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Math Tutorial - 4

Q1] The number of accident in a year attributed to taxi driver in a city follow poisson distribution with mean 3 out of 1000 taxi drivers. Find approximately the number of drivers with
① No accidents in a year
② more than 3 accident in a year

$$P(x=0) = \frac{e^{-3} \cdot 3^0}{0!}$$

$$= \frac{e^{-3} \cdot 1}{1}$$

$$= 0.0497$$

No. of taxi drivers have no accidents =
$NP = (1000)(0.0497)$
$= 49.7 \approx 50$

$1 - P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$$1 - \left[0.0497 + \frac{e^{-3} \cdot 3^1}{1!} + \frac{e^{-3} \cdot 3^2}{2!} + \frac{e^{-3}(3)^3}{3!}\right]$$

$1 - [0.0497 + 0.1493 + 0.2240 + 0.2240]$
$1 - [0.647]$
$= 0.353$

No of taxi drivers $= (1000) \times (0.353)$
$= 353$

2] Fit a poisson distribution to the following data

No. of death	0	1	2	3	4
frequencies	123	59	14	3	1

→ mean $= m = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$$= \frac{(123)(0) + (59)(1) + (14)(2) + (3)(3) + (4)(1)}{200}$$

$$= \frac{0 + 59 + 28 + 9 + 4}{200} = 0.5$$

for $P(0)$; $P(1)$; $P(2)$, $P(3)$, $P(4)$

① $P(0) = \dfrac{e^{-0.5}(0.5)^0}{0!} = 0.6065 = 121.3 \approx 121$

② $P(1) = \dfrac{e^{-0.5}(0.5)^1}{1!} = 0.303 \cong 61$

③ $P(2) = \dfrac{e^{-0.5}(0.5)^2}{3!} = 0.758 = 15.16 \cong 15$

④ $P(3) = \dfrac{e^{-0.5}(0.5)^3}{3!} = 0.0126 \doteq 2.52 \cong 3$

⑤ $P(4) = \dfrac{e^{-0.5}(0.5)^4}{4!} = 0.0015 = 03 \doteq 0.3$

x_i	0	1	2	3	4
f_i	121	61	15	3	0.3

3] If x is a normal variable which mean 10 and standard deviation 4

find ① $P(|x - 14| \leq 1)$
② $P(15 \leq x \leq 18)$
③ $P(x \leq 12)$

→ mean = 10
SD = 4
for = 14 → $z = \frac{x-m}{\sigma}$

$= \frac{14-10}{4} = \frac{4}{4} = 1$

$P(|x-14| \leq 1) = P(|z| \leq 1)$

= Area between $z=0$ to $z=-1$
$= 0.5 +$ area betw $z0$ to $z=1$
$= 0.5 + 0.3413$
$= 0.8413$

$P(5 \leq x \leq 18)$

for $n = 5 \rightarrow z = \frac{x-m}{\sigma} = \frac{5-10}{4} = \frac{-5}{4} = -1.2$

for $x = 18 - 1$ $z = \frac{x-m}{\sigma} = \frac{18-10}{4} = \frac{8}{4} = 2$

$P(5 \leq x \leq 18) = P(-1.25 \leq z \leq 2)$

Area between $z=0$ to $z=1.25$ + area between $z=0$ to $z=2$

$= 0.3944 + 0.49772$

$= 0.8716$

$P(x \le 12) = z = \frac{x-m}{\sigma} = \frac{12-10}{4} = 0.5$

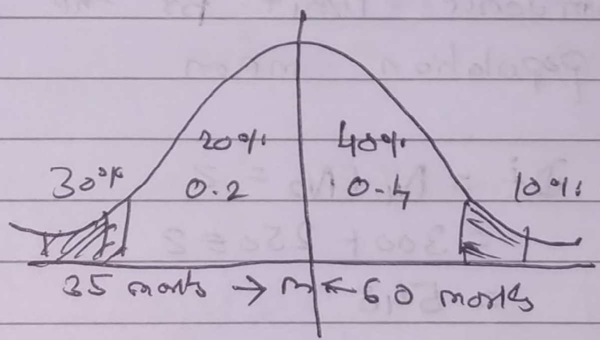
$P(x \le 12) = P(z \le 0.5)$

$= 0.5 +$ area between $z=0$ to $z=0.5$

$= 0.5 + 1915$

$= 0.6915$

4] Marks obtained by students in a exam follow normal distribution if 30 % of students got below 35 marks and 10 % got above 60 marks find mean.



Area 0.2 corresponds to $z=0$ to $z=-0.52$

Area 0.4 corresponds to $z=0$ to $z=1.28$

$-0.52 = \frac{35-m}{\sigma}$

$= 0.52\sigma = 35 - m$

$m - 0.52\sigma = 35$ —— ①

$$1.28 = \frac{60-m}{\sigma}$$

$$1.28\sigma = 60 - m$$

$$m + 1.28\sigma = 60 \quad — (2)$$

Solving eqⁿ (1) & (2)
we get
$m = 42.22$
$\sigma = 13.88$

5] Two samples drawn from two different population gave the following results

	Size	mean	S.D
Sample 1	300	87	10
Sample 2	250	84	08

Find 95% confidence limit for the difference between the population mean

$m_1 = 87$ $\qquad Df = N_1 + N_2 - 2$
$n_1 = 300$ $\qquad = 300 + 250 - 2$
$s_1 = 10$ $\qquad = 548$
$m_2 = 84$
$n_2 = 250$ $\qquad Df_1 = n_1 - 1 = 299$
$s_2 = 8$ $\qquad Df_2 = n_2 - 1 = 249$

$\mu_1 - \mu_2 = (m_1 - m_2) = 3$ and 95% confidence limit i.e (1.96).

The difference between the two population means $(\mu_1 - \mu_2)$ lies in between $(1.46, 4.54)$

Pooled variance

$$S^2p = \frac{(df_1)(S_1)^2 + (df_2)(S_2)^2}{dF}$$

$$= \frac{(299)(100) + (249)(64)}{548}$$

$$= 83.638 \simeq 83.64$$

Standard error

$$S(m_1 - m_2) = \sqrt{\frac{S^2p}{n_1} + \frac{S^2p}{n_2}}$$

$$= \sqrt{\frac{83.64}{300} + \frac{83.64}{250}}$$

$$= \sqrt{0.27 + 0.33}$$

$$= \sqrt{0.6}$$

$$\therefore S(m_1 - m_2) = 0.774.$$

Confidence Interval

$$\mu_1 - \mu_2 = (m_1 - m_2)\, I + (S) \cdot (m_1 - m_2)$$

$$= 3I \cdot (1.96 + 0.77)$$

$$\therefore \mu_1 - \mu_2 = 3 + 1.5092$$

Q6] A random sample of 50 items gives the mean 6.2 and ~~xx~~ variance 10.24 can it be regarded as drawn from normal population with mean 5.4 at 5% level of significance

→ Step I : Null hypothesis $H_0 \rightarrow \mu = 5.4$

Alternative hypothesis $H_a \neq 5.4$

$\mu \neq 5.4$

Step II Calculation of z

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$\bar{x} = 6.2$

$n = 50$

$\mu = 5.4$

$\sigma = 3.2$

$$z = \frac{6.2 - 5.4}{3.2/\sqrt{50}}$$

$$z = \frac{0.8}{3.2/7.07} = \frac{0.8}{0.4526} = 1.767$$

Step III : level of significance 5% (α)

Step IV = critical value $(z\alpha)$ at $c = 0.5$

5% critical value $Z\alpha = 1.96$

Step IV ; Decision

here $z_c < z_\alpha$

$= 1.767 < 1.96$

Null hypothesis accepted

Q 7] A certain drug administrated to 12 patient resulted in following changes to blood pressure

n	$di = xi - A$	di
5	2	4
2	-1	1
8	5	25
-1	-4	16
3-A	0	0
0	-3	9
6	3	9
-2	-5	25
1	-2	4
5	2	4
0	-3	9
4	1	1

$\Sigma di = -5$ $\quad\quad \Sigma di^2 = 107$

$\therefore \bar{x} = 9 + \frac{\Sigma di}{n} = 3 + \left(\frac{-5}{12}\right)^2 = 2.88$

$\Sigma (x_i - \bar{x}) = \Sigma di - \frac{(\Sigma di)^2}{n} = 107 - \frac{25}{12}$

$\Sigma (x_i - \bar{x}) = 104.92$

Step 1 : Consider null hypothesis . $H_0 = \mu = 0$
Alternative hypothesis $H_a \neq \mu$

Step 2: Calculation

$$\therefore s = \sqrt{\frac{\sum (x' - \bar{x})^2}{n}} = 2.96$$

$$\therefore T = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

$$= \frac{2.58}{0.892} = 2.89$$

Step 3: level of significance $\alpha = 5\%$

Step 4: Critical value

$z_{\gamma} = 2.201$

$z_c > z_{\alpha}$

$2.89 > 2.20$

Null hypothesis is rejected
i.e there is Increase in blood pressure

8] In a survey of 200 boys of which in 75 were Intelligent 40 had educated father while 85 of the unintelligent boys had uneducated father. Do this figures support the hypothesis that educated father has intelligent boys.

→ Table for given Information

	Intelligent boys	Unintelligent boys	Total
Educated father	40	40	80
uneducated father	35	85	120
Total	75	125	200

table for expected frequency

	Intelligent Boys	Uninteligent boys	Total
Educated father	$\frac{A_1 \times B_1}{200} = \frac{75 \times 80}{200} = 30$	50	80
Uneducated father	45	75	120
Total	75	125	200

table for $[O-E]^2$

O	E	O-E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
40	30	10	100	3.33
35	45	-10	100	2.22
40	50	-10	100	2
85	75	10	100	1.33

$$\sum \frac{(O-E)^2}{E} = 8.88$$

$$\chi^2_c = 8.88$$

Step 1:. Null hypothesis:
There is a no association
Alternative hypothesis
There is an association

Step 2:- Calculated value
$\chi_c^2 = 8.88$

Step 3: level of significance = 5%.

Step 4: Table value of χ^2_α
χ^2_α at 50 % los for $(r-1)(c-1)$
$(2-2)(2-1)$
$\chi_\alpha^2 = 2$
$\chi^2_\alpha = 3.84$

Step 5: decision
$\chi^2_c > \chi^2_4$
$8.8 > 3.84$
Null hypothesis rejected.
There is on association

q] A Dies was thrown 132 times and the following frequency were observed ~~No obtained 1~~

No. obtained	1	2	3	4	5	6	total
Frequency	15	20	25	15	29	28	132

Test the hypothesis that the dice is unbiased

→ To use x^2 distribution

$$x^2 = \Sigma \frac{(O-E)^2}{E}$$

$\frac{Total}{6} = \frac{132}{6} = 22$

table for x^2

O	E	O.E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
15	22	-7	49	E. 0. 222
20	22	-2	4	0.18
25	22	3	9	0.40
15	22	7	49	2.22
29	22	7	49	2.22
28	22	6	36	1.63

$$\Sigma \frac{(O-E)^2}{E} = 8.87$$

Step : I

Null hypothesis

There is a no association (Dice is unbiased)

Alternative hypothesis

there is on association

Step III :

level of significance . 50 %

Step II . calculate value

$x^2_c = 8.87$

Step IV : Table value of $x^2 \alpha$

$x^2 \alpha$ at 5% Los for $(n-1) = (6-1) = 5$

$= 11.07$