

# Conflict-Driven Clause Learning

Sasha Fedchin<sup>1</sup>

<sup>1</sup>Department of Computer Science  
Tufts University

# Motivation: why DPLL is not enough

Looking for satisfying interpretation of:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (b \vee f) \\ & \wedge (b \vee g) \\ & \wedge (\neg f \vee h \vee b) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \end{aligned}$$

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Implication Graph:

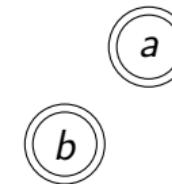


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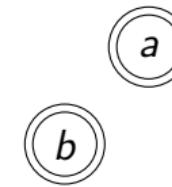


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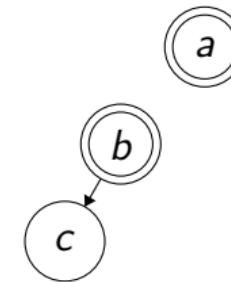


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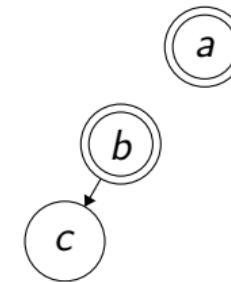


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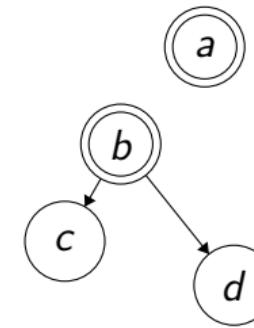


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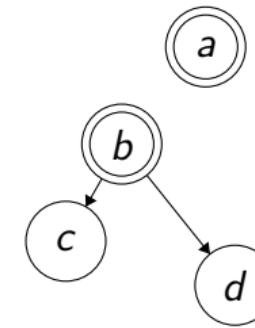


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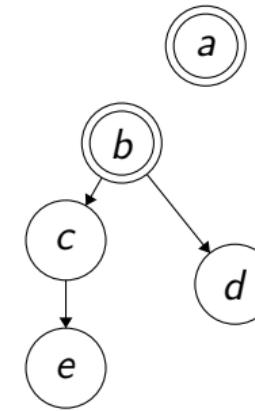


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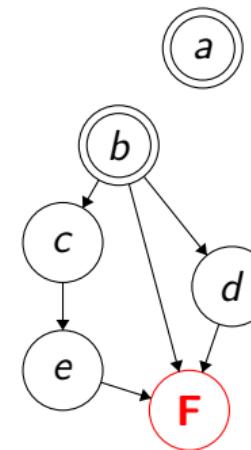


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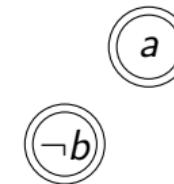


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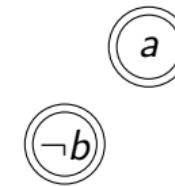


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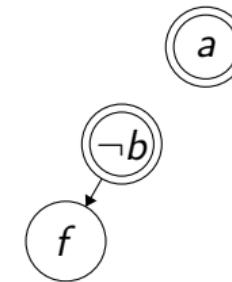


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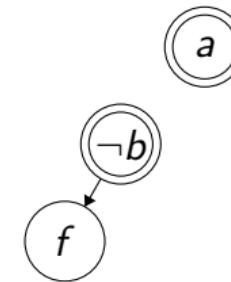


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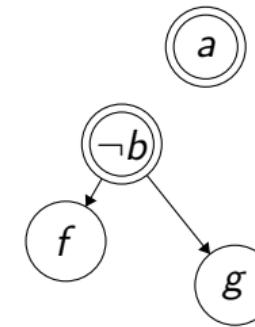


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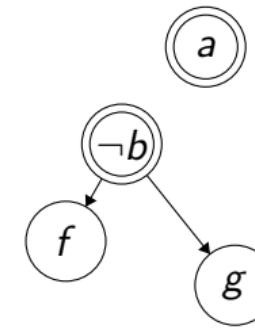


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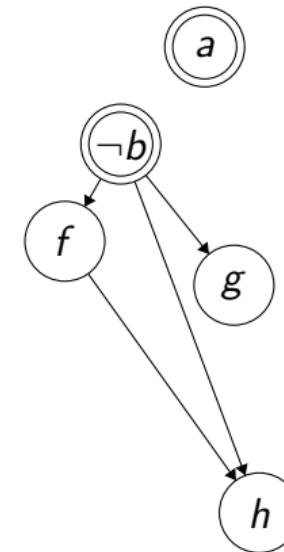


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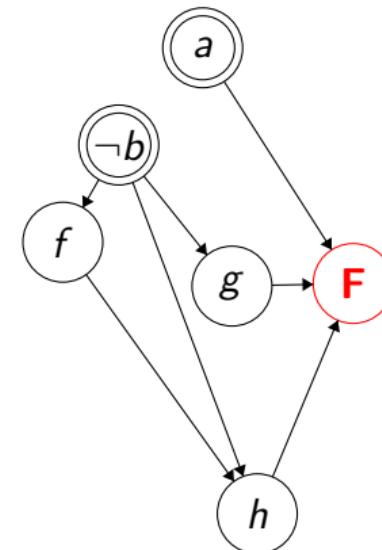


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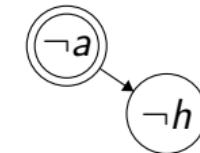


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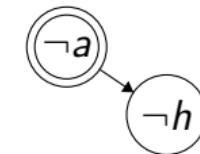


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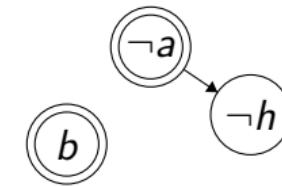


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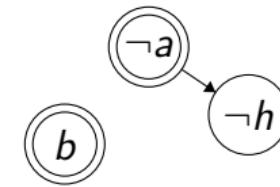


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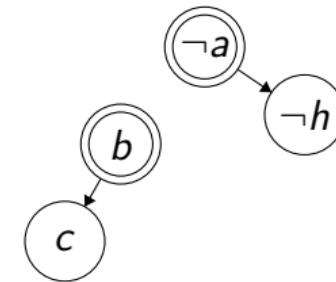


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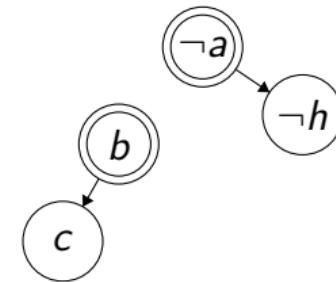


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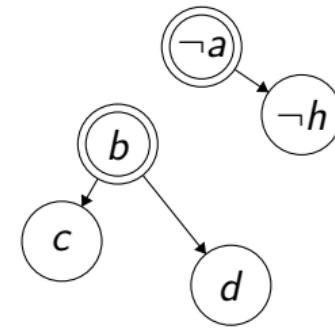


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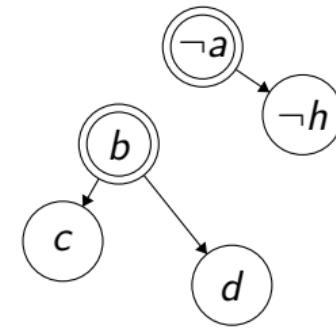


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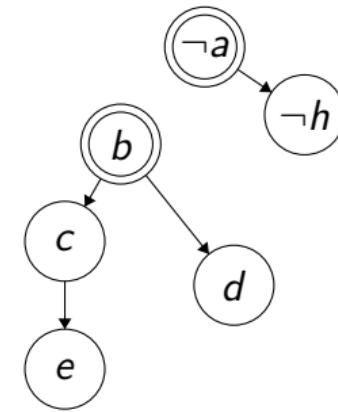


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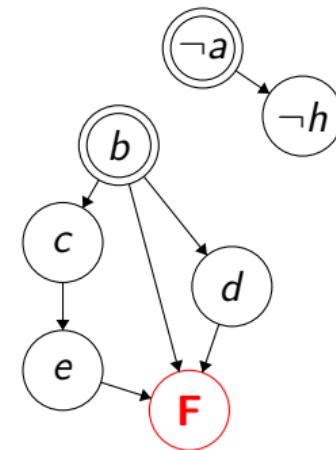


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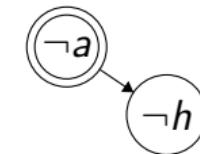


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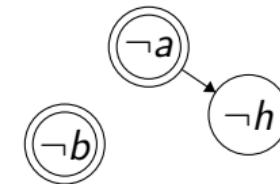


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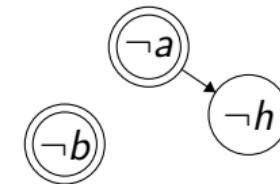


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Implication Graph:

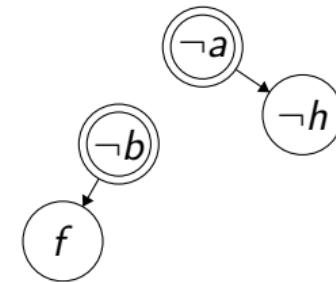


# Motivation: why DPLL is not enough

Looking for satisfying interpretation of:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (\textcolor{red}{b} \vee \textcolor{blue}{f}) \\ & \wedge (\textcolor{red}{b} \vee g) \\ & \wedge (\neg \textcolor{red}{f} \vee h \vee b) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \end{aligned}$$

Implication Graph:

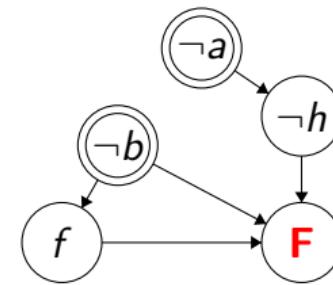


# Motivation: why DPLL is not enough

Looking for satisfying interpretation of:

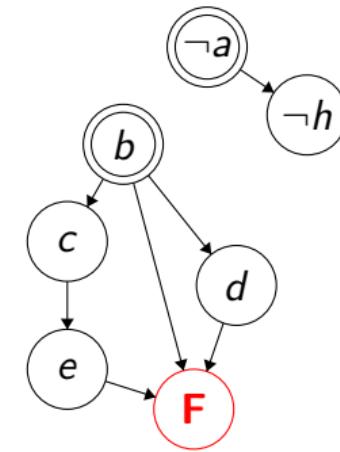
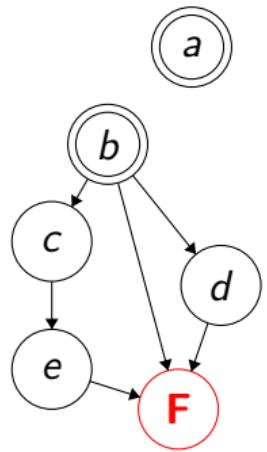
$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (\textcolor{red}{b} \vee \textcolor{blue}{f}) \\ & \wedge (\textcolor{red}{b} \vee g) \\ & \wedge (\boxed{\neg f \vee h \vee b}) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \end{aligned}$$

Implication Graph:



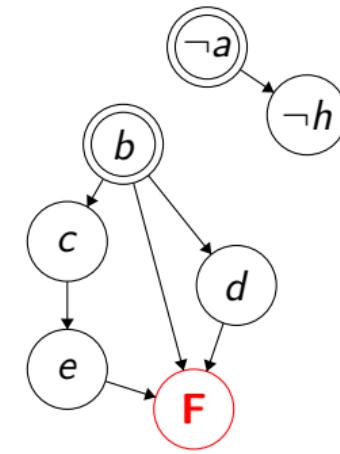
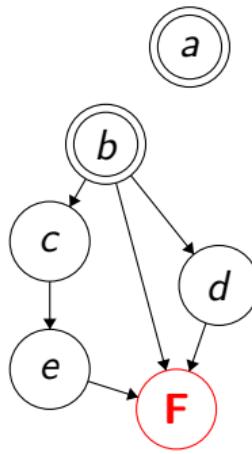
# Conflict Driven Clause Learning - Motivation

- ▶ We have encountered two conflicts that had similar implication graphs:



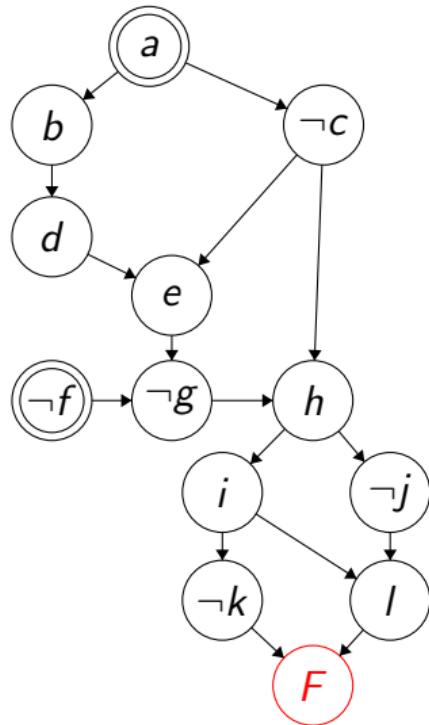
# Conflict Driven Clause Learning - Motivation

- ▶ We have encountered two conflicts that had similar implication graphs:

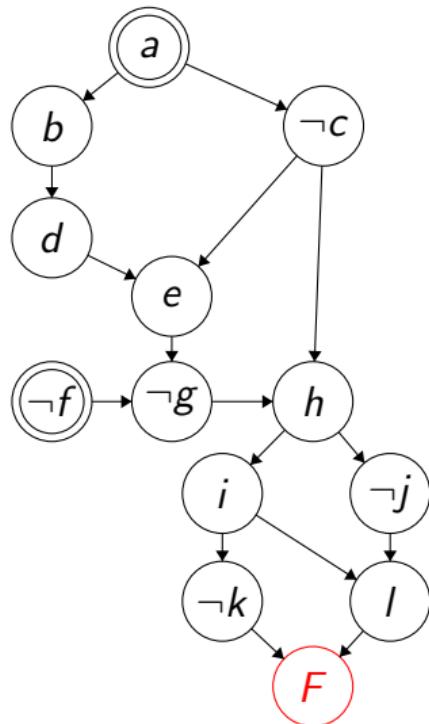


- ▶ We can learn the  $\neg b$  clause from the conflict on the left and then avoid deriving it for the second time. This is **conflict-driven clause learning**.

# Learning Clauses from Conflicts

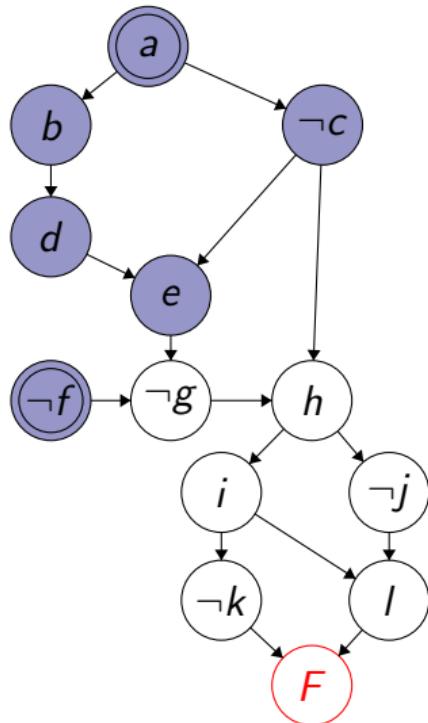


# Learning Clauses from Conflicts



To learn a clause:

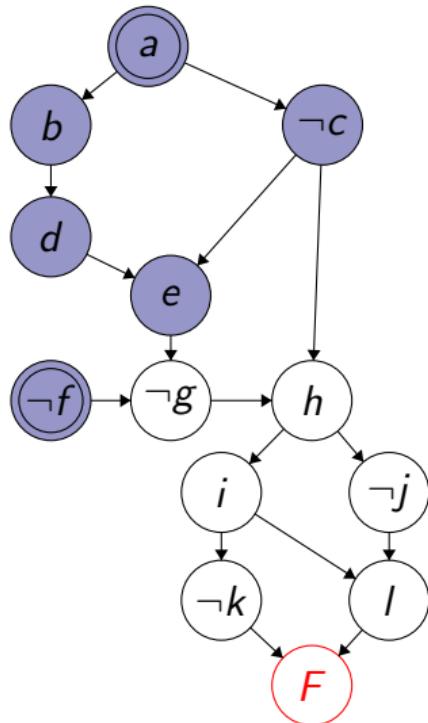
# Learning Clauses from Conflicts



To learn a clause:

- ▶ Cut the graph in two sets  $A$  and  $B$ .

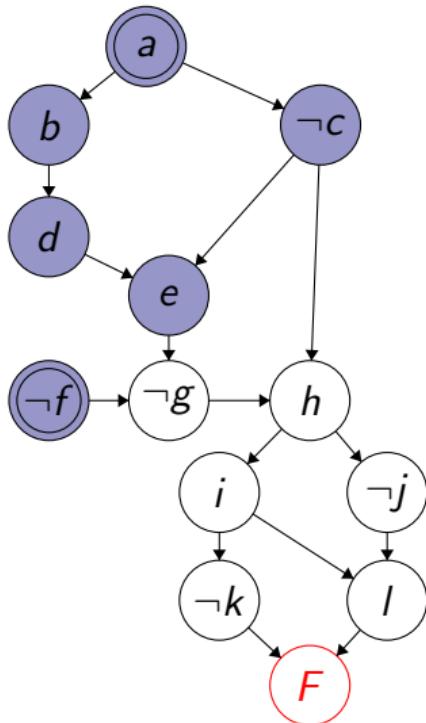
# Learning Clauses from Conflicts



To learn a clause:

- ▶ Cut the graph in two sets  $A$  and  $B$ .
- ▶ Decision vertices  $a$  and  $\neg f$  must belong to  $A$ .

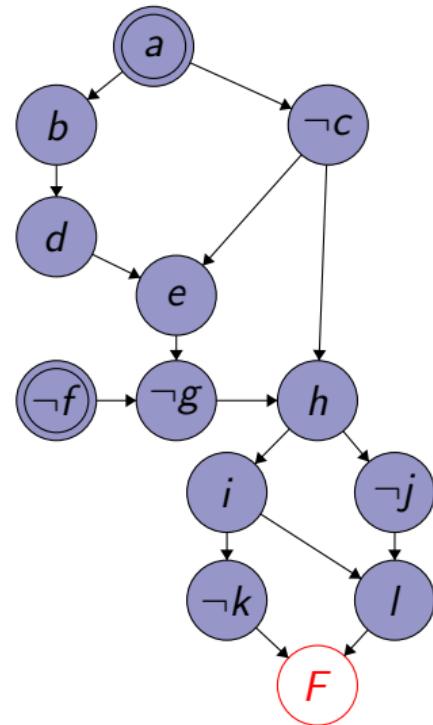
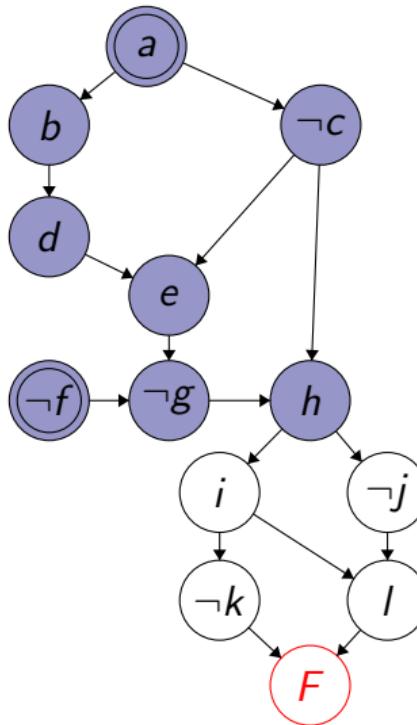
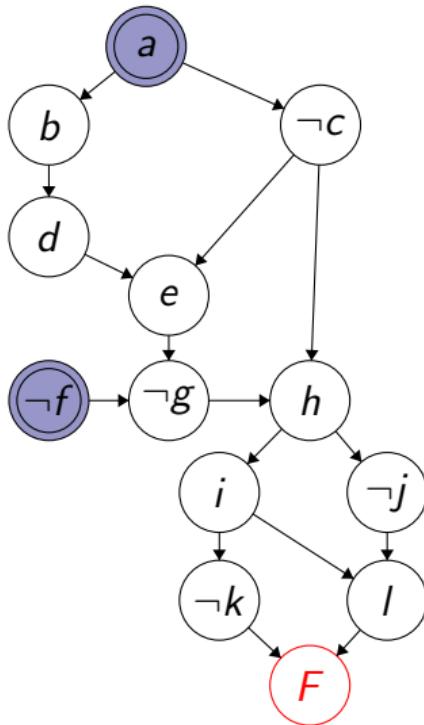
# Learning Clauses from Conflicts



To learn a clause:

- ▶ Cut the graph in two sets  $A$  and  $B$ .
- ▶ Decision vertices  $a$  and  $\neg f$  must belong to  $A$ .
- ▶ Vertices in  $A$  that have edges into  $B$  form the literals of a new clause ( $f \vee \neg e \vee c$ ).

# Multiple cuts are possible



# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (b \vee f) \\ & \wedge (b \vee g) \\ & \wedge (\neg f \vee h \vee b) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \end{aligned}$$

# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

Implication Graph:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (b \vee f) \\ & \wedge (b \vee g) \\ & \wedge (\neg f \vee h \vee b) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \end{aligned}$$



# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

Implication Graph:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (b \vee f) \\ & \wedge (b \vee g) \\ & \wedge (\neg f \vee h \vee b) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \end{aligned}$$

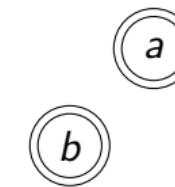


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

Implication Graph:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (b \vee f) \\ & \wedge (b \vee g) \\ & \wedge (\neg f \vee h \vee b) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \end{aligned}$$

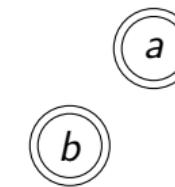


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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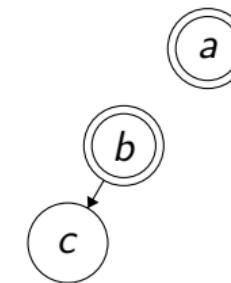


# CDCL: Motivating Example Revisited

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Implication Graph:

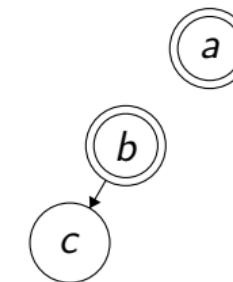


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (b \vee f) \\ & \wedge (b \vee g) \\ & \wedge (\neg f \vee h \vee b) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \end{aligned}$$

Implication Graph:

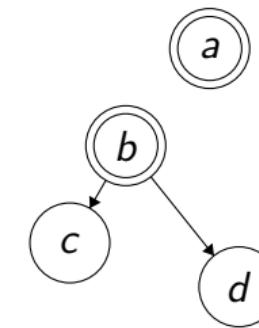


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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Implication Graph:

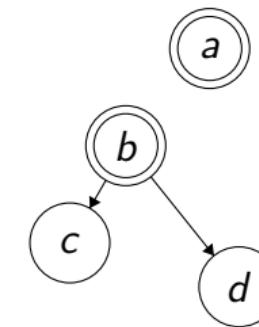


# CDCL: Motivating Example Revisited

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Implication Graph:

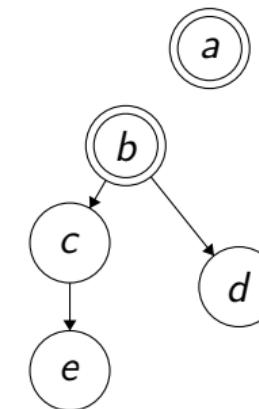


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Looking for satisfying interpretation of:

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Implication Graph:

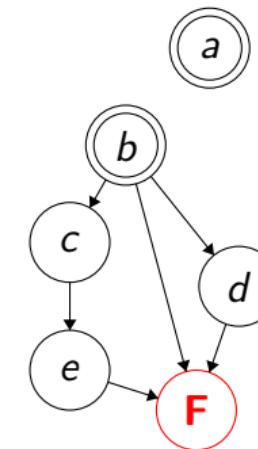


# CDCL: Motivating Example Revisited

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Implication Graph:



# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

Implication Graph:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (b \vee f) \\ & \wedge (b \vee g) \\ & \wedge (\neg f \vee h \vee b) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \\ & \wedge \neg b \end{aligned}$$

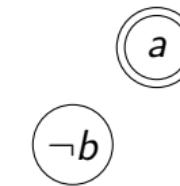


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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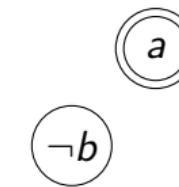


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (\textcolor{red}{b} \vee f) \\ & \wedge (\textcolor{red}{b} \vee g) \\ & \wedge (\neg f \vee h \vee \textcolor{red}{b}) \\ & \wedge (\neg h \vee \neg g \vee \neg \textcolor{red}{a}) \\ & \wedge \neg b \end{aligned}$$

Implication Graph:

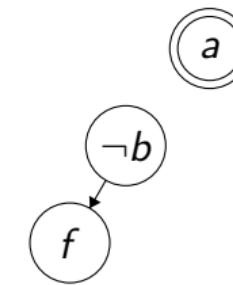


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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Implication Graph:

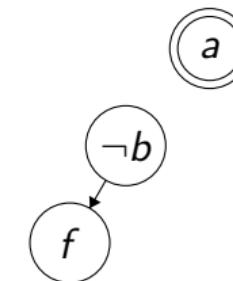


# CDCL: Motivating Example Revisited

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Implication Graph:

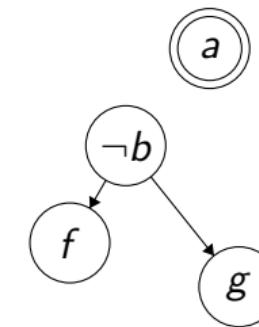


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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Implication Graph:

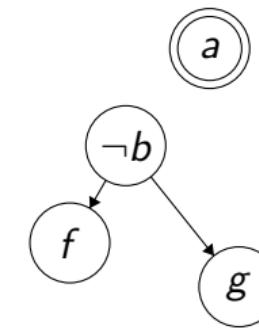


# CDCL: Motivating Example Revisited

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Implication Graph:

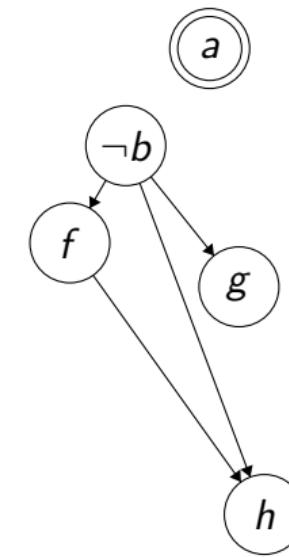


# CDCL: Motivating Example Revisited

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Implication Graph:

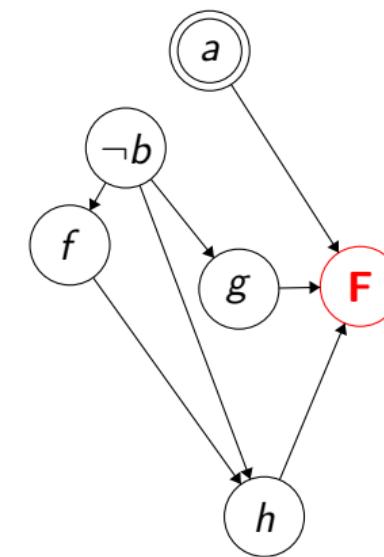


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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Implication Graph:



# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

Implication Graph:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (b \vee f) \\ & \wedge (b \vee g) \\ & \wedge (\neg f \vee h \vee b) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \\ & \wedge \neg b \end{aligned}$$

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# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

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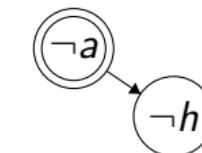


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Looking for satisfying interpretation of:

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Implication Graph:

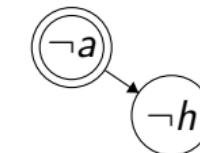


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

Implication Graph:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (b \vee f) \\ & \wedge (b \vee g) \\ & \wedge (\neg f \vee \textcolor{red}{h} \vee b) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \\ & \wedge \neg b \end{aligned}$$

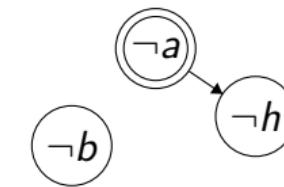


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (b \vee f) \\ & \wedge (b \vee g) \\ & \wedge (\neg f \vee h \vee b) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \\ & \wedge \neg b \end{aligned}$$

Implication Graph:

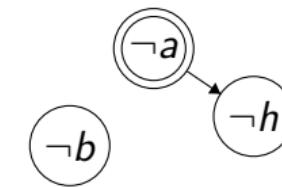


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Implication Graph:

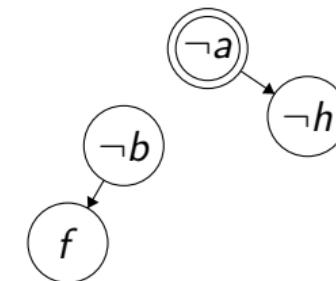


# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (\textcolor{red}{b} \vee \textcolor{blue}{f}) \\ & \wedge (\textcolor{red}{b} \vee g) \\ & \wedge (\neg f \vee \textcolor{red}{h} \vee \textcolor{red}{b}) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \\ & \wedge \neg b \end{aligned}$$

Implication Graph:



# CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\begin{aligned} & \wedge (a \vee \neg h) \\ & \wedge (\neg b \vee c) \\ & \wedge (\neg b \vee d) \\ & \wedge (\neg c \vee e) \\ & \wedge (\neg e \vee \neg d \vee \neg b) \\ & \wedge (\textcolor{red}{b} \vee \textcolor{blue}{f}) \\ & \wedge (\textcolor{red}{b} \vee g) \\ & \wedge (\boxed{\neg f \vee h \vee b}) \\ & \wedge (\neg h \vee \neg g \vee \neg a) \\ & \wedge \neg b \end{aligned}$$

Implication Graph:

