Probabilistic Alternating-Time μ-Calculus

Abstract

In this paper, we propose *probabilistic alternating-time μ-calculus* (PAMC), a *probabilistic* extension of the alternating-time μ-calculus, for expressing properties of multi-agent systems with uncertain behaviors. The semantics of PAMC formulae is defined on *probabilistic concurrent game structures* (PCGSs), a natural model of multi-agent systems with *uncertain behaviors*. We investigate the model-checking problem for PAMC and its two fragments PAMC_0 and PAMC_1 over PCGSs under different setting of strategies, namely, whether strategies are deterministic or randomized and whether strategies have full memory or memoryless. We present a **2NEXPTIME** model-checking algorithm for PAMC under memoryful setting and **2EXPTIME** model-checking algorithm for PAMC under deterministic memoryless setting. We show that the problem for PAMC_1 is in $(\mathbf{UP} \cap \mathbf{co{-}UP})^{\mathbf{NP} \cap \mathbf{co{-}NP}}$ under randomized memoryful setting and deterministic memoryless setting. For PAMC_0, the problem is in $\mathbf{UP} \cap \mathbf{co{-}UP}$ and tractable under randomized memoryful setting when the alternation depth of PAMC formulae is bounded.

1 Introduction

A *multi-agent system* (MAS), in a nutshell, is a computerized system composed of autonomous agents, in which the behavior of each agent is determined by its observed information of the system. MASs have been successfully used to solve problems that are difficult or inefficient for an individual agent to tackle. However, the coexistence of multiple autonomous agents in a MAS makes it extremely difficult to precisely analyze the system behaviors.

Model-checking is an approach for automatically verifying whether a model meets a given specification. Recently, it has been extended to verify MASs (Alur, Henzinger, and Kupferman 1997; 2002). In these works, MASs are modeled as *concurrent game structures* (CGSs) and specifications are expressed in *alternating-time temporal logics*, e.g., ATL, ATL* and alternating-time μ-calculus (AMC), for which model-checking algorithms were also given therein and later implemented in the model-checker MOCHA (Alur et al. 1998). Starting from these works, both the formal methods community and the AI community have placed increasing emphasis on model-checking MASs including specification logics, MAS models and their model-checking algorithms.

In order to make accurate analysis of more complex MASs in which agents (e.g., environment) may have random or uncertain behaviors due to unpredictable physical conditions, Chen and Lu extended CGSs, ATL and ATL* with *probability* leading to *probabilistic CGSs* (PCGSs), *probabilistic* alternating-time temporal logics: *PATL* and *PATL** (Chen and Lu 2007). Later, Chen et al. proposed an extension of PATL with *reward operators*. These techniques were implemented in a model-checker PRISM-games which were successfully applied to analyze probabilistic MASs (Chen et al. 2012; 2013). Schnoor studied the bisimulations contained in PATL* and the complexity of the model-checking problem under incomplete information (i.e., agents can only observe partial information of systems) and memoryless (i.e., decision made by agents only depend on the last state rather than the full history) setting (Schnoor 2010). Huang et al. investigated the PATL* model-checking problem over probabilistic interpreted systems with incomplete information and synchronous perfect recall (Huang, Su, and Zhang 2012). Unfortunately, this problem is undecidable.

This paper follows the direction of (Chen and Lu 2007). We introduce *probabilistic alternating-time μ-calculus* (PAMC) which can be seen as a *probabilistic* extension of AMC or an *alternating-time* generalization of probabilistic μ-calculus (Castro, Kilmurray, and Piterman 2015). The semantics of PAMC is defined over PCGSs in which strategies of agents and transition functions are respectively equipped with probability distribution on decisions and successor states. PAMC can express interesting properties that cannot be expressed by PATL*. For example, let us consider the following property:

> Agents 1 and 2 have strategies such that whatever other agents do, the probability of occurrence of p at every even position is greater than 0.9.

This property cannot be expressed by any PATL* formula, but can be expressed in PAMC as the formula $\langle\{1,2\}\rangle^{>0.9}\nu Z.(p \wedge \mathbf{XX}\, Z)$.

We investigate the *model-checking* problem for PAMC and its two fragments PAMC_0 and PAMC_1 under different setting of strategies, namely, whether strategies are deterministic or randomized and whether strategies have full memory or memoryless. PAMC_0 and PAMC_1 are obtained by restricting syntax of *path formulae*. We show that PAMC is strictly more expressive than AMC and PATL*, and is able to encode several well known logics such as PμTL, PCTL, PCTL* and probabilistic μ-calculus (Liu et al. 2015; Castro, Kilmurray, and Piterman 2015). In addition, PAMC$_0$ and PAMC$_1$ are able to encode AMC and PATL, respectively.

We showed that PAMC does not have memoryless property, but PAMC$_1$ has. PAMC enjoys determinacy property under memoryful setting rather than memoryless.

Concerning on the model-checking problem, we show that the problem for PAMC is in **2NEXPTIME** under randomized/deterministic memoryful setting and in **2EXPTIME** under deterministic memoryless setting. For PAMC$_1$, the problem is in $(\mathbf{UP} \cap \mathbf{co\text{-}UP})^{\mathbf{NP} \cap \mathbf{co\text{-}NP}}$ under randomized memoryful setting and deterministic memoryless setting. For PAMC$_0$, the problem is in $\mathbf{UP} \cap \mathbf{co\text{-}UP}$ and can be decided in polynomial time of the size of PCGSs.

Due to lack of space, expressiveness of PAMC and proofs are given in the supplemental material.

2 Concurrent Games

We fix a countable set **AP** of *atomic propositions*. Let $[k]$ denote the set $\{1, ..., k\}$ for a given natural number $k \in \mathbb{N}$. For a countable set X, a *probability distribution* on X is a function $\mathsf{Pr} : X \to [0, 1]$ such that $\sum_{x \in X} \mathsf{Pr}(x) = 1$. Let $\Upsilon(X)$ denote the set of probability distributions on X. A *σ-algebra* over a set Ω is a subset $\mathcal{F} \subseteq 2^\Omega$ such that $\mathcal{F}$ is closed under complement and countable union, in which each element of $\mathcal{F}$ is called an *event*. A *probability space* is a triple $(\Omega, \mathcal{F}, \mathsf{Pr})$, where Ω is a *sample space*, $\mathcal{F} \subseteq 2^\Omega$ is a σ-algebra over Ω, $\mathsf{Pr} : \mathcal{F} \to [0, 1]$ is a *probability measure* such that $\mathsf{Pr}(\Omega) = 1$ and for each two disjoint events $X, Y \in \mathcal{F}$, $\mathsf{Pr}(X \cup Y) = \mathsf{Pr}(X) + \mathsf{Pr}(Y)$.

Probabilistic Concurrent Game Structures

Definition 1 (Probabilistic Concurrent Game Structures)**.** *A probabilistic concurrent game structure (PCGS) is a tuple $\mathcal{G} = (\mathsf{Ag}, \mathsf{Act}, Q, \delta, \lambda, q_0)$, where*

- $\mathsf{Ag} = \{1, ..., n\}$ *is a finite set of* agents *(a.k.a. players),*
- Act *is a finite set of* actions *which can be made by agents, we use $\mathcal{D}$ to denote the set of* decisions $\mathbf{d} = \langle a_1, ..., a_n \rangle$ *such that $\forall i \in [n], a_i \in \mathsf{Act}$, denoted by $\mathbf{d}(i)$, is the action made by the agent i,*
- *Q is a finite set of* states *with $q_0 \in Q$ as the initial state,*
- $\delta : Q \times \mathcal{D} \to \Upsilon(Q)$ *is a* probability transition function*, we use $\delta(q, \mathbf{d}, q')$ to denote $\delta(q, \mathbf{d})(q')$,*
- $\lambda : \mathbf{AP} \to 2^Q$ *is the* labelling function *assigning to each atomic proposition a set of states.*

Given two states $q_1, q_2 \in Q$, q_2 is a *possible successor* of q_1 if there exists a decision $\mathbf{d} \in \mathcal{D}$ such that $\delta(q_1, \mathbf{d}, q_2) > 0$. Intuitively, if $\delta(q_1, \mathbf{d}, q_2) > 0$ and the PCGS $\mathcal{G}$ is at the state q_1, then the probability of the PCGS $\mathcal{G}$ moves from the state q_1 to the state q_2 when the agents Ag in the PCGS $\mathcal{G}$ cooperatively make the decision $\mathbf{d}$ is $\delta(q_1, \mathbf{d}, q_2)$.

A *track* $\pi \in Q^+$ (resp. *path* $\rho \in Q^\omega$) is a finite (resp. an infinite) sequence of states $q_0q_1...q_m$ (resp. $q_0q_1...$) such that for every $i \in [m]$ (resp. $i \geq 1$), q_i is a possible successor of q_{i-1}. Given a track $\pi = q_0q_1...q_m$ (resp. a path $\pi = q_0q_1...$), we use π_i to denote the state q_i, $\pi_{\geq i}$ to denote the suffix starting from q_i, and $\pi_{\leq i}$ to denote the prefix $q_0...q_i$. Let $\mathsf{Trks}_\mathcal{G} \subseteq Q^+$ (resp. $\mathsf{Paths}_\mathcal{G} \subseteq Q^\omega$) denote the set of tracks (resp. paths), and $\mathsf{Trks}_{\mathcal{G},q}$ (resp. $\mathsf{Paths}_{\mathcal{G},q}$) denote the set of all the tracks (resp. paths) that start from the state $q \in Q$. The subscript $\mathcal{G}$ is dropped from $\mathsf{Trks}_\mathcal{G}$, $\mathsf{Trks}_{\mathcal{G},q}$, $\mathsf{Paths}_\mathcal{G}$ and $\mathsf{Paths}_{\mathcal{G},q}$ when it is clear from the context.

Strategies and Probability Measure of PCGSs

A *randomized strategy* for an agent $i \in \mathsf{Ag}$ in the PCGS $\mathcal{G}$ is a function $\theta : \mathsf{Trks} \to \Upsilon(\mathsf{Act})$ that assigns to each track (i.e., the history the agent saw so far) a probability distribution on Act. A randomized strategy $\theta : \mathsf{Trks} \to \Upsilon(\mathsf{Act})$ is *deterministic* if for every track $\pi \in \mathsf{Trks}$, $\theta(\pi)$ is a δ-distribution, namely, for every $\pi \in \mathsf{Trks}$, there is a unique action $a \in \mathsf{Act}$ such that $\theta(\pi)(a) = 1$. A randomized/deterministic strategy $\theta : \mathsf{Trks} \to \Upsilon(\mathsf{Act})$ is *memoryless* if for every $\pi, \pi' \in \mathsf{Trks}$ and $q \in Q$, $\theta(\pi q) = \theta(\pi' q)$, otherwise is *memoryful*. Memoryless strategy can be simplified as a function $\theta : Q \to \Upsilon(\mathsf{Act})$. Let Θ denote the set of all the possible strategies.

A *coalition* is a set of agents $A \subseteq \mathsf{Ag}$, a *coalition strategy* of A is a function $\upsilon_A : A \to \Theta$ that assigns to each agent $i \in A$ a strategy $\upsilon_A(i)$. We use $\overline{A}$ to denote the set of agents $\mathsf{Ag} \setminus A$. A *response strategy* of A is a function $\upsilon_{\overline{A}} : \overline{A} \to \Theta$ which assigns to each agent $i \in \overline{A}$ (i.e., agent i out of A) a strategy $\upsilon_{\overline{A}}(i)$. Let V_A and $V_{\overline{A}}$ respectively denote the sets of all the possible coalition strategies and response strategies of A. Given a track π, a decision $\mathbf{d} \in \mathcal{D}$, a coalition strategy υ_A and a response strategy $\upsilon_{\overline{A}}$ of A, let $\mathsf{Pr}^{\upsilon_A, \upsilon_{\overline{A}}}(\pi, \mathbf{d})$ denote the probability:

$$\prod_{i \in A} \upsilon_A(i)(\pi)(\mathbf{d}(i)) \cdot \prod_{i \in \overline{A}} \upsilon_{\overline{A}}(i)(\pi)(\mathbf{d}(i)),$$

that is the probability of the decision $\mathbf{d}$ made by the agents Ag with respect to the track π.

Given a coalition $A \subseteq \mathsf{Ag}$, a coalition strategy υ_A and a response strategy $\upsilon_{\overline{A}}$ of A, a path ρ is *compatible* with the coalition strategy υ_A and the response strategy $\upsilon_{\overline{A}}$, if for every $i \geq 0$, there is a decision $\mathbf{d}_i \in \mathcal{D}$ such that $\delta(\rho_i, \mathbf{d}_i, \rho_{i+1}) > 0$, and $\mathsf{Pr}^{\upsilon_A, \upsilon_{\overline{A}}}(\rho_{\leq i}, \mathbf{d}_i) > 0$. Intuitively, the compatible condition of paths rules out infeasible paths under the coalition strategy υ_A and the response strategy $\upsilon_{\overline{A}}$.

Let $\mathsf{Paths}_q^{\upsilon_A, \upsilon_{\overline{A}}}$ denote the set of paths starting from q that are compatible with respect to υ_A and $\upsilon_{\overline{A}}$. Formally,

$$\mathsf{Paths}_q^{\upsilon_A, \upsilon_{\overline{A}}} = \{ \rho \in \mathsf{Paths}_q \mid \rho \text{ is compatible with } \upsilon_A \text{ and } \upsilon_{\overline{A}} \}.$$

Given the coalition strategy υ_A and response strategy $\upsilon_{\overline{A}}$ of a set A of agents, a *cylinder set* (i.e., *event*) of a track π is defined as the set:

$$\mathsf{Cyl}^{\upsilon_A, \upsilon_{\overline{A}}}(\pi) = \{ \rho \in \mathsf{Paths}_{\pi_0}^{\upsilon_A, \upsilon_{\overline{A}}} \mid \pi \text{ is a prefix of } \rho \}.$$

Let $\mathsf{Cyl}_q^{\upsilon_A, \upsilon_{\overline{A}}}$ denote the set of cylinder sets $\mathsf{Cyl}^{\upsilon_A, \upsilon_{\overline{A}}}(\pi)$, where $\pi \in \mathsf{Trks}_q$. Given a coalition strategy υ_A, a response strategy $\upsilon_{\overline{A}}$ and a state $q \in Q$, the probability space associated with the PCGS $\mathcal{G}$ is the triple

$$(\mathsf{Paths}_q^{\upsilon_A, \upsilon_{\overline{A}}}, \mathsf{Cyl}_q^{\upsilon_A, \upsilon_{\overline{A}}}, \mathsf{Pr}_q^{\upsilon_A, \upsilon_{\overline{A}}}),$$

in which the probability measure $\mathsf{Pr}_q^{\upsilon_A,\upsilon_{\overline{A}}}$ is defined as: $\mathsf{Pr}_q^{\upsilon_A,\upsilon_{\overline{A}}}(\mathsf{Cyl}^{\upsilon_A,\upsilon_{\overline{A}}}(q)) = 1$ and $\mathsf{Pr}_q^{\upsilon_A,\upsilon_{\overline{A}}}(\mathsf{Cyl}^{\upsilon_A,\upsilon_{\overline{A}}}(q_0 q_1 ... q_m)) =$

$$\prod_{i=0}^{m-1} \sum_{\mathbf{d}\in\mathcal{D}} \mathsf{Pr}_q^{\upsilon_A,\upsilon_{\overline{A}}}(q_0 q_1 ... q_i, \mathbf{d}) \cdot \delta(q_i, \mathbf{d}, q_{i+1}),$$

where $q_0 = q$. Notice that $\mathsf{Cyl}^{\upsilon_A,\upsilon_{\overline{A}}}(q) = \mathsf{Paths}_q^{\upsilon_A,\upsilon_{\overline{A}}}$.

Stochastic Parity Games

Stochastic parity games are two-player turn-based concurrent games. We review stochastic parity games which will be used in our techniques.

Definition 2 (Stochastic Parity Games)**.** *A* stochastic parity game *is a tuple $\mathcal{G} = (V, E, (V_0, V_1, V_2), F, \Delta)$, where (V, E) is a directed graph and (V_0, V_1, V_2) is a partition of the set V of vertexes, $F : V \to \{0, ..., k\}$ is a* rank function*, and $\Delta : V_2 \to \Upsilon(V)$ is a* probabilistic transition function*.*

The vertexes in V are called *states*, in V_0 are called *player-0 states*, in V_1 are called *player-1 states* and in V_2 are called *probabilistic states*. Tracks and paths of stochastic parity games are defined similar to PCGSs. Given a path ρ, let $\mathsf{inf}(\rho)$ denote the set of infinite often visited states in ρ. A *strategy* for player-i for $i \in \{0, 1\}$ is a function $\theta : V^* V_i \to \Upsilon(V)$ that assigns to every track ending with a state in V_i a distribution on the set of states V. Let Θ_i with $i \in \{0, 1\}$ denote the set of all strategies for player-i. Given two strategies $\theta_0 \in \Theta_0$ and $\theta_1 \in \Theta_1$, the path ρ is (θ_0, θ_1)-*possible* for the game $\mathcal{G}$ if for every $k \geq 0$ the following conditions hold:

- if $\rho_k \in V_i$ for $i \in \{0, 1\}$, then $\theta_i(\rho_{\leq k}, \rho_{k+1}) > 0$;
- if $\rho_k \in V_3$, then $\delta(\rho_k, \rho_{k+1}) > 0$.

Given a state v, let $\mathsf{Outcomes}(v, \theta_0, \theta_1)$ denote the set of (θ_0, θ_1)-possible paths starting from v. Similar to PCGSs, the strategies $\theta_0 \in \Theta_0$, $\theta_1 \in \Theta_1$ and the state $v \in V$ induce the probability space $(\mathsf{Outcomes}(v, \theta_0, \theta_1), \mathsf{Cyl}_v^{\theta_0,\theta_1}, \mathsf{Pr}_v^{\theta_0,\theta_1})$ (details refer to (Chatterjee and Henzinger 2012)).

A *Markov chain* is a special case of stochastic parity game $\mathcal{G} = (V, E, (V_0, V_1, V_2), F, \Delta)$ in which $V_0 = V_1 = \emptyset$.

Given a state v and a number $r \in [0, 1]$, player-0 *wins* the game from the state v with respect to a condition $\sim r$ if there exists a strategy $\theta_0 \in \Theta_0$ such that for all $\theta_1 \in \Theta_1$, the following condition holds: where $\sim \in \{\geq, >\}$,

$$\mathsf{Pr}_v^{\theta_0,\theta_1}(\{\rho \in \mathsf{Outcomes}(v, \theta_0, \theta_1) \mid \min_{v' \in \mathsf{inf}(\rho)} F(v') \text{ is } \mathit{even}\}) \sim r.$$

The winning for player-1 is defined analogously. Memoryless, memoryful, randomized and deterministic strategies in stochastic parity games are defined similar to PCGSs.

Theorem 1. *(Chatterjee, Jurdzinski, and Henzinger 2004; Chatterjee and Henzinger 2012) Given a stochastic parity game $\mathcal{G}$, a state v and a condition $\sim r$, whether player-i for $i \in \{0, 1\}$ can win the game or not can be decided in* $\mathbf{NP} \cap \mathbf{co{-}NP}$ *or exponential time. The problem can be decided in polynomial time for Markov chain. Moreover, deterministic memoryless strategies are sufficient to determine the problem.*

3 Probabilistic Alternating-Time μ-Calculus

In this section, we introduce *probabilistic alternating-time μ-calculus*, a *probabilistic* extension of AMC (Alur, Henzinger, and Kupferman 2002). In probabilistic alternating-time μ-calculus, coalition quantifiers $\langle A \rangle$ are equipped with probabilistic constraints $\sim c$ for $\sim \in \{\geq, >\}$ and $c \in [0, 1]$.

Definition 3 (Probabilistic Alternating-Time μ-Calculus)**.** *Let $\mathcal{Z}$ be a finite set of proposition variables. The syntax of probabilistic alternating-time μ-calculus (PAMC for short) formulae is depicted as follows: where Φ are* state formulae*, Ψ are* path formulae*,*

$$\begin{aligned} \Phi ::= &\ \ p \mid \neg p \mid Z \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid \mu Z.\Phi \mid \nu Z.\Phi \mid \langle A \rangle^{\sim c}\Psi \\ \Psi ::= &\ \ \Phi \mid \Psi \wedge \Psi \mid \Psi \vee \Psi \mid \mu Z.\Psi \mid \nu Z.\Psi \mid \mathbf{X}\Psi \end{aligned}$$

where $p \in \mathbf{AP}$, $Z \in \mathcal{Z}$, $c \in [0, 1]$, $\sim \in \{\geq, >\}$ and $A \subseteq \mathsf{Ag}$.

A PAMC formula ϕ is *closed* if each variable $Z \in \mathcal{Z}$ used in ϕ is in the scope of a fixpoint operator μZ or νZ. A PAMC formula is called a *principal formula* if it is in the form of $\langle A \rangle^{\sim c}\psi$ and no subformula of ψ is in the form of $\langle A' \rangle^{\sim' c'}\psi'$. The *alternation depth* of a formula ϕ, denoted by $dep(\phi)$, is the number of alternations in the nesting of least and greatest fixpoints. Let Ag^ϕ denote the set of agents appeared in the formula ϕ and $true \equiv p \vee \neg p$ for $p \in \mathbf{AP}$.

The semantics of PAMC state formulae ϕ is given w.r.t. a PCGS and a valuation $\xi : \mathcal{Z} \to 2^Q$. We use $\xi[Z \mapsto S]$ to denote the valuation which is equal to ξ except for $\xi[Z \mapsto S](Z) = S$. Given a PCGS $\mathcal{G} = (\mathsf{Ag}, \mathsf{Act}, Q, \delta, \lambda, q_0)$ and a valuation $\xi : \mathcal{Z} \to 2^Q$, the denotation function $||\circ||_{\mathcal{G}}^{\xi}$ that maps state formulae to sets of states is inductively defined as follows:

- $||p||_{\mathcal{G}}^{\xi} = \lambda(p)$, $||\neg p||_{\mathcal{G}}^{\xi} = Q \setminus \lambda(p)$, $||Z||_{\mathcal{G}}^{\xi} = \xi(Z)$;
- $||\langle A \rangle^{\sim c}\psi||_{\mathcal{G}}^{\xi} = \{\, q \in Q \mid \exists \upsilon_A \in V_A,\ \forall \upsilon_{\overline{A}} \in V_{\overline{A}} :$ $\mathsf{Pr}_q^{\upsilon_A,\upsilon_{\overline{A}}}(\{\rho \in \mathsf{Paths}_q^{\upsilon_A,\upsilon_{\overline{A}}} \mid 0 \in ||\psi||_{\mathcal{G}}^{\xi,\rho}\}) \sim c \,\}$;
- $||\phi_1 \wedge \phi_2||_{\mathcal{G}}^{\xi} = ||\phi_1||_{\mathcal{G}}^{\xi} \cap ||\phi_2||_{\mathcal{G}}^{\xi}$;
- $||\phi_1 \vee \phi_2||_{\mathcal{G}}^{\xi} = ||\phi_1||_{\mathcal{G}}^{\xi} \cup ||\phi_2||_{\mathcal{G}}^{\xi}$;
- $||\mu Z.\phi||_{\mathcal{G}}^{\xi} = \bigcap\{Q' \subseteq Q \mid ||\phi||_{\mathcal{G}}^{\xi[Z\mapsto Q']} \subseteq Q'\}$;
- $||\nu Z.\phi||_{\mathcal{G}}^{\xi} = \bigcup\{Q' \subseteq Q \mid ||\phi||_{\mathcal{G}}^{\xi[Z\mapsto Q']} \supseteq Q'\}$.

Given an ω-word $w = \alpha_0\alpha_1...$ over $2^{\mathbf{AP}}$, the denotation function $||\circ||_{\mathcal{G}}^{\xi,w}$ that maps path formulae to a set of positions defined as follows:

- $||\phi||_{\mathcal{G}}^{\xi,w} = \{\, i \in \mathbb{N} \mid \alpha_i \in ||\phi||_{\mathcal{G}}^{\xi}\}$ if ϕ is a state formula;
- $||\psi_1 \wedge \psi_2||_{\mathcal{G}}^{\xi,w} = ||\psi_1||_{\mathcal{G}}^{\xi,w} \cap ||\psi_2||_{\mathcal{G}}^{\xi,w}$;
- $||\psi_1 \vee \psi_2||_{\mathcal{G}}^{\xi,w} = ||\psi_1||_{\mathcal{G}}^{\xi,w} \cup ||\psi_2||_{\mathcal{G}}^{\xi,w}$;
- $||\mathbf{X}\psi||_{\mathcal{G}}^{\xi,w} = \{\, i \in \mathbb{N} \mid i + 1 \in ||\psi||_{\mathcal{G}}^{\xi,w}\}$;
- $||\mu Z.\phi||_{\mathcal{G}}^{\xi,w} = \bigcap\{N \subseteq \mathbb{N} \mid ||\phi||_{\mathcal{G}}^{\xi[Z\mapsto N],w} \subseteq N\}$;
- $||\nu Z.\phi||_{\mathcal{G}}^{\xi,w} = \bigcup\{N \subseteq \mathbb{N} \mid ||\phi||_{\mathcal{G}}^{\xi[Z\mapsto N],w} \supseteq N\}$.

Given a path ρ and a path formula ψ, $||\phi||_{\mathcal{G}}^{\xi,\rho}$ denotes the set $||\phi||_{\mathcal{G}}^{\xi,w}$, where w is the ω-word $\alpha_0\alpha_1...$ induced by ρ, namely, for every $i \geq 0$, $\alpha_i = \{p \in \mathbf{AP} \mid \rho_i \in \lambda(p)\}$.

Notice that universal coalition quantifier $[A]^{\sim c}\psi$ can be derived as $\langle A\rangle^{\overline{\sim}1-c}\neg\psi$, where $\overline{\geq} \equiv >$ and $\overline{>} \equiv \geq$.

For every PAMC formula $\langle A\rangle^{\sim c}\psi$, every coalition strategy υ_A and response strategy $\upsilon_{\overline{A}}$ of A, the set of paths $\{\rho \in \mathsf{Paths}_q^{\upsilon_A,\upsilon_{\overline{A}}} \mid 0 \in ||\psi||_{\mathcal{G}}^{\xi,\rho}\}$ is measurable (c.f. Remark 10.57 in (Baier and Katoen 2008)). We sometime drop the superscript ξ from $||\phi||_{\mathcal{G}}^{\xi}$ and $||\phi||_{\mathcal{G}}^{\xi,\rho}$ if ϕ is a closed formula.

Let PAMC$_1$ denote the sublogic of PAMC in which path formulae are given by the following rule:

$$\Psi ::= \mathbf{X}\Phi \mid \Phi\mathbf{U}\Phi \mid \Phi\mathbf{R}\Phi$$

where Φ are state formulae, $\phi_1\mathbf{U}\phi_2 \equiv \mu Z.\phi_2 \vee (\phi_1 \wedge \mathbf{X}Z)$ and $\phi_1\mathbf{R}\phi_2 \equiv \nu Z.(\phi_1 \wedge \phi_2) \vee (\phi_2 \wedge \mathbf{X}Z)$.

Let PAMC$_0$ denote the sublogic of PAMC$_1$ in which path formulae are only in the form of $\mathbf{X}\Phi$.

Proposition 1. *PAMC$_1$ has memoryless property, but PAMC does not. PAMC enjoys determinacy property under memoryful setting, but not under memoryless setting.*

Linear-time μ-calculus (μTL for short) is a sublogic of PAMC by disallowing $\langle A\rangle^{\sim c}\Psi$. Formally, formulae of μTL are generated by the following rule:

$$\Psi ::= p \mid \neg p \mid Z \mid \Psi \wedge \Psi \mid \Psi \vee \Psi \mid \mu Z.\Psi \mid \nu Z.\Psi \mid \mathbf{X}\Psi$$

where $p \in \mathbf{AP}$ and $Z \in \mathcal{Z}$. The semantics of μTL formulae is defined similar to path formulae of PAMC.

Deterministic Parity Automata

Definition 4 (Deterministic Parity Automata)**.** *A deterministic parity automaton (DPA) $\mathcal{P}$ is a tuple $(G, \Sigma, \delta, g^0, F)$, where G is a finite set of states, Σ is the input alphabet, $\delta : G \times \Sigma \to G$ is a transition function, $g^0 \in G$ is the initial state and $F : G \to \{0, ..., k\}$ is a rank function.*

A *run* ρ of $\mathcal{P}$ over an ω-word $\alpha_0\alpha_1... \in \Sigma^\omega$ is an infinite sequence of states $\rho = g_0g_1...$ such that $g_0 = g^0$, and for every $i \geq 0$, $g_{i+1} = \delta(g_i, \alpha_i)$. Let $\mathsf{inf}(\rho)$ be the set of states visited infinitely often in ρ. A run ρ is *accepting* iff $\min_{g\in\mathsf{inf}(\rho)} F(g)$ is *even*.

Theorem 2. *(Dax 2006; Piterman 2007) For every μTL sentence ψ, we can construct a DPA $\mathcal{P} = (G, 2^{\mathbf{AP}}, \delta, g^0, F)$ in* **2EXPTIME** *such that $\mathcal{P}$ recognizes all of the ω-words that satisfy ϕ, the size of $\mathcal{P}$ is doubly-exponential in the size of ψ.*

4 Model-Checking Algorithms

In this section, we propose model-checking algorithms for PAMC, PAMC$_1$ and PAMC$_0$ under different settings.

Let us fix a PCGS $\mathcal{G} = (\mathsf{Ag}, \mathsf{Act}, Q, \delta, \lambda, q_0)$ and a closed PAMC formula ϕ. To compute the set $||\phi||_{\mathcal{G}}$, we will iteratively compute the set of states of subformulae from the innermost subformulae. The main challenge is how to compute $||\phi||_{\mathcal{G}}^{\xi}$ for state formulae ϕ of the form $\langle A\rangle^{\sim c}\psi$.

For a principal formula $\phi = \langle A\rangle^{\sim c}\psi$, we first show that checking whether $q \in ||\phi||_{\mathcal{G}}^{\xi}$ can be reduced to the winning problem of stochastic parity games which can be solved in **NP** $\cap$ **co−NP** (cf. Theorem 1). For non principal formula of the form $\phi = \langle A\rangle^{\sim c}\psi$, we first compute the set of satisfaction states of the principal subformulae φ of ψ and then construct a new formula ϕ' obtained by replacing these principal subformulae φ with fresh atomic propositions p_φ in ϕ, where p_φ is assigned by the set $||\varphi||_{\mathcal{G}}^{\xi}$ in λ. We continue applying the above two steps on ϕ' until ϕ' becomes a principal formula. Finally, $||\phi'||_{\mathcal{G}}^{\xi}$ can be computed which is $||\phi||_{\mathcal{G}}^{\xi}$.

Satisfaction States for Principal Formulae

Given a PCGS $\mathcal{G} = (\mathsf{Ag}, \mathsf{Act}, Q, \delta, \lambda, q_0)$, a valuation $\xi : \mathcal{Z} \to 2^Q$ and a principal formula $\phi = \langle A\rangle^{\sim c}\psi$, we reduce the problem of computing $||\phi||_{\mathcal{G}}^{\xi}$ to the winning problem of a stochastic parity game.

Let $\mathcal{P}_\psi = (G, 2^{\mathbf{AP}}, \delta', g^0, F)$ be the DPA recognizing all the ω-words satisfying ψ. We define the stochastic parity game $\mathcal{G}_\psi = (V, E, (V_0, V_1, V_2), F', \Delta)$, where

- $V_0 = Q \times G \times \{\bot\}$, $V_1 = Q \times G \times \{f : A \to \mathsf{Act}\}$, $V_2 = Q \times G \times \{f : \mathsf{Ag} \to \mathsf{Act}\}$,
- F' is the rank function such that $F'(q, g, f) = F(g)$, for every $(q, g, f) \in V$,
- Δ and E are defined as follows:
  - $((q, g, \bot), (q, g, f)) \in E$ for every $q \in Q$, $g \in G$ and $f : A \to \mathsf{Act}$,
  - $((q, g, f), (q, g, f')) \in E$ for every $q \in Q$, $g \in G$, $f : A \to \mathsf{Act}$ and $f' : \mathsf{Ag} \to \mathsf{Act}$ such that $f'(i) = f(i)$ for all $i \in A$,
  - $\Delta((q, g, f), (q', \delta'(g, P), \bot)) = \delta(q, \mathbf{d}, q')$ for every $q \in Q$, $g \in G$, $f : \mathsf{Ag} \to \mathsf{Act}$, where $\mathbf{d}(i) = f(i)$ for all $i \in \mathsf{Ag}$ and $P = \{p \in \mathbf{AP} \mid q \in \lambda(p)\} \cup \{Z \in \mathcal{Z} \mid q \in \xi(Z)\}$.

Lemma 1. *For every state $(q, g^0, \bot) \in V_0$, player-0 can win the game $\mathcal{G}_\psi$ from $(q, g^0, \bot)$ with respect to the condition $\sim c$ iff $q \in ||\langle A\rangle^{\sim c}\psi||_{\mathcal{G}}^{\xi}$.*

Intuitively, given a coalition strategy υ_A and a response strategy $\upsilon_{\overline{A}}$ of the set A of agents, there is a one-to-one correspondence between the path $\rho = q_0q_1...$ in the PCGS $\mathcal{G}$ and the path $\rho' = (q_0, g^0, \bot)(q_0, g^0, f_0)(q_0, g^0, f_0')(q_1, g^1, \bot)(q_1, g^1, f_1)(q_1, g^1, f_1')...$ in $\mathcal{G}_\psi$, where for every $i \geq 0$, $g^{i+1} = \delta'(g^i, P_i)$ for $P_i = \{p \in \mathbf{AP} \mid q_i \in \lambda(p)\} \cup \{Z \in \mathcal{Z} \mid q_i \in \xi(Z)\}$, and $\delta(q_i, \mathbf{d}_i, q_{i+1}) = \Delta((q_i, g^i, f_i'), (q_{i+1}, g^{i+1}, \bot))$ with $\mathbf{d}_i(j) = f_i'(j)$ for all $j \in \mathsf{Ag}$. Moreover, there are corresponding strategies θ_0, θ_1 for player-0 and player-1 such that for every prefix $\pi = q_0...q_k$ of ρ, the joint distribution of actions Act on the prefix π in υ_A is same as the distribution $\theta_0(\rho'_{\leq 3k})$ over V_1, and the joint distribution of actions Act on the prefix π in $\upsilon_{\overline{A}}$ is same as the distribution $\theta_1(\rho'_{\leq 3k+1})$ over V_2.

We mention that this approach works only for memoryful setting rather than memoryless setting. Indeed, there may exist two positions i, j in ρ such that $\rho_i = \rho_j$, but the DPA reaches different states, i.e., $g^i \neq g^j$. In this situation, each agent should select the same action at the positions i and j under memoryless setting. However, the stochastic parity game can select different actions at the states $(q_i, g^i, \bot)$ and $(q_j, g^j, \bot)$ due to $g_i \neq g_j$.

Lemma 2. *For every principal formula ϕ, $\|\phi\|^{\xi}_{\mathcal{G}}$ can be computed in* **2NEXPTIME** *under randomized/determinisitic memoryful setting.*

If we consider the problem under deterministic memoryless setting, we can reduce the problem of computing $\|\phi\|^{\xi}_{\mathcal{G}}$ to determine whether player-0 can win in the Markov chain $\mathcal{G}_\psi$. We do this as follows. First, we nondeterministically choose a coalition strategy υ_A for the agents A by selecting a subset $\delta' \subseteq \delta$ of transition relation such that for every states $q, q_1, q_2 \in Q$ and decisions $\mathbf{d}_1, \mathbf{d}_2 \in \mathcal{D}$, if $\delta'(q, \mathbf{d}_1, q_1) > 0$ and $\delta'(q, \mathbf{d}_2, q_2) > 0$, then $\mathbf{d}_1(i) = \mathbf{d}_2(i)$ for all agents $i \in A$.

Then, we check response strategies $\upsilon_{\overline{A}}$ of A one-by-one by selecting a subset $\delta'' \subseteq \delta'$ of transition relation such that for states $q, q_1, q_2 \in Q$ and decisions $\mathbf{d}_1, \mathbf{d}_2 \in \mathcal{D}$, if $\delta''(q, \mathbf{d}_1, q_1) > 0$ and $\delta''(q, \mathbf{d}_2, q_2) > 0$, then $\mathbf{d}_1(i) = \mathbf{d}_2(i)$ for all agents $i \in \overline{A}$. The PCGS using δ'' is indeed a Markov chain which is induced by the coalition strategy υ_A and the response strategy $\upsilon_{\overline{A}}$. Therefore, we can determine whether $\mathsf{Pr}_q^{\upsilon_A,\upsilon_{\overline{A}}}(\{\rho \in \mathsf{Paths}_q^{\upsilon_A,\upsilon_{\overline{A}}} \mid 0 \in \|\psi\|^{\xi,\rho}_{\mathcal{G}}\}) \sim c$ or not in the induced Markov chain in **2EXPTIME**. If there exists a coalition strategy υ_A (i.e., $\delta' \subseteq \delta$) such that for all response strategies $\upsilon_{\overline{A}}$ (i.e., $\delta'' \subseteq \delta'$), $\mathsf{Pr}_q^{\upsilon_A,\upsilon_{\overline{A}}}(\{\rho \in \mathsf{Paths}_q^{\upsilon_A,\upsilon_{\overline{A}}} \mid 0 \in \|\psi\|^{\xi,\rho}_{\mathcal{G}}\}) \sim c$ holds. We get that $q \in \|\phi\|^{\xi}_{\mathcal{G}}$. There are exponential many of δ' and δ''.

Lemma 3. *For every principal formula ϕ, $\|\phi\|^{\xi}_{\mathcal{G}}$ can be computed in* **2EXPTIME** *under deterministic memoryless setting.*

Model-Checking Algorithm for PAMC

Given a PCGS $\mathcal{G} = (\mathsf{Ag}, \mathsf{Act}, Q, \delta, \lambda, q_0)$ and a closed PAMC state formula ϕ, we assume that each variable Z in ϕ is only bound at most once. $\mathsf{eval}(\phi, \xi)$ with $\xi(Z) = \emptyset$ for $Z \in \mathcal{Z}$ computes the set of states that satisfy ϕ, in which the auxiliary functions *parent* and *OpenSub* are defined as follows. $parent(\sigma Z.\psi)$ returns the minimal state formula of the form $\sigma' Z'.\psi'$ such that $\sigma \neq \sigma'$, $\sigma Z.\psi$ is a subformula of ψ' and Z' is used in ψ. $OpenSub(\sigma Z.\psi)$ returns a set of state subformulae $\sigma' Z'.\psi'$ of ψ such that $\sigma \neq \sigma'$ and Z is used in ψ'. The function eval is similar to that proposed in (Castro, Kilmurray, and Piterman 2015) for model-checking μ-PCTL, in which state formulae including fixed points are calculated in the standard way. To compute $\mathsf{eval}(\phi, \xi)$ for $\phi = \langle A \rangle^{\sim c} \psi$, we recursively compute $\mathsf{eval}(\phi_i, \xi)$ for every maximal state subformula ϕ_i in ψ and replace ϕ_i by a fresh atomic proposition p_{ϕ_i} whose value is $\mathsf{eval}(\phi_i, \xi)$ in λ, until ϕ becomes a principal and can be computed by applying Lemma 2 or Lemma 3.

Theorem 3. *The PAMC model-checking problem on PCGSs is in* **2NEXPTIME** *under* randomized/deterministic memoryful *setting and in* **2EXPTIME** *under* deterministic memoryless *setting.*

Function eval(state formula: ϕ, $\mathcal{Z} \to 2^Q : \xi$)

```
switch φ do
    case φ1 ∧ φ2 : return eval(φ1, ξ) ∩ eval(φ2, ξ) ;
    case φ1 ∨ φ2 : return eval(φ1, ξ) ∪ eval(φ2, ξ) ;
    case p : return λ(p) ;
    case ¬p : return Q \ λ(p) ;
    case Z return ξ(Z) ;
    case σZ.φ1 :
        Q' := (σ = μ) ? Q : ∅;
        if parent(σZ.φ1) = σ'Z'.φ' then
            foreach σ'Z1.φ2 ∈ OpenSub(σZ.φ1) do
                ξ := ξ[Z1 ↦ Q'];
        Q_new := (σ = μ) ? ∅ : Q;
        repeat
            Q_old := Q_new;
            Q_new := eval(φ1, ξ[Z ↦ Q_new]);
        until Q_new = Q_old;
        return Q_new;
    case ⟨A⟩^{~c}ψ :
        φ := ⟨A⟩^{~c}ψ;
        while φ is not principal do
            Let {φ1, ..., φm} be the set of maximal state
            subformulae in φ;
            φ := φ[p_φ1/φ1, ..., p_φm/φm];
            λ(p_φi) := eval(φi, ξ), ∀i ∈ [m];
        return ||φ||^ξ_G by applying Lemma 2 or Lemma 3;
```

Model-Checking Algorithm for PAMC$_1$

The double exponential blowup in Theorem 3 comes from the translating of μTL into DPAs. When PAMC$_1$ formulae are considered, the sizes of DPAs for μTL formulae are bounded by a constant. Indeed, for each principal formula $\langle A \rangle \phi$, ϕ must be in the form of $\mathbf{X}\phi_1$, $\phi_1 \mathbf{U} \phi_2$ and $\phi_1 \mathbf{R} \phi_2$. In the function `eval`, ϕ_1 and ϕ_2 can be replaced by two fresh atomic propositions p_{ϕ_1} and p_{ϕ_2}. This provides us the formula of the form $\mathbf{X} p_{\phi_1}$, $p_{\phi_1} \mathbf{U} p_{\phi_2}$ and $p_{\phi_1} \mathbf{R} p_{\phi_2}$, which can be translated into a DPA in linear time and its size is bounded (in fact, the DPA is a deterministic Büchi automaton). This implies that the size of the stochastic parity game $\mathcal{G}_\psi$ underlying Lemma 2 is polynomial of the size of the PCGS and formula, as well as the size of the Markov chain underlying Lemma 3.

Theorem 4. *The PAMC$_1$ model-checking problem on PCGSs is in* $(\mathbf{UP} \cap \mathbf{co{-}UP})^{\mathbf{NP} \cap \mathbf{co{-}NP}}$ *under* randomized/deterministic memoryful *setting and* deterministic memoryless *setting.*

Model-Checking Algorithm for PAMC$_0$

Let us fix a PCGS $\mathcal{G} = (\mathsf{Ag}, \mathsf{Act}, Q, \delta, \lambda, q_0)$ and a closed PAMC$_0$ formula ϕ. To compute the set $\|\phi\|_{\mathcal{G}}$, our approach follows the model-checking algorithm for PAMC on PCGSs. We iteratively compute the set of satisfaction states for subformulae from the inner-most subformulae, in which formulae of the form $\phi = \langle A \rangle^{\sim c} \mathbf{X} \psi$ can be computed in polynomial time based on the following proposition.

Proposition 2. *Given a state $q \in Q$ and a PAMC$_0$ principal formula $\phi = \langle A \rangle^{\sim c}\mathbf{X}\psi$ the following property holds:* $q \in ||\psi||^{\xi}_{\mathcal{G}}$ *iff* $\max\ \{\min\ \{\sum_{q_1 \in ||\phi||^{\xi}_{\mathcal{G}}} \delta(q, \mathbf{d}, q_1) \mid \forall i \in \overline{A}, \mathbf{d}(i) \in \mathsf{Act}\ \} \mid \forall j \in A, \mathbf{d}(j) \in \mathsf{Act}\} \sim c$.

Proof. Since $||\phi||^{\xi}_{\mathcal{G}} = \{q \in Q \mid \exists \upsilon_A \in V_A,\ \forall \upsilon_{\overline{A}} \in V_{\overline{A}} : \mathsf{Pr}_q^{\upsilon_A, \upsilon_{\overline{A}}}(\{\rho \in \mathsf{Paths}_q^{\upsilon_A, \upsilon_{\overline{A}}} \mid \rho_1 \in ||\psi||^{\xi}_{\mathcal{G}}\}) \sim c\}$, then $q \in ||\phi||^{\xi}_{\mathcal{G}}$ iff there exists $\upsilon_A \in V_A$ such that for all $\upsilon_{\overline{A}} \in V_{\overline{A}}$,

$$\mathsf{Pr}_q^{\upsilon_A, \upsilon_{\overline{A}}}(\{\rho \in \mathsf{Paths}_q^{\upsilon_A, \upsilon_{\overline{A}}} \mid \rho_1 \in ||\psi||^{\xi}_{\mathcal{G}}\}) \sim c.$$

It is easy to get that $q \in ||\phi||^{\xi}_{\mathcal{G}}$ iff there exists $\upsilon_A \in V_A$ such that for all $\upsilon_{\overline{A}} \in V_{\overline{A}}$, $\sum_{q_1 \in ||\psi||^{\xi}_{\mathcal{G}}, \mathbf{d} \in \mathcal{D}} \mathsf{Pr}^{\upsilon_A, \upsilon_{\overline{A}}}(q, \mathbf{d}) \cdot \delta(q, \mathbf{d}, q_1) \sim c$. This later statement can be reformulated as: $\max\{\ \min\{\ \sum_{q_1 \in ||\psi||^{\xi}_{\mathcal{G}}} \delta(q, \mathbf{d}, q_1) \mid \forall i \in \overline{A}, \mathbf{d}(i) \in \mathsf{Act}\ \} \mid \forall j \in A, \mathbf{d}(j) \in \mathsf{Act}\ \} \sim c$. □

Given a principal formula $\phi = \langle A \rangle^{\sim c}\mathbf{X}\psi$, if $||\psi||^{\xi}_{\mathcal{G}}$ is known, then $||\phi||^{\xi}_{\mathcal{G}}$ can be computed in polynomial time by applying Proposition 2. On the other hand, $||\psi||^{\xi}_{\mathcal{G}}$ without using $\langle A \rangle$ in ψ can be computed in time $\mathbf{UP} \cap \mathbf{co{-}UP}$ (Liu et al. 2015), we immediately get that:

Theorem 5. *The model-checking problem for PAMC$_0$ on PCGSs is in* $\mathbf{UP} \cap \mathbf{co{-}UP}$ *and can be decided in* $\mathbf{O}((|\phi| \cdot |Q|)^{dep(\phi)})$ *time under randomized memoryful setting.*

5 Related Work

The related work on MAS models, specification logics and their model-checking problems were discussed in the introduction section. In model-checking probabilistic MASs context, we introduced PAMC and its two fragments PAMC$_1$ and PAMC$_0$, where PAMC and PAMC$_1$ are respectively more expressive than PATL* and PATL (Chen and Lu 2007) because of fixpoint operators. (Chen and Lu 2007) proposed a model-checking algorithm for PATL on turn-based PCGSs that cannot verify PATL* formulae or PCGSs. One of our model-checking algorithms is aimed at PAMC$_0$ on PCGSs which are more expressive than theirs both in terms of specification logic and MAS model, but without increasing complexity.

(Schnoor 2010) showed that the PATL* model-checking problem is in **3EXPTIME** and **2EXPTIME**-hard under imperfect information and memoryless setting, and the problem becomes **PSPACE**-complete under the deterministic memoryless setting. The problem is undecidable under incomplete information and synchronous perfect recall (Huang, Su, and Zhang 2012), in which a **2EXPTIME** model-checking algorithm was given for a fragment of PATL* by dropping the until operator **U**. In this paper, we only consider the model-checking problem under perfect information setting with more expressive logics. Knowledge operators and probabilistic knowledge operators were introduced into PCTL by (Wan et al. 2016; Wan, Bentahar, and Hamza 2013) but without coalition modalities.

For reasoning about agents' knowledge, or belief, desire and intention, extensions of ATL with *knowledge operators* (van der Hoek and Wooldridge 2002) and *belief, desire, and intention operators* were proposed (Bourahla and Benmohamed 2005), in which CGSs are extended with *accessibility relations*. To express cooperation and enforcement of agents, a more expressive logic, called *strategy logic* (SL), was proposed in (Mogavero, Murano, and Vardi 2010). Generally, its model-checking problem is undecidable. Therefore, several decidable fragments of SL were investigated (Cermák, Lomuscio, and Murano 2015; Mogavero, Murano, and Sauro 2013; 2014; Mogavero et al. 2014). To reason about timed MASs, timed alternating-time temporal logics and timed CGSs were investigated (Henzinger and Prabhu 2006; Brihaye et al. 2007).

Besides these works, a novel MAS model called *pushdown game structures* were introduced to model pushdown multi-agent systems, a kind of *infinite-state* multi-agent systems (Murano and Perelli 2015). The model-checking problems of pushdown game structures against alternating-time temporal logics (such as ATL, ATL* and AMC) and variant fragments of strategy logic were studied in (Murano and Perelli 2015; Chen, Song, and Wu 2016a; 2016b).

Stochastic games (module checking) are quite close to model-checking PCGSs. Many approaches were proposed to quantitatively and/or qualitatively analyze two-player or multi-player stochastic games, such as (Chatterjee, Jurdzinski, and Henzinger 2004; de Alfaro and Majumdar 2004; Chatterjee, Henzinger, and Horn 2009; Chatterjee and Henzinger 2012; Chatterjee, Doyen, and Henzinger 2013). However, as discussed in (Jamroga and Murano 2014), multiplayer games (module checking) and model-checking CGSs are incomparable, hence in probabilistic setting.

6 Conclusion

We proposed PAMC, a probabilistic extension of AMC, and its two fragments PAMC$_1$ and PAMC$_0$ for reasoning about multi-agent systems with uncertain behaviors. We showed that PAMC is strictly more expressive than PATL*, while PAMC$_1$ (resp. PAMC$_0$) is able to encode PATL and μ-PCTL (resp. AMC).

We proved that PAMC does not have memoryless property, but PAMC$_1$ has. PAMC enjoys determinacy property under memoryful setting, but not under memoryless setting. We showed that the model-checking problem for PAMC is in **2NEXPTIME** under randomized/deterministic memoryful setting and in **2EXPTIME** under deterministic memoryless setting. The problem for PAMC$_1$ becomes $(\mathbf{UP} \cap \mathbf{co{-}UP})^{\mathbf{NP} \cap \mathbf{co{-}NP}}$ under randomized memoryful setting and deterministic memoryless setting. For PAMC$_0$, the problem is in $\mathbf{UP} \cap \mathbf{co{-}UP}$ and can be decided in polynomial time of the size of PCGSs.

References

Alur, R.; Henzinger, T. A.; Mang, F. Y. C.; Qadeer, S.; Rajamani, S. K.; and Tasiran, S. 1998. MOCHA: modularity in model checking. In *Proceedings of the 10th International Conference on Computer Aided Verification*, 521–525.

Alur, R.; Henzinger, T. A.; and Kupferman, O. 1997. Alternating-time temporal logic. In *Proceedings of the 38th*

IEEE Symposium on Foundations of Computer Science, 100–109.

Alur, R.; Henzinger, T. A.; and Kupferman, O. 2002. Alternating-time temporal logic. *Journal of the ACM* 49(5):672–713.

Baier, C., and Katoen, J. 2008. *Principles of model checking*. MIT Press.

Bourahla, M., and Benmohamed, M. 2005. Model checking multi-agent systems. *Informatica (Slovenia)* 29(2):189–198.

Brihaye, T.; Laroussinie, F.; Markey, N.; and Oreiby, G. 2007. Timed concurrent game structures. In *Proceedings of the 18th International Conference on Concurrency Theory*, 445–459.

Castro, P. F.; Kilmurray, C.; and Piterman, N. 2015. Tractable probabilistic mu-calculus that expresses probabilistic temporal logics. In *Proceedings of the 32nd Symposium on Theoretical Aspects of Computer Science*, 211–223.

Cermák, P.; Lomuscio, A.; and Murano, A. 2015. Verifying and synthesising multi-agent systems against one-goal strategy logic specifications. In *Proceedings of the 29th AAAI Conference on Artificial Intelligence*, 2038–2044.

Chatterjee, K., and Henzinger, T. A. 2012. A survey of stochastic ω-regular games. *Journal of Computer and System Science* 78(2):394–413.

Chatterjee, K.; Doyen, L.; and Henzinger, T. A. 2013. A survey of partial-observation stochastic parity games. *Formal Methods in System Design* 43(2):268–284.

Chatterjee, K.; Henzinger, T. A.; and Horn, F. 2009. Stochastic games with finitary objectives. In *Proceedings of the 34th International Symposium on Mathematical Foundations of Computer Science*, 34–54.

Chatterjee, K.; Jurdzinski, M.; and Henzinger, T. A. 2004. Quantitative stochastic parity games. In *Proceedings of the 25th Annual ACM-SIAM Symposium on Discrete Algorithms*, 121–130.

Chen, T., and Lu, J. 2007. Probabilistic alternating-time temporal logic and model checking algorithm. In *Proceedings of the 4th International Conference on Fuzzy Systems and Knowledge Discovery*, 35–39.

Chen, T.; Forejt, V.; Kwiatkowska, M. Z.; Parker, D.; and Simaitis, A. 2012. Automatic verification of competitive stochastic systems. In *Proceedings of the 18th International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, 315–330.

Chen, T.; Forejt, V.; Kwiatkowska, M. Z.; Parker, D.; and Simaitis, A. 2013. Prism-games: A model checker for stochastic multi-player games. In *Proceedings of the 19th International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, 185–191.

Chen, T.; Song, F.; and Wu, Z. 2016a. Global model checking on pushdown multi-agent systems. In *Proceedings of the 30th AAAI Conference on Artificial Intelligence*, 2459–2465.

Chen, T.; Song, F.; and Wu, Z. 2016b. Verifying pushdown multi-agent systems against strategy logics. In *Proceedings of the 25th International Joint Conference on Artificial Intelligence*.

Dax, C. 2006. *Games for the Linear Time μ-Calculus*. Ph.D. Dissertation, Ludwig-Maximilians-University Munich.

de Alfaro, L., and Majumdar, R. 2004. Quantitative solution of omega-regular games. *Journal of Computer and System Science* 68(2):374–397.

Henzinger, T. A., and Prabhu, V. S. 2006. Timed alternating-time temporal logic. In *Proceedings of the 4th International Conference on Formal Modeling and Analysis of Timed Systems*, 1–17.

Huang, X.; Su, K.; and Zhang, C. 2012. Probabilistic alternating-time temporal logic of incomplete information and synchronous perfect recall. In *Proceedings of the 26th AAAI Conference on Artificial Intelligence*.

Jamroga, W., and Murano, A. 2014. On module checking and strategies. In *Proceedings of the 13th International Joint Conference on Autonomous Agents and Multiagent Systems*, 701–708.

Liu, W.; Song, L.; Wang, J.; and Zhang, L. 2015. A simple probabilistic extension of modal mu-calculus. In *Proceedings of the 24th International Joint Conference on Artificial Intelligence*, 882–888.

Mogavero, F.; Murano, A.; Perelli, G.; and Vardi, M. Y. 2014. Reasoning about strategies: On the model-checking problem. *ACM Transations on Computational Logic* 15(4):34:1–34:47.

Mogavero, F.; Murano, A.; and Sauro, L. 2013. On the boundary of behavioral strategies. In *Proceedings of the 28th IEEE Symposium on Logic in Computer Science*, 263–272.

Mogavero, F.; Murano, A.; and Sauro, L. 2014. A behavioral hierarchy of strategy logic. In *Proceedings of the 15th International Workshop on Computational Logic in Multi-Agent Systems*, 148–165.

Mogavero, F.; Murano, A.; and Vardi, M. Y. 2010. Reasoning about strategies. In *Proceedings of the 30th Conference on Foundations of Software Technology and Theoretical Computer Science*, 133–144.

Murano, A., and Perelli, G. 2015. Pushdown multi-agent system verification. In *Proceedings of the 24th International Joint Conference on Artificial Intelligence*, 1090–1097.

Piterman, N. 2007. From nondeterministic Büchi and streett automata to deterministic parity automata. *Logical Methods in Computer Science* 3(3).

Schnoor, H. 2010. Strategic planning for probabilistic games with incomplete information. In *Proceedings of the 9th International Joint Conference on Autonomous Agents and Multiagent Systems*, 1057–1064.

van der Hoek, W., and Wooldridge, M. 2002. Tractable multiagent planning for epistemic goals. In *Proceedings of the 1st International Joint Conference on Autonomous Agents and Multiagent Systems*, 1167–1174.

Wan, W.; Bentahar, J.; and Hamza, A. B. 2013. Model checking epistemic-probabilistic logic using probabilistic interpreted systems. *KBS* 50:279–295.

Wan, W.; Bentahar, J.; Yahyaoui, H.; and Hamza, A. B. 2016. Verifying concurrent probabilistic systems using probabilistic-epistemic logic specifications. *Applied Intelligence* 1–30.