Modern Portfolio Optimization using Artificial Intelligence and Machine Learning

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Introduction

- **Portfolio Optimization** aims to allocate stock assets/equities to
  - *maximize* returns for a given level of risk or
  - *minimize* risk for a given level of return.
- Classical approaches like **Modern Portfolio Theory** (MPT) rely on solving Non-Linear (Quadratic) optimization problems.
- Modern **Artificial Intelligence / Machine Learning** methods can be utilized to enhance portfolio optimization by identifying complex patterns and adapting to market dynamics.
- This project compares standard MPT strategies against these modern AI/ML methods on two stock groups:
  - Magnificent Seven (MAG7)
  - Dow Jones 30 (DJ30).

Louis Bachelier: The Theory of Speculation

Developed over a century, modern financial theory traces its origin back to the early 1900s with **Louis Bachelier's** pioneering work.

- In his dissertation, *The Theory of Speculation*, Bachelier argued that future prices cannot be projected with any certainty.
- Mostly overlooked at the time, his ideas shaped the *Efficient Market Hypothesis* (EMH).
- EMH states that markets use all the information at hand to determine prices; challenges investors' attempts to "beat the market".

Alfred Cowles III: Investigating Professional Investors

In the 1930s and 1940s, **Alfred Cowles III** studied the ability of professional investors to select winning stocks.

- Little proof was found that professional investors could consistently outperform the market.
- His studies confirmed EMH's hypothesis that known information shapes stock prices.
- This supported the case for passive investing versus the attempt to "beat the market."

Benjamin Graham: The Birth of Value Investing

Around the same time, **Benjamin Graham** emphasized the identification of undervalued stocks, creating the foundation for **value investing**.

- His techniques influenced significant investors, such as Warren Buffet.
- His approach was to find securities that were priced below their intrinsic value: *"Buy low, sell high"*.

Harry Markowitz: Modern Portfolio Theory (MPT)

In 1952, **Harry Markowitz** revolutionized finance with *Portfolio Selection*, introducing his **Modern Portfolio Theory** (MPT).

- MPT linked risk and return, showing that **diversification** is key to controlling risk.
- He introduced the **Mean-Variance Optimization** (MVO) which involves solving a problem Non-Linear (Quadratic) optimization to choose portfolios that maximize returns for a given level of risk.

Portfolio Return and Variance

For a given portfolio, described by the weight vector w, the (total) **portfolio return** is the weighted sum of individual asset returns:

$$R_p = \sum_{i=1}^{n} w_i R_i \tag{1}$$

where R_i is the return of asset i.
The **expected return** of the portfolio is:

$$E[R_p] = \mu^T w \tag{2}$$

The **portfolio variance** is given by:

$$\sigma_p^2 = w^T \Sigma w \tag{3}$$

Markowitz's Optimization Model

Markowitz's MVO is a quadratic optimization problem:

$$\max_{w} f(w) = \mu^T w - \alpha w^T \Sigma w \tag{4}$$

subject to the *linear* and *positivity* constraints:

$$\sum_{i} w_i = 1, \quad w_i \geq 0, \quad \forall i \tag{5}$$

where:

- w represents the portfolio weights.
- μ is the vector of expected returns.
- Σ is the covariance matrix of the asset returns.
- α is a *risk-aversion* parameter.

The Sharpe Ratio

Introduced by William Sharpe in 1966, the **Sharpe Ratio** compares the portfolio's *expected excess return* to its *volatility*:

$$S = \frac{\mu^T w - R_f}{\sqrt{w^T \Sigma w}} \tag{6}$$

where:

- $\mu^T w$ is the expected return of the portfolio.
- R_f is the risk-free rate.
- The denominator represents the volatility of the portfolio (which measures the risk).

An optimal portfolio maximizes the Sharpe ratio, balancing risk and return.

Capital Asset Pricing Model (CAPM)

In the 1960s, **William Sharpe** introduced the **CAPM**, expanding MPT by adding **Beta** (β):

$$E[r_i] = r_f + \beta_i(E[r_m] - r_f) \tag{7}$$

where:

- $E[r_i]$ is the expected return of asset i.
- r_f is the risk-free rate.
- $E[r_m]$ is the market return.
- β_i represents the asset's *sensitivity* to market fluctuations.

CAPM proposes that asset returns are driven only by market risk.

Fama-French Three-Factor Model

In 1993, **Eugene Fama** and **Kenneth French** expanded CAPM by adding **size** and **value** factors:

$$E[r_i] - r_f = \alpha_i + \beta_{MKT}\left(E[r_m] - r_f\right) + \beta_{SMB} \cdot SMB + \beta_{HML} \tag{8}$$

where:

- SMB is the *size* premium (Small Minus Big).
- HML is the *value* premium (High Minus Low).

This model offers a more advanced approach to risk management and portfolio diversification.

The Role of AI and Machine Learning in Portfolio Optimization

Recent developments in **AI/ML** have drastically improved portfolio optimization:

- Techniques like regression analysis, ARIMA, and LSTMs are used for risk prediction.
- Reinforcement learning and clustering algorithms improve asset selection and portfolio construction.
- AI-driven models offer greater flexibility to navigate complex market conditions.

The future of portfolio optimization is expected to integrate AI-driven models for even more dynamic portfolio strategies.

Portfolio Optimization: Introduction

The primary goal of **portfolio optimization** is to allocate stock assets / equities so that the portfolio's *expected return* is maximized for a certain degree of *risk*, or conversely, risk is minimized for a given level of return.

- This project will explore several modern portfolio optimization techniques, including:
  - Traditional methods: Equally Weighted (Naive), Maximum Sharpe Ratio, Minimum Volatility.
  - Advanced methods: Principal Component Analysis (PCA), Hierarchical Risk Parity (HRP).
  - Multi-factor models: Capital Asset Pricing Model (CAPM), Fama-French Three-Factor Model.
- These conventional *benchmark* approaches provide a robust framework for modern portfolio optimization by accounting for various risk-return profiles.

Monte Carlo Simulations

Monte Carlo simulations are used to visualize the portfolio **feasibility space** and the **Efficient Frontier**.

- A large number of portfolio weight vectors are generated randomly.
    - Corresponding expected returns, volatility, and Sharpe ratios are calculated for each random portfolio.
- The results are plotted, showing the *Efficient Frontier* as the upper boundary of the feasibility space.
- This simulation helps evaluate the return/risk profile of each benchmark portfolio.

Equally Weighted (Naive) Portfolio

The **Equally Weighted** Portfolio assigns equal weights to all assets in the portfolio.

$$w_i = \frac{1}{n} \tag{9}$$

where n is the number of assets in the portfolio.

- This strategy is simple and often used as a benchmark.
- An investor may chose this strategy if they do not have any reason to prefer any stock over any other stock in the portfolio.

Maximum Sharpe Ratio Portfolio

The **Maximum Sharpe Ratio** Portfolio identifies the portfolio with the highest risk-adjusted return, as measure by the Sharpe ratio defined as:

$$S = \frac{E[R_p] - R_f}{\sigma_p} \tag{10}$$

where:

- $E[R_p]$ is the expected return of the portfolio.
- R_f is the risk-free rate.
- σ_p is the standard deviation (volatility) of the portfolio.

Maximizing this ratio provides an optimal balance between risk and return.

Minimum Volatility Portfolio

The **Minimum Volatility** portfolio aims to minimize portfolio risk/volatility (as measure by the asset's variance).

$$\min_{w} \sigma_p^2 = w^T \Sigma w \tag{11}$$

subject to the linear constraint:

$$\sum_{i=1}^{n} w_i = 1 \tag{12}$$

where Σ is the covariance matrix of asset returns.

The portfolio with the least risk lies at the leftmost point of the Efficient Frontier.

Capital Asset Pricing Model (CAPM)

The CAPM predicts an asset's expected return based on its correlation with the market.

$$E[r_i] = r_f + \beta_i(E[r_m] - r_f) \tag{13}$$

where:

- $E[r_i]$ is the expected return of asset i.
- r_f is the risk-free rate.
- $E[r_m]$ is the market return.
- β_i represents the asset's *sensitivity* to market fluctuations.

CAPM helps evaluate expected returns based on market exposure, informing the investor to base their decisions on knowledge of this market risk factor.

Fama-French Three-Factor Model

The **Fama-French Three-Factor Model** extends CAPM by including **size** and **value** factors. The portfolio's excess returns are modeled as:

$$E([r_i) - r_f = \alpha_i + \beta_{MKT} \cdot MKT + \beta_{SMB} \cdot SMB + \beta_{HML} \cdot HML$$

where:

- SMB is the size factor (*Small Minus Big* market cap).
- HML is the value factor (*High Minus Low* book-to-market ratio).

This model helps to explain asset returns by considering factors beyond just market risk.

Principal Component Analysis (PCA)

Principal Component Analysis (PCA) is a popular dimensionality reduction technique that transforms correlated asset returns into uncorrelated components.

- PCA helps identify the most significant *empirical* risk factors affecting asset returns within the portfolio.
- It reduces the dimensionality of data, keeping the most important variance-explaining components.
- PCA aids in constructing portfolios based on principal components, improving risk and return optimization.

The goal is to find the principal components that account for the most variance in asset returns while reducing the complexity of the portfolio allocation process.

PCA: Covariance Matrix Calculation

PCA starts by calculating the covariance matrix S of asset returns. The covariance matrix captures the relationships between the assets, showing how their returns co-vary over time.
The sample covariance matrix S is computed as:

$$S = \frac{1}{N}\sum_{n=1}^{N}(R_n - \bar{R})(R_n - \bar{R})^T \tag{14}$$

where:

- R_n is the return vector for the n-th observation.
- $\bar{R}$ is the mean return vector.
- N is the number of observations.

This covariance matrix is symmetric and reflects the variance of each asset on the diagonal, and the covariances between pairs of assets off the diagonal.

PCA: Eigenvalue Decomposition

The next step in PCA is performing an **Eigen-Decomposition** of the *sample covariance matrix* S. This process identifies the directions of maximum variance, represented by eigenvectors, and the amount of variance explained by each direction, represented by eigenvalues.
We solve the eigenvalue problem:

$$Su_i = \lambda_i u_i \tag{15}$$

where:

- u_i is the i-th eigenvector.
- λ_i is the corresponding eigenvalue.

The eigenvectors u_i are orthogonal and represent the directions along which the variance is maximized. The eigenvalues λ_i represent the amount of variance explained by each eigenvector.

PCA: Portfolio Weight Calculation from Eigenvectors

In PCA portfolio optimization, the portfolio weights are determined from the eigenvectors corresponding to the principal components. The portfolio weight w_i for asset i is proportional to the i-th component of the first eigenvector u_1 (the direction of maximum variance).
The weight w_i for asset i is given by:

$$w_i = \frac{|u_{1i}|}{\sum_{i=1}^{D} |u_{1i}|} \tag{16}$$

where:

- u_{1i} is the i-th component of the first eigenvector u_1.
- D is the number of assets.

The PCA portfolio is based on the direction of maximum variance, typically the most important *empirical* risk factor in the portfolio.

PCA: Portfolio Construction with Multiple PCs

If the goal is to project the data onto an M-dimensional subspace, where $M < D$, the portfolio weights are constructed by selecting the eigenvectors corresponding to the largest M eigenvalues.

The steps are as follows:

1. Compute the covariance matrix S of the asset returns.
2. Perform eigenvalue decomposition to obtain the eigenvectors $u_1, u_2, \ldots, u_M$ corresponding to the largest eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_M$.
3. Construct the portfolio weights by normalizing the eigenvectors $u_1, u_2, \ldots, u_M$ and combining them based on their respective eigenvalues.

The weights are proportional to the eigenvalues, emphasizing the principal components with larger eigenvalues. This approach produces a portfolio that captures the most important risk factors influencing asset returns.

PCA: Mathematical Representation of Portfolio Weights

The portfolio weights for each asset are determined from the normalized eigenvector components. The weight w_i for asset i is calculated as:

$$w_i = \frac{|u_{1i}|}{\sum_{i=1}^{D} |u_{1i}|} \tag{17}$$

This equation guarantees that the total portfolio weight sums to 1, ensuring proper allocation across assets. The weight is proportional to the component of the first principal component corresponding to asset i.

Hierarchical Risk Parity (HRP)

Hierarchical Risk Parity (HRP) combines hierarchical clustering with risk parity to optimize portfolio construction.

- Step 1: Hierarchical clustering using asset correlations.
- Step 2: Quasi-diagonalization of the covariance matrix.
- Step 3: Recursive bisection to split the portfolio into sub-portfolios.

HRP helps create diversified and robust portfolios that are less sensitive to volatility.

HRP: Hierarchical Asset Clustering

The first step in HRP is hierarchical asset clustering using the correlation matrix to group assets with similar behaviors.

- **Distance Metric:** The distance $d_{i,j}$ between two assets i and j is defined as:
$$d_{i,j} = \sqrt{\frac{1}{2}(1 - \rho_{i,j})} \tag{18}$$
where $\rho_{i,j}$ is the correlation between assets i and j.
- **Clustering:** Using the distance matrix, we perform hierarchical clustering with a single linkage criterion, merging assets or clusters based on the least pairwise distance.
- **Dendrogram:** The clustering process is visualized through a dendrogram, showing the hierarchical relationships among assets.

This step ensures that assets with similar risk profiles are grouped together, improving portfolio diversification.

HRP: Quasi-Diagonalization of Covariance Matrix

After clustering, we quasi-diagonalize the covariance matrix Σ to improve portfolio allocation efficiency by reducing the complexity of the covariance structure.

- **Reordering:** The covariance matrix is reordered using a permutation matrix P to position assets within the same cluster closer to the diagonal:

$$\Sigma_{QD} = P \Sigma P^T \tag{19}$$

  where Σ_{QD} is the quasi-diagonalized covariance matrix.

- **Objective:** This step aims to reduce the covariance values for assets that belong to different clusters, simplifying the optimization process for the subsequent stages.

The quasi-diagonalization improves risk management by reflecting asset relationships in the covariance structure more accurately.

HRP: Recursive Bisection and Inverse-Variance Allocation

The final phase in HRP involves recursive bisection, where the portfolio is split into sub-portfolios, and risk is allocated inversely to the variance.

- **Portfolio Splitting:** The portfolio is split into two sub-portfolios P_1 and P_2, with covariance matrices Σ_1 and Σ_2, respectively.

$$\mathrm{Var}(P_1) = w_1^T \Sigma_1 w_1, \quad \mathrm{Var}(P_2) = w_2^T \Sigma_2 w_2 \tag{20}$$

- **Inverse-Variance Allocation:** Each asset in a sub-portfolio is weighted inversely to its variance, ensuring risk is evenly distributed.

$$w_i^{(j)} = \frac{1}{\mathrm{diag}[\Sigma_j]^{-1}} \cdot \frac{1}{\mathrm{trace}(\mathrm{diag}[\Sigma_j]^{-1})} \tag{21}$$

  where $w_i^{(j)}$ is the weight of asset i in sub-portfolio j, and Σ_j is the covariance matrix of sub-portfolio j.

Recursive bisection continues splitting sub-portfolios at each level to further reduce risk concentration, ensuring optimal diversification at every hierarchical level.

HRP: Total Portfolio Weight Calculation

The total weight w_i of an asset i in the entire portfolio is the weighted sum of its contributions from all recursive bisections.

$$w_i = \sum_{j=1}^{K} w_i^{(j)} \tag{22}$$

where $w_i^{(j)}$ is the weight of asset i in sub-portfolio j, and K is the total number of sub-portfolios.

Reinforcement Learning for Portfolio Optimization

Reinforcement Learning (RL) is applied to portfolio optimization to dynamically adjust asset weights and maximize risk-adjusted returns over time by training **agents** to learn a **policy function** based on **rewards**.

- **Objective:** Learn optimal portfolio allocations by interacting with market data.
- **Goal:** Maximize cumulative rewards, often measured using Sharpe ratio or total return.
- **RL Framework:** The RL model uses a *Deep Q-Learning* architecture to determine portfolio weights based on state-action pairs.

Reinforcement Learning in Portfolio Optimization

Reinforcement Learning (RL) is applied to portfolio optimization to dynamically adjust asset weights and maximize risk-adjusted returns over time by training **agents** to learn a **policy function** based on **rewards** where we define the following components of RL:

- Agent: The portfolio manager or algorithm.
- State: The market condition or financial system.
- Action: The decision on how to allocate assets.
- Reward: The return from the portfolio.
- Environment: The financial market where the agent operates.

RL uses techniques like Q-learning and policy gradients to maximize long-term portfolio performance.

Mathematical Foundations of RL in Portfolio Optimization

The goal of RL is to find the optimal policy π^* that maximizes the expected cumulative reward over time.

- **Total Cumulative Reward**: The total reward G_t from time t is the discounted sum of future rewards:

$$G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \tag{23}$$

  where γ is the discount factor (ranging from 0 to 1) and R_{t+k+1} is the reward at time $t+k+1$.

- **Value Function**: The value function $V(s)$ estimates the long-term reward of being in a given state s:

$$V(s) = \mathbb{E}[G_t | S_t = s] \tag{24}$$

- **Q-value Function**: The Q-value function $Q(s, a)$ evaluates the expected reward from performing action a in state s:

$$Q(s, a) = \mathbb{E}[G_t | S_t = s, A_t = a] \tag{25}$$

RL: Bellman Equation for Q-Learning

The **Bellman Equation** provides a recursive way to compute the value function or Q-value.

- **Value Function** (for value-based RL):

$$V(s) = \mathbb{E}[R_{t+1} + \gamma V(S_{t+1}) | S_t = s]$$

- **Q-value Function** (for Q-learning):

$$Q(s, a) = \mathbb{E}[R_{t+1} + \gamma \max_{a'} Q(S_{t+1}, a') | S_t = s, A_t = a]$$

The goal is to maximize the Q-value for each state-action pair.

RL: Deep Q-Learning (DQN) Update Rule

In Deep Q-Learning, the Q-value function is approximated using a deep neural network.

- The loss function is minimized to update the Q-values:

$$L(\theta) = \mathbb{E}[(r + \gamma \max_{a'} Q(S', a'; \theta^-) - Q(S, a; \theta))^2]$$

  where r is the immediate reward, γ is the discount factor, and θ^- represents the weights of the target network.

This approach ensures stable learning by minimizing the difference between the predicted and target Q-values.

RL: Portfolio Rebalancing and Action Selection

The RL agent dynamically changes the portfolio weights based on state observations, optimizing the portfolio for maximum risk-adjusted return.

- **Action Selection**: Actions (portfolio rebalancing) are selected using an epsilon-greedy strategy, where ϵ decays over time to shift from exploration to exploitation.
- **Portfolio Rebalancing**: The RL model uses the Q-values to determine the best allocation of assets at each time step, adjusting weights as market conditions change.

The RL agent aims to maximize cumulative returns while controlling risk.

Benchmark Portfolios

Our benchmark portfolios are built from the *Magnificent Seven* (MAG7) stocks:

- Amazon (AMZN)
- Google (GOOGL)
- Apple (AAPL)
- Microsoft (MSFT)
- Tesla (TSLA)
- Nvidia (NVDA)
- Meta (META)

Data spans from January 1, 2021, to December 31, 2023, with historical adjusted closing prices retrieved from Yahoo Finance.

- Daily returns were calculated from these prices and annualized to compute expected returns.
- The covariance matrix of returns was calculated to capture the correlation and variance across the assets.

We evaluate three portfolios: **Equal Weights Portfolio, Maximum Sharpe Ratio Portfolio, and Minimum Volatility Portfolio**.

Performance Metrics for Portfolio Weights (2021-2023)

Portfolio	Return	Volatility	Sharpe Ratio
Equal Weights Portfolio	23.49%	32.03%	73.34%
Maximum Sharpe Ratio Portfolio	47.72%	44.38%	107.54%
Minimum Volatility Portfolio	19.67%	26.11%	75.34%

Table: Performance Metrics for Portfolio Weights (2021-2023)

Portfolio Feasibility Space and Efficient Frontier

Monte Carlo (MC) simulations help visualize the **Feasibility Space** and the **Efficient Frontier**.

- The *Efficient Frontier* is the curve of optimal portfolios with the best tradeoff between risk and return.
  - Portfolios below the *Efficient Frontier* are suboptimal.
- The *Feasibiltiy Space* is visualized with random portfolios and optimal portfolios are positioned along the frontier.

This MC simulation helps investors choose a portfolio that best matches their risk tolerance and return objectives.

Efficient Frontier with Portfolio Allocations

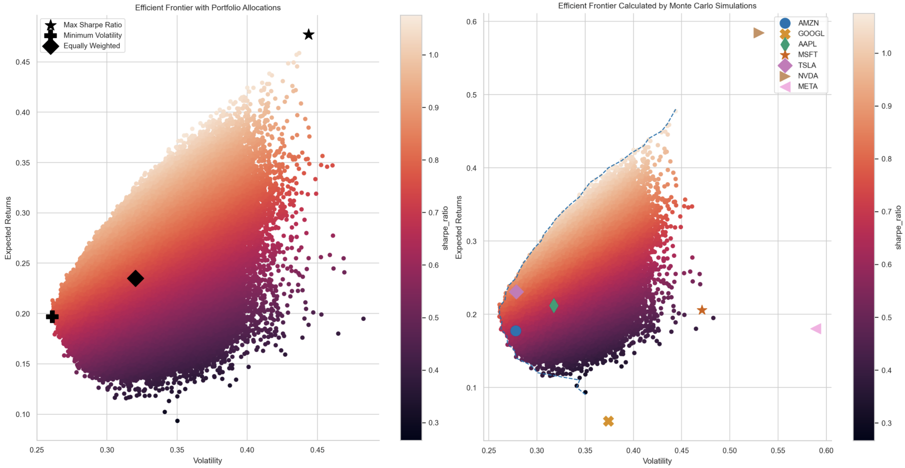

Figure: Efficient Frontier of 3 Portfolios with Portfolio Allocations (Left) and Efficient Frontier of MAG 7 Stocks with Portfolio Allocations (Right)

Risk-Return Tradeoff

- The Efficient Frontier illustrates the best possible tradeoff between risk and return.
- Portfolios below the Efficient Frontier are considered suboptimal, offering lesser returns for the same level of risk.
- The Maximum Sharpe Ratio Portfolio lies at the ideal frontier point, offering the highest return for a given level of risk.
- The Minimum Volatility Portfolio minimizes risk but offers a lower return.
- The Equal Weights Portfolio offers a balanced but less efficient risk-return tradeoff.

Equally Weighted Portfolio

The Equally Weighted Portfolio allocates an equal share of capital to each of the seven assets.

$$w_i = \frac{1}{7}, \quad \forall i \tag{26}$$

The asset allocations for each asset are:

- AMZN: 14.29%
- GOOGL: 14.29%
- AAPL: 14.29%
- MSFT: 14.29%
- TSLA: 14.29%
- NVDA: 14.29%
- META: 14.29%

- Simple to implement, with no optimization required.
- Suitable for investors seeking a balanced, diversified portfolio.
- Does not account for individual asset returns or risk variations.
- May be suboptimal in terms of risk-adjusted performance.

Maximum Sharpe Ratio Portfolio

The Maximum Sharpe Ratio Portfolio maximizes the risk-adjusted return, calculated as:

$$S = \frac{E[R_p] - R_f}{\sigma_p} \tag{27}$$

- Maximizes the Sharpe ratio by optimizing the risk-return tradeoff.
- The portfolio lies at the ideal point on the Efficient Frontier.
- Typically involves higher volatility but provides the best risk-adjusted return.

Maximum Sharpe Ratio Portfolio

The Maximum Sharpe Ratio Portfolio seeks to maximize the risk-adjusted return by investing more in assets that provide the highest return for a given level of risk.

- Maximize the portfolio's expected return while minimizing its risk (volatility).
- Allocate more to assets with higher risk-adjusted returns, like Nvidia.

This ensures that the portfolio achieves the highest return for the least risk.

Portfolio Weights

The Maximum Sharpe Ratio Portfolio allocates weights to assets based on their risk-return profiles. The portfolio weights for the MAG7 stocks are as follows:

- **AMZN**: 0.89%
- **GOOGL**: 2.09%
- **AAPL**: 12.77%
- **MSFT**: 4.27%
- **TSLA**: 6.37%
- **NVDA**: 72.04%
- **META**: 1.56%

Minimum Volatility Portfolio

The Minimum Volatility Portfolio minimizes risk (volatility) by solving:

$$\text{Minimize } \sigma_p^2 = w^T \Sigma w \tag{28}$$

Where Σ is the covariance matrix of asset returns. The constraint is:

$$\sum_{i=1}^{n} w_i = 1 \tag{29}$$

- Focuses on reducing volatility, making it suitable for conservative investors.
- Results in a portfolio with lower return but significantly lower risk.
- Positioned toward the lower-left side of the Efficient Frontier.

Minimum Volatility Portfolio

The Minimum Volatility Portfolio aims to reduce the portfolio's overall risk by investing in less volatile assets.

- The strategy sacrifices returns in order to minimize portfolio risk.
- It is suitable for risk-averse investors willing to forgo some return to lower risk.
- The portfolio aims to achieve a favorable risk-return tradeoff by prioritizing less volatile assets.

The portfolio weights are allocated based on asset volatility and their contribution to overall risk.

Portfolio Weights for Minimum Volatility

The portfolio weights for the Minimum Volatility Portfolio are as follows:

- **AMZN**: 41.50%
- **GOOGL**: 5.45%
- **AAPL**: 16.72%
- **MSFT**: 3.04%
- **TSLA**: 32.18%
- **NVDA**: 0.53%
- **META**: 0.58%

Risk-Return Tradeoff in Minimum Volatility Strategy

The Minimum Volatility Portfolio sacrifices higher returns in favor of reducing overall risk. The allocation is heavily weighted toward lower-risk assets like Amazon (AMZN) and Apple (AAPL).

- The portfolio minimizes exposure to high-volatility assets like Nvidia (NVDA) and Tesla (TSLA).
- By reducing volatility, the strategy provides more stable but slower growth compared to riskier portfolios.
- Suitable for risk-averse investors looking for stability, even at the expense of higher returns.

This approach aims for consistent, lower-risk growth, making it ideal for conservative investors.

Forward Testing Results

The portfolios were forward-tested using market data from 2024, applying the weights derived from historical data.

- The performance of each portfolio was tracked from January to December 2024.
- The graph below shows the cumulative returns of the 3 Portfolios

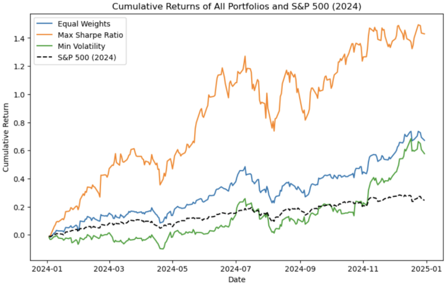

Figure: Portfolios' Cumulative Returns in 2024 (Naive, Max Sharpe, Min Volatility)

Portfolio Performance in 2024

The cumulative return for each portfolio in 2024 was:

- **Equal Weights Portfolio Value**: $16,717.46
- **Portfolio Value with Maximum Sharpe Ratio**: $24,028.55
- **Value of Minimum Volatility Portfolio**: $13,349.75

Discrete Allocation Simulation

The real-world asset purchases were simulated using the Discrete Allocation approach.

- **Naive Portfolio**: The portfolio is equally split across the seven assets, with a small cash reserve ($14.96).
- **Maximum Sharpe Ratio Portfolio**: Allocated 52 shares to Nvidia (NVDA), with $26.12 remaining cash.
- **Minimum Volatility Portfolio**: Allocated 17 shares to Amazon (AMZN) and 8 shares to Tesla (TSLA), with $26.21 remaining cash.

Multi-Factor Models

Multi-factor models explain the variation in asset returns by considering multiple factors beyond the market return alone.

- The Capital Asset Pricing Model (CAPM) uses market risk as the sole factor to explain returns.
- The Fama-French 3-Factor Model expands on CAPM by adding two additional factors: size (SMB) and value (HML).

In this section, we explore the CAPM and Fama-French 3-Factor Model regressions for three portfolio strategies:

- Maximum Sharpe Ratio
- Minimum Volatility
- Equally Weighted

CAPM Results for Three Portfolios

The CAPM regression for the portfolios shows how the market factor drives portfolio returns.

Portfolio	Beta	R-squared	P-value	Alpha
Maximum Sharpe Ratio	1.7494	0.391	0.000	0.0012
Minimum Volatility	1.4749	0.482	0.000	-0.0024
Equally Weighted	1.4238	0.614	0.000	0.0072

Table: CAPM Results for the Three Portfolios

CAPM Regression Insights

- **Maximum Sharpe Ratio Portfolio**:
  - Has the highest beta (1.7494), indicating high sensitivity to market fluctuations.
  - Alpha (0.0012) is not statistically significant, suggesting no abnormal returns after adjusting for market risk.
- **Minimum Volatility Portfolio**:
  - Beta of 1.4749 shows some market exposure but is more stable than the Maximum Sharpe Ratio Portfolio.
  - Alpha (-0.0024) is negative, suggesting the portfolio did not outperform the market.
- **Equally Weighted Portfolio**:
  - Beta of 1.4238 shows it closely tracks the market.
  - Alpha (0.0072) is positive but not statistically relevant, indicating no significant outperformance.

The CAPM results show that market risk is the dominant factor influencing portfolio returns, with no significant alpha for any of the portfolios.

Fama-French 3-Factor Model Regression Results

The Fama-French 3-Factor Model adds the size (SMB) and value (HML) factors to the market (MKT) factor in explaining portfolio returns.

Portfolio	Intercept	MKT	SMB	HML	R-squared
Maximum Sharpe Ratio	0.0204	1.3216	-0.5790	-0.8752	0.527
Minimum Volatility	0.0111	1.1196	-0.5368	-0.6194	0.741
Equally Weighted	0.0177	1.3427	-0.1244	-0.8263	0.820

Table: Fama-French 3-Factor Model Regression Results

Fama-French 3-Factor Model Insights

- **Maximum Sharpe Ratio Portfolio**:
  - R-squared = 0.527, indicating that the model explains 52.7% of the portfolio's returns variation.
  - The market factor is statistically significant (p-value < 0.0001).
  - The negative coefficients for SMB (-0.5790) and HML (-0.8752) suggest that small-cap and value stocks hurt the portfolio's performance.
- **Minimum Volatility Portfolio**:
  - R-squared = 0.741, showing that 74.1% of the variation in the portfolio's returns is explained by the factors.
  - The market factor is also significant, with a positive correlation of 1.1196.
  - The negative coefficients for SMB (-0.5368) and HML (-0.6194) indicate that small and value stocks negatively impacted performance.
- **Equally Weighted Portfolio**:
  - R-squared = 0.820, indicating that the market and the additional factors explain 82% of the portfolio's return variation.
  - The portfolio's performance is driven by the market factor (MKT =

Conclusion

- The CAPM and Fama-French 3-Factor Model show that market movements drive the returns of all three portfolios.
- None of the portfolios produced significant alpha, indicating that the market largely explained their returns and the factors considered.
- The Maximum Sharpe Ratio Portfolio is the most volatile, but the market heavily influences it.
- The Minimum Volatility and Equally Weighted Portfolios also track the market but with different sensitivity levels.
- Further Exploration of factors like size and value reveals their impact on portfolio performance, with the portfolios underperforming when small and value stocks dominate.

These findings suggest that while the market heavily influences the portfolios, strategies incorporating additional risk factors might improve performance in the future.

PCA Portfolio: Introduction

Principal Component Analysis (PCA) is a dimensionality reduction technique that seeks to reduce the factors influencing asset returns while retaining the key components of variability.

- PCA focuses on capturing the most influential factors contributing to stock returns.
- The portfolio aims to allocate more capital to assets responsible for the greatest variance in the data.
- By identifying the primary components that explain market movements, PCA simplifies the portfolio allocation process.

The PCA-optimized portfolio for the MAG7 stocks shows low volatility and a balanced approach to risk management while attempting to capture critical market dynamics.

PCA Portfolio Volatility and Sharpe Ratio

The PCA portfolio's volatility and Sharpe ratio were computed as follows:

- **Volatility**: 2.79% (relatively low compared to other portfolios like Max Sharpe Ratio Portfolio (44.38%) and Minimum Volatility Portfolio (26.11%)).
- **Sharpe Ratio**: 0.04, which is lower than the Maximum Sharpe Ratio Portfolio's (1.08) but still indicates a return adjusted for risk.
- Although the PCA portfolio's Sharpe ratio is lower, it offers a simpler, effective portfolio optimization strategy.
- It provides reasonable returns while keeping volatility low, making it suitable for conservative investors.

PCA Portfolio Weights

The PCA model decomposes the covariance matrix of asset returns, assigning weights based on the most significant principal components.

- **AAPL**: 6.69% (Stable, high-cap stock with moderate volatility)
- **AMZN**: 5.99% (Less volatile than TSLA and NVDA, with less impact on the first principal component)
- **GOOGL**: 2.44% (Lower volatility, resulting in a smaller weight)
- **META**: 2.68% (Lower volatility, smaller weight compared to more volatile stocks)
- **MSFT**: 3.67% (Similar volatility to AAPL, smaller weight in portfolio)
- **NVDA**: 26.06% (High volatility and significant market influence)
- **TSLA**: 52.45% (Dominates the portfolio due to high volatility and market influence)
- TSLA and NVDA are the most heavily weighted due to their strong variance and market movement correlation.
- AAPL, AMZN, and other lower volatility stocks received smaller allocations.

Forward Testing Results

The PCA-optimized portfolio was forward-tested using actual market data from 2024.

- The cumulative returns of the PCA portfolio showed strong performance in the second half of the year, reflecting the significant influence of TSLA and NVDA.

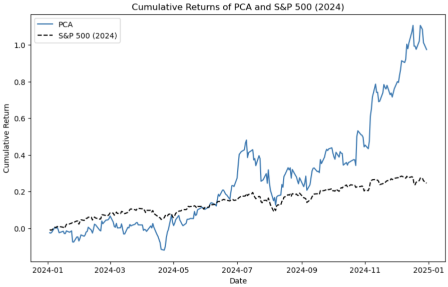

Figure: Portfolios' Cumulative Returns in 2024 (PCA)

PCA Portfolio Performance in 2024

The PCA-optimized portfolio achieved notable performance, ending 2024 with the following results:

- **Portfolio Value at Year-End**: \$19,750.25 (97.5% growth from the initial investment of \$10,000)
- The PCA portfolio showed consistent growth, driven by the significant performance of TSLA and NVDA.

Discrete Allocation Simulation

To simulate real-world portfolio construction, the PCA portfolio was subjected to the Discrete Allocation approach, ensuring efficient use of capital.

- **Shares Allocated**:
  - AAPL: 3 shares
  - AMZN: 3 shares
  - GOOGL: 1 share
  - MSFT: 1 share
  - NVDA: 18 shares
  - TSLA: 13 shares
- **Leftover Cash**: $64.41 (minor, indicating effective capital utilization)
- The simulation reveals that PCA provides a highly efficient portfolio, allocating funds across the assets with minimal leftover cash.
- The discrete allocation approach closely mirrors the portfolio's optimization while considering transaction limits and fractional shares.

PCA Conclusion

- The PCA portfolio provides a balanced risk-return profile with relatively low volatility compared to other portfolios.
- The volatility and market influence of TSLA and NVDA drive the portfolio's success.
- Despite its lower Sharpe ratio, the PCA portfolio offers steady returns with reduced risk, making it suitable for conservative investors.
- The discrete allocation results show that the PCA-optimized portfolio is well-suited for real-world implementation, with effective capital utilization.

The PCA model demonstrates its value in portfolio optimization by simplifying the allocation process while maintaining a diversified and lower-risk portfolio. Though not achieving the highest returns, it offers a steady, more predictable growth path.

Hierarchical Risk Parity (HRP) Portfolio Optimization

Portfolio optimization aims to balance returns and minimize risk. Traditional methods like MVO often struggle with large, complex datasets, leading to issues like overfitting and inadequate diversification.

- HRP optimizes portfolios by focusing on diversification and risk management.
- HRP uses hierarchical clustering to group assets by their risk profiles.
- Risk parity ensures risk is distributed evenly across assets, enhancing risk-adjusted returns.
- HRP improves resilience and diversification, making it a more effective solution than traditional methods.

HRP Portfolio Weights for DJ30 Stocks

The HRP model optimizes portfolio weights for the Dow Jones Industrial Average (DJ30) stocks. Below are the calculated optimal weights:

- **AAPL**: 0.01799
- **AMGN**: 0.04726
- **AXP**: 0.01334
- **BA**: 0.01851
- **CAT**: 0.01604
- **CRM**: 0.01787
- **CSCO**: 0.03526
- **CVX**: 0.02496
- **DIS**: 0.02065
- **DOW**: 0.01631
- **GS**: 0.01645
- **JNJ**: 0.06650
- **JPM**: 0.01934
- **KO**: 0.05521
- **MCD**: 0.06407
- **MMM**: 0.02088
- **MRK**: 0.04697
- **MSFT**: 0.01794
- **NKE**: 0.01620
- **PG**: 0.04763
- **TRV**: 0.04963
- **UNH**: 0.04597

Portfolio Performance Metrics

- **Annual Volatility:** 13.1%
- **Sharpe Ratio:** 0.73

Risk-Managed Allocation in HRP Portfolio

The pie chart below shows the distribution of the HRP portfolio, with larger allocations given to lower-volatility stocks and smaller allocations to higher-volatility ones.

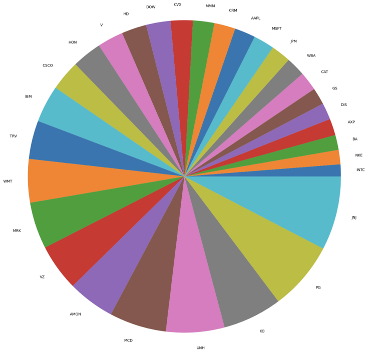

Figure: Pie Chart of Weight Distributions for HRP

High-Weighed Stocks (Weight > 0.05)

Stocks with lower volatility and weaker correlations with other assets are given higher weights in the HRP portfolio. These stocks are:

- **VZ** (Verizon Communications), **JNJ** (Johnson & Johnson), **MCD** (McDonald's), **KO** (Coca-Cola), **WMT** (Walmart)
- These companies provide steady returns with lower risk profiles.
- Their stability and lower correlation with other assets make them ideal candidates for larger portfolio weights.

These high-weighted stocks help to balance the portfolio and reduce overall volatility, ensuring a more secure return stream.

Mid-Weighed Stocks ($0.03 < \text{Weight} \leq 0.05$)

Stocks like **TRV** (Travelers Companies), **PG** (Procter & Gamble), **IBM** (International Business Machines), **MRK** (Merck & Co.), **AMGN** (Amgen), **UNH** (UnitedHealth Group), and **CSCO** (Cisco Systems) receive moderate weights. These stocks:

- Exhibit moderate volatility and stronger correlations with the broader market.
- Provide diversification across various sectors but require balancing to avoid overexposure to riskier assets.

HRP assigns these stocks moderate weights to balance risk and return while ensuring diversification across sectors.

Low-Weighed Stocks (Weight ≤ 0.03)

Stocks such as **CVX** (Chevron), **HD** (Home Depot), **HON** (Honeywell International), **WBA** (Walgreens Boots Alliance), **AAPL** (Apple), **AXP** (American Express), **BA** (Boeing), **CAT** (Caterpillar), **CRM** (Salesforce), **DIS** (Walt Disney), **DOW** (Dow Chemical), **GS** (Goldman Sachs), **INTC** (Intel), **JPM** (JPMorgan Chase), **MMM** (3M), **NKE** (Nike), **MSFT** (Microsoft), and **V** (Visa) receive lower weights due to:

- Higher volatility and stronger correlations with market or sector trends.
- These stocks contribute diversity to the portfolio and increase overall risk.
- By assigning them lower weights, HRP minimizes their impact on portfolio volatility.

This approach ensures a more balanced risk-return tradeoff by reducing the exposure to higher volatility assets.

Dendrogram of DJ30 Stocks

The dendrogram visualizes how the DJ30 stocks are grouped based on their correlations.

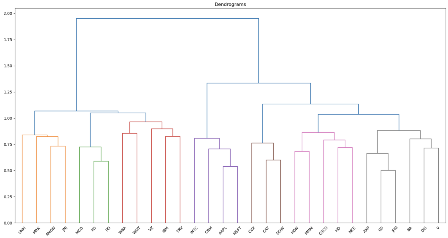

Figure: Dendrogram of DJ30 Stocks

- Shorter branches indicate stronger correlations, while longer branches indicate weaker correlations.
- The HRP approach ensures no group becomes overexposed by balancing allocations across these clusters.

Correlation Heatmap of DJ30 Stocks

The heatmap represents the correlation coefficients between pairs of stocks.

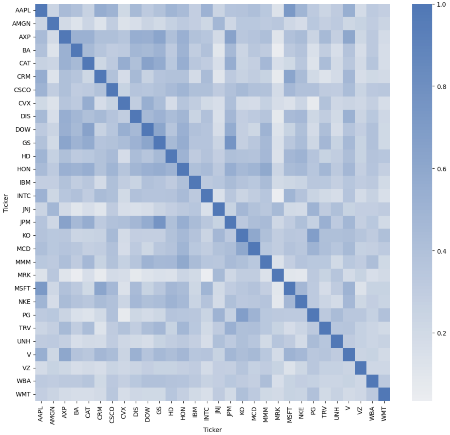

Figure: Heatmap of DJ30 Stock Correlations

Heatmap Dendrogram

The heatmap dendrogram further illustrates how stocks are grouped based on correlation.

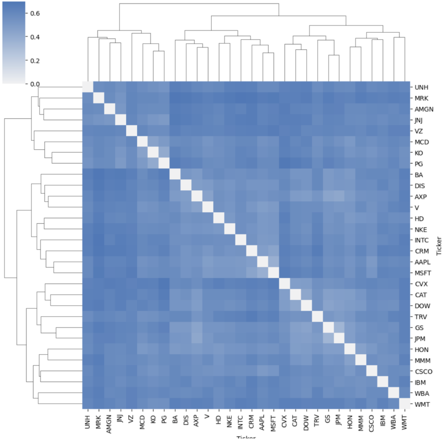

Figure: Heatmap Dendrogram

Bar Chart of HRP Portfolio Weights

The bar chart below shows the distribution of weights for the DJ30 stocks in the HRP portfolio.

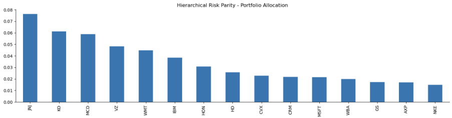

Figure: Bar Chart of Weights for DJ30 Stocks

- Stocks like **JNJ**, **KO**, and **MCD** receive larger allocations due to their low volatility.
- Stocks such as **GS**, **AXP**, and **NKE** receive smaller allocations due to their higher risk.
- The HRP approach ensures that risk is evenly distributed, improving diversification and stability.

HRP Portfolio Cumulative Returns

The HRP-optimized portfolio's cumulative returns over 2024 show steady growth.

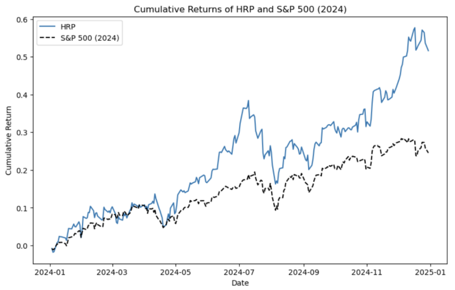

Figure: 2024 Cumulative Returns for HRP Portfolio

Portfolio Allocation and Results (2024)

- **HRP Portfolio Value at the end of 2024 (Cumulative Return):** \$11,010.16
- **Discrete Allocation:**
  - AAPL: 1, AMGN: 2, BA: 1, CRM: 1, CSCO: 6, CVX: 2, DIS: 2, DOW: 5
  - HD: 1, HON: 1, IBM: 2, INTC: 8, JNJ: 5, JPM: 1, KO: 9, MCD: 2
  - MMM: 2, MRK: 5, NKE: 2, PG: 3, TRV: 2, UNH: 1, V: 1, VZ: 20
  - WBA: 31, WMT: 7
- **Leftover Cash:** \$12.22

HRP Conclusion

- The HRP portfolio optimization method effectively balances risk and return by focusing on diversification.
- By clustering assets based on correlations, HRP reduces risk concentration and improves the portfolio's stability.
- The portfolio's steady growth and efficient risk management performance demonstrate HRP's potential for portfolio optimization.
- The HRP approach is well-suited for investors seeking a diversified and balanced portfolio with controlled risk.

HRP offers a sophisticated method to optimize portfolios, improving resilience and capturing growth while minimizing risk.

Reinforcement Learning for Portfolio Optimization

We apply RL with an actor-critic architecture to optimize portfolio allocations for the MAG 7 stocks.

- **Objective:** Learn dynamic, risk-adjusted portfolio allocations by interacting with the market environment.
- **Actor Network:** Proposes portfolio weights (actions).
- **Critic Network:** Evaluates the value of state-action pairs to guide learning.
- The model adapts to market changes and captures nonlinear asset relationships.

Data Preparation and Rolling Window Dataset Usage

- Historical adjusted closing prices for MAG 7 stocks are converted to daily returns.
- Data is structured using a **rolling window** approach:
  - Each RL state includes L days of historical returns for all assets.
  - Allows the agent to learn from temporal patterns and evolving trends.
- Returns are used for both input features and reward calculation.

State Construction and Feature Engineering

- Each state is a matrix of asset returns over a lookback window (e.g., 50 days).
- Optionally includes indicators like momentum or volatility.
- The state matrix (assets $\times$ time) is flattened before passing into the neural network.

Model Architecture and Parameter Selection

- Actor and Critic networks use dense layers with:
  - **ELU activation**, **L2 regularization**, and **dropout**.
- **Actor output:** portfolio weights via tanh (supports long/short positions).
- **Critic output:** scalar value estimate via linear activation.
- Key hyperparameters:
  - Lookback window size, learning rate, γ, batch size, memory size, ϵ decay.

Training Process and Policy Optimization

- At each step:
  - Agent observes state and selects action (portfolio weights).
  - Environment returns reward based on weighted asset returns.
- Both Actor and Critic updated using mean squared error (MSE) loss.
- Optimized with the Adam optimizer for stable convergence.

Action Mapping and Portfolio Constraints

- Model scores are mapped to actions: *buy*, *sell*, or *hold*.
- Long-only strategy: negative weights are shifted to zero.
- Long-short strategy: weights may be negative.
- All weight vectors are normalized to sum to 1, preserving capital constraints.

Exploration vs. Exploitation and ϵ-Greedy Policy

- The agent balances:
  - **Exploration :** discovering new strategies through random actions.
  - **Exploitation:** leveraging the learned policy to maximize returns.
- An ϵ-greedy policy governs action selection.
- The exploration rate ϵ decays over time to favor exploitation in later training stages.

Experience Replay for Stable Learning

- Experience replay stores transitions: $(s, a, r, s', \texttt{done})$ in a memory buffer.
- During training:
  - Mini-batches are sampled randomly to break temporal correlation.
  - Target Q-values are calculated using observed and predicted rewards.
- This stabilizes learning and improves sample efficiency in volatile markets.

Evaluation and Deployment

- After training, the agent is tested on out-of-sample data.
- Portfolio performance is assessed using:
  - **Cumulative return**, **Sharpe ratio**, **drawdown**, and volatility.
- The workflow: data processing $\rightarrow$ state construction $\rightarrow$ training $\rightarrow$ evaluation.

Key Parameter Selection

- **Lookback window:** Number of previous days included in each state.
- **Window size:** Length of the rolling window for extended simulations.
- **Episode count:** Number of passes through the training dataset.
- **Batch size:** Number of samples used in each Q-learning update.
- **Rebalance period:** Frequency of portfolio allocation updates (typically every 30 trading days).

Performance Evaluation: RL vs Equal Weights

We evaluate the RL agent's performance using various financial metrics: volatility, Sharpe ratio, alpha, and beta.

- **Volatility:** The RL agent has lower volatility (0.0109) compared to the Equal Weights Portfolio (0.0211).
- **Sharpe Ratio:** The RL portfolio outperforms with a Sharpe ratio of 0.7148 compared to 0.5841 for the Equal Weights Portfolio.
- **Alpha:** The RL agent has a positive alpha (0.0002), indicating slight outperformance.
- **Beta:** The RL portfolio has a beta of 0.3937, indicating lower market sensitivity.

The RL model excels in risk-adjusted returns and volatility control.

RL Agent-Generated Portfolio Weights

The Reinforcement Learning (RL) agent produced portfolio weights for the MAG 7 stocks based on a policy acquired from previous interactions with the market environment. The agent assign long (positive) and short (negative) positions, with the constraint that all weights sum to 1. The generated action weights for the MAG 7 are:

$$\text{Action Weights} = [0.8204, -0.0308, -0.3362, 0.1531, 0.0114, 0.0173, 0.3647]$$

Interpretation of RL Portfolio Weights (1/2)

- **AMZN (0.8204)**: A significant positive weight suggests strong preference, likely due to Amazon's projected high return and relatively lower risk. Amazon is a major contributor to the portfolio's expected return.
- **GOOGL (-0.0308)**: A small negative weight indicates a mild short position, implying that the agent anticipates lower returns or higher volatility from Google than other assets.
- **AAPL (-0.3362)**: The agent assigns a substantial short position to Apple, suggesting lower risk-adjusted returns or a conservative forecast based on historical patterns.
- **MSFT (0.1531)**: A moderate long position reflects the agent's balanced outlook on Microsoft—reasonable return potential with medium risk.

Interpretation of RL Portfolio Weights (2/2)

- **TSLA (0.0114)**: A small allocation to Tesla implies a cautious approach toward its high volatility. Despite its growth potential, its erratic price movements limit its portfolio weight.
- **NVDA (0.0173)**: Although the allocation is low, Nvidia remains in the portfolio, likely due to its role in tech sector momentum or AI-related trends.
- **META (0.3647)**: A high weight indicates strong confidence in Meta's performance, possibly due to favorable risk-return characteristics or anticipated upside.

Cumulative Returns: RL Portfolio vs S&P 500 (2024)

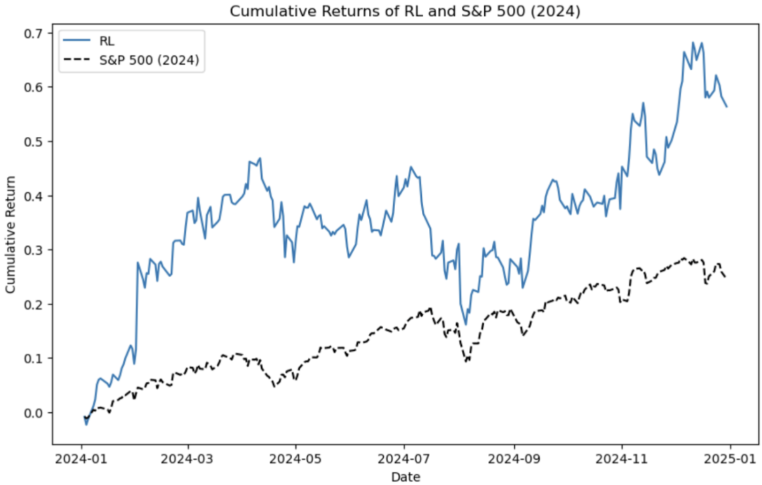

Figure: Cumulative Returns of RL and S&P 500 (2024)

RL Portfolio Value and Discrete Allocation (End of 2024)

Portfolio Value Outcomes:

- **RL Portfolio Value:** $15,633.64

Final Discrete Allocation:

- **AMZN:** 27 shares
- **MSFT:** 2 shares
- **NVDA:** 1 share
- **META:** 5 shares
- **GOOGL:** –2 shares (short)
- **AAPL:** –13 shares (short)
- **Leftover cash:** $98.85

Interpretation:

- The agent concentrated capital in high-conviction positions (AMZN, META).
- Modest long allocations in MSFT and NVDA reflect measured confidence.

Summary of Portfolio Strategy

- The RL agent shows a clear bias toward high-volatility, growth-oriented stocks such as Amazon and Meta.
- Conservative or negative positions are taken in more stable, large-cap stocks like Apple and Google.
- This reflects a strategy prioritizing potential upside while minimizing exposure to stocks perceived as underperforming or risk-heavy.
- The learned policy demonstrates the agent's ability to balance risk and return dynamically through long and short allocations.

Cumulative Return: RL vs Equal Weights

The RL portfolio does not outperform the Equal Weights Portfolio in cumulative.

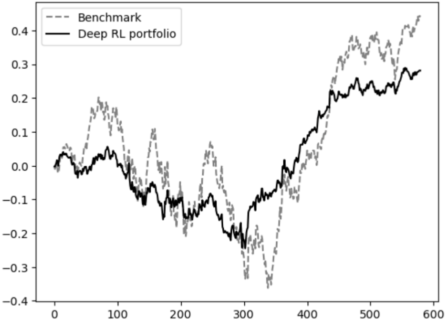

Figure: Cumulative Returns RL Agent vs. Equal Weights

The RL portfolio adapts to market conditions, showing sharper growth and better risk management.

RL Conclusion

- The RL model effectively learns optimal portfolio allocations and adapts to market conditions.
- By dynamically adjusting portfolio weights, the RL model has a comparable performance to the Equal Weights Portfolio in terms of risk-adjusted returns.
- Reinforcement Learning offers a powerful tool for portfolio optimization, providing superior performance in volatile markets.

The RL-based portfolio optimization strategy provides better returns and risk management than traditional static allocation methods.

Conclusion: Evolution of Portfolio Optimization

- Portfolio optimization has evolved from foundational theories like Markowitz's Modern Portfolio Theory (MPT) to advanced AI and machine learning approaches.
- Traditional techniques like Equal Weights, Maximum Sharpe Ratio, and Minimum Volatility portfolios provide different ways of balancing risk and return.
- Multi-factor models, such as CAPM and Fama-French, offer a more in-depth analysis of systematic risks influencing portfolio performance.

Conclusion: Contemporary and AI-Based Techniques

- Advanced methods such as Principal Component Analysis (PCA) and Hierarchical Risk Parity (HRP) enable effective risk diversification and dimensionality reduction.
- Reinforcement Learning (RL) offers dynamic, data-driven optimization, enabling real-time adjustment to market conditions and enhanced adaptability in volatile environments.
- These strategies indicate great promise for resilient, varied, and flexible portfolio building.

Analysis of Cumulative Returns (2024): Graph

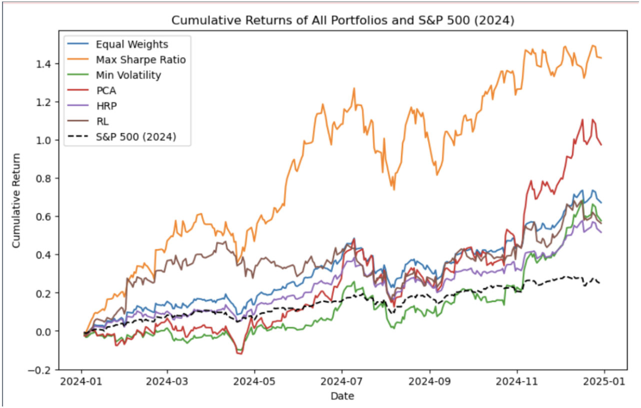

Figure: Cumulative Returns of All Portfolios and S&P 500 (2024)

Analysis of Cumulative Returns (2024)

- **Max Sharpe Ratio portfolio** achieved the highest cumulative return but with elevated volatility and drawdowns, highlighting the risk-return trade-off.
- Showcasing the promise of advanced, adaptive optimization methods, **PCA and RL portfolios** produced strong returns, often exceeding the S&P 500.
- Ideal for risk-averse investors, **Min Volatility and HRP** portfolios showed consistent growth and fewer drawdowns.
- **Equal Weights portfolio** portfolio provided a comparable performance and volatility, acting as a reasonable baseline.
- Most optimized portfolios outperformed S&P 500, highlighting the advantage of dynamic and systematic portfolio strategies.

Outlook and Final Remarks

- Classical optimization models remain valuable foundations, but integrating AI and ML techniques brings new flexibility and robustness to portfolio management.
- Combining economic theory with modern computational tools enables creating resilient, efficient, and adaptive portfolios.
- Ongoing advancements in AI/ML will continue to shape the future of financial market analysis and portfolio optimization.

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