

SPA Theory and Results Interpretation

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1 Introduction

This report presents a statistical evaluation of multiple trading strategies based on Bollinger Bands. The strategies are constructed by varying two parameters: the lookback period ($\text{period} \in \{5, 10, 15, 20, 25, 30\}$) and the bandwidth multiplier ($k \in \{1, 1.5, 2\}$), yielding 18 distinct strategies. These are evaluated using the Superior Predictive Ability (SPA) test.

2 Definitions and Key Concepts

- **Significance Level (α):** The probability of rejecting the null hypothesis when it is actually true (Type I error). A significance level of 5% ($\alpha = 0.05$) means there is a 5% chance of concluding a strategy is better than the benchmark when it is not.
- **Benchmark Model:** A reference model (model 0 in this case) against which all other models are compared.
- **Alternative Models:** The remaining strategies being tested for potential improvement over the benchmark.
- **Loss Function:** In this context, the negative return from a strategy, used as a measure of performance.
- **Loss Differential d_j :** The difference in loss between an alternative model j and the benchmark, $d_j = L_j - L_0$.
- **Superior Predictive Ability (SPA) Test:** A statistical test used to evaluate whether any model in a set of competing models outperforms a benchmark model in terms of predictive accuracy or profitability.

3 Hypotheses for the SPA Test

The SPA test is used to determine whether any of the candidate models offer statistically superior performance over a benchmark model (model 0).

Let:

- L_0 be the loss (negative return) from the benchmark model,
- L_j be the loss from the j -th alternative model,
- $d_j = L_j - L_0$ be the loss differential.

The hypotheses are defined as:

- **Null Hypothesis (H_0):** The benchmark model performs at least as well as all alternatives.

$$H_0 : \max_j \mathbb{E}[d_j] \leq 0$$

- **Alternative Hypothesis (H_1):** At least one alternative model performs better than the benchmark.

$$H_1 : \max_j \mathbb{E}[d_j] > 0$$

4 Results and Interpretation

4.1 SPA Test Results

The SPA test was conducted at a 5% significance level. The following models were found to significantly outperform the benchmark model (model 0):

Models: 5, 9, 10, 11, 15, 16, 17

Interpretation: These results indicate that the above strategies yield statistically superior returns compared to the benchmark strategy.

5 Conclusion

The SPA test results indicate that several Bollinger Band-based strategies significantly outperform the benchmark model. These findings provide a strong basis for selecting high-performing strategies for further validation or deployment.