Combining Qualitative and Quantitative Reasoning for Logic-based Games — Case for Support

Michael Wooldridge and **Enrico Marchioni**

University of Oxford

Research Track Record

The project will be undertaken within the **Department of Computer Science** at the **University of Oxford**, with **Professor Michael Wooldridge** acting as PI and **Dr Enrico Marchioni** as the named RA and co-investigator. Wooldridge will provide expertise on logic-based multi-agent systems in general, and Boolean games in particular; Marchioni will provide expertise on the particular formalism used throughout the project (Łukasiewicz logics). Wooldridge and Marchioni already have a successful track record of relevant collaboration, having published work directly relevant to the proposed project [19]. ***Note that Wooldridge is fully funded through the closely connected ERC Advanced Grant 291528 ("RACE") for the period of the proposed project, and hence requests no funding for it.***

The Department of Computer Science at the University of Oxford (formerly known as the Computer Laboratory) has grown rapidly over the past decade, and now has a complement of approx 55 full-time academic staff and 130 postdoctoral research assistants. The Department has strengths in several areas related to the proposed project: logical foundations of computing (Professors Abramsky and Ong), model checking/verification (Professors Kwiatkowska, Kroening, Ouaknine, Worrell) and game theory/computation (Professors Goldberg and Koutsoupias).

Michael Wooldridge has been a Professor in the Department of Computer Science at the University of Oxford since 2012. Before this he was for 12 years a Professor at the University of Liverpool. He gained his PhD in 1992 for research in the logical foundations of multi-agent systems, and since then has published more than 300 articles on this and related topics. In 2006 he was the recipient of the *ACM Autonomous Agents Research Award* (given annually by ACM to an individual whose research has been particularly influential in the area of autonomous agents and multi-agent systems). He was elected an ECCAI Fellow in 2007 and a AAAI Fellow in 2008. He was general chair for the AAMAS-2003 conference, program chair for AAMAS-2005, program chair for ECAI-2010, and will be general chair for IJCAI-2015. He has served as editor-in-chief of the journal *Autonomous Agents and Multi-Agent Systems*, as an associate editor for *Journal of AI Research* (JAIR), and as an editor of *Artificial Intelligence* journal, *Journal of Logic & Computation*, *Journal of Applied Logic*, *Computational Intelligence*, and *Knowledge & Information Systems*. Wooldridge has authored/co-authored seven books, including: *An Introduction to Multi-Agent Systems* (Chinese translation 2003, Greek translation 2008, second edition 2009); *Computational Aspects of Cooperative Game Theory* (Morgan-Claypool 2011; co-authored with Chalkiadakis and Elkind); and *Principles of Automated Negotiation* (Cambridge UP 2014; co-authored with Fatima and Kraus). Wooldridge has been co-recipient of the AAMAS conference best paper award three times: in 2008, 2009, and 2011.

Wooldridge's work has more than 44,000 citations on Google Scholar[1]; Microsoft Academic Search ranks him in the top 15 most cited AI researchers[2], and in the top 100 most cited computer scientists overall[3]. In 2011, he was awarded a five-year ERC Advanced Investigator Grant (project 291528: "RACE"). This project is investigating logical methods for reasoning about economic equilibrium properties of game-like concurrent systems/multi-agent systems, and as such is very closely related with the proposed project. The proposed project will benefit enormously from the existing team of postdoc RAs working on the RACE project in the area of logic, verification, and game theory (Julian Gutierrez, Paul Harrenstein, Tomasz Michalak).

[1] http://tinyurl.com/d9y73zj
[2] http://tinyurl.com/ojsz2lc
[3] http://tinyurl.com/m7ncbhw

Enrico Marchioni is a Marie Curie Postdoctoral Research Fellow in Engineering and Information Sciences at the Institut de Recherche en Informatique de Toulouse (IRIT), Paul Sabatier University, France. He obtained a PhD from the University of Salamanca (Spain) in 2006. Prior to his current appointment at IRIT, he was a lecturer at the Open University of Catalonia, Spain, (2007-2008) and a postdoctoral research fellow at the Artificial Intelligence Research Institute (IIIA) of the Spanish National Research Council (2008-2012). His research at IIIA was funded by a three-year Juan de la Cierva Research Fellowship in Computer and Information Technologies awarded by the Spanish Ministry of Science and Innovation after his application was ranked first in the area. In 2012 he was awarded a Marie Curie Intra-European Research Fellowship in the area of Engineering and Information Sciences to carry out the individual research project Non-Additive Axiomatic Models of Strategic Interaction.

SELECTED RECENT PUBLICATIONS

Wooldridge:

- J. Gutierrez and M. Wooldridge. Equilibria of Concurrent Games on Event Structures. In *LICS–2014*, Vienna 2014.
- J. Gutierrez, P. Harrenstein, and M. Wooldridge. Reasoning about Equilibria in Game-like Concurrent Systems. In *KR–2014*, July 2014.
- J. Grant, S. Kraus, M. Wooldridge, I. Zuckerman. Manipulating games by sharing information. In *Studia Logica*, 102:267–295, 2014.
- J. Gutierrez, P. Harrenstein, and M. Wooldridge. Iterated Boolean Games. In *IJCAI-2013*, August 2013.
- W. van der Hoek, C. Witteveen, M. Wooldridge. Program equilibrium–a program reasoning approach. In *International Journal of Game Theory*, 42(3)639–671, 2013.
- M. Wooldridge, U. Endriss, S. Kraus, J. Lang. Incentive engineering for Boolean games. In *Artificial Intelligence*, 195:418–439, 2013.
- N. Hazon, Y. Aumann, S. Kraus, M. Wooldridge. On the evaluation of election outcomes under uncertainty. In *Artificial Intelligence*, 189:1–18, 2012.
- G. Chalkiadakis, E. Elkind, M. Wooldridge. Computational Aspects of Cooperative Game Theory. *Morgan & Claypool*, 2011.
- N. Troquard, W. van der Hoek, M. Wooldridge. Reasoning About Social Choice Functions. In *Journal of Philosophical Logic*, 40:473–498, 2011.
- P. E. Dunne, P. McBurney, S. Parsons, M. Wooldridge. Weighted Argument Systems: Basic Definitions, Algorithms, and Complexity Results. In *Artificial Intelligence*, 175:457-486, 2011.
- W. van der Hoek, D. Walther, M. Wooldridge. Reasoning about the Transfer of Control. In *JAIR*, 37:437-477, 2010.
- P. E. Dunne, S. Kraus, E. Manisterski, M. Wooldridge. Solving Coalitional Resource Games. In *Artificial Intelligence*, 174:20-50, 2010.
- J. Ruan, W. van der Hoek, and M. Wooldridge. Verification of Games in the Game Description Language. In *Journal of Logic and Computation*. 19(6):1127-1156, 2009.
- E. Elkind, L. Goldberg, P. Goldberg, and M. Wooldridge. On the computational complexity of weighted voting games. In *Annals of Mathematics and Artificial Intelligence*, 56(2):109-131, June 2009.
- T. Ågotnes, W. van der Hoek, and M. Wooldridge. Reasoning about Coalitional Games. In *Artificial Intelligence*, 173(1):45-79, January 2009.

Marchioni:

- E. Marchioni and M. Wooldridge. Łukasiewicz Games. *Proc. AAMAS 2014*. Paris, France, 2014.
- G. Lenzi and E. Marchioni. An algebraic characterization of o-minimal and weakly o-minimal MV-chains. *Journal of Pure and Applied Algebra*, 218:90–100, 2014.
- T. Flaminio, L. Godo, E. Marchioni. On the logical formalization of possibilistic counterparts of states over n-valued Łukasiewicz events. *Journal of Logic and Computation*, 21(3): 429–446, 2011.
- E. Marchioni. On computational complexity of semilinear varieties. *Journal of Logic and Computation*, 18(6): 941–958, 2008.
- E. Marchioni and F. Montagna. Complexity and definability issues in $\text{Ł}\Pi\frac{1}{2}$. *Journal of Logic and Computation*, 17(2): 311–331, 2007.

Background

In the past two decades, logic-based automated verification techniques such as model checking have made the transition from being of purely theoretical interest to being of practical applicability for the analysis of real-world computer systems. A standard question asked in model checking is whether a given system can in principle behave in such a way that it satisfies a property, represented as a temporal logic formula. In our work, we are interested in extending these techniques to the specification and verification of systems in which *the system components are self-interested, and strive to achieve personal goals that may not be compatible with the goals of others*. If we aim to analyse such a system, then whether a particular system behaviour is feasible *in principle* is not the most important question. Instead, we need to ask *whether the behaviour will result under the assumption that system participants act rationally in pursuit of their personal goals*. The standard mathematical framework within which to formulate such questions is that of *game theory*: the mathematical theory of self-interested interacting agents. Game theory suggests that, in the analysis of a system of self-interested individuals, the most important question is whether a particular behaviour represents an *equilibrium* of the system: a condition that obtains because no participant has any rational incentive to deviate from it. To this end, game theory has defined a range of different equilibrium concepts, of which Nash equilibrium is probably the best-known and most widely applied. Now, while several logical formalisms have been successfully introduced to model and reason about the strategic interactions between players and coalitions of players in game-like systems [1, 6, 23], these formalisms have focussed on players' strategies and their outcomes, and largely ignore players' preferences and the equilibria that may result from players acting in pursuit of these. One reason for this is that the standard logics used in the analysis of systems (e.g., LTL, CTL, ATL, ...) are well suited to express *qualitative* properties of systems, but are not well suited to express *quantitative* properties. They can only easily capture situations where agents have dichotomous preferences, i.e., where a player is simply attempting to achieve a goal, and is satisfied if their goal is achieved, and unsatisfied otherwise. This contrasts with the standard approach in game theory, in which participant preferences are specified through real-valued utility functions. Thus, while in game-theoretic models players' objectives are typically expressed *quantitatively*, logical models based on Boolean logic are inherently *qualitative* in nature.

The primary aim of this project is to bridge this gap between utility-based game theoretic models and logical models by defining and evaluating new mathematical and computational frameworks that combine the quantitative and qualitative aspects of both approaches in a unified framework.

Some proposals to enrich logical models by adding numerical values to represent payoffs and their maximisation have already been put forward (see [5]). The logical systems defined are essentially expansions of known modal logics for multi-player games, obtained by adding arithmetical constraints to the language. These logics make it possible to express the fact that certain (coalitions of) agents bring about an outcome with a guaranteed payoff, represented by a classical modal two-valued formula. There have also been many attempts to formulate preference logics, typically as variants of modal logic. However, it is generally acknowledged that existing proposals have substantial limitations [4].

Our approach is radically different. Instead of starting with logics built on classic two-valued formulas, we aim at building our theories on *many-valued Łukasiewicz logics* [7, 21]. Łukasiewicz logics are multiple-valued logics which can be understood as generalisations of classical two-valued logics, and yet which have the following enormously valuable property: *every continuous piecewise linear polynomial functions with integer coefficients can be represented as a formula of Łukasiewicz logic*. Looking at our interests, we can thus interpret a Łukasiewicz formula as a compact, logical specification of a utility function; and in particular, Łukasiewicz logics allow for much richer utility functions than is the case if we use classical logics. **It is the fact that Łukasiewicz formulae can be understood as logical specifications of continuous piecewise linear polynomial functions with integer coefficients that makes them of particular interest to the present project: other multiple valued logics are certainly available, but Łukasiewicz is by far the best understood logic with this property.**

Overall, the aim of this project is to develop a novel approach to the representation of strategic abilities of agents in game-like scenarios. By encoding quantitative properties of the game-theoretic models with many-valued Łukasiewicz logics, we can construct a far richer logical theory of game-theoretic multi-agent systems, which will offer greater expressive power while retaining the elegance and simplicity of the logical approach.

Related work: Two bodies of work are related to the proposed project: the first is that which attempts to extend logics with an apparatus for quantitative reasoning; the second is work which tries to provide a logical framework for reasoning about preferences. With respect to the first, an exemplar formalisms is the quantitative μ-calculus [16], a language for reasoning about probabilistic transition systems. Basically this is a logical language that describes the transitions between states and can be given a game-theoretic interpretation in terms of probabilistic transitions between the stages of a game. The motivation for this work is fundamentally different to our own. We are not working with stochastic transition systems, but aim to use Łukasiewicz formulae to represent utility functions, and then aim to define temporal and strategy modal logics based on Łukasiewicz logics. Also somewhat related is the Łukasiewicz μ calculus [20], which extends the μ-calculus with Łukasiewicz operators. Again, the intent is different to ours. With respect to preference logics, many such formalisms have been developed in AI and philosophy: a typical appropach is to use modal expressions $P\phi$ to mean "ϕ is true in all worlds preferred to the current one". Such logics have well known limitations, and it is generally accepted that there is no entirely satisfactory logic of preferences [4]. Our approach is fundamentally different, using Łukasiewicz formulae as logical specifications of payoff functions.

National Importance

The national importance of this project stems from two points:

- **First**, the proposed project fits well with several EPSRC priority themes. Most obviously, in the ICT theme, the project clearly connects with artificial intelligence technologies, theory of computation, and verification and correctness. In the Digital Economy theme, the project connects with social computing, and more generally, the project is at the heart of the multi-agent systems domain, which represents one of the key paradigms for thinking about the future of interconnected systems. The game-theoretic aspect of our proposal explicitly embraces the idea of thinking about such systems as *economic* systems.

- **Second**, the proposed research is in a domain (multi-agent systems) that the UK has very significant research strengths, with large and highly successful research groups in this area for example at Southampton, Liverpool, Imperial College, Edinburgh, Aberdeen, and other institutions. The PI is internationally leading in this area, as clearly demonstrated by the awards and fellowships he has received. *Investment in this project will help to consolidate the UK's leading position in this increasingly important research area*.

Research Hypothesis & Objectives

Our **basic research hypothesis** is that Łukasiewicz logics can provide a powerful and natural logical framework within which we can compactly and naturally represent utility/objective functions for game theoretic settings; and that this will permit the logical analysis of a much wider and more interesting class of problems than is possible using logics based on classical operators.

Our **objectives** for the project are as follows:

1. To define and develop a full theory of compact representations of cooperative and non-cooperative models of games, called Łukasiewicz games, where players' payoffs are given by continuous piecewise linear polynomial functions with integer coefficients encoded by Łukasiewicz formulas. Our investigation will cover the logical, mathematical and game-theoretic properties of both static and dynamic Łukasiewicz games.

2. To develop an extensive computational study of decision problems for Łukasiewicz games. The goal is to obtain theoretical complexity results as well as to develop algorithms.

3. Building on the above, to define modal logics built over Łukasiewicz logics for reasoning about static and dynamic games in which participants' payoffs are encoded through Łukasiewicz formulas. These models will make it possible to represent at the symbolic level strategies and payoffs as well as game-theoretic properties of Łukasiewicz games.

4. To investigate model checking over the new logical models, determine the computational complexity of the satisfiability problem, and develop practical algorithms for the verification of their properties.

Programme & Methodology

The research programme is broken down in two **work packages**, each further divided into discrete tasks; in addition, we have one **cross-cutting theme**, relating to developing demonstrators and applications of the formalisms and models we develop.

WP1: Game Theoretic Models

The aim of this work package is to introduce and study a theory of compact representations of games based on Łukasiewicz logics, called Łukasiewicz games. In these games players' payoff functions are represented through Łukasiewicz formulas, whose semantic interpretation corresponds to continuous piecewise linear polynomial functions over (a subset of) $[0,1]^m$. The theory of Łukasiewicz games will significantly extend and generalise the theory of Boolean games [15], whose models of strategic interaction offer a natural symbolic representation of games, but for which preferences are dichotomous (or quasi-dichotomous).

The first part of the project will be primarily theoretical, and will focus on the definition and the game-theoretic and logical properties of Łukasiewicz games. The work will encompass both static and dynamic non-cooperative games, as well as cooperative games. We will then turn to the computational analysis of Łukasiewicz games. We will first investigate the complexity of the satisfiability problem for quantified Łukasiewicz formulas and then apply the results to the computational study of game-theoretic solutions problems, such as the existence of equilibria, dominant strategies, etc.

The theory of Łukasiewicz games will offer symbolic and compact models to represent strategic interactions in which numerical payoffs are encoded through Łukasiewicz formulas. This theory will be the basis upon which the logical models for specification and verification of multi-agent systems will be introduced in the second work package.

Task 1.1 : Non-cooperative games. The starting point of the first task will be the study of static non-cooperative **Łukasiewicz Games (ŁGs, for short)**. A first definition of these games was first introduced by Marchioni and Wooldridge in [19]. Given a Łukasiewicz logic $\mathcal{L}$, denote by $L \subseteq [0,1]$ be the set of truth-values of $\mathcal{L}$. Non-cooperative Łukasiewicz games can be defined as types of games based on $\mathcal{L}$ that involve a finite set of players $\mathsf{P} = \{P_1, \ldots, P_n\}$ and a finite set of propositional variables V. Each player P_i is in control of a subset of variables $\mathsf{V}_i \subseteq \mathsf{V}$; thus the sets V_i form a partition of V. Being in control of a set V_i of variables means that P_i assigns to the variables in V_i values from L. So, a strategy for an agent P_i is a function $s_i : \mathsf{V}_i \to L$ that corresponds to a valuation of the variables controlled by P_i. Each agent is assigned an $\mathcal{L}$-formula ϕ_i, with propositional variables from V, which specifies a continuous piecewise linear polynomial function f_{ϕ_i}, which defines the utility for player P_i. We will start by defining solution concepts for Łukasiewicz games both for pure strategies and mixed strategies. A first logical characterisation of the existence of pure strategy Nash Equilibria for ŁGs on finite-valued Łukasiewicz logics was given in [19]. We will explore whether such a characterisation can be generalised and adapted to mixed strategies. We will also study different classes of games such as symmetric, coordination, and constant sum games (among others) and investigate whether different solution concepts can be given a simpler characterisation for those special cases. Since strategies in ŁGs correspond to particular sets of points in $(L)^n \subseteq [0,1]^n$ on the graphs of the payoff functions, we will also study the geometric properties of the sets of equilibria, dominant strategies, as well as those sets of strategies that are cost-efficient. The goal is to find conditions that guarantee the non-emptiness of such sets and in which cases they have minimal and maximal elements. We will conduct this study for the whole class of ŁGs and for special classes of games as well. Since Łukasiewicz formulas define special kinds of linear polynomial functions, we expect some of the results and techniques developed for polynomial games over $[0,1]$ to be useful [10]. In the second part of the first task, the focus will be on dynamic games, called dynamic Łukasiewicz games, i.e., games where choices are not made at once but sequentially between players. We will first start with the simpler case of two-player games with perfect information, i.e., where players at each stage of the game know exactly the state of the game and which moves have been made. Players' payoffs can be encoded through a pair of Łukasiewicz formulas as in ŁGs. At each move, a player assigns a value to one of the variables under her control. At each assignment, the state of the game is represented as a new pair of formulas obtained by replacing the chosen variable with a constant representing the assigned value. In this sense, dynamic Łukasiewicz games can be seen as sequences of ŁGs. Starting from the two-player case, we plan to lay out a precise definition of dynamic ŁGs, and study the notion of subgame perfect equilibrium, later generalising these notions to games with n players. We will also investigate games with imperfect information, where players are not always aware of the choices made by the others and thus might not know at certain stages which values have been assigned to certain variables. The study of dynamic ŁGs will leads us to investigate iterated games, where players repeat the same game finitely or infinitely many times.

Task 1.2: Cooperative Games. Here we will study ŁGs in which players can form coalitions. We will define cooperative Łukasiewicz games as extensions of ŁGs where groups of agents may decide to jointly select certain strategies to maximise their individual payoff and/or minimise the costs of their actions. In this framework we will define the notions of a core and stable set as sets of valuations (i.e., strategies) preferred by all coalitions, i.e., valuations that guarantee a better payoff, and so a higher value of the Łukasiewicz payoff functions. We will investigate the properties of the core and stable sets and find a characterisation of their non-emptiness. We will also explore a different approach to cooperative games with Łukasiewicz functions based on that proposed in [8] for infinite-valued Łukasiewicz logic. A cooperative ŁG for the players $\{P_1, \ldots, P_n\}$ can be defined as continuous piecewise polynomial function $f : L^n \to L$ such that $f(\overline{0}) = 0$ for the constant function $\overline{0}$. In the classical case a cooperative game is represented by a function $v : 2^n \to \mathbb{R}$ from the set of all coalitions into the set of real numbers. Since a coalition is a subset of the set of players, it can be identified with its characteristic function. Following an idea proposed by Aubin [3] we can interpret each vector $(x_1, \ldots, x_n) \in L^n$ as representing the degree to which each player contributes to or participates in the coalition. Therefore the function f encodes the value assigned to each configuration of degrees of participation of the players. In this framework, we plan to study the notions and properties of core and stable sets. Our investigation, however, will mainly focus on the first approach, which is directly based on the notion of ŁGs introduced in the first task.

Task 1.3: Quantified Łukasiewicz Formulas. Here, we plan to study the problem of solving quantified Łukasiewicz formulas. This will help us investigate computational problems for ŁGs, since it is safe to assume that several of their properties and solution concepts, such as the existence of equilibria, can be expressed through this kind of formulas. A quantified Łukasiewicz formula has the form $Q_1, \ldots Q_m\phi$ where ϕ is a Łukasiewicz formula and each Q_j is a quantified propositional variable, $\forall p_j$ or $\exists p_j$ ranging over the set of truth-values. We are interested in determining the complexity of checking satisfiability of a quantified Łukasiewicz formula. For finitely-valued Łukasiewicz formulas we expect the problem to be in PSPACE. Another approach to studying satisfiability of quantified Łukasiewicz formula is through quantifier elimination. Every formula $Q_1, \ldots Q_m\phi$ is (in most cases) equivalent to a formula ψ without quantifiers [17]. Therefore we plan to study the complexity of eliminating the quantifiers to reduce satisfiability for $Q_1, \ldots Q_m\phi$ to the satisfiability of ψ (which is NP-complete). While quantifier elimination for Łukasiewicz logics can be reduced to eliminating quantifiers in the theories of the group of integers and the group of reals [17, 18], we plan to develop algorithms that are tailored to Łukasiewicz logics. An additional aim of this task will be the study of theorem proving for Łukasiewicz logics with quantification. Reduction of tableaux systems to Mixed Integer Programming [14], resolution methods [22], and reduction to Satisfiability Modulo Theories [2] are some of the proof methods that have been developed for theorem proving for propositional Łukasiewicz logics (i.e., without quantifiers). We will explore the possibility of adapting those methods to quantified formulas or whether other approaches are needed. We expect the approach laid out in [12] for solving quantified verification conditions using Satisfiability Modulo Theories to be our main reference.

Task 1.4: Complexity of Game Solutions. Based on the results and techniques developed in task 1.3, this task will focus on computational problems for Łukasiewicz games. For non-cooperative games, we will study the complexity of computing the solution concepts introduced in task 1.1. Among those, we will focus on computing best responses, the existence of dominant strategies, the existence of equilibria, as well as the existence of efficient strategies that minimise costs. For cooperative games our goal will be to determine the complexity of computing membership to and existence of the core and stable sets. We will examine whether certain specific classes of games (coordination, constant sum, symmetric, etc.) yield tractable decision problems. We will identify useful classes of games for which solution concepts can be easily computed. The goal is to find which conditions a ŁG must satisfy to be effectively tractable. On the other hand we will also investigate whether it is possible to modify some of the solutions concepts introduced in task 1.1 so as to obtain tractable computational problems. We expect game-theoretic properties of Łukasiewicz games to be expressible through quantified Łukasiewicz formulas and therefore use the techniques developed in task 1.3 for satisfiability.

WP2: Modal Logics for Games

The main goal of this work package is to define and study new logics for the specification and verification of multi-agent systems that act selfishly to achieve their own goals. These models will be based on Łukasiewicz logics, whose formulas can be interpreted as continuous piecewise linear polynomial functions over (a subset of) $[0, 1]^m$. The use of these logics will provide formal systems to reason about numerical payoffs that are represented through Łukasiewicz formulas. This approach will encode the numerical aspects of game theory into symbolic logical systems and also make it possible to represent and reason about the properties of Łukasiewicz games. We will start by defining linear and branching temporal Łukasiewicz logics in order to reason about the evolution of payoffs over time. We will then expand these systems with modal operators for coalitions of players, strategic choices and solution concepts so as to represent the strategic interactions between agents. We call these logics strategy Łukasiewicz logics. For all the systems introduced, our aim will be to provide sound and complete axiomatic systems. We will then consider the verification of properties of the logical systems previously studied. We will examine the theoretical computational complexity as well as explore practical algorithms for performing model checking and determining satisfiability for both temporal and strategy Łukasiewicz logics.

Task 2.1: Temporal Łukasiewicz Logics. In order to define modal logics to specify the strategic behaviour of competing agents in a dynamic environment, we need a way to express how the values of Łukasiewicz formulas might change over time (i.e., the associated payoff, in a game-theoretic setting). Here we will use temporal logics to represent and reason about the dynamic evolution of a system. Our first step in this task will then be to define expansions of Łukasiewicz logic with temporal operators for both linear and branching time temporal systems. No study of this kind has been carried our so far. We will introduce in the language of Łukasiewicz logics LTL operators X ("in the next state"), G ("always from now on"), F ("sometime in the future"), and U ("until"), and CTL-like path quantifiers E ("on some path path"), and A ("on every path"). The many-valued nature of Łukasiewicz logics will require a semantic interpretation of those modalities that differs from the classical one. As an example, consider the operator G. In classical Linear Temporal Logic, the formula $\mathsf{G}\psi$ is true at a certain time t in a model $\mathcal{M}$ if, in all the following moments $t' > t$, ψ is true. When introduced in the Łukasiewicz language, G can have two different semantic interpretations. The first is Boolean: a formula $\mathsf{G}\phi$ is true at a certain time t in a model $\mathcal{M}$ if, in all the following moments $t' > t$, the value of ϕ is non-decreasing, i.e., it is at least as great as the value in t. Alternatively, G can be given a many-valued interpretation: the truth-value of $\mathsf{G}\phi$ is the infimum of the all the future values, i.e., for all $t' > t$. We plan to explore these and other possible interpretations for G and the other temporal operators mentioned above. The goal of this first task is then to define linear time and branching time temporal Łukasiewicz modal logics for which we intend to provide a sound and complete axiomatization and study their logical properties.

Task 2.2: Strategy Łukasiewicz Logics. Building on the results of the first task, the main goal in this second task is to define modal logics to reason about strategies and abilities of agents in game-like scenarios. The first step will be to introduce modalities that express cooperation between players. The main reference will be the ATL logic [1] (and its expansions), where coalition operators such as $\langle\langle \mathsf{C} \rangle\rangle$ make it possible to express the fact that the coalition C can bring about a goal ψ in some or all future states. Therefore, we plan to expand Łukasiewicz temporal logics with similar modalities $\langle\langle \mathsf{C} \rangle\rangle$. Their interpretation in conjunction with temporal operators will vary depending on the chosen temporal semantics (as explained in the previous task). As an example, following the Boolean interpretation, a formula such as $\langle\langle \mathsf{C} \rangle\rangle\mathsf{X}\phi$ can be regarded as true if in the next state the payoff generated by the coalition C, (i.e., the value of ϕ), does not decrease. Under a many-valued interpretation, the truth-value of $\langle\langle \mathsf{C} \rangle\rangle\mathsf{X}\phi$ can be simply interpreted as the value of ϕ at the next state, or, in other words, the payoff obtained at the following state after a strategic choice made by the coalition C. As the second step in our task, we will investigate new modal operators that will make it possible to *explicitly specify* the strategic choices certain coalitions and players. In addition, we will study how to define new operators to express concepts such as best response and equilibria, to explicitly represent the game-theoretic interactions of multi-agent systems. The concept of a best response could be encoded through a new operator B, so that $\mathsf{B}\langle\langle \mathsf{C} \rangle\rangle_{\vec{s}}\phi$ represents the maximum payoff obtained by some agent when C chooses $\vec{s}$. We will consider these and other options to define sound and complete logics for the specification of multi-agent interactions in which we can explicitly reason about the game-theoretic properties of the system.

Task 2.3: Complexity and Algorithms. Here, we plan to study the computational complexity of the process of model checking for the logics introduced in the previous module [9], as well as the problem of checking whether a formula is satisfiable or if it is a tautology. We will investigate whether any of the classical methods such as dynamic programming algorithms, Büchi automata, and tableau techniques can be adapted to our case. We believe the use of Łukasiewicz logics will require the development of new algorithms and techniques for determining the theoretical complexity of the newly introduced logics. On the other hand, given the connection between Łukasiewicz logics and linear polynomial functions, we can also approach these problems with techniques from linear and mixed integer programming.

Task 2.4: Practical Model Checking. We expect several of the problems to be analysed in the previous task to be intractable. Therefore, in this second task, we will focus on practical model checking for the verification of properties of the temporal and strategy Łukasiewicz logics we plan to develop.

We will explore whether any of the techniques that are prominently used for classical temporal and strategy logics such as partial order reductions, symmetry reductions, ordered-binary decision diagrams, bounded and unbounded model checking, and (predicate) abstraction can be adapted to our systems based on Łukasiewicz logics. Once again, given the connection between Łukasiewicz logics and linear polynomial functions, we expect to use some tools and techniques from linear and mixed integer programming.

Cross Cutting Theme: Demonstrators & Applications

The results of this project will be largely theoretical, but our work is driven by the need for models, formalisms, and algorithms that will ultimately be practicably applicable. Thus, throughout the project, we will test the models, formalisms, and algorithms we develop against exemplar case studies and applications. Our preliminary work in this area [19] indicates that the approach we advocate can be used for the analysis of significant game theoretic scenarios. Throughout the project, we will expand this collection of demonstration scenarios, with the basic evaluation criteria of the extent to which the scenario can be **naturally** and **compactly** captured within the Łukasiewicz logic frameworks we develop. We will work with scenarios from the game theory literature, and also with computational settings. With respect to the latter, we will for example consider auction settings, "selfish routing"-type problems, resource allocation protocols, and other scenarios in which the self interest of participants plays a role. This cross-cutting theme will be active throughout the life of the project.

References

[1] R. Alur, T. A. Henzinger, O. Kupferman. Alternating-Time Temporal Logic. *Journal of the ACM*, 49, 672–713, 2002.

[2] C. Ansótegui, M. Bofill, F. Manyà, M. Villaret. Building Automated Theorem Provers for Infinitely-Valued Logics with Satisfiability Modulo Theory Solvers. In *42nd IEEE Int. Symp. on Multiple-Valued Logic* (ISMVL), 25–30, 2012.

[3] J. P. Aubin. Cooperative Fuzzy Games. *Mathematics of Operations Research*, 6(1): 1–13, 1981.

[4] J. van Benthem. *Logic in Games*. MIT Press, 2014.

[5] N. Bulling, V. Goranko. How to Be Both Rich and Happy: Combining Quantitative and Qualitative Strategic Reasoning about Multi-Player Games. In *Proc. Strategic Reasoning 2013*, Rome, April 2013

[6] K. Chatterjee, T. A. Henzinger, N. Piterman. Strategy logic. *Information and Computation*, 208, 677–693, 2010.

[7] R. Cignoli, I. M. L. D'Ottaviano, D. Mundici. *Algebraic Foundations of Many-valued Reasoning*, V Kluwer, 2000.

[8] P. Cintula, T. Kroupa. Simple games in Łukasiewicz calculus and their cores. *Kybernetika*, 49(3): 404–419, 2013.

[9] E. M. Clarke, O. Grumberg, D. A. Peled. *Model Checking*. MIT Press, 1999.

[10] M. Dresher, S. Karlin, L. S. Shapley. Polynomial games. In *Contributions to the Theory of Games*, Annals of Mathematics Studies, no. 24. Princeton UP, 161–180, 1950.

[11] P. E. Dunne, S. Kraus, W. van der Hoek, and M. Wooldridge. Cooperative boolean games. In *Proc. AAMAS-2008*.

[12] Y. Ge, C. Barrett, C. Tinelli. Solving Quantified Verification Conditions using Satisfiability Modulo Theories. *Annals of Math. and AI*, 55(1-2): 101–122, 2009.

[13] J. Grant, S. Kraus, M. Wooldridge, and I. Zuckerman. Manipulating games by sharing information. In *Studia Logica*, 2014.

[14] R. Hähnle. *Automated Deduction in Multiple-Valued Logics*. Oxford UP, 1993.

[15] P. Harrenstein et al. Boolean games. In J. van Benthem, editor, *Proc. TARK VIII*, 2001.

[16] M. Huth and M. Z. Kwiatkowska. Quantitative analysis and model checking. In *Proc. LICS*, 1997.

[17] G. Lenzi and E. Marchioni. An algebraic characterization of o-minimal and weakly o-minimal MV-chains. *Journal of Pure and Applied Algebra*, 218: 90–100, 2014.

[18] E. Marchioni Amalgamation through quantifier elimination for varieties of commutative residuated lattices. *Archive for Math. Logic*, 51(1): 15–34, 2012.

[19] E. Marchioni, M. Wooldridge. Łukasiewicz Games. In *Proc. AAMAS-2014*, Paris, France, 2014.

[20] M. Mio and A. Simpson Łukasiewicz μ-Calculus. In *Proc. FICS*, 2013

[21] D. Mundici. *Advanced Łukasiewicz Calculus and MV-algebras*. Trends in Logic, Vol. 35, Springer, 2011.

[22] H. Wagner. A new resolution calculus for the infinite-valued propositional logic of Łukasiewicz. In *Proc. FTP-98*.

[23] D. Walther, C. Lutz, F. Wolter, and M. Wooldridge. ATL satisfiability is indeed ExpTime-complete. *Journal of Logic and Comp.*, 2006.