

# How to formally reason about strategic behaviour

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AI is about building agents to  
do tasks we think require  
intelligence

# What is an agent?

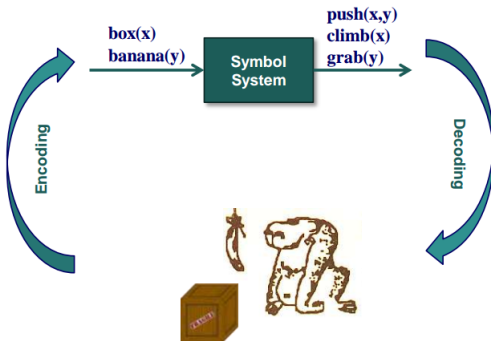
- An agent is something that *acts*, based on sensing the world.
- An agent is *rational* if it acts to achieve the best (expected) outcome.

Russell, Norvig

# What is an agent?

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# ...Multi-agent systems are everywhere!

Many real and imagined systems consist of multiple agents:

1. **distributed**, i.e., agents have their own **view** of the world.
2. **goal-directed**, i.e., agents **strategise** to achieve their own goals.

E.g.

- multiplayer games (winning tasks)
- robot assisted search-and-rescue (co-ordination tasks)
- autonomous transport vehicles (safety tasks)
- voting and auction protocols (fairness tasks)
- rational distributed computing (secret-sharing tasks)
- ...

# Research Challenge

How to understand, predict  
and control agent behaviour?

# Lots of disciplines involved

1. Humanities: How do we want agents to behave?
2. Computer engineering: How to engineer such agents?
3. **Computer science**: How to make sure agents behave as we want?
  - 3.1 **Theoretical foundations**
  - 3.2 Academic tools
  - 3.3 Industrial applications

# Research Challenge

How to understand, predict and control agent behaviour?

- **Methodology**

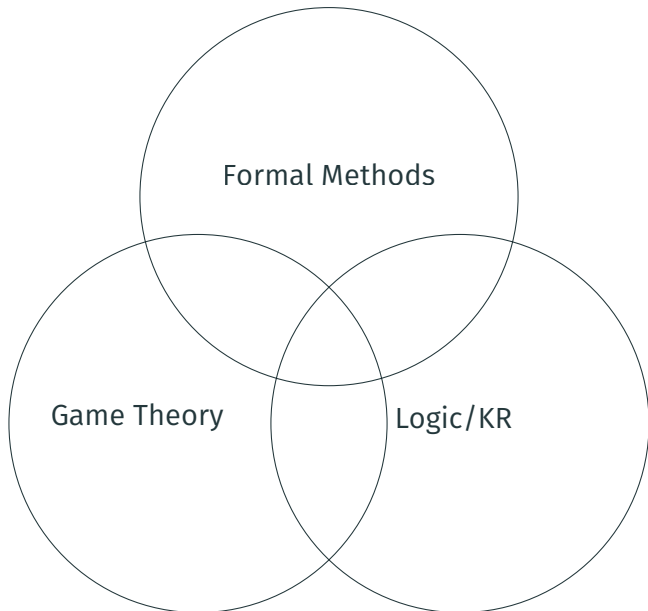
Model agent tasks in logic (for understanding), and devise algorithms (for prediction/control).

- **Difficulty**

Agents are distributed and goal directed.



# Methodology



# Formal Methods 101

$$\text{Model-checking paradigm}^1 = \begin{cases} \text{model} & M \\ \text{specify} & \varphi \\ \text{algorithm for} & M \models \varphi \end{cases}$$

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<sup>1</sup>If formula  $\varphi$  talks about strategies, model-checking can usually be adapted to do control (not just prediction).

# Formal Methods 101

$$\text{Model-checking paradigm}^1 = \begin{cases} \text{model} & M \\ \text{specify} & \varphi \\ \text{algorithm for} & M \models \varphi \end{cases}$$

Classical model-checking **can't handle the complexities** of multi-agent systems.

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<sup>1</sup>If formula  $\varphi$  talks about strategies, model-checking can usually be adapted to do control (not just prediction).

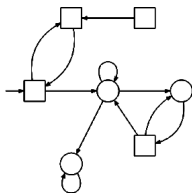
# My current research goals

$$\text{Model-checking paradigm} = \begin{cases} \text{model} & M \\ \text{specify} & \varphi \\ \text{algorithm for} & M \models \varphi \end{cases}$$

1. Devise **models** that capture essential aspects of multi-agent systems.
2. Devise **logics** that can specify complex agent/system goals.
3. Devise **algorithms** for model-checking, or prove that none exists.

**Note.** Steps 1 and 2 are useful in themselves! (Knowledge Representation philosophy)

# Models



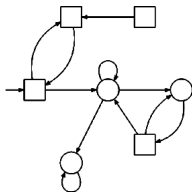
$M = \langle S, \text{tr}, (Obs_a)_{a \in Ag} \rangle$  is a transition system modeling dynamics

$S$	set of states
$\text{tr} : S \times \text{Act} \rightarrow S$	transition function
$Obs_a : S \rightarrow \Omega$	observation function

Special case:  $Obs_a(s) = s$

Full observation

# Models



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Special case:  $Obs_a(s) = s$

## Full observation

- **Strategy** tells an agent how to act based on its observations
- Strategy for each agent determines a **path** in  $M$

# Logics

New logic: Strategy Logic for agents with partial observation

$$\begin{array}{c} \text{Boolean} \qquad \text{Temporal} \qquad \text{Strategic} \\ \hline p \mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \mid X \varphi \mid \varphi_1 \text{ U } \varphi_2 \mid (\exists s) \varphi \mid (a_i \mapsto s) \varphi \end{array}$$

# Logics

New logic: Strategy Logic for agents with partial observation

Boolean			Temporal		Strategic							
$p$	$ $	$not\ \varphi$	$ $	$\varphi_1\ or\ \varphi_2$	$ $	$next\ \varphi$	$ $	$\varphi_1\ until\ \varphi_2$	$ $	$(exists\ s)\ \varphi$	$ $	$(a\ uses\ s)\ \varphi$



# Logics

$$\overbrace{p \mid \textit{not } \varphi \mid \varphi_1 \textit{ or } \varphi_2}^{\text{Boolean}} \mid \overbrace{\textit{next } \varphi \mid \varphi_1 \textit{ until } \varphi_2}^{\text{Temporal}} \mid \overbrace{(\textit{exists } s) \varphi \mid (a \textit{ uses } s) \varphi}^{\text{Strategic}}$$

Boolean part expresses state properties:

- $\textit{Crashed}(a_1), \textit{not Crashed}(a_1), \textit{InRoom}(a_1, r_2), \dots$

# Logics

$$\begin{array}{c} \text{Boolean} \qquad \qquad \text{Temporal} \qquad \qquad \text{Strategic} \\ \hline p \mid \textit{not } \varphi \mid \varphi_1 \textit{ or } \varphi_2 \mid \textit{next } \varphi \mid \varphi_1 \textit{ until } \varphi_2 \mid (\textit{exists } s) \varphi \mid (a \textit{ uses } s) \varphi \end{array}$$

Temporal part expresses agent goals:

- *eventually InRoom( $a_1, r_2$ ), never Crashed( $a_1$ ), ...*

# Logics

$$\overbrace{p \mid \text{not } \varphi \mid \varphi_1 \text{ or } \varphi_2}^{\text{Boolean}} \mid \overbrace{\text{next } \varphi \mid \varphi_1 \text{ until } \varphi_2}^{\text{Temporal}} \mid \overbrace{(\text{exists } s) \varphi \mid (a \text{ uses } s) \varphi}^{\text{Strategic}}$$

Strategic part expresses game-theoretic properties:

- $(\text{exists } s_1) (a_1 \text{ uses } s_1) \text{ goal}_1$   
“I have a strategy to win” (solitaire)
- $(\text{exists } s_i)_i (a_i \text{ uses } s_i)_i \text{ goal}$   
“We have strategies that win” (coordination)
- $(\text{exists } s_1) (\text{forall } s_2) (a_i \text{ uses } s_i)_i \text{ goal}_1$   
“I have a strategy that beats any of yours” (tic-tac-toe)
- $(\exists s_i)_i (a_i \mapsto s_i)_i \&_i \neg [\neg \text{goal}_i \wedge (\exists s') (a_i \mapsto s') \text{goal}_i]$   
“We can play NE” (rational secret sharing, fair division, poker)

# Logics

$$\overbrace{p \mid \text{not } \varphi \mid \varphi_1 \text{ or } \varphi_2}^{\text{Boolean}} \mid \overbrace{\text{next } \varphi \mid \varphi_1 \text{ until } \varphi_2}^{\text{Temporal}} \mid \overbrace{(\text{exists } s) \varphi \mid (a \text{ uses } s) \varphi}^{\text{Strategic}}$$

Natural, and very expressive

It can express

- state properties,
- (Boolean) agent goals,
- game-theoretic properties (NE, SPE, ESS, Pareto optimality, ...).

# Algorithms for $M \models \varphi$

**Distributed agents make things  
hard...**

# Distributed agents make things hard...

- Multi-player games in TCS (70s) undecidable
- Distributed synthesis in FM (90s) undecidable
- Finite horizon DEC-POMDPs in AI (00s) NEXP-complete

**Insight.** The **source of these difficulties** is the ability of agents, each with their own observations of the world, to communicate privately with each other.

# Distributed agents make things hard...

What can be done?

1. Restrict observability
2. Restrict private communication



# Distributed agents make things hard...

What can be done? Restrict observability

How? Hierarchical observation/information

# Distributed agents make things hard...

What can be done? Restrict observability

How? Hierarchical observation/information

- Peterson, Reif ('79)
- Pnueli, Rosner ('90)
- Kupferman, Vardi ('01)
- Finkbeiner, Schewe ('05)
- vdMeyden, Wilke ('05)
- Berwanger, Mathew, vdBogaard ('16)
- Berthon, Maubert, Murano, R., Vardi (LICS '17)

**Observation.** Mathematically elegant, but none of the examples mentioned have hierarchical observation

# Distributed agents make things hard...

What can be done? Restrict private communication

How? Broadcast-communication/public-actions

# Distributed agents make things hard...

What can be done? Restrict private communication

How? Broadcast-communication/public-actions

- Lomuscio, vdMeyden, Ryan, Wilke ('00, '05) Broadcasting
- vDitmarsch, vdHoek ... Public Announcements Logic (DEL)
- Kominis, Geffner ('15,'17) Public Actions (Planning)
- Belardinelli, Lomuscio, Murano, R. (AAMAS '17, IJCAI '17)

# Models

**Model:** Public action transition system  $M$  — the last action of each agent is observable to all agents.

E.g.,

- community-card games (bridge, poker)
- broadcasting distributed systems (secret-sharing protocols)
- epistemic puzzles (muddy children)
- auctions (open-outcry)

Many examples

# Algorithms for $M \models \varphi$

**Theorem.**  $M \models \varphi$  is decidable if  $M$  is a public-action transition system.

Complexity?

- In  $(k + 2)$ -exptime in the model and the formula, and  $(k - 1)$ -expspace-hard in the formula, where  $k =$  quantifier-block depth of formula.

**In fact.** Computational complexity is similar to fully-observable transition systems!

Middle ground.

Fully observable TS < Public Action TS < Unrestricted TS

# Proofs

- Q. How did you come up with the algorithms?
- A. We used the automata-theoretic approach to model-checking.<sup>2</sup>
- Q. But what do the algorithms actually look like?
- A. Operations on automata that run on trees.
- Q. ??
- A. Think of it like manipulating **regular expressions** that match trees instead of strings!

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<sup>2</sup>for which Vardi and Wolper won the Gödel prize for outstanding papers in TCS.

# Summary

- **Current research goal**  
Devise models, logics and algorithms for understanding, predicting, and controlling behaviour of agents in multi-agent systems.
- **Challenge**  
Combination of partial observability and private communication.
- **Solutions**  
Impose hierarchical observation, or restrict private communication, and use automata.



# **Future research goals**

Capture quantitative aspects

# Future research goals

## Capture quantitative aspects

1. Agents: Probabilistic (instead of just deterministic)
2. Tasks: Optimisation (instead of just Boolean)

# Future research goals

**Context.** Many protocols rely on coin-flips.

**Project.** Develop *Probabilistic Strategy Logic* for reasoning about stochastic multi-agent transition systems.

**Methodology.** Study restrictions on memory of probabilistic agents and reduce to reasoning about polynomial inequalities.<sup>3</sup>

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<sup>3</sup>Formally, reduce  $M \models \varphi$  to the first-order theory of real arithmetic, which is solvable in EXPSPACE by work of Tarski/Seidenberg.

# Future research goals (speculative)

**Context.** MDPs, POMDPs, DEC-POMDPs are important models.

**Challenge.** Agent goal is to optimise expected-reward, which is a very “low-level” specification.

**Project.** Unify reward- and declarative-specifications.

**Methodology.** ???

# Overview of recent research lines

Formal methods ...

...for strategising agents with qualitative temporal goals

AAMAS'16/'17/'18, IJCAI'17, LICS'17

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Formal methods ...

...for strategising agents with **quantitative** temporal goals

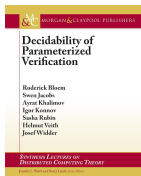
IJCAI'17, CSL'18

# Overview of recent research lines

Formal methods ...

...when the number of agents is **not known** apriori

CONCUR'14, VMCAI'14, ICALP'15, M&C'15 (book), IJCAR'16, VMCAI'18

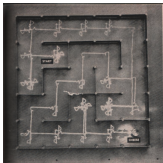


# Overview of recent research lines

Formal methods ...

...when the environment is **partially-known**

PRIMA'15 (best-paper), AAMAS'15/'16, IJCAI'16/'17





# What sort of researcher am I?

- I'm part of a growing group bringing insights from formal methods to bear on problems in AI.
- I'm rooted in Formal Methods and Knowledge Representation philosophies.
- My focus has broadened over time:
  1. Automata for reasoning about mathematical structures
  2. Logic/automata in Formal Methods
  3. Formal Methods and Artificial Intelligence

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# **Future research goals**

Academic Tools

# Future research goals

**Context.** Existing tools for multi-agent systems (*MCK* vdMeyden et. al.; *Eve* Wooldridge et. al.; *MCMAS* Lomuscio et. al.) can't handle logics and models above.

**Challenge.** High computational complexity, complex constructions.

**Project.** Engineer practical algorithms by translating to optimised tools, such as planners.<sup>4</sup>

**Methodology.** Give new semantics for logics based on finite paths of  $M$ .

De Giacomo, Vardi IJCAI '13/'15/'16

McIlraith,... AAMAS/IJCAI/KR '18

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<sup>4</sup>Planning in AI is a form of synthesis.