

$$A \subseteq A' \Rightarrow \text{pre}_\exists(A) \subseteq \text{pre}_\exists(A')$$

(Solving Model Checking Problem)

- Let define:

$$\text{pre}_\exists(Y) = \{s \in S \mid \exists s' \in Y \text{ s.t. } (s, s') \in \mathcal{R}\}$$

$$\text{pre}_\forall(Y) = \{s \in S \mid \mathcal{R}(s) \subseteq Y\}$$

- Compute $\llbracket \varphi \rrbracket = \{s \in S \mid s \models \varphi\}$

$$\llbracket p \rrbracket = \{s \in S \mid p \in L(s)\}$$

$$\llbracket \neg \varphi \rrbracket = S \setminus \llbracket \varphi \rrbracket$$

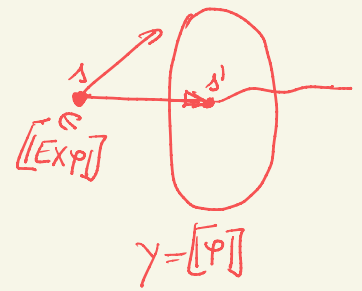
$$\llbracket \varphi_1 \vee \varphi_2 \rrbracket = \llbracket \varphi_1 \rrbracket \cup \llbracket \varphi_2 \rrbracket$$

$$\llbracket EX \varphi \rrbracket = \text{pre}_\exists(\llbracket \varphi \rrbracket)$$

$$\llbracket AF \varphi \rrbracket = MC_{CTL}^{AF}(\varphi)$$

$$\llbracket E(\varphi_1 \mathcal{U} \varphi_2) \rrbracket = MC_{CTL}^{EU}(\varphi_1, \varphi_2)$$

- Test if the input state $s \in \llbracket \varphi \rrbracket$



- $MC_{CTL}^{EU}(\varphi_1, \varphi_2)$ is computed as:

- $Y := \emptyset; Z := \llbracket \varphi_2 \rrbracket$

- while $Z \not\subseteq Y$ do:

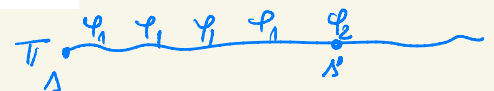
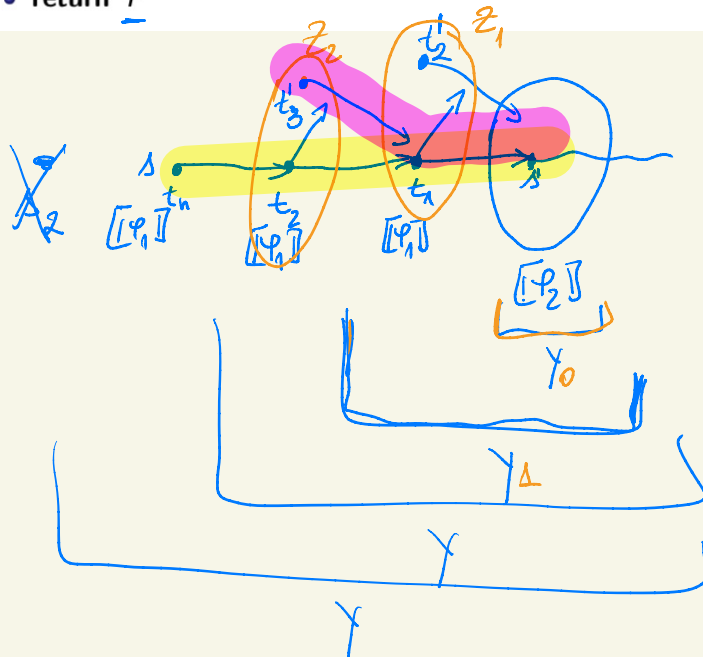
$$Y = Y \cup Z;$$

$$Z = \text{pre}_\exists(Y) \cap \llbracket \varphi_1 \rrbracket$$

- return Y

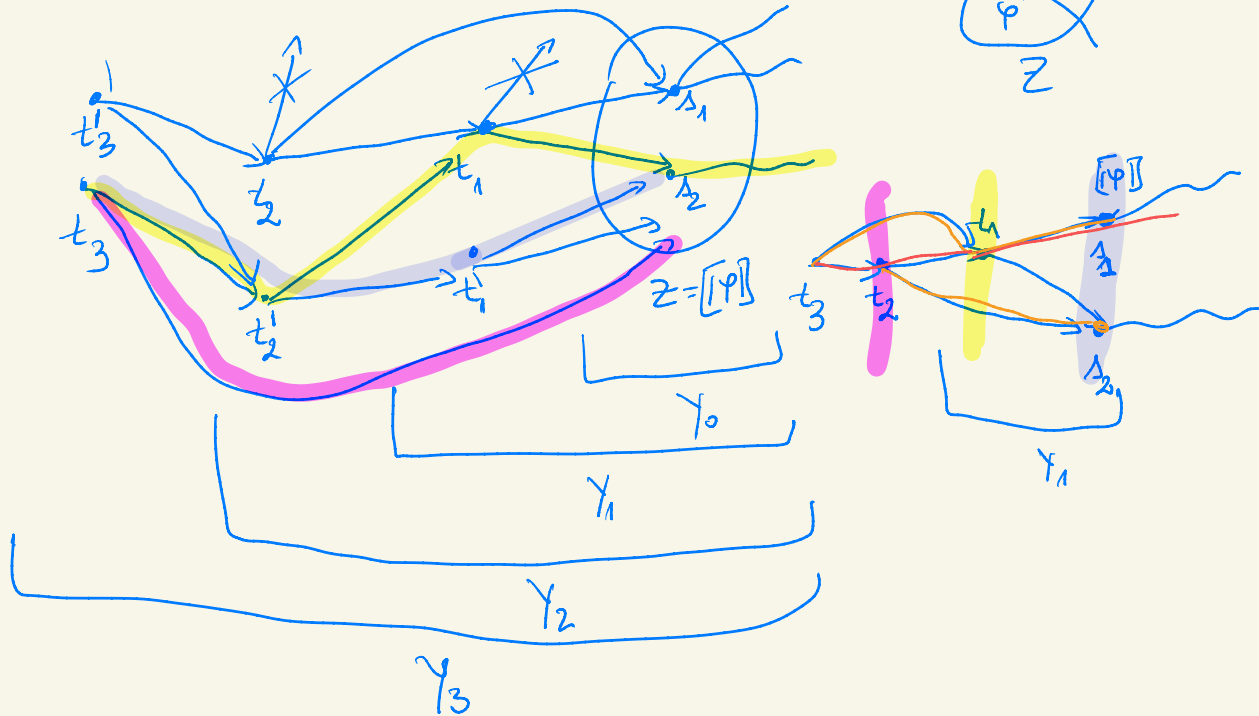
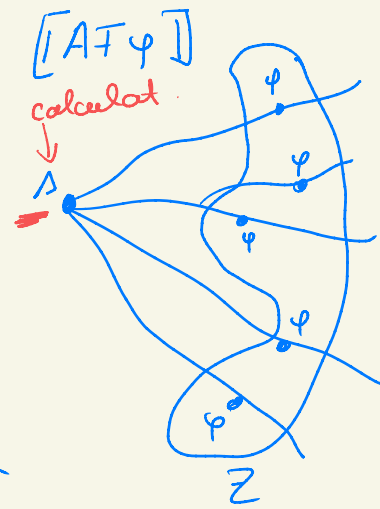
Stop when cannot add nodes in Y!

$$\llbracket E \varphi_1 \mathcal{U} \varphi_2 \rrbracket$$



• $MC_{CTL}^{AF}(\varphi)$ is computed as:

- $Y := S; Z := \llbracket \varphi \rrbracket;$
- **while** $Y \neq Z$ **do**:
 $Y := Z$
 $Z = \underline{Z} \cup \text{pre}_V(Y)$
- **return** Y



$$AX\varphi \equiv \neg EX\neg\varphi$$

$$A(\varphi_1 U \varphi_2) \equiv \neg(E(\neg\varphi_2 U (\neg\varphi_1 \wedge \neg\varphi_2)) \vee EG\neg\varphi_2)$$

$$EF\varphi \equiv E(TU\varphi)$$

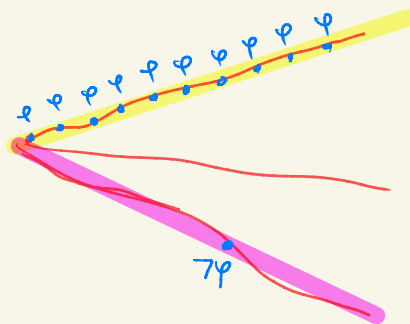
$$EG\varphi \equiv \neg AF\neg\varphi$$

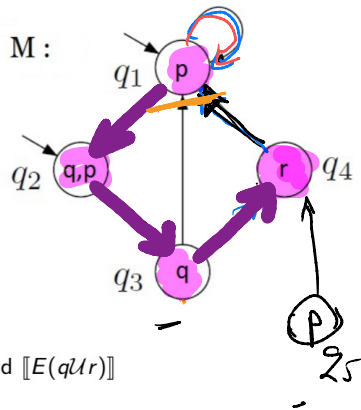
$$AG\varphi \equiv \neg EF\neg\varphi$$

$$G\varphi \equiv \neg F\neg\varphi$$

← definim alg, similar cu
EU & AF

$$\begin{aligned} \llbracket AX\varphi \rrbracket &= \llbracket \neg EX\neg\varphi \rrbracket = S \setminus \llbracket EX\neg\varphi \rrbracket = S \setminus \text{pre}_Z(\llbracket \neg\varphi \rrbracket) \\ &= S \setminus \text{pre}_Z(S \setminus \llbracket \varphi \rrbracket) \end{aligned}$$





$$\llbracket AXq \rrbracket = \llbracket \neg EX \neg q \rrbracket =$$

$$= S \setminus \text{pre}_3(S \setminus \llbracket q \rrbracket)$$

$$\llbracket q \rrbracket = \{q_2, q_3\}$$

$$S \setminus \llbracket q \rrbracket = \{q_1, q_4\}$$

$$\text{pre}_3(S \setminus \llbracket q \rrbracket) = \{q_3, q_1, q_4\}$$

$$\llbracket AXq \rrbracket = \{q_2\}$$

$$\llbracket EXp \rrbracket = \text{pre}_3(\llbracket p \rrbracket)$$

$$\llbracket p \rrbracket = \{q_1, q_2\}$$

$$\llbracket EXp \rrbracket = \{q_1, q_3, q_4\}$$

$$\llbracket E(q \wedge r) \rrbracket$$

$$Y = \emptyset \quad Z = \llbracket q \rrbracket = \{q_2, q_3\}$$

$$Y = Y \cup Z = \{q_2, q_3\}$$

$$Z = \{q_3\} = \text{pre}_3(q_4) \cap \llbracket q \rrbracket$$

$$\{q_3, q_5\} \cap \{q_2, q_3\}$$

$$Y = Y \cup Z = \{q_2, q_3, q_4\}$$

$$Z = \{q_2, q_3\}$$

$$Y = Y \cup Z = \{q_2, q_3, q_4\}$$

$$Z = \{q_2, q_3, \cancel{q_4}\} = \text{pre}_3(Y) \cap \llbracket q \rrbracket$$

$$\{q_2, q_3, q_4\} \cap \{q_2, q_3\}$$

$$Z \subseteq Y \text{ stop.}$$

$MC_{CTL}^{EU}(\varphi_1, \varphi_2)$ is computed as:

- $Y := \emptyset; Z := \llbracket \varphi_2 \rrbracket$
 - while $Z \not\subseteq Y$ do:
 - $Y = Y \cup Z;$
 - $Z = \text{pre}_3(Y) \cap \llbracket \varphi_1 \rrbracket$
 - return Y
- Stop when cannot add nodes in Y!

$$\llbracket q \rrbracket = \{q_2, q_3\}$$