Complexities of Automata, Games, Synthesis

Sasha

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1 Model Checking

LTL, CTL* PSPACE
CTL, PTIME

2 Satisfiability

LTL, PSPACE
CTL*, ??
CTL, ??

3 Solving games

States n, Transitions m

3.1 Perfect Information

Reachability $O(m)$
Buchi $O(mn)$
Parity $O(n^{d-1}m)$, $O((\frac{2n}{d})^{\lfloor \frac{d}{2} \rfloor} m)$
LTL $2^n 2^{2^{O(|\psi|)}}$

3.2 Imperfect Information

Reachability EXPTIME [2, ?], belief-space
Buchi EXPTIME [2], belief-space
Parity ?? nondeterministic exptime ??
LTL 2EXPTIME, [?]

4 Formulas to automata

LTL to ABW, linear
LTL to NBW,UCW $2^{O(|\phi|)}$
CTL, alternation-free μ-calculus to AWT, linear (KVW00)
CTL* to ART, exponentially many states, ??linearly?? many pairs [1]
CTL* to HAT, exponential states, linear depth KVW00
μ-Calculus to ART, linear (EJ)

μ-C to APT, linear size, number of colors is alternation depth, KVW00

5 Automata to Automata

n states, l priorities, r pairs
NBW to DPW, 2^n states, n priorities [4]
NBWs to DRW, 2^n states, $O(n)$ pairs (Safra)
APT (or 2APT) to NPT, $2^{nl\log(nl)}$ states, $O(nl)$ priorities [3, **?**]
ART to NRT, $(nr)^{O(nr)}$ states and $O(nr)$ pairs [3]
ABT to NBT, ??Exp?? [3]

6 Membership of Automata

NPT, $\text{NP} \cap co - \text{NP}$, equivalent to solving parity games, so $O(n^{d-1}m)$.
ART, NP[**?**]
AWT, linear [**?**]

7 Emptiness of Automata

n states, m size transition, d degree of trees, p priorities, l input letters
DPW = parity game, $O(n^{p-1}m)$
NRW costs $(np)^{O(p)}$ [5]
NRT $(lnp)^{O(p)}$ [5]
NPT $O(dm(n+m)^p)$, [**?**]
NPT $(|\Sigma|n^{O(d)})^{O(p)}$, [**?**]

7.1 Alternating

APT = satisfiability of μ-calculus, EXPTIME, (Chapter 9 of Automata, Logic, Games)
APT,AWT,AHT, fixed degree d, EXPTIME-c (survey by Vardi, Esparza; MSS86, MS95, FL79, Sei90)

7.1.1 1-letter

These reduce to emptiness of 1-letter alternating word automata with same condition, i.e., 2 player games.
1APW = parity game = NPintersect CO-NP
1AWW = linear time
1AHW = linear time, space $O(a \log^2 b)$ where a is depth and b is size (see survey)

8 Synthesis Perfect-Information

Alphabets: Input I, Output O, Hidden environment E.

8.1 LTL

2EXPTIME Lower Bound [6]
2EXPTIME Upper Bound [5]
Method 1 - Orna-Moshe?

- Translate Φ to a UCW by dualising the Vardi Wolper. This UCW has 1EXP many states.
- Translate the UCW to a UCT by running it on all paths. No blowup.
- Test the UCT for emptiness. The total cost is 2EXP.

Method 2 - Pnueli-Rosner

- Translate Φ to a UBW by Vardi Wolper. This UBW has 1EXP many states.
- Translate the UBW to a DRW by Safra. This DRW has 2EXP states, 1EXP rabin-pairs.
- Translate the DRW to a DRT by running it on all paths. No blowup.
- Test the DRT for emptiness. The total cost is $(2EXP1EXP)^{O(1EXP)} = 2EXP$.

8.2 CTL*

8.3 CTL

EXPTIME Lower from satisfiability [?]
EXPTIME Upper [?]

9 Synthesis Imperfect-Information

9.1 LTL

2EXPTIME Lower Bound from Perfect-Information.
2EXPTIME Upper Bound [?]
Build ABT that accepts strategy $\sigma : I^+ \to O$ iff every play consistent with σ satisfies Φ.

1. Translate Φ to a UBW by Vardi Wolper. This UBW has 1EXP many states.
2. Build UBT that runs on t_σ, stores in its memory the belief-state, and universally launches the UBW in all directions. Blowup by size of largest belief-state, $2^{|E|}$ or $|E|^B$.
3. Emptiness of ABT is exponential. Total cost $2^{2^{O(|\varphi|+|E|)}}$ or $2^{2^{O(|\varphi|)}+|E|^B}$.

9.2 CTL*

9.3 CTL

10 Automata Problem = Game Problem

10.1 Emptiness

?? Right??
?? It seems these are equivalence. i.e., games can be thought of as emptiness problems of automata ??

1. Nondet word emptiness = solving one player game.
2. Alt word emptiness = solving two player blind game ($\exists$-player does not see the state of the automaton)
3. Nondet tree emptiness = solving two player game.
4. Alt tree emptiness = solving two player game of imperfect information in which observation sets correspond to directions (i.e., $\exists$-player does not see automaton-state, only tree-direction).

10.2 Membership

Membership of Alternating Tree Automata are two player games.

References

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[3] D. E. Muller and P. E. Schupp. Simulating alternating tree automata by nondeterministic automata: New results and new proofs of the theorems of Rabin, McNaughton and Safra. *Theoretical Computer Science*, 141:69–107, 1995.

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[6] R. Rosner. *Modular Synthesis of Reactive Systems*. PhD thesis, Weizmann Institute of Science, 1992.