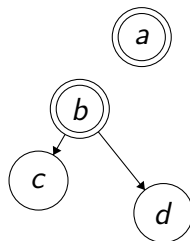
$$\wedge (b \vee f)$$

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Implication Graph:



CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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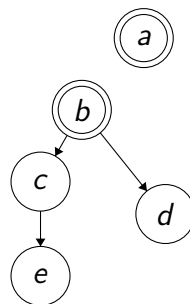
$$\wedge (b \vee f)$$

$$\wedge (b \vee g)$$

$$\wedge (\neg f \vee h \vee b)$$

$$\wedge (\neg h \vee \neg g \vee \neg a)$$

Implication Graph:

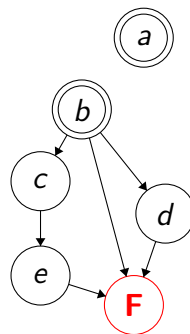


CDCL: Motivating Example Revisited

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$$\begin{aligned} &\wedge (a \vee \neg h) \\ &\wedge (\neg b \vee c) \\ &\wedge (\neg b \vee d) \\ &\wedge (\neg c \vee e) \\ &\wedge (\neg e \vee \neg d \vee \neg b) \\ &\wedge (b \vee f) \\ &\wedge (b \vee g) \\ &\wedge (\neg f \vee h \vee b) \\ &\wedge (\neg h \vee \neg g \vee \neg a) \\ &\wedge \neg b \end{aligned}$$

Implication Graph:



CDCL: Motivating Example Revisited

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 &\wedge (b \vee f) \\
 &\wedge (b \vee g) \\
 &\wedge (\neg f \vee h \vee b) \\
 &\wedge (\neg h \vee \neg g \vee \neg a) \\
 &\wedge \neg b
 \end{aligned}$$

Implication Graph:

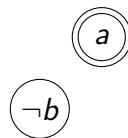


CDCL: Motivating Example Revisited

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Implication Graph:

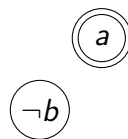


CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\begin{aligned} &\wedge (a \vee \neg h) \\ &\wedge (\neg b \vee c) \\ &\wedge (\neg b \vee d) \\ &\wedge (\neg c \vee e) \\ &\wedge (\neg e \vee \neg d \vee \neg b) \\ &\wedge (\textcolor{red}{b} \vee f) \\ &\wedge (\textcolor{red}{b} \vee g) \\ &\wedge (\neg f \vee h \vee \textcolor{red}{b}) \\ &\wedge (\neg h \vee \neg g \vee \textcolor{red}{\neg a}) \\ &\wedge \neg b \end{aligned}$$

Implication Graph:

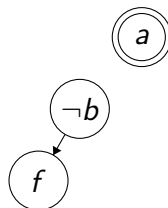


CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\begin{aligned} &\wedge (a \vee \neg h) \\ &\wedge (\neg b \vee c) \\ &\wedge (\neg b \vee d) \\ &\wedge (\neg c \vee e) \\ &\wedge (\neg e \vee \neg d \vee \neg b) \\ &\wedge (\textcolor{red}{b} \vee \textcolor{blue}{f}) \\ &\wedge (\textcolor{red}{b} \vee g) \\ &\wedge (\neg \textcolor{red}{f} \vee h \vee \textcolor{red}{b}) \\ &\wedge (\neg h \vee \neg g \vee \neg \textcolor{red}{a}) \\ &\wedge \neg b \end{aligned}$$

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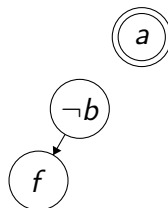


CDCL: Motivating Example Revisited

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CDCL: Motivating Example Revisited

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$$\wedge (b \vee f)$$

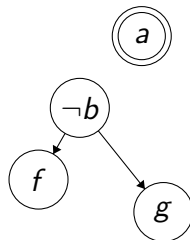
$$\wedge (\textcolor{red}{b} \vee \textcolor{blue}{g})$$

$$\wedge (\neg \textcolor{red}{f} \vee h \vee \textcolor{red}{b})$$

$$\wedge (\neg h \vee \neg \textcolor{red}{g} \vee \neg \textcolor{red}{a})$$

$$\wedge \neg b$$

Implication Graph:



CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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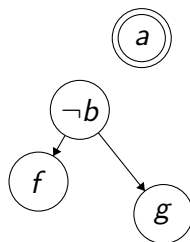
$$\wedge (b \vee g)$$

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$$\wedge \neg b$$

Implication Graph:

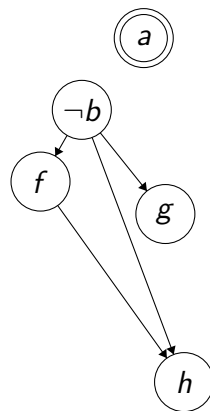


CDCL: Motivating Example Revisited

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Implication Graph:

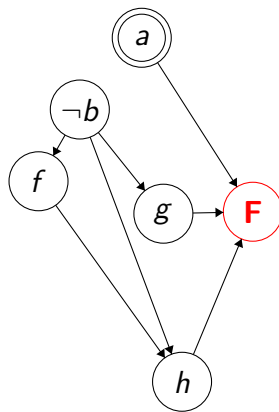


CDCL: Motivating Example Revisited

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Implication Graph:



CDCL: Motivating Example Revisited

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Implication Graph:

CDCL: Motivating Example Revisited

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Implication Graph:



CDCL: Motivating Example Revisited

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Implication Graph:



CDCL: Motivating Example Revisited

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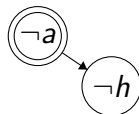
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Implication Graph:

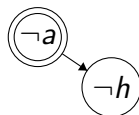


CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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Implication Graph:

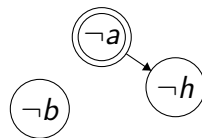


CDCL: Motivating Example Revisited

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Implication Graph:



CDCL: Motivating Example Revisited

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$$\wedge (\textcolor{red}{b} \vee f)$$

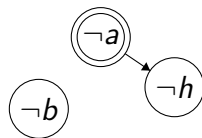
$$\wedge (\textcolor{red}{b} \vee g)$$

$$\wedge (\neg f \vee \textcolor{red}{h} \vee \textcolor{red}{b})$$

$$\wedge (\neg h \vee \neg g \vee \neg a)$$

$$\wedge \neg b$$

Implication Graph:

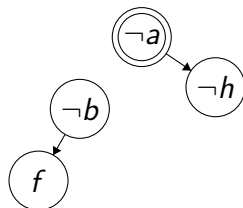


CDCL: Motivating Example Revisited

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Implication Graph:



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Implication Graph:

