Relativistic Energy

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Abstract

In this experiment, the kinetic energy and momentum of electrons emitted by Strontium-90 is computed with the centroids of histogram peaks, by measuring the amplitude of light pulses proportional to the kinetic energy of the electron, and the diameters of the electron paths through a magnetic field. This experimental data is more consistent with the relativistic expressions for kinetic energy and momentum than with the Newtonian expressions for kinetic energy and momentum, thereby showing the relativistic expressions are correct for masses with speeds approaching the speed of light.

Introduction

In Newtonian mechanics, a particle of mass *m* and speed *v* has kinetic energy *K* given in terms of its momentum *p:*

$$K = \frac{mv^2}{2} = \frac{p^2}{2m}$$

Equation 1.

According to Einstein's special relativity, however, Eq. (1) is an approximation accurate in the limit $v << c$. The correct formula for *K* for any particle speed is:

$$K = E - mc^2 \text{ with } E^2 = (mc^2)^2 + (pc)^2$$

Equation 2.

In this equation, *c* is the speed of light, *mc^2* is the rest energy of the particle, and *E* is the total particle energy. In this lab, *K* will be measured as a function of *p* for the electrons (beta rays) emitted by the radioactive decay of Strontium-90, which have speeds up to 0.98*c*. The momentum of these electrons can be computed with the magnetic field *B* and a path radius *r*:

$$r = \frac{p}{eB}$$

Equation 3.

Apparatus

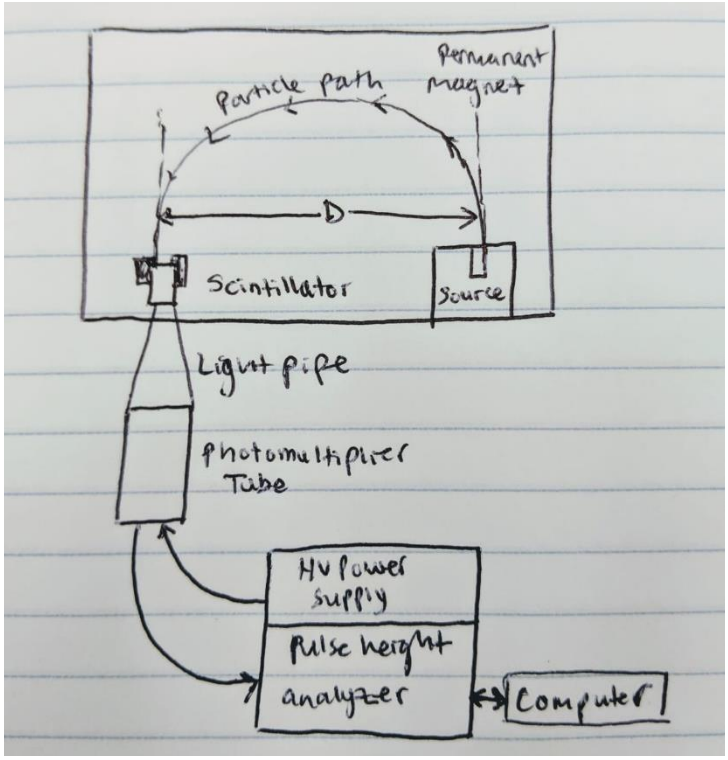

Figure 1. Schematic of the apparatus (top view).

The apparatus components consist of a source of relativistic electrons, a magnetic field, a scintillation detector and pulse height analysis.

Two sources will be used for the relativistic electrons. The first is Bismuth-207 to calibrate the energy measurements with its well-defined kinetic energy $K = 0.975$ MeV, the second is the Strontium-90 source which emits a wide range of energies.

The momentum p is measured using a magnetic field which bends the electron path with radius by Eq. (3). In the diagram, $r = D/2$ away from the source, such that p can be varied by changing D.

The electrons are detected by a small scintillator which converts some K into a flash of light which is guided by the plastic "light pipe" to a photomultiplier tube, requiring a voltage of 1100 volts, which converts the light into an electrical pulse. Therefore, the output of the photomultiplier is an electrical pulse with amplitude proportional to K of the detected electron. Finally, the electrical pulses are fed to a pulse height analyzer, which measures the height of the pulses, creating a histogram viewable on the connected computer. If all electrons have similar K, there will be a peak in the histogram.

Procedure:

First, the Bismuth-207 source is used to calibrate the kinetic energy measurement, since it has a well-defined kinetic energy of 0.975 MeV. The source is placed facing the magnet, and the detector is positioned at the location D = 7.5 cm, which is calculated in the section below, measured from the center of the source to the center of the detector. Lead is placed next the detector to prevent gamma rays from reaching the detector. The background spectrum is first measured (5-minute duration) by placing a thick piece of Aluminum blocking the hole on the source, to absorb all beta rays. The aluminum is then removed to measure the spectrum. From this spectrum, the background spectrum is subtracted to observe the 0.975 MeV beta peak. The ROI function is used to estimate the channel number for the 0.975 MeV peak and its uncertainty by noting the approximate difference between three measurements of the channel centroid.

Next, the Strontium-90 source is used to measure the kinetic energy as a function of momentum. The detector is placed near the middle of the magnet, and the source is blocked with a thick piece of Aluminum and the background spectrum is measured (3-minute duration). 5 values of D (7.3-11.3 cm) over the entire usable range of D are selected for the spectrum measurements without the Aluminum block. The background spectrum is subtracted from each of these measurements to observe the peaks. The ROI function is used to estimate the channel number for the peaks and their uncertainties by noting the approximate difference between three measurements of the channel centroid.

Preliminary calculations:

1. Momentum calculations

Relativistic:

$$p = \frac{1}{c}\sqrt{E^2 - (mc^2)^2}$$

$$E = k + mc^2 = 0.975MeV + 0.511MeV = 1.486\ MeV$$

$$p = \frac{1}{c}\sqrt{(1.486MeV)^2 - (0.511MeV)^2} = 1.395MeV/c$$

The value D that corresponds to this momentum is 7.5 cm and was found by the Detector Separation vs. Momentum D-avg line on the graph provided in the experimental handout. This is the distance used for the Bismuth-207 source.

Non-Relativistic:

$$p = \sqrt{2m_eK} = \sqrt{2 * 0.511\left(\frac{MeV}{c^2}\right) * 0.975MeV} = 0.9982\ MeV/c$$

2. By the right-hand rule, the direction of the magnetic field is down since the charge is negative and moving counterclockwise.
3. Strength of the magnetic field calculations (relativistic)

$$r = \frac{p}{eB} = \frac{D}{2}$$

$$\frac{2p}{eD} = B = 2 * \frac{(1.395 * 10^6)\left(\frac{eV}{c}\right)}{1eV * (7.5 * 10^{-2})m} = 0.124\ Tesla$$

4. **Theory Curves**

For a range of momentum values, Eq. (1) and Eq. (2) were used to calculate the Newtonian prediction the relativistic prediction of *K*, respectively, then, plotted in the figure below.

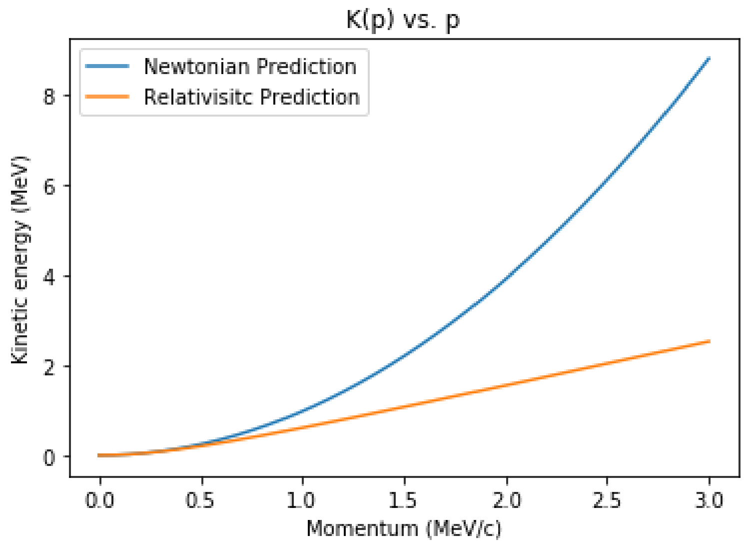

Figure 2. Kinetic energy theory curves.

Preliminary Data

On the oscilloscope, it was observed that the heights of the pulses are smaller when the detector is near the source, and larger when it is far from the source. Also, it was observed that the rate of pulses decreases for large separations, and that it decreases significantly at the largest separation. A sketch of a pulse on the oscilloscope is below, with the horizontal scale by 10ns and the vertical scale by 10mV.

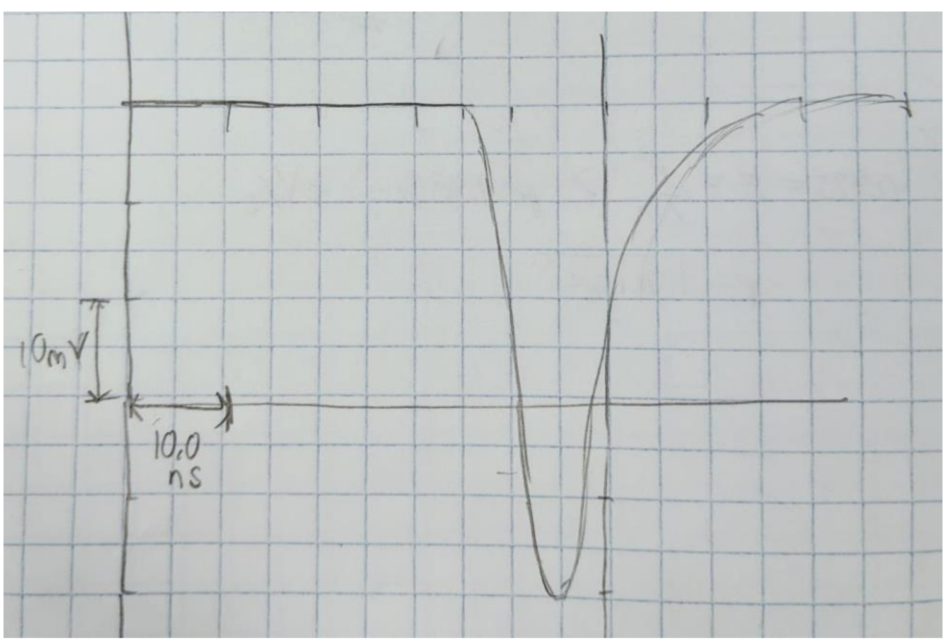

Figure 3. Pulse on oscilloscope.

Next, the kinetic energy peak on the pulse height analyzer was observed with the Strontium-90 source. The scintillator was placed about D = 8 cm from the source. The histogram of the background subtracted data was very scattered with a higher centroid than expected (~360 channels) and showed that the overall experiment was not working. The setup was checked and did not seem to have issues. Because of this, data was provided for this experiment.

Bismuth-207 Data

7.5 cm

	Channel Range	Channel Centroid
1	(125,245)	184
2	(150,220)	183
3	(170,200)	184

Table 1.

Channel Centroid Average: 183 +/- 2 Channels

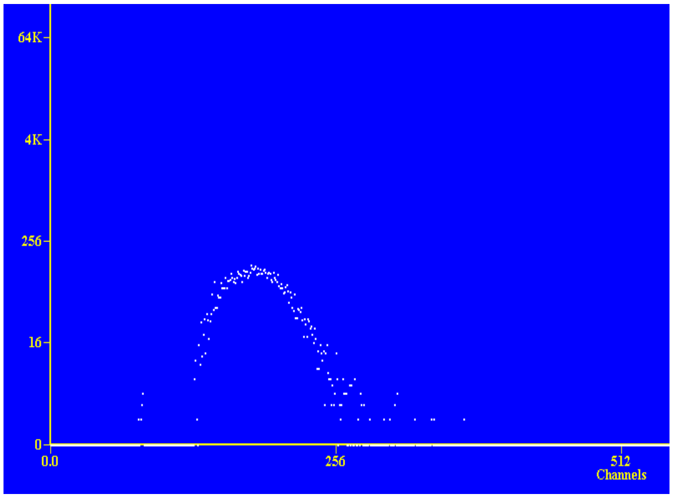

Figure 4. Peak for D = 7.5 cm.

Strontium-90 Data

7.3 cm

	Channel Range	Channel Centroid
1	(140,220)	178
2	(150,210)	178
3	(160,200)	179

Table 2.

Channel Centroid Average: 178 +/- 2 Channels

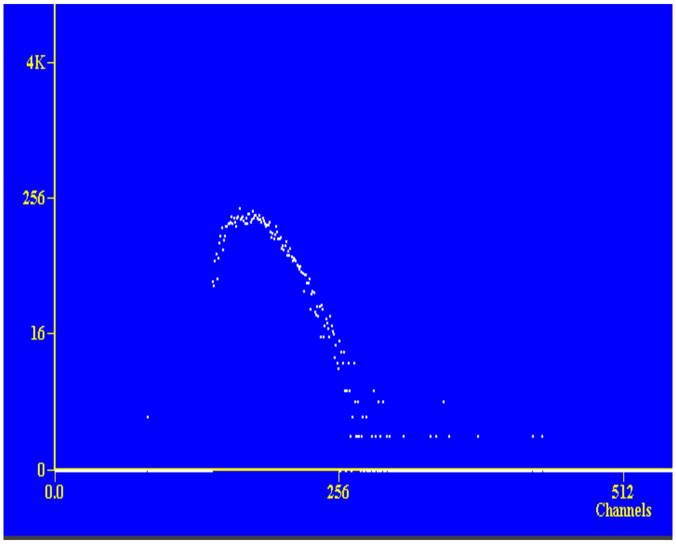

Figure 5. Peak for D = 7.3 cm.

8.3 cm

	Channel Range	Channel Centroid
1	(150,260)	205
2	(165,245)	204
3	(180,230)	204

Table 3.

Channel Centroid Average:
204 +/- 2 Channels

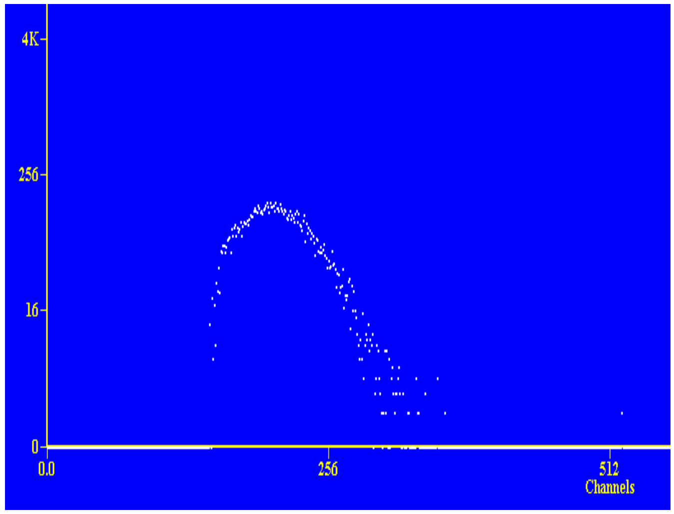

Figure 6. Peak for D = 8.3 cm.

9.3 cm

	Channel Range	Channel Centroid
1	(160,360)	252
2	(220,290)	254
3	(240,270)	254

Table 4.

Channel Centroid Average:
253 +/- 3 Channels

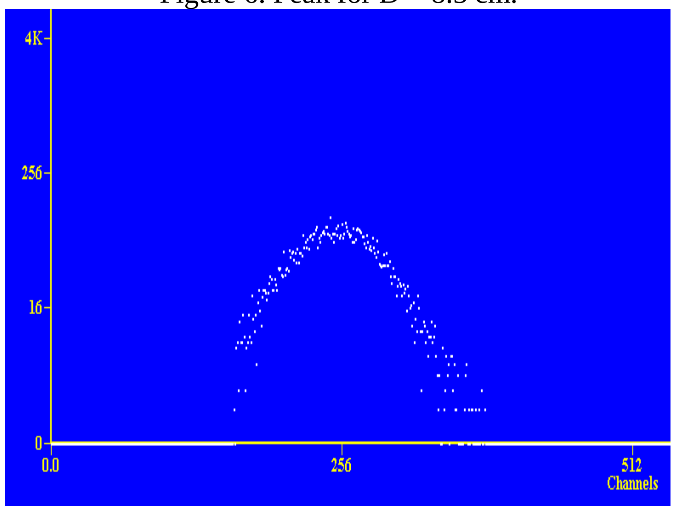

Figure 7. Peak for D = 9.3 cm.

10.3 cm

	Channel Range	Channel Centroid
1	(210,390)	297
2	(245,355)	297
3	(270,330)	298

Table 5.

Channel Centroid Average:
297 +/-2 Channels

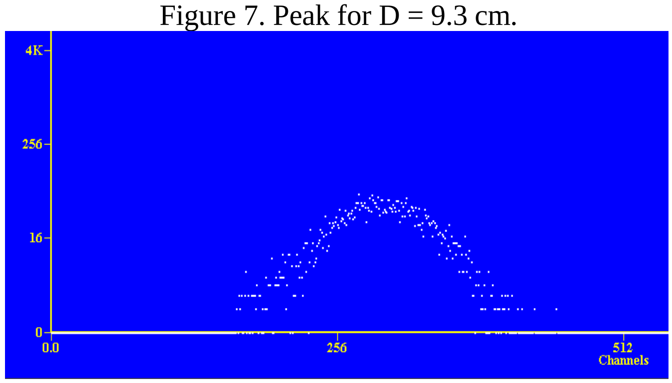

Figure 8. Peak for D = 10.3 cm.

11.3 cm

	Channel Range	Channel Centroid
1	(230,400)	317
2	(260,380)	318
3	(290,350)	319

Table 6.

Channel Centroid Average:
317 +/- 2 Channels

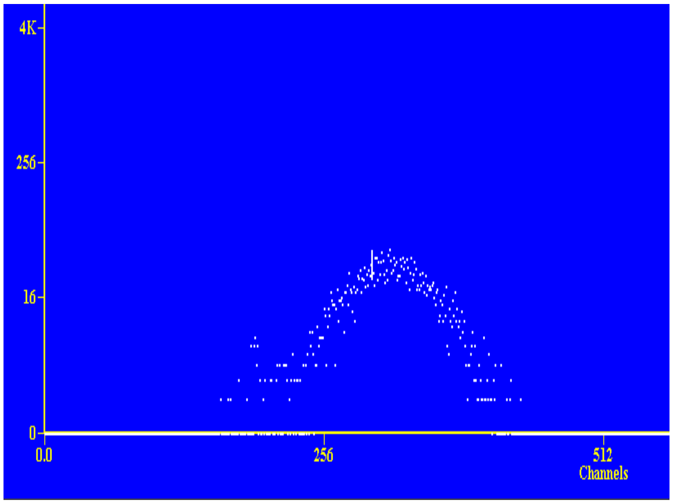

Figure 9. Peak for D = 11.3 cm.

Analysis:

Calculating momentum from data: The calculated magnetic field is used in Eq. (3) to compute the momentum *p* for each path diameter *D* in units kg*m/s, and is then converted to units MeV/c.

$$B = 0.124\,Tesla = 0.124\left(\frac{kg}{C*s}\right)$$

$$p = reB = \frac{D}{2}(m) * 1.602 * 10^{-19}(C) * 0.124\left(\frac{kg}{C*s}\right)$$

$$[p] = \frac{kg*m}{s}$$

$$\frac{1MeV}{c} = 5.36 * 10^{-22}\left(\frac{kg*m}{s}\right)$$

Calculating kinetic energy from data: The Bismuth-207 centroid is used to compute the energy to channel ratio which is then used to compute the kinetic energies respective to each of the Strontium-90 centroids. The estimated error on each of the centroids propagated through this calculation by single-variable error propagation to compute the error on each value of *K*.

Bismuth-207 centroid: 183 channels

$$\frac{Energy}{Channel} = \frac{0.75MeV}{183\ channels} \cong 0.0053\frac{MeV}{channel}$$

Strontium-90 centroids: $c_1 = 178, c_2 = 204, c_3 = 253, c_4 = 297, c_5 = 317\ channels$

Strontium-90 centroid errors: $s_1, s_2, s_4, s_5 = \pm 2,\ \ s_3 = \pm 3\ channels$

$$K_1 = c_1 channels * 0.0053\frac{MeV}{channel}$$

$$s_{K_1} = (c_1 + s_1)channels * 0.0053\frac{MeV}{channel} - K_1$$

Results:

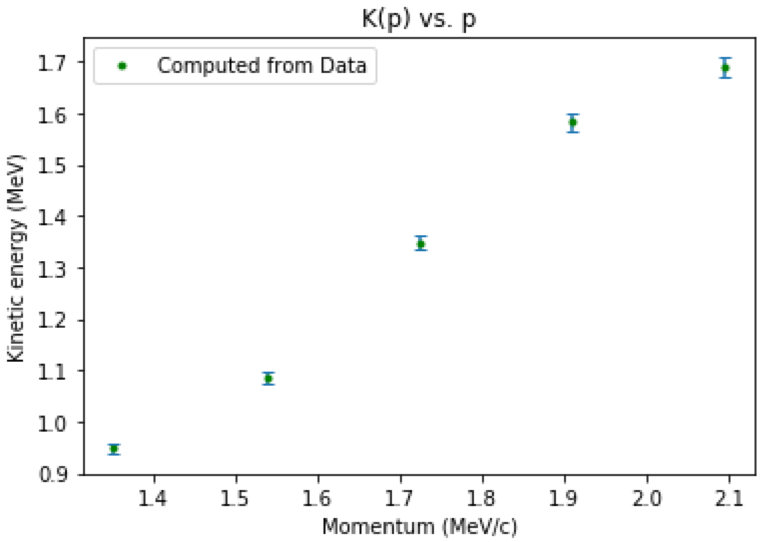

Figure 10. Computed Kinetic Energy vs. Momentum from the Strontium-90 data.

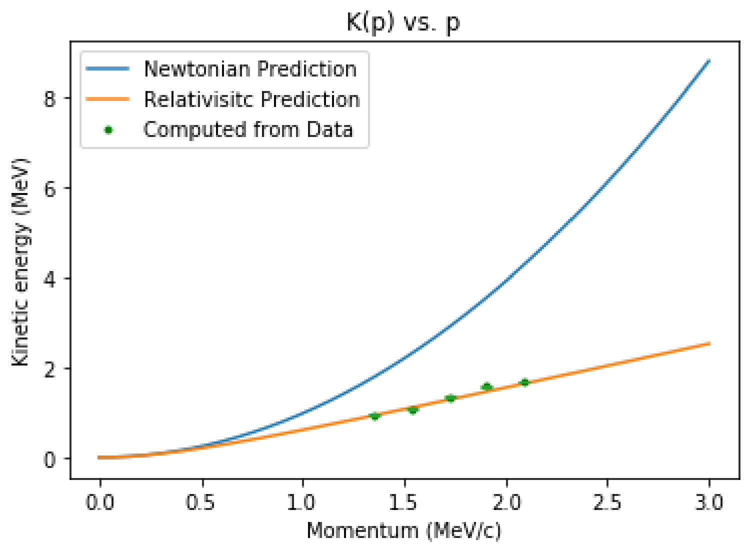

Figure 11. The Computed Kinetic Energy vs. Momentum data compared with the theory curves.

	D = 7.3 cm	D = 8.3 cm	D = 9.3 cm	D = 10.3 cm	D = 11.3 cm
Path radius 'r' (m)	0.0365	0.0415	0.0465	0.0515	0.0565
Momentum (MeV/c)	1.353	1.538	1.723	1.909	2.094
Kinetic Energy (MeV)	0.948	1.087	1.348	1.582	1.689
+/- *K* error (MeV)	0.010	0.012	0.015	0.017	0.019

Table 7. Computed *K* and *p* from data.

Conclusion:

The experimental data of the kinetic energy and momentum is more consistent with the relativistic expression for kinetic energy. As seen in Fig. (11), the theory curves match for $v << c$ such that the kinetic energies are the same for Newtonian and relativistic momentum predictions for momentum less than 0.5 MeV/c. For momentums larger than 0.5 MeV/c, the predictions deviate, and the deviation increases as particle velocity approaches c. As mentioned in the introduction, the electrons emitted by the radioactive decay of Strontium-90 have speeds up to $0.98c$ which is very close to the speed of light. Therefore, it was anticipated that the Kinetic energy vs. Momentum calculated from the data would be more consistent with the relativistic expression for K. This is confirmed by the data in Fig. (11) because the points computed from the data, including their errors, fall along the relativistic prediction trend when K of the relativistic prediction obviously deviates from the Newtonian prediction. Therefore, because the significant deviation of the data from the Newtonian prediction, the Newtonian expression for K cannot be used in calculations when particle speed v approaches c, such as for these beta decay electrons. All in all, the data confirms the necessity of Eq. (2) to precisely compute K and p for mass travelling at a speed approaching the speed of light. And, since the data falls so neatly on the relativistic prediction, Einstein's Eq. (2) is correct.