Page No.
Date

Tutorial No: 1

Shubham Mankar
A-73
SE-IT-IV
EM-IV

Q1] Find eigen values & vectors

1) $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

→ $(A - \lambda I)x = 0$

$$\begin{bmatrix} 2-\lambda & -2 & 3 \\ 1 & 1-\lambda & 1 \\ 1 & 3 & -1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -2 & 3 \\ 1 & 1-\lambda & 1 \\ 1 & 3 & -1-\lambda \end{vmatrix} = 0$$

$\lambda^3 - (\text{sum of DE})\lambda^2 + (\text{sum of minors of DE})\lambda - |A| = 0$

$\lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0$ —— (C.E)

$\lambda = 3, -2, 1$ —— (eigen values)

b) For eigen vectors

For $\lambda = 3$

$$\begin{vmatrix} 2-3 & -2 & 3 \\ 1 & 1-3 & 1 \\ 1 & 3 & -1-3 \end{vmatrix} = 0$$

$$\begin{vmatrix} -1 & -2 & 3 \\ 1 & -2 & 1 \\ 1 & 3 & -4 \end{vmatrix} = 0$$

$$-1x_1 - 2x_2 + 3x_3 = 0$$
$$1x_1 - 2x_2 + x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} -2 & 3 \\ -2 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & -2 \\ 1 & -2 \end{vmatrix}}$$

$$x_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda = -2$

$$\begin{vmatrix} 4 & -2 & 3 \\ 1 & 3 & 1 \\ 1 & 3 & 1 \end{vmatrix} = 0$$

$$x_1 + 3x_2 + x_3 = 0$$

Page No.
Date

$$\frac{x_1}{-11} = \frac{-x_2}{1} = \frac{x_3}{14}$$

$$x_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -11 \\ -1 \\ 14 \end{bmatrix}$$

for $\lambda = 1$

$$\begin{vmatrix} 1 & -2 & 3 \\ 1 & 0 & 1 \\ 1 & 3 & -2 \end{vmatrix} = 0$$

$$x_1 + 0x_2 + x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} -2 & 3 \\ 0 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & -2 \\ 1 & 0 \end{vmatrix}}$$

$$\frac{x_1}{-2} = \frac{-x_2}{-2} = \frac{x_3}{2}$$

$$x_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Page No.
Date

Q3] Find the eigen values of following

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

→ $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 0 & -1 \\ 0 & 2-\lambda & 0 \\ -1 & 0 & 2-\lambda \end{vmatrix} = 0$$

$\therefore \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\lambda = 3, 1, 2$

Q4] If $A = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$ find eigen value of $6A^{-1} + A^3 + 2I$

→ I) eigen values

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 2-\lambda & 0 \\ 4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$|A - \lambda I| = 0$

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda = 2, 3$

eigen values of A are 2, 3.

Put $\lambda = 2$

$\frac{6}{2} + (2)^3 + 2 = 3 + 8 + 2$
$= 13$

Put $\lambda = 3$

$\frac{6}{3} + (3)^3 + 2 = 2 + 27 + 2 = 31$

$\therefore$ eigen values for $6A^{-1} + A^3 + 2I$ are 13, 31

Q5] Verify caley Hamilton th^m for

$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and hence find A^{-1} and A^4

1) $(A - \lambda I)X = 0$

$$\begin{bmatrix} 1-\lambda & 2 & -2 \\ -1 & 3-\lambda & 0 \\ 0 & -2 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\lambda^3 - 5\lambda^2 + 9\lambda - 1 = 0$$

Caley Hamilton thm

$$A^3 - 5A^2 + 9A - 1I = 0$$

$$\begin{bmatrix} A^3 \end{bmatrix} - 5\begin{bmatrix} A^2 \end{bmatrix} + 9\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} + 1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence CHT proved.

b) To find A^{-1}

$$A^3 - 5A^2 + 9A - 1I = 0$$

Multiply by A^{-1}

$$\therefore A^{-1}A^3 - 5A^2 \cdot A^{-1} + 9A \cdot A^{-1} - 1I \cdot A^{-1} = 0$$

$$A^2 \cdot I - 5A \cdot I + 9I - 1A^{-1} = 0$$

$A^{-1} = A^2 - 5A + 9I$

$$A^{-1} = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}^2 - 5\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

i] To find A^4

$A^3 - 5A^2 + 9A - 1I = 0$

multiplying by A

$A^4 - 5A^3 + 9A^2 - 2A = 0$

$$A^4 = \begin{bmatrix} -55 & 102 & 26 \\ -19 & -17 & 32 \\ 32 & -38 & -23 \end{bmatrix}$$

Q6] Verify Cayley Hamilton thm

$A^6 - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I$

where $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

$\rightarrow$ $|A - \lambda I| = 0$

$$\begin{bmatrix} 3-\lambda & 10 & 5 \\ -2 & -3-\lambda & -4 \\ 3 & 5 & 7-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} 3-\lambda & 10 & 5 \\ -2 & -3-\lambda & -4 \\ 3 & 5 & 7-\lambda \end{vmatrix} = 0$$

$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0 \longleftarrow (CE)$

CHT

$A^3 - 7A^2 + 16A - 12I$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence proved.

Page No. ____ Date ____

for $A^6 - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I$

Put $A = \lambda$

$\lambda^6 - 6\lambda^5 + 9\lambda^4 + 4\lambda^3 - 12\lambda^2 + 2\lambda = 1$

$$
\begin{array}{r}
\lambda^3 + \lambda^2 \\
\lambda^3 - 7\lambda^2 + 16\lambda - 12 \,\big)\, \lambda^6 - 6\lambda^5 + 9\lambda^4 + 4\lambda^3 - 12\lambda^2 + 2\lambda - 1 \\
\lambda^6 - 7\lambda^5 + 16\lambda^4 - 12\lambda^3 \\
- \quad + \quad - \quad + \\
\hline
\lambda^5 - 7\lambda^4 + 16\lambda^3 - 12\lambda^2 \\
\lambda^5 - 7\lambda^2 + 16\lambda^3 - 12\lambda^2 \\
- \quad + \quad - \quad + \\
\hline
0 + 2\lambda - 1
\end{array}
$$

Divendent = Divisor x Quotient + Remainder

$= (\lambda^3 - 7\lambda^2 + 16\lambda - 12) \times (\lambda^3 + \lambda^2) + (2\lambda - 1)$

Put $\lambda = A$

$= (A^3 - 7\lambda^2 + 16A - 12) \times (A^3 + A^2) + (2\lambda - I)$

$= 0 \times (A^3 + \lambda^2) + (2\lambda - I)$

$$= 2\begin{bmatrix} 3 & 16 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 20 & 16 \\ -4 & -7 & -8 \\ 6 & 10 & 13 \end{bmatrix}$$

Q7] Show that matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalizable

Find the transforming matrix and diagonal mat

→ a) To find eigen values

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$\lambda = 15, 0, 3$ —— (Distinct)

Here,

∵ eigen values are distinct, the matrix is always diagonalizable

b) Eigen vectors

For $\lambda = 15$

$$\begin{vmatrix} 8-15 & -6 & 2 \\ -6 & 7-15 & -4 \\ 2 & -4 & 3-15 \end{vmatrix} = 0$$

$$\begin{vmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{vmatrix} = 0$$

$$-7x_1 - 6x_2 + 2x_3 = 0$$
$$-6x_1 - 8x_2 - 4x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ -8 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -7 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -7 & -6 \\ -6 & -8 \end{vmatrix}}$$

$$\frac{x_1}{40} = \frac{-x_2}{40} = \frac{x_3}{20}$$

$$x_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} = 0$$

$$5x_1 - 6x_2 + 2x_3 = 0$$
$$-6x_1 + 4x_2 - 4x_2 = 0$$

$$\frac{x_1}{-16} = \frac{-x_2}{-8} = \frac{x_3}{-16}$$

$$\frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

$$x_2 = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$$

$\lambda = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} = 0$$

$$8x_1 - 6x_2 + 2x_3 = 0$$
$$-6x_1 + 7x_2 + 4x_3 = 0$$

$$\frac{x_1}{10} = \frac{-x_2}{-20} = \frac{x_3}{20}$$

Page No.
Date

$$x_3 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$\therefore M = [x_1 \; x_2 \; x_3]$

$$= \begin{bmatrix} 2 & -2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Q8] Show that matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 4 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is a diagonilizable

$\rightarrow$ $(A - \lambda I)x = 0$

$$\begin{bmatrix} -9-\lambda & 4 & 4 \\ -8 & 3-\lambda & 4 \\ -16 & 8 & 7-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

eigen values,

$$\lambda^3 - 1\lambda^2 - 5\lambda - 3 = 0$$

$$\lambda = 3, -1, -1 \quad \text{—— (Repeated)}$$

eigen vectors

$\lambda = 3$

$$\begin{bmatrix} -12 & 4 & 4 \\ -8 & 0 & 4 \\ -16 & 8 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\frac{x_1}{\begin{vmatrix} 4 & 4 \\ 0 & 4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -12 & 4 \\ -8 & 4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -12 & 4 \\ -8 & 0 \end{vmatrix}}$$

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

for $\lambda = -1$

$$\begin{array}{l} R_1 \rightarrow \\ R_2 \rightarrow \\ R_3 \rightarrow \end{array} \begin{bmatrix} -8 & 4 & 4 \\ -8 & 4 & 4 \\ -16 & 8 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

AM = No. of time (-1) appears = 2

GM = n - r

GM = 3 - ?

$$\frac{R_1}{-4} ; \frac{R_2}{-4} ; \frac{R_3}{-8}$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 2 & -1 & -1 \\ 2 & -1 & -1 \\ 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2 - R_1 \;,\; R_3 - R_1$

$$N \begin{bmatrix} 2 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$GM = n - r$
$= 3 - 1$
$GM = 2$
$GM = AM$

matrix is diagonalizable

$2x_1 - x_2 - x_3 = 0$

Assume,
$x_1 = s$
$x_2 = t$
$x_3 = 2s - t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} s \\ t \\ 2s-t \end{bmatrix} = \begin{bmatrix} s+0t \\ 0s+t \\ 2s-t \end{bmatrix}$$

$$m^{-1} = \begin{bmatrix} -2 & 1 & 1 \\ 3 & -1 & -1 \\ 2 & 0 & -1 \end{bmatrix}$$

Page No.
Date

$D = M^{-1} A M$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Q9] Find the eigen values and eigen vector of

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$(A - \lambda I) X = 0$

$$\begin{bmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Here,
matrix A is triangular matrix
$\therefore \lambda = 2, 2, 2$ ← (Repeated)

$\lambda = 2$
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$0x_1 + x_2 + 0x_3 = 0$
$0x_1 + 0x_2 + x_3 = 0$

Page No. Date

$$\frac{x_1}{\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}}$$

$$\frac{x_1}{1} = \frac{-x_2}{0} = \frac{x_3}{0}$$

$$x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$