This is the supplemental material for the paper "Probabilistic Alternating-Time μ-Calculus", containing omitted proofs due to space restriction.

A Comparison with AMC, PATL* and μ-PCTL

The logics we compared in this section are defined over concurrent game structures (CGSs). Therefore, we first recall the definition of CGSs.

Concurrent Game Structures

A PCGS $\mathcal{G} = (\mathsf{Ag}, \mathsf{Act}, Q, \delta, \lambda, q_0)$ is a *concurrent game structure* (CGS), if δ is defined as $Q \times \mathcal{D} \to \Upsilon(Q)$ such that for every $q \in Q$ and $\mathbf{d} \in \mathcal{D}$, there exists exactly one state $q' \in Q$ such that $\delta(q, \mathbf{d}, q') = 1$. We will use $\delta(q, \mathbf{d})$ to denote the unique state q' such that $\delta(q, \mathbf{d}, q') = 1$. Strategies used by agents in CGSs are also deterministic. In this setting, given a coalition strategy $\upsilon_A \in V_A$ and a response strategy $\upsilon_{\overline{A}} \in V_{\overline{A}}$ of agents A, for every state $q \in Q$, $\mathsf{Paths}_q^{\upsilon_A, \upsilon_{\overline{A}}}$ is a singleton set. We use $\rho_q^{\upsilon_A, \upsilon_{\overline{A}}}$ to denote the path in $\mathsf{Paths}_q^{\upsilon_A, \upsilon_{\overline{A}}}$.

A PCGS $\mathcal{G} = (\mathsf{Ag}, \mathsf{Act}, Q, \delta, \lambda, q_0)$ is a *Markov decision process* (MDP) if $\mathcal{G}$ is a one-agent PCGS, i.e., $|\mathsf{Ag}| = 1$.

Alternating-time μ-Calculus

Alternating-time μ-calculus (AMC) is a powerful alternating-time temporal logic that is strictly more expressive than ATL and ATL*. We show that PAMC_0 is strong enough to express AMC over CGSs. AMC is an extension of μ-calculus with coalition modalities (Alur, Henzinger, and Kupferman 2002).

Definition 1. *(Alur, Henzinger, and Kupferman 2002) AMC formulae are given by the following grammar:*

$$\phi ::= p \mid \neg p \mid Z \mid \phi \wedge \phi \mid \phi \vee \phi \mid \langle A \rangle \mathbf{X}\phi \mid [A]\mathbf{X}\phi \mid \mu Z.\phi \mid \nu Z.\phi$$

where $p \in \mathbf{AP}$, $Z \in \mathcal{Z}$ and $A \subseteq \mathsf{Ag}$.

The semantics of AMC formulae is defined by the denotation function $\|\circ\|_{\mathcal{G}}^{\xi}$ that maps AMC formulae to sets of states of a CGS $\mathcal{G}$. Formally, given a CGS $\mathcal{G} = (\mathsf{Ag}, \mathsf{Act}, Q, \delta, \lambda, q_0)$, an AMC formula ϕ and a valuation $\xi : \mathcal{Z} \to 2^Q$, $\|\circ\|_{\mathcal{G}}^{\xi}$ is inductively defined as follows:

- $\|p\|_{\mathcal{G}}^{\xi} = \lambda(p)$, $\|\neg p\|_{\mathcal{G}}^{\xi} = Q \setminus \lambda(p)$, $\|Z\|_{\mathcal{G}}^{\xi} = \xi(Z)$;
- $\|\langle A \rangle \mathbf{X}\psi\|_{\mathcal{G}}^{\xi} = \{\, q \in Q \mid \exists \upsilon_A \in V_A,\ \forall \upsilon_{\overline{A}} \in V_{\overline{A}}, \delta(q, \mathbf{d}) \in \|\psi\|_{\mathcal{G}}^{\xi}\}$, where $\upsilon_A(i)(q, \mathbf{d}(i)) = 1$ for every $i \in A$ and $\upsilon_{\overline{A}}(j)(q, \mathbf{d}(j)) = 1$ for every $j \in \overline{A}$;
- $\|[A]\mathbf{X}\psi\|_{\mathcal{G}}^{\xi} = \{\, q \in Q \mid \forall \upsilon_A \in V_A,\ \exists \upsilon_{\overline{A}} \in V_{\overline{A}}, \delta(q, \mathbf{d}) \in \|\psi\|_{\mathcal{G}}^{\xi}\}$, where $\upsilon_A(i)(q, \mathbf{d}(i)) = 1$ for every $i \in A$ and $\upsilon_{\overline{A}}(j)(q, \mathbf{d}(j)) = 1$ for every $j \in \overline{A}$;
- $\|\mu Z.\phi\|_{\mathcal{G}}^{\xi} = \bigcap\{Q' \subseteq Q \mid \|\phi\|_{\mathcal{G}}^{\xi[Z \mapsto Q']} \subseteq Q'\}$;
- $\|\nu Z.\phi\|_{\mathcal{G}}^{\xi} = \bigcup\{Q' \subseteq Q \mid \|\phi\|_{\mathcal{G}}^{\xi[Z \mapsto Q']} \supseteq Q'\}$;
- $\|\phi_1 \wedge \phi_2\|_{\mathcal{G}}^{\xi} = \|\phi_1\|_{\mathcal{G}}^{\xi} \cap \|\phi_2\|_{\mathcal{G}}^{\xi}$;
- $\|\phi_1 \vee \phi_2\|_{\mathcal{G}}^{\xi} = \|\phi_1\|_{\mathcal{G}}^{\xi} \cup \|\phi_2\|_{\mathcal{G}}^{\xi}$.

Given an AMC formula ϕ, let $enc(\phi)$ denote the PAMC_0 formula obtained from ϕ by the following recursive transformation: where $\circ \in \{\wedge, \vee\}$,

$$\begin{array}{ll} enc(p) = p & enc(\phi_1 \circ \phi_2) = enc(\phi_1) \circ enc(\phi_2) \\ enc(\neg p) = \neg p & enc(\langle A \rangle \mathbf{X}\phi) = \langle A \rangle^{>0} \mathbf{X} enc(\phi) \\ enc(Z) = Z & enc([A]\mathbf{X}\phi) = [A]^{>0} \mathbf{X} enc(\phi) \\ enc(\mu Z.\phi) = \mu Z.enc(\phi) & enc(\nu Z.\phi) = \nu Z.enc(\phi) \end{array}$$

Theorem 1. *For every CGS $\mathcal{G}$ and every AMC formula ϕ, $\|\phi\|_{\mathcal{G}}^{\xi} = \|enc(\phi)\|_{\mathcal{G}}^{\xi}$.*

It is known that AMC is more expressive than ATL and ATL* (Alur, Henzinger, and Kupferman 2002). It follows that PAMC_0 is strong enough to express all standard alternating-time temporal logics.

Probabilistic Alternating-time Temporal Logic

Probabilistic alternating-time temporal logic (PATL*) is the first probabilistic extension of the alternating-time temporal logic ATL* (Chen and Lu 2007; Schnoor 2010; Huang, Su, and Zhang 2012).

Definition 2. *PATL* formulae are given by the following grammar: where Φ are state formulae, Ψ are path formulae,*

$$\begin{array}{rl} \Phi ::= & p \mid \neg p \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid \langle A \rangle^{\sim c} \Psi \\ \Psi ::= & \Phi \mid \Psi \wedge \Psi \mid \Psi \vee \Psi \mid \mathbf{X}\Psi \mid \Psi \mathbf{U} \Psi \mid \mathbf{R}\Psi \end{array}$$

where $p \in \mathbf{AP}$, $Z \in \mathcal{Z}$, $A \subseteq \mathsf{Ag}$, $\sim \in \{\geq, >\}$ and $c \in [0, 1]$.

Remark 1. *The original definition of PATL* in (Chen and Lu 2007) allows $\sim$ to be $=$, which is dropped in the later works (Schnoor 2010; Huang, Su, and Zhang 2012). PAMC disallows $\sim$ to be $=$. Therefore, we drop $=$ in the definition of PATL*.*

The semantics of path formulae and state formulae of PATL* are given by the denotation function $\|\circ\|_{\mathcal{G}}^{\xi}$ and $\|\circ\|_{\mathcal{G}}^{\xi,\rho}$ which are inductively defined similar as PAMC, in which $\|\psi_1 \mathbf{U} \psi_2\|_{\mathcal{G}}^{\xi,\rho} \equiv \|\mu Z.\psi_2 \vee (\psi_1 \wedge \mathbf{X}Z)\|_{\mathcal{G}}^{\xi,\rho}$ and $\|\psi_1 \mathbf{R} \psi_2\|_{\mathcal{G}}^{\xi,\rho} \equiv \|\nu Z.(\psi_1 \wedge \psi_2) \vee (\psi_2 \wedge \mathbf{X}Z)\|_{\mathcal{G}}^{\xi,\rho}$.

Given a PATL* formula ϕ, let $enc(\phi)$ denote the PAMC formula obtained from ϕ by the following recursive transformation:

$$\begin{array}{ll} enc(p) = p & enc(\phi_1 \wedge \phi_2) = enc(\phi_1) \wedge enc(\phi_2) \\ enc(\neg p) = \neg p & enc(\phi_1 \vee \phi_2) = enc(\phi_1) \vee enc(\phi_2) \\ enc(\mathbf{X}\phi) = \mathbf{X} enc(\phi) & enc(\langle A \rangle^{\sim c} \mathbf{X}\phi) = \langle A \rangle^{\sim c} \mathbf{X} enc(\phi) \end{array}$$

$$enc(\phi_1 \mathbf{U} \phi_2) = \mu Z.enc(\phi_2) \vee (enc(\phi_1) \wedge \mathbf{X}Z)$$
$$enc(\phi_1 \mathbf{R} \phi_2) = \nu Z.enc(\phi_1 \wedge \phi_2) \vee (enc(\phi_2) \wedge \mathbf{X}Z)$$

PATL is a subclass of PATL* in which path formulae are given by: $\Psi ::= \mathbf{X}\Phi \mid \Phi \mathbf{U} \Phi \mid \mathbf{R}\Phi$, where Φ are state formulae. Similar to the transformation from PATL* to PAMC, PATL formulae can be encoded into PAMC_1.

Lemma 1. *Every PATL* (resp. PATL) state formula ϕ, we can construct a PAMC (resp. PAMC_1) formula $enc(\phi)$ such that $\|\phi\|_{\mathcal{G}} = \|enc(\phi)\|_{\mathcal{G}}$, for all PCGSs $\mathcal{G}$.*

Similar to the relation between AMC and ATL*, fixpoint operators cannot be expressed in PATL*, we get that:

Theorem 2. *PAMC is strictly more expressive than PATL*, and PAMC_1 is strictly more expressive than PATL.*

μ-PCTL

μ-PCTL was introduced by (Castro, Kilmurray, and Piterman 2015) for reasoning about probabilistic systems, like Markov chain and Markov decision processes. μ-PCTL is able to encode several well-known probabilistic such as PCTL and PμTL (Liu et al. 2015).

Definition 3 (μ-PCTL)**.** *The syntax of probabilistic μ-PCTL is given by the following grammar, where Φ are* state formulae*, Ψ are* path formulae*,*

$$\Phi ::= p \mid \neg p \mid Z \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid \mu Z.\Phi \mid \nu Z.\Phi \mid [\Psi]^{\sim c}$$
$$\Psi ::= \mathbf{X}\Phi \mid \Phi\mathbf{U}\Phi \mid \Phi\mathbf{R}\Phi$$

where $p \in \mathbf{AP}$, $Z \in \mathcal{Z}$, $c \in [0,1]$ and $\sim \in \{\geq, >\}$.

The semantics of μ-PCTL formulae is defined over MDPs (note that the semantics is defined over Markov chains in (Castro, Kilmurray, and Piterman 2015)). Given a MDP $\mathcal{G} = (\mathsf{Ag}, \mathsf{Act}, Q, \delta, \lambda, q_0)$, a state formula ϕ and a valuation $\xi : Z \to 2^Q$, the denotation function $\| \circ \|^{\xi}_{\mathcal{G}}$ that maps state formulae to sets of states is inductively defined as same as PAMC. The semantics of path formulae is defined similar to PAMC$_1$.

Given a μ-PCTL formula ϕ, it is easy to transform ϕ into an equivalent PAMC$_1$ formula $enc(\phi)$ using $enc([\psi]^{\sim c}) = \langle \mathsf{Ag} \rangle^{\sim c} enc(\psi)$.

Theorem 3. *For every MDP $\mathcal{G}$ and every μ-PCTL state formula ϕ, $\|\phi\|^{\xi}_{\mathcal{G}} = \|enc(\phi)\|^{\xi}_{\mathcal{G}}$.*

B Memorability and Determinacy

In this section, we show that the memoryless property of PAMC$_1$, hence PAMC$_0$ as well. However, in general, PAMC does not have memoryless property. For determinacy, we show that there is no difference between randomized and deterministic under memoryful setting for PAMC. However, allowing $\sim \in \{\geq, >, =\}$ or only considering memoryless strategies will lose this determinacy property.

Memoryless vs. Memoryful

Proposition 1. *Given a principal formula $\phi = \langle A \rangle^{\sim c}\mathbf{X}\psi$, and the coalition strategy υ_A and the response strategy $\upsilon_{\overline{A}}$ of A, for every state $q \in Q$, we have the following property:* $\mathsf{Pr}^{\upsilon_A,\upsilon_{\overline{A}}}_q(\{\rho \in \mathsf{Paths}^{\upsilon_A,\upsilon_{\overline{A}}}_q \mid \rho_1 \in \|\psi\|^{\xi}_{\mathcal{G}}\}) = \sum_{\mathbf{d}\in\mathcal{D}, q'\in\|\psi\|^{\xi}_{\mathcal{G}}} \mathsf{Pr}^{\upsilon_A,\upsilon_{\overline{A}}}(q,\mathbf{d}) \cdot \delta(q,\mathbf{d},q')$.

Proof. The result immediately follows from the fact that $\{\rho \in \mathsf{Paths}^{\upsilon_A,\upsilon_{\overline{A}}}_q \mid \rho_1 \in \|\psi\|^{\xi}_{\mathcal{G}}\} = \bigcup_{q'\in\|\psi\|^{\xi}_{\mathcal{G}}} \mathsf{Cyl}^{\upsilon_A,\upsilon_{\overline{A}}}(qq')$. □

Proposition 2. *Given a PCGS $\mathcal{G} = (\mathsf{Ag}, \mathsf{Act}, Q, \delta, \lambda, q_0)$ and a closed PAMC$_1$ state formula ϕ, let $\|\phi\|^R_{\mathcal{G}}$ and $\|\phi\|^r_{\mathcal{G}}$ respectively denote the set of states satisfying ϕ under memoryful and memoryless settings. Then, $\|\phi\|^R_{\mathcal{G}} = \|\phi\|^r_{\mathcal{G}}$.*

Proof. By applying induction on structure, it is sufficient to show the cases for $\phi = \langle A \rangle^{\sim c}\mathbf{X}\psi$, $\phi = \langle A \rangle^{\sim c}\psi_1\mathbf{U}\psi_2$ and $\phi = \langle A \rangle^{\sim c}\psi_1\mathbf{R}\psi_2$.

Let us consider the case $\phi = \langle A \rangle^{\sim c}\mathbf{X}\psi$. Since $q \in \|\langle A \rangle^{\sim c}\mathbf{X}\psi\|^{\xi}_{\mathcal{G}}$ iff there exists $\upsilon_A \in V_A$ such that for all $\upsilon_{\overline{A}} \in V_{\overline{A}}$:

$$\mathsf{Pr}^{\upsilon_A,\upsilon_{\overline{A}}}_q(\{\rho \in \mathsf{Paths}^{\upsilon_A,\upsilon_{\overline{A}}}_q \mid \rho_1 \in \|\phi\|^{\xi}_{\mathcal{G}}\}) \sim c.$$

By applying Proposition 1, we get that $p \in \|\langle A \rangle^{\sim c}\mathbf{X}\psi\|^{\xi}_{\mathcal{G}}$ iff there exists $\upsilon_A \in V_A$ such that for all $\upsilon_{\overline{A}} \in V_{\overline{A}}$:

$$\sum_{\mathbf{d}\in\mathcal{D}, q'\in\|\psi\|^{\xi}_{\mathcal{G}}} \mathsf{Pr}^{\upsilon_A,\upsilon_{\overline{A}}}(q,\mathbf{d}) \cdot \delta(q,\mathbf{d},q') \sim c,$$

in which only the distributions $\upsilon_A(i)(q)$ and $\upsilon_A(j)(q)$ for $i \in A,\ j \in \overline{A}$ make sense. The result immediately follows.

To prove the case $\phi = \langle A \rangle^{\sim c}\psi_1\mathbf{U}\psi_2$, we first consider $\phi = \langle A \rangle^{\sim c}\psi_1\mathbf{U}^{\leq k}\psi_2$, where $\mathbf{U}^{\leq k}$ denotes that the state formula ϕ_2 should be fulfilled after at most k steps. Since $q \in \|\langle A \rangle^{\sim c}\psi_1\mathbf{U}\psi_2\|^{\xi}_{\mathcal{G}}$ iff $v = \max\limits_{\upsilon_A\in V_A} \min\limits_{\upsilon_{\overline{A}}\in V_{\overline{A}}} \mathsf{Pr}^{\upsilon_A,\upsilon_{\overline{A}}}_q(\{\rho \in \mathsf{Paths}^{\upsilon_A,\upsilon_{\overline{A}}}_q \mid \rho_0 \in \|\psi_1\mathbf{U}^{\leq k}\psi_2\|^{\xi}_{\mathcal{G}}\}) \sim c$. We have that

1. $v = 1$ if $q \in \|\langle A \rangle^{\sim c}\psi_2\|^{\xi}_{\mathcal{G}}$,
2. $v = 0$ if $q \notin \|\langle A \rangle^{\sim c}\psi_1\|^{\xi}_{\mathcal{G}} \cup \|\langle A \rangle^{\sim c}\psi_2\|^{\xi}_{\mathcal{G}}$,
3. $v = 0$ if $q \notin \|\langle A \rangle^{\sim c}\psi_1\|^{\xi}_{\mathcal{G}} \setminus \|\langle A \rangle^{\sim c}\psi_2\|^{\xi}_{\mathcal{G}}$ and $k = 0$,
4. $v = \max\limits_{\upsilon_A\in V_A} \min\limits_{\upsilon_{\overline{A}}\in V_{\overline{A}}} \sum_{\mathbf{d}\in\mathcal{D}, q'\in\|\psi\|^{\xi}_{\mathcal{G}}} \mathsf{Pr}^{\upsilon_A,\upsilon_{\overline{A}}}(q,\mathbf{d}) \cdot \delta(q,\mathbf{d},q') \cdot \mathsf{Pr}^{\upsilon_A,\upsilon_{\overline{A}}}_{q'}(\{\rho \in \mathsf{Paths}^{\upsilon_A,\upsilon_{\overline{A}}}_{q'} \mid \rho_0 \in \|\psi_1\mathbf{U}^{\leq k-1}\psi_2\|^{\xi}_{\mathcal{G}}\})$, otherwise.

By applying the induction hypothesis, $\|\psi_1\|^R_{\mathcal{G}} = \|\psi_1\|^r_{\mathcal{G}}$ and $\|\psi_2\|^R_{\mathcal{G}} = \|\psi_2\|^r_{\mathcal{G}}$. If agents $\overline{A}$ can make actions such that either Item 2 or Item 3 holds for a given coalition strategy υ_A, then $\upsilon_{\overline{A}}$ uses these actions under the track q. While agents A try to make actions such that Item 1 holds for any $\upsilon_{\overline{A}}$. In these two cases, the result follows. Otherwise, we iteratively expand $\mathsf{Pr}^{\upsilon_A,\upsilon_{\overline{A}}}(q,\mathbf{d}) \cdot \delta(q,\mathbf{d},q') \cdot \mathsf{Pr}^{\upsilon_A,\upsilon_{\overline{A}}}_{q'}(\{\rho \in \mathsf{Paths}^{\upsilon_A,\upsilon_{\overline{A}}}_{q'} \mid \rho_0 \in \|\psi_1\mathbf{U}^{\leq k-1}\psi_2\|^{\xi}_{\mathcal{G}}\}$ in Item 4 and continue select actions according Items 1, 2 and 3. This gives us memoryless strategies.

For the unbounded case $\phi = \langle A \rangle^{\sim c}\psi_1\mathbf{U}\psi_2$, memoryless strategies are also sufficient by considering $\lim\limits_{k\to\infty} \max\limits_{\upsilon_A\in V_A} \min\limits_{\upsilon_{\overline{A}}\in V_{\overline{A}}} \mathsf{Pr}^{\upsilon_A,\upsilon_{\overline{A}}}_q(\{\rho \in \mathsf{Paths}^{\upsilon_A,\upsilon_{\overline{A}}}_q \mid \rho_0 \in \|\psi_1\mathbf{U}^{\leq k}\psi_2\|^{\xi}_{\mathcal{G}}\}) \sim c$.

The case $\phi = \langle A \rangle^{\sim c}\psi_1\mathbf{R}\psi_2$ can be seen as $\phi = \langle A \rangle^{\sim c}\neg(\neg\psi_1\mathbf{U}\neg\psi_2)$ where $\neg$ at the beginning of ψ_1 and ψ_2 can be pushed inside as usual. The proof follows the lines for $\mathbf{U}$. □

Remark 2. *The memoryless property in Proposition 2 only holds for model-checking rather than* strategy synthesis *which is the problem of computing strategies of agents to fulfill the given formula. Consider the PAMC$_1$ formula $\phi = \langle\{1\}\rangle^{\geq 1}\mathbf{X}(p \wedge \langle\{1\}\rangle^{\geq 1}\mathbf{X}\langle\{1\}\rangle^{\geq 1}\mathbf{X}\neg p)$ and the PCGS $\mathcal{G} = (\{1\}, \{a_1, a_2\}, \{q_0, q_1\}, \delta, \lambda, q_0)$, where*

- $\delta(q_0, a_1, q_1) = 1$,
- $\delta(q_0, a_2, q_0) = 1$,
- $\delta(q_1, \sigma, q_0) = 1$ *for* $\sigma \in \{a_1, a_2\}$,
- $\lambda(p) = \{q_1\}$.

It is easy to see that $q_0 \in ||\phi||_{\mathcal{G}}$ under memoryless setting, as agent 1 can choose difference action at state q_0 to fulfill ϕ and the subformula $\langle\{1\}\rangle^{\geq 1}\mathbf{X}\langle\{1\}\rangle^{\geq 1}\mathbf{X}\neg p$. However, it is impossible to synthesize a memoryless strategy for agent 1 to fulfill ϕ.

We now show that PAMC does not have memoryless property even under deterministic setting by the following example.

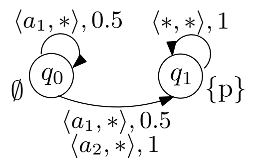

Figure 1: Example for memoryless property on PAMC, where $* \in \{a_1, a_2\}$.

Example 1. *Let us consider the PCGS $\mathcal{G} = (\{1,2\}, \{a_1, a_2\}, \{q_0, q_1\}, \delta, \lambda, q_0)$ as shown in Figure 1, where for every $* \in \{a_1, a_2\}$,*

- $\delta(q_0, \langle a_1, *\rangle, q_0) = 0.5$ *and* $\delta(q_0, \langle a_1, *\rangle, q_1) = 0.5$,
- $\delta(q_0, \langle a_2, *\rangle, q_1) = 1$,
- $\delta(q_1, \mathbf{d}, q_1) = 1$ *for every* $\mathbf{d} \in \{a_1, a_2\}^2$,
- $\lambda(p) = \{q_1\}$.

Consider the closed PAMC state formula $\phi = \langle\{1\}\rangle^{\geq 0.5}\mathbf{X}(\neg p \wedge \mathbf{X}p)$ which states that agent 1 has a strategy such that whatever agent 2 does, the probability to achieve the goal $\mathbf{X}(\neg p \wedge \mathbf{X}p)$ is no less than 0.5. It is easy to see that there is no memoryless strategy for agent 1 to achieve the goal, i.e., $||\phi||^r_{\mathcal{G}} = \emptyset$. While $||\phi||^R_{\mathcal{G}} = \{q_0\}$, as agent 1 can use the strategy $\upsilon_{\{1\}}$ such that $\upsilon_{\{1\}}(q_0)(a_1) = 1$ and $\upsilon_{\{1\}}(q_0q_0)(a_2) = 1$.

Randomized vs. Deterministic

Theorem 4. *Given a closed PAMC state formula ϕ, let $||\phi||^p_{\mathcal{G}}$ and $||\phi||^d_{\mathcal{G}}$ respectively denote the set of states satisfying ϕ under randomized and deterministic settings. Then, $||\phi||^p_{\mathcal{G}} = ||\phi||^d_{\mathcal{G}}$ under memoryful setting.*

Proof. The proof is straightforward by induction on the structure of ϕ. We only need to consider principal formulae of the form $\langle A\rangle^{\sim c}\psi$. The proof for the case $\langle A\rangle^{\sim c}\psi$ follows from Theorem 2 and Lemma 1. □

Let PAMC $^=$ be the logic by setting $\sim \in \{\geq, >, =\}$ in the PAMC. Then, Theorem 4 will not hold for PAMC $^=$. We show this by the following example.

Example 2. *Let us consider the PCGS $\mathcal{G} = (\{1,2\}, \{a_1, a_2\}, \{q_0, q_1\}, \delta, \lambda, q_0)$ as shown in Figure 1 and the closed PAMC $^=$ state formula $\phi = \langle\{1\}\rangle\mathbf{X}^{=0.75}p$.*

In randomized setting, agent 1 has a strategy $\upsilon_{\{1\}}$ such that for all strategies $\upsilon_{\{2\}}$ of agent 2, $\mathsf{Pr}^{\upsilon_{\{1\}},\upsilon_{\{2\}}}_{q_0}(\{\rho \in \mathsf{Paths}^{\upsilon_{\{1\}},\upsilon_{\{2\}}}_{q_0} \mid \rho_1 \in ||p||^{\xi}_{\mathcal{G}}\}) = 0.75$, where $\upsilon_{\{1\}}(\pi)(a_1) = 0.5$ and $\upsilon_{\{1\}}(\pi)(a_2) = 0.5$. Therefore $q_0 \in ||p||^{\xi}_{\mathcal{G}}$. While, in deterministic setting, it is easy to see that $q_0 \notin ||p||^{\xi}_{\mathcal{G}}$.

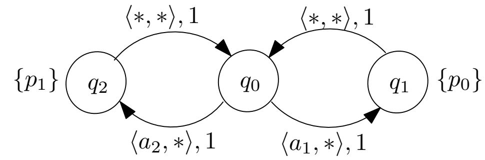

Figure 2: Example for determinacy, where $* \in \{a_1, a_2\}$.

Under memoryless setting, PAMC does not have determinacy property as shown by the following example.

Example 3. *Let us consider the PCGS $\mathcal{G} = (\{1,2\}, \{a_1, a_2\}, \{q_0, q_1, q_3\}, \delta, \lambda, q_0)$ as shown in Figure 2 and the closed PAMC state formula $\phi = \langle\{1\}\rangle^{>0}(\mu Z.p_1 \wedge \mu Y.p_0)$. Under randomized memoryless setting, $q_0 \in ||\phi||^{\xi}_{\mathcal{G}}$, while $q_0 \notin ||\phi||^{\xi}_{\mathcal{G}}$ under deterministic memoryless setting.*

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