Reasoning about Strategic Abilities: Agents with Truly Perfect Recall

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In alternating-time temporal logic $\mathsf{ATL}^\star$, agents with perfect recall assign choices to sequences of states, i.e., to possible finite histories of the game. However, when a nested strategic modality is interpreted, the new strategy does not take into account the previous sequence of events. It is as if agents collect their observations in the nested game again from scratch, thus effectively forgetting what they observed before. Intuitively, it does not fit the assumption of agents having perfect recall of the past.

In this paper, we investigate the alternative semantics for $\mathsf{ATL}^\star$ where the past is not forgotten in nested games. We show that the standard semantics of $\mathsf{ATL}^\star$ coincides with the "truly perfect recall" semantics in case of agents with perfect information. On the other hand, the two semantics differ significantly if agents have imperfect information about the state of the game. The same applies to the standard vs. "truly perfect recall" semantics of $\mathsf{ATL}^\star$ with persistent strategies. We compare the relevant variants of $\mathsf{ATL}^\star$ by looking at their their expressive power, sets of validities, and feasibility of model checking.

CCS Concepts:•**Theory of computation** → **Modal and temporal logics;** *Logic and verification;*•**Computing methodologies** → **Multi-agent systems;**

Additional Key Words and Phrases: alternating-time temporal logic, multi-player games, reasoning about knowledge and time

1. INTRODUCTION

The *alternating-time temporal logic* $\mathsf{ATL}^\star$ and its fragment ATL [Alur et al. 1997; 2002] are logics which allow for reasoning about strategic interactions in multi-agent systems (MAS). The main idea is to extend the framework of temporal logic with the game-theoretic notion of *strategic ability*. Hence, $\mathsf{ATL}^\star$ enables to express statements about what agents (or groups of agents) can achieve. For example, $\langle\!\langle a\rangle\!\rangle\Diamond\mathsf{win_a}$ says that agent a has the ability to eventually win no matter what the other agents do, while $\langle\!\langle a, b\rangle\!\rangle\Box\mathsf{safe}$ expresses that agents a and b together can force the system to always remain in a safe state. Such properties can be useful for specification, verification and reasoning about interaction in agent systems. They have become especially relevant due to active development of algorithms and tools for verification where the "correctness" property is given in terms of strategic ability [Alur et al. 2000; Alur et al. 2001; Kacprzak and Penczek 2004; Lomuscio and Raimondi 2006; Chen et al. 2013; Huang and van der Meyden 2014; Busard et al. 2014; Pilecki et al. 2014; Lomuscio et al. 2015]. Still, when verifying a system, one must first of all have a clear idea *what* is to be verified. An important challenge for model checking MAS is to define the correctness property in the right way. This means choosing the right language and the right semantics, one which accurately captures agents' abilities in a given context.

The challenge is not only theoretical. When designing a system, specifying requirements, or verifying its properties, one must choose between many semantic variants of $\mathsf{ATL}^\star$ that start from different assumptions about the capabilities of agents. For instance, agents may be able to observe the full state of the system or only parts of it (perfect vs. imperfect information), and they may base their decisions on the current state only, or on the entire history of the game (perfect vs. imperfect

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recall) [Schobbens 2004; Jamroga and van der Hoek 2004]. Intermediate cases of finite-memory strategies have also been studied [Vester 2013]. Moreover, agents can have objective or subjective ability to achieve their goals [Bulling and Jamroga 2014], their strategies can come with or without long-term commitment [Ågotnes et al. 2007; Brihaye et al. 2009], they can be assumed to play rationally [Bulling et al. 2008], be equipped with bounded resources [Alechina et al. 2004; Alechina et al. 2009; Alechina et al. 2010; 2011; Bulling and Farwer 2010b; 2010a], a cost-free mechanism for broadcasting information within a team [Dima et al. 2010; Guelev et al. 2011], and so on.

In this paper, we focus on the commonly accepted perfect recall semantics of strategic ability, proposed in [Alur et al. 2002; Schobbens 2004]. We point out that, for nested strategic modalities, it interprets formulae of $\mathsf{ATL}^\star$ in a counterintuitive way. We show how to modify the semantics so that it avoids the problem, and, most importantly, we formally investigate the difference between the standard and the new semantics in terms of valid sentences, expressive power, and complexity of model checking.

1.1. Contribution: Analysis of Perfect Recall in Logics of Strategic Ability

We begin our analysis by observing that the standard perfect recall semantics of $\mathsf{ATL}^\star$ has a counterintuitive flavor: despite using perfect recall strategies, agents may not have access to all of their past observations. More precisely, agents *forget* their past observations once they proceed to realize a sub-goal in the game. As an example, consider the formula $\langle\!\langle b, c\rangle\!\rangle \Diamond \langle\!\langle a, b\rangle\!\rangle \bigcirc \mathsf{married_{ab}}$ which expresses that Bob and Charles have a joint strategy to ensure that, at some point in the future, Alice and Bob will be able to get married. Agents' abilities rely on their knowledge; in case of perfect recall, one would assume that each agent can use all their past observations to determine their subsequent actions. However, the semantics of $\mathsf{ATL}^\star$ interprets the subformula $\langle\!\langle a, b\rangle\!\rangle \bigcirc \mathsf{married_{ab}}$ *in the original model*. This amounts to assuming that Bob, when looking for his best strategy to make $\langle\!\langle a, b\rangle\!\rangle \bigcirc \mathsf{married_{ab}}$ true, must ignore (or forget) all the observations that he has made while executing his strategy for $\langle\!\langle b, c\rangle\!\rangle \Diamond \langle\!\langle a, b\rangle\!\rangle \bigcirc \mathsf{married_{ab}}$.

In this paper, we study a modified semantics of $\mathsf{ATL}^\star$ where formulae are interpreted in finite sequences of states rather than single states of the system. We call the semantics *truly perfect recall* to emphasize that decisions within a strategy can refer to the whole history of observations made by the agents. We show that the new semantics offers a significantly different view of agents' abilities from the original semantics of $\mathsf{ATL}^\star$. More precisely, we prove that if agents have imperfect information then $\mathsf{ATL}^\star$ with truly perfect recall differs from $\mathsf{ATL}^\star$ with standard perfect recall in terms of expressive power as well as valid sentences. The same can be shown for variants of $\mathsf{ATL}^\star$ that allow agents to irrevocably commit to their strategies [Ågotnes et al. 2007; Brihaye et al. 2009]. We also point out that truly perfect recall makes model checking $\mathsf{ATL}^\star$ harder than in the standard semantics.[1] Analogously to [Bulling and Jamroga 2014], we conclude that the truly perfect recall semantics corresponds to a different *class of games*, and allows for expressing different properties of those games, than the "classic" variants of $\mathsf{ATL}^\star$ from [Alur et al. 2002; Schobbens 2004].

1.2. Motivating Examples

The main point that we raise concerns the semantics of formulae that involve nested strategic modalities. That is, formulae of $\mathsf{ATL}^\star$ that specify agents' ability to endow someone with (or prevent from) the ability to achieve a given goal. In other words, we are concerned with specifications that address the existence of strategies which enable (or disable) other strategies. At the first glance, such formulae may seem rather esoteric, and not likely to be encountered in specifications of actual systems. Below we present a number of examples which demonstrate the opposite, namely that the ability to influence abilities can be extremely important. Hence, the right semantics of nested strategic modalities is also of utmost importance for the soundness of reasoning and verification.

[1] That is, the complexity significantly increases for decidable fragments of the problem.

Example 1.1 (*Security policy*). Consider designing and implementing a web banking infrastructure. Typically, it should enable some functionalities while at the same time preserving relevant security objectives. We claim that, in many cases, functionality can be understood in terms of a lower bound on the abilities of authorised agents, while security can be seen as an upper bound on the abilities of intruders. For example, a bank manager may allow a credit line on accounts of certain clients. This implies that clients can use credit on their own account, but not on others. The ability of the manager can be specified as:

$$\langle\langle manager\rangle\rangle\Box \bigwedge_{u\in\mathbb{A}\mathrm{gt}} \Big(\langle\langle u\rangle\rangle\Diamond \mathsf{credit_u} \wedge \bigwedge_{u'\in\mathbb{A}\mathrm{gt}\setminus\{u\}} \neg\langle\langle u\rangle\rangle\Diamond \mathsf{credit_{u'}}\Big)$$

Example 1.2 (*Social fairness*). The Ministry of Education should strive to ensure fair access to university education across the population, e.g., by funding scholarships and stipends for students, subsidizing infrastructure in underdeveloped regions, etc. The objective is that everybody should be given an opportunity to study, provided that they choose to do so, and can conceivably make it. Let $\mathsf{E}\Diamond\psi$ mean "there is at least one possible path where ψ eventually holds," i.e., it is at least conceivable that ψ can become true at some future moment. Now, the requirement that "the Ministry should be able to provide fair access to university education" can be captured by the following formula:

$$\langle\langle ministry\rangle\rangle\Box \bigwedge_{a\in\mathbb{A}\mathrm{gt}} \Big(\mathsf{E}\Diamond \mathsf{completeStudies_a} \rightarrow \langle\langle a\rangle\rangle\Diamond \mathsf{study_a}\Big).$$

Example 1.3 (*Robot soccer*). A possible task specification for a member of a RoboCup team is:

$$\langle\langle a\rangle\rangle\Diamond\Big(\mathsf{possession_b} \wedge \langle\langle b\rangle\rangle\Diamond \mathsf{goal_{ab}}\Big),$$

i.e., the robot a should seek a strategy to pass the ball to its teammate b in such a way that b can score a goal for their team.

Example 1.4 (*TCP*). The following specification describes an objective of the congestion control mechanism in the TCP protocol:

$$\langle\langle sender\rangle\rangle\Box(\mathsf{send\_pkt} \rightarrow \langle\langle receiver\rangle\rangle\Diamond \mathsf{ack\_pkt})$$

expressing that a sender can maintain that whenever a packet is sent, the receiver has a strategy to eventually acknowledge it. Such a strategy may include actions such as replying with a delay, so that the receiver buffer does not overflow.

Example 1.5 (*Distribution of cryptographic keys*). Public key cryptography is based on generation of a pair of keys (sk_a, pk_a) where sk_a is agent a's secret key, and pk_a is his public key. The secret key is known only by a, whereas the public key is openly available to everybody (e.g., posted on the web). They serve dual cryptographic functions, i.e., messages encrypted by pk_a can be decrypted only using sk_a, and vice versa. The keys can be used for either communication or authentication. If another agent wants to send a private message to a, she can encrypt the message with pk_a and send it to a (who is the only agent possessing the key to decrypt it). If a wants to authenticate himself to another agent, he sends her a message plus its copy encrypted with his secret key (typically the communication is supposed to be private, so both parts are additionally encrypted with b's public key). When b decrypts the second part with a's public key and it matches the first part, then the message must have originated from a. However, this only works when the process of key distribution is trustworthy, i.e., when a and b know that the public keys pk_b, pk_a indeed come from b and a, respectively. If we are mainly interested in communication then the goal of key exchange between agents a and b can be specified as:

$$\langle\langle a,b\rangle\rangle\Diamond\Box \bigwedge_{m} \bigwedge_{c\neq a,b} \Big(\langle\langle a\rangle\rangle(\Diamond K_b m \wedge \Box\neg K_c m) \wedge \langle\langle b\rangle\rangle(\Diamond K_a m \wedge \Box\neg K_c m)\Big),$$

where $K_i m$ denotes that agent i knows the content of message m.

In what follows, we will use a simplified working example to illustrate the main definitions in an intuitive way. We point out, however, that the working example shares the most important feature with the above motivating scenarios, in the sense that it asks about existence of a strategy to provide (or deprive) an agent with/of specific strategic ability.

1.3. Related Work

An important strand in research on ATL* emerged in quest of the "right" semantics for strategic ability for a specific setting. ATL was combined with epistemic logic [van der Hoek and Wooldridge 2003; Jamroga and van der Hoek 2004], and several semantic variants were defined for various assumptions about agents' memory [Schobbens 2004; Jamroga and van der Hoek 2004; Ågotnes and Walther 2009; Vester 2013] and available information [Schobbens 2004; Jamroga and van der Hoek 2004; Ågotnes 2006; Jamroga and Ågotnes 2007], cf. also [Bulling and Jamroga 2014; Ågotnes et al. 2015] for a broader discussion. Moreover, many conceptual extensions have been considered, e.g., with explicit reasoning about strategies [van der Hoek et al. 2005; Walther et al. 2007; Chatterjee et al. 2007b; Mogavero et al. 2010a], bounded resources [Alechina et al. 2009; Alechina et al. 2010; Bulling and Farwer 2010b], rationality assumptions in the form of game-theoretic solution concepts [Bulling et al. 2008], mechanisms for coordination within teams [Hawke 2010; van Ditmarsch and Knight 2014], and persistent commitment to strategies [Ågotnes et al. 2007; Brihaye et al. 2009]. Another strand of papers considered variants of coalitional ability where members of the team were assumed to have a unified view of the state of the system, either by sharing their information at no cost throughout the game [Guelev and Dima 2008; Dima et al. 2010; Guelev et al. 2011], or by aggregating their uncertainty at each step [Diaconu and Dima 2012].

Several papers have come close to what we propose and investigate here. A very similar, history-based semantics for ATL* with imperfect information and perfect recall (called ATEL-R*) was in fact considered as early as [Jamroga and van der Hoek 2004], but it was not studied further. Later, [Bulling and Dix 2010] also used a semantic relation that referred to the history of events. The focus of that work, however, was coalition formation, and the history-based semantics was used to keep track of the satisfaction of agents' goals. Furthermore, [Mogavero et al. 2010b] used history-based semantics to study a variant of ATL* where coalition A can enforce property ψ in state q if, on all plays enforced by A from q, ψ is true when evaluated *from the beginning of the game*. This differs from our work as follows. First, we only use histories to propagate past observations to strategies that are witnesses to nested strategic modalities, and not to evaluate the "winning condition" (our path subformulae are purely future-oriented). Secondly, [Mogavero et al. 2010b] look only at the perfect information setting, whereas we consider the cases of both perfect and imperfect information (with and without strategic commitment). Accordingly, our results are very much different. While [Mogavero et al. 2010b] show that their "relentful ATL*" has the same expressive power and model checking complexity as standard ATL*, our "ATL* with truly perfect recall" has incomparable expressive power and different model checking complexity. Moreover, it generates a different set of validities than standard ATL*.

History-based semantics for ATL* with imperfect information was also used in several papers by Dima and his coauthors [Dima et al. 2010; Guelev et al. 2011; Diaconu and Dima 2012], and the focus of those papers is very close to what we study here. The differences are as follows. First, we look at the "truly perfect recall" semantics of ATL* in abstract concurrent game structures, whereas Dima et al. define the semantics in interpreted systems of infinite runs with perfect recall. Secondly, the semantics of coalitional play in [Dima et al. 2010; Guelev et al. 2011; Diaconu and Dima 2012] is seriously restricted by assuming that each coalition uses a single, aggregated indistinguishability relation; in fact, it can be argued that this amounts to treating coalitions as single agents in disguise, cf. [Kaźmierczak et al. 2014]. Thirdly, we focus on *comparing* the new semantics to the standard semantics, while Dima et al. concentrate on development of axiomatic systems and model checking algorithms for their variants of ATL*. Fourthly, we also investigate the impact of truly perfect recall in the case when persistent strategic commitment is allowed.[2]

1.4. Structure of the Article

The paper is structured as follows. We introduce the classic variants of alternating-time temporal logic with perfect recall and perfect/imperfect information in Section 2. In Section 3, we point out the "forgetting" phenomenon in the classical semantics of ATL*, and present the truly perfect recall semantics that avoids it. In Section 4 we compare the expressive powers of the variants of ATL* with the standard vs. truly perfect recall semantics. In Section 5 we do the same with respect to the validity sets induced by the two semantics. Section 6 looks at the difference between standard perfect recall and truly perfect recall in the presence of persistent strategies. Finally, in Section 7, we briefly address the impact of truly perfect recall on the computational complexity of model checking for decidable fragments of the problem. We conclude our work in Section 8, and discuss some directions for future research.

The material presented in this article is based on the conference papers [Bulling et al. 2013; 2014]. It extends the conference versions with detailed proofs, carefully constructed motivating and working examples, and formal analysis of truly perfect recall under strategic commitment. It also adds a brief discussion on the impact of truly perfect recall on the effectiveness and complexity of model checking ATL*.

2. REASONING ABOUT STRATEGIC ABILITY

In this section, we briefly recall the main concepts behind ATL* and its variants.

2.1. Syntax of Alternating-Time Temporal Logic

ATL* [Alur et al. 1997; 2002] can be seen as a generalization of the branching time logic CTL*, with the path quantifiers E and A being replaced by *strategic modalities* $\langle\!\langle A\rangle\!\rangle$. The formula $\langle\!\langle A\rangle\!\rangle\gamma$ expresses that group A has a *collective strategy* to enforce the temporal property γ where γ can include the temporal operators $\bigcirc$ ("next"), and $\mathcal{U}$ ("until"). Formally, let Π be a countable set of atomic propositions, and $\mathbb{A}\mathrm{gt}$ be a finite nonempty set of agents. The language of ATL* is given by the following grammar:

$$\begin{array}{l}\varphi ::= p \mid \neg\varphi \mid \varphi\wedge\varphi \mid \langle\!\langle A\rangle\!\rangle\gamma,\\ \gamma ::= \varphi \mid \neg\gamma \mid \gamma\wedge\gamma \mid \bigcirc\gamma \mid \gamma\,\mathcal{U}\,\gamma, \qquad \text{where } A\subseteq\mathbb{A}\mathrm{gt} \text{ and } p\in\Pi.\end{array}$$

We define "sometime in the future" as $\Diamond\gamma \equiv \top\,\mathcal{U}\,\gamma$ and "always in the future" as $\Box\gamma \equiv \neg\Diamond\neg\gamma$. Formulae φ and γ are called *state* and *path formulae* of ATL*, respectively. State formulae constitute the language of ATL*. By requiring that each temporal operator is immediately preceded by a strategic modality, we obtain the sub-language ATL; for example, $\langle\!\langle A\rangle\!\rangle\Diamond p$ is an ATL formula but $\langle\!\langle A\rangle\!\rangle\Diamond\Box p$ and $\langle\!\langle A\rangle\!\rangle(\Diamond p\wedge\Diamond r)$ are not.

2.2. Models: Imperfect Information Concurrent Game Structures

We interpret ATL* formulae over *imperfect information concurrent game structures* (iCGS) [van der Hoek and Wooldridge 2003; Schobbens 2004]. An iCGS is given by a tuple $M = \langle\mathbb{A}\mathrm{gt}, St, \Pi, \pi, Act, d, o, \{\sim_a \mid a\in\mathbb{A}\mathrm{gt}\}\rangle$ consisting of a nonempty finite set of all agents $\mathbb{A}\mathrm{gt} = \{1,\dots,k\}$, a nonempty set of states St, a set of atomic propositions Π and their valuation $\pi : \Pi \to 2^{St}$, and a nonempty finite set of (atomic) actions Act. Function $d : \mathbb{A}\mathrm{gt}\times St \to 2^{Act}$ defines nonempty sets of actions available to agents at each state; we will usually write $d_a(q)$ instead of $d(a,q)$. Function o is a (deterministic) transition function that assigns the outcome state $q' = o(q,\alpha_1,\dots,\alpha_k)$ to each state q and tuple of actions $\langle\alpha_1,\dots,\alpha_k\rangle$ such that $\alpha_i\in d_i(q)$ for

[2]A variant of ATL* that combines imperfect information, truly perfect recall, and persistent strategies has been considered in [Guelev and Dima 2012]. We discuss the relationship to our work in Section 6.2.

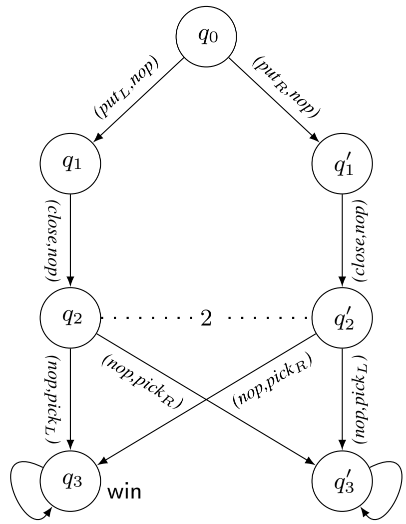


Fig. 1. The iCGS M_1 describing the *shell* game. Tuples (α_1, α_2) represent the action profiles. α_1 denotes an action of player 1—the shuffler—and action α_2 of player 2—the guesser. The dotted line represents 2's indistinguishability relation; State q_3 is labelled with the only proposition win. For example, when the guesser plays action $pick_R$ in state q_2 the game proceeds to state q_3'. nop indicates the "do nothing" action.

$1 \leq i \leq k$. Finally, each $\sim_a \subseteq St \times St$ is an equivalence relation that represents the indistinguishability of states from agent a's perspective.[3] We assume that agents have identical choices in indistinguishable states ($d_a(q) = d_a(q')$ whenever $q \sim_a q'$). We also assume that collective knowledge is interpreted in the sense of "everybody knows', i.e., $\sim_A = \bigcup_{a \in A} \sim_a$. We will use $[q]_A = \{q' \mid q \sim_A q'\}$ to refer to A's epistemic image of state q. Note that the perfect information concurrent game structures (CGS) from [Alur et al. 2002] can be seen as a specific type of iCGS that assumes each $\sim_a$ to be the minimal reflexive relation.

Example 2.1 (*Shell game*). Consider model M_1 in Figure 1 that depicts a simple version of the *shell game*. There are two players: the *shuffler* 1 and the *guesser* 2. Initially, the shuffler places a ball in one of two shells (the left or the right). The shells are open, and the guesser can see the location of the ball. Then the shuffler turns the shells over, so that the ball becomes hidden. The guesser wins if he picks up the shell containing the ball. Formally: $\mathbb{A}\mathrm{gt} = \{1, 2\}$, $St = \{q_0, q_1, q_2, q_3, q_1', q_2', q_3'\}$, $\Pi = \{\mathsf{win}\}$, $\pi(q_3) = \{\mathsf{win}\}$, $Act = \{put_L, put_R, pick_L, pick_R, close, nop\}$, $d_1(q_0) = \{put_L, put_R\}$, $d_1(q_1) = d_1(q_1') = \{close\}$, $d_1(q_2) = d_1(q_2') = d_2(q_0) = d_2(q_1) = d_2(q_1') = \{nop\}$, $d_1(q_2) = d_1(q_2') = \{pick_L, pick_R\}$, $d_i(q_3) = d_i(q_4) = \{nop\}$ for $i \in \mathbb{A}\mathrm{gt}$. $q \sim_i q$ for all $q \in St$ and $i \in \mathbb{A}\mathrm{gt}$. Also, $q_2 \sim_2 q_2'$. The function o is illustrated in Figure 1. Obviously, this is a very simplified version of the shell game as the shuffler does not even shuffle the shells; he simply places the ball in one of them and closes them. However, the example is rich enough to point out the limitations of the standard semantics of $\mathsf{ATL}^\star$.

Two remarks are in order. First, the relation $\sim_a$ encodes a's (in)ability to distinguish pairs of states, based on the qualities encapsulated in those states. That is, $q \sim_a q'$ iff q and q' *look* the same to a, independent of the history of events that led to them. If one assumes that the agent has external memory that allows her to remember the history of past events, this must be represented by

[3]The relations capture *observational* indistinguishability. The *knowledge* that an agent collects by means of subsequent observations is not encoded in the model but rather in the constraints on strategies that the agent is allowed to play, see also the remark after Example 2.1.

an indistinguishability relation on *histories*, introduced in the next paragraph. Secondly, in order to describe an actual game, we also need to fix the initial state of an iCGS. A pair (M, q) consisting of an iCGS M and a state of M is called a *pointed iCGS*.

A *history* h is a finite sequence of states $q_0 q_1 \dots q_n \in St^+$ which results from the execution of subsequent transitions; that is, there must be an action profile connecting q_i with q_{i+1} for each $i = 0, \dots, n-1$. Two histories $h = q_0 q_1 \dots q_n$ and $h' = q'_0 q'_1 \dots q'_m$ are *indistinguishable for agent* a (denoted $h \approx_a h'$) iff $n = m$ and $q_i \sim_a q'_i$ for $i = 0 \dots n$. This corresponds to the notion of *synchronous perfect recall* in temporal-epistemic logic [Fagin et al. 1995]. We also extend the indistinguishability relation over histories $\approx_a$, to groups: $\approx_A = \bigcup_{a \in A} \approx_a$. We write $h \circ h'$ to refer to the concatenation of the histories h, h' and *last*(h) to refer to the first and last state from history h, respectively. $\Lambda_M^{fin}(q)$ is the set of all histories in model M starting from state q, and $\Lambda_M^{fin} = \bigcup_{q \in St} \Lambda_M^{fin}(q)$ is the set of all histories in model M.

A *path* $\lambda = q_0 q_1 q_2 \dots$ is an infinite sequence of states such that there is a transition from each q_i to q_{i+1}. We write $h \circ \lambda$, where $h = q'_0 q'_1 \dots q'_n$ to refer to the path $q'_0 q'_1 \dots q'_n q_0 q_1 q_2 \dots$ obtained by concatenating h and λ, provided that there is a transition from q'_n to q_0. We use $\Lambda_M(q)$ to refer to the set of paths in M that start in state q, and define $\Lambda_M := \bigcup_{q \in St_M} \Lambda_M(q)$ to be the set of paths in M. We use $\lambda[i]$ to denote the ith position on path λ (starting from $i = 0$), $\lambda[i, j]$ (with $j \geq i$) to denote the history $q_i \dots q_j$, and $\lambda[i, \infty]$ to denote the subpath of λ starting from i. Whenever the model is clear from context, we shall omit the subscript. As we will see later, the semantics of formulae is defined over paths. The truth, however, essentially depends on the sequence of propositional labels of each state and not on the name of the state. Hence, we say that two paths λ and λ' are *propositionally equivalent*, in notation $\lambda \equiv \lambda'$ if, and only if, $\lambda[i] \in \pi(p)$ iff $\lambda'[i] \in \pi(p)$ for all $i \in \mathbb{N}$ and $p \in \Pi$.

2.3. Strategies and Their Outcomes

A *strategy* of agent a is a conditional plan that specifies what a is going to do in each situation. It makes sense, from a conceptual and computational point of view, to distinguish between two types of strategies: an agent may base its decision on the current state or on the whole *history* of events that have happened. In this paper, we consider only the latter case. A *perfect information* strategy (*I-strategy* for short) is a function $s_a : St^+ \to Act$ such that $s_a(q_0 \dots q_n) \in d_a(q_n)$ for all $q_0 \dots q_n \in St^+$. An *imperfect information strategy* (*i-strategy*) must be additionally *uniform*, in the sense that $h \approx_a h'$ implies $s_a(h) = s_a(h')$. A *collective x-strategy* s_A with $x \in \{I, i\}$, is a tuple of *x*-strategies, one per agent in A. In particular, for imperfect information, each individual strategy s_a of s_A must be uniform. We also note that uniformity restricts each strategy component of s_A with respect to the corresponding individual indistinguishability relation $\sim_a$ and not to the one corresponding to the group: $\sim_A$. We use $s_A|_a$ to denote agent a's part of the collective strategy s_A, and $s_\emptyset$ to denote the empty profile which is the only strategy of the empty coalition.

The function $out_M(h, s_A)$ returns the set of all paths in M starting with history h, that can occur when s_A is executed. Formally:

> $out_M(h, s_A) = \{h \circ \lambda = q_0 q_1 q_2 ... \mid$ such that for each $i \geq |h|$ there exists $\langle \alpha_{a_1}^{i-1}, \dots, \alpha_{a_k}^{i-1} \rangle$ such that $\alpha_a^{i-1} \in d_a(q_{i-1})$ for every $a \in \mathbb{A}$gt, $\alpha_a^{i-1} = s_A|_a(q_0 q_1 \dots q_{i-1})$ for every $a \in A$, and $o(q_{i-1}, \alpha_{a_1}^{i-1}, \dots, \alpha_{a_k}^{i-1}) = q_i\}$.

Example 2.2 (*Strategies and outcomes*). Consider model M_1 from Example 2.1, coalition $A = \{1, 2\}$, and its collective strategy $s_A = (s_1, s_2)$ where $s_1(q_0) = put_L$, $s_1(q_0 q_1) = s_1(q_0 q'_1) = close$ and $s_2(q_0 q_1 q_2) = pick_L$. The values of $s_a(h)$ are unimportant for all the other combinations of $h \in \Lambda_{M_1}^{fin}$ and $a \in A$. If s_A is executed in state q_0, we have $out_M(q_0, s_A) = \{q_0 q_1 q_2 q_3^\omega\}$.

In the same model, consider strategy $s'_A = (s'_1, s'_2)$, where s'_1 assigns an arbitrary permissible action to each history, $s'_2(q_0 q_1 q_2) = s'_2(q_0 q'_1 q'_2) = pick_L$ and $s'_2(h) = nop$ for all other histories. If $q_0 q_1 q_2$ is the current game development (the initial state was q_0, followed by q_1 and q_2), then

$out_M(q_0q_1q_2, s'_A) = \{q_0q_1q_2q_3^\omega\}$. At the same time, if $s'_2(q_2) = s'_2(q'_2) = pick_L$ and $s'_2(h) = nop$ for all other histories, then — if the game starts in q_2 — we have: $out_M(q_2, s'_A) = \{q_2q_3^\omega\}$.

Function $plays_M^x(h, s_A)$ returns the set of paths which agents from A consider possible if the game started with h and strategy s_A is executed. For perfect information, $plays_M^I(h, s_A) = out_M(h, s_A)$. For imperfect information, $plays_M^i(h, s_A)$ includes also the paths that A *think* might occur, i.e., ones starting from histories that are indistinguishable from A's point of view:

$$plays_M^x(h, s_A) = \begin{cases} out_M(h, s_A) & \text{for } x = I \\ \bigcup_{h \approx_A h'} out_M(h', s_A) & \text{for } x = i \end{cases}$$

Example 2.3 (*Subjective outcome of a strategy*). Let us revisit the strategies presented in Example 2.2. Recall that $out_M(q_2, s'_A) = \{q_2q_3^\omega\}$, and observe that $out_M(q'_2, s'_A) = \{q'_2q_3'^\omega\}$. Hence $plays_M^i(q_2, s'_A) = \{q_2q_3^\omega, q'_2q_3'^\omega\}$: the history of the game is q_2 and since coalition A cannot distinguish q_2 from q'_2, both the "*winning*" and "*losing*" paths are considered possible when s'_A is executed.

Note that the above definitions of functions *out* and *plays* are slightly more general than the ones from [Alur et al. 2002; Schobbens 2004; Bulling and Jamroga 2014]: outcome paths are constructed given an initial *sequence of states* rather than a single state. This will prove convenient when we define the truly perfect recall semantics of $\mathsf{ATL}^\star$ in Section 3.

2.4. Standard Perfect Recall Semantics

Let M be an iCGS and $\lambda \in \Lambda_M$. The *(standard perfect recall) semantics* of $\mathsf{ATL}^\star$ $\models_x$, parameterized with $x \in \{i, I\}$, can be defined as follows:

$M, \lambda \models_x p$ iff $\lambda[0] \in \pi(p)$ (where $p \in \Pi$);
$M, \lambda \models_x \neg\varphi$ iff $M, \lambda \not\models_x \varphi$;
$M, \lambda \models_x \varphi_1 \wedge \varphi_2$ iff $M, \lambda \models_x \varphi_1$ and $M, \lambda \models_x \varphi_2$;
$M, \lambda \models_x \langle\langle A \rangle\rangle\varphi$ iff there is a collective x-strategy s_A such that, for each $\lambda' \in plays_M^x(\lambda[0], s_A)$, we have $M, \lambda' \models_x \varphi$;
$M, \lambda \models_x \bigcirc\varphi$ iff $M, \lambda[1, \infty] \models_x \varphi$;
$M, \lambda \models_x \varphi_1 \mathcal{U} \varphi_2$ iff there is $i \in \mathbb{N}_0$ such that $M, \lambda[i, \infty] \models_x \varphi_2$ and for all $0 \le j < i$, we have that $M, \lambda[j, \infty] \models_x \varphi_1$.

Also, for a state q and a state formula φ, we define $M, q \models_x \varphi$ iff $M, \lambda \models_x \varphi$ for any $\lambda \in \Lambda_M(q)$.[4] We refer to the logic obtained by combining $\models_x$ with the language of $\mathsf{ATL}^\star$, i.e. all state formulae, as $\mathsf{ATL}_\mathsf{x}^\star$. A state formula φ is *valid* in $\mathsf{ATL}_\mathsf{x}^\star$ iff $M, q \models_x \varphi$ for all M and states q in M.

Example 2.4 (*Shell game ctd.*). Consider the iCGS M_1 from Figure 1, and assume q_2 is the initial state of the game. It is easy to see that $M_1, q_2 \models_I \langle\langle 2 \rangle\rangle\Diamond\mathsf{win}$: under perfect information, the guesser can win by choosing the left shell in q_2. On the other hand, $M_1, q_2 \not\models_i \langle\langle 2 \rangle\rangle\Diamond\mathsf{win}$: under imperfect information, the guesser has no *uniform* strategy that succeeds from both q_2 and q'_2. Finally, if the game begins in q_0 then the guesser can win ($M_1, q_0 \models_i \langle\langle 2 \rangle\rangle\Diamond\mathsf{win}$) by using the i-strategy: "play $pick_L$ (resp. $pick_R$) after history $q_0q_1q_2$ (resp. $q_0q'_1q'_2$)". The strategy is uniform as both histories are distinguishable for the guesser[5].

Remark 2.5. Informally, $M, \lambda \models_I \langle\langle A \rangle\rangle\varphi$ holds iff there exists a collective I-strategy s_A such that φ holds on all outcome paths that result from executing s_A after history $\lambda[0]$. Thus, in the case

[4]Note that we can equivalently define $M, q \models_x \varphi$ as $M, \lambda \models_x \varphi$ for all $\lambda \in \Lambda_M(q)$.
[5]We note that the guesser has no memoryless strategy (i.e. a strategy that assigns actions to states only) to win, as such a strategy had to assign the same choices to q_2 and q'_2.

of standard $\mathsf{ATL}^\star$, the history is always limited to the *current state*, and thus the previous states of the play are completely ignored.

Note also that $M, q \models_i \langle\langle A\rangle\rangle\varphi$ requires A to have a single strategy that is successful in *all* states indistinguishable from q for *any member* of the coalition. Moreover, standard epistemic operators can be expressed in $\mathsf{ATL}^\star_\mathsf{i}$ as follows. Let $\mathcal{N}\varphi \equiv \varphi\,\mathcal{U}\,\varphi$. It is easy to see that $\mathcal{N}\varphi$ expresses that φ holds "now," i.e., at the initial state of the given path.[6] Then, $K_a\varphi$ can be defined as $\langle\langle a\rangle\rangle\mathcal{N}\varphi$. It is easy to check that $M, q \models_i K_a\varphi$ iff for all q' such that $q \sim_a q'$ and $M, q' \models_i \varphi$.

3. STRATEGIES WITH TRULY PERFECT RECALL

We have already seen that, in $\mathsf{ATL}^\star$, strategies are synthesised with respect to the current state of the game ($\lambda[0]$), and that "*previous events*" influence neither the strategy selection nor the resulting paths. In this section we illustrate how this leads to the forgetting phenomenon. We also introduce a "*no-forgetting*" semantics for $\mathsf{ATL}^\star$.

3.1. Agents with Standard Perfect Recall Forget

In the standard semantics of $\mathsf{ATL}^\star$ agents "forget" information about the past, even if they are assumed to have perfect recall. This can be seen in the case of nested cooperation modalities, as illustrated by Example 2.4 and Remark 2.5. For instance, in formula $\langle\langle a\rangle\rangle\Diamond\langle\langle b\rangle\rangle\Box\mathsf{p}$, agent b must start collecting observations from scratch when executing her strategy to bring about $\Box\mathsf{p}$. The history of the game determined by the first cooperation modality is not taken into account. This leads to counterintuitive effects, as the following example shows.

Example 3.1 (*Forgetting in perfect recall*). On one hand, $M_1, q_0 \models_i \langle\langle 2\rangle\rangle\Diamond\mathsf{win}$, that is, the guesser has a uniform strategy to win the shell game starting in q_0. On the other hand, $M_1, q_2 \models_i \neg\langle\langle 2\rangle\rangle\Diamond\mathsf{win}$. As the shuffler in q_0 can easily enforce the future state to be q_2, we obtain that $M_1, q_0 \models_i \langle\langle 1\rangle\rangle\Diamond\neg\langle\langle 2\rangle\rangle\Diamond\mathsf{win}$. Thus, in (M_1, q_0), the guesser has the ability to win no matter what the shuffler does, and *at the same time* the shuffler has a strategy to deprive the guesser of the ability no matter what the guesser does.

3.2. ATL* with Truly Perfect Recall

To get rid of this "forgetting" behavior, we will use the *truly perfect recall semantics* of $\mathsf{ATL}^\star$, captured by relation $\models^{nf}_x$, where $x \in \{i, I\}$, i.e., two different variants which are used again for the perfect (I) and imperfect information (i) cases. Formulae are interpreted over triples consisting of a model, a path and an index $k \in \mathbb{N}_0$ which indicates the current position on the infinite path. Intuitively, the subhistory of the path up to k encodes the past, and the subpath starting after k, the future. The crucial part of this semantics is that the agents always remember the sequence of the past events — and they can *learn* from those events.

Definition 3.2 (*Truly perfect recall semantics for* $\mathsf{ATL}^\star$). Let M be an iCGS, $\lambda \in \Lambda_M$ and $k \in \mathbb{N}_0$. The *truly perfect recall semantics* of $\mathsf{ATL}^\star$ $\models_x$, parameterized with $x \in \{i, I\}$, is defined as follows:

$M, \lambda, k \models^{nf}_x p$ iff $\lambda[k] \in \pi(p)$ for $p \in \Pi$;
$M, \lambda, k \models^{nf}_x \neg\varphi$ iff $M, \lambda, k \not\models^{nf}_x \varphi$;
$M, \lambda, k \models^{nf}_x \varphi_1 \wedge \varphi_2$ iff $M, \lambda, k \models^{nf}_x \varphi_1$ and $M, \lambda, k \models^{nf}_x \varphi_2$;
$M, \lambda, k \models^{nf}_x \langle\langle A\rangle\rangle\varphi$ iff there exists a collective x-strategy s_A such that, for all $\lambda' \in$ *plays*$^x_M(\lambda[0,k], s_A)$, we have $M, \lambda', k \models^{nf}_x \varphi$;
$M, \lambda, k \models^{nf}_x \bigcirc\varphi$ iff $M, \lambda, k+1 \models^{nf}_x \gamma$
$M, \lambda, k \models^{nf}_x \varphi_1\,\mathcal{U}\,\varphi_2$ iff there exists $i \geq k$ such that $M, \lambda, i \models^{nf}_x \varphi_2$ and $M, \lambda, j \models^{nf}_x \varphi_1$ for all $k \leq j < i$.

[6]Equivalently, we can define $\mathcal{N}\varphi \equiv \bot\,\mathcal{U}\,\varphi$.

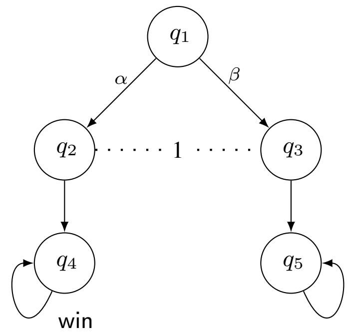


Fig. 2. The iCGS M_f with a single player (1) and two possible actions (α and β) in state q_1, which lead to q_2 and q_3, respectively. States q_2 and q_3 are indistinguishable. The player forgets her previous action if a subformula is evaluated in q_2 or q_3. This happens because, traditionally, paths record only states and not actions executed by the player.

We use $\mathsf{ATL}^\star_{\mathsf{nf,x}}$ to refer to the logic that combines the syntax of $\mathsf{ATL}^\star$ with the semantic relation $\models^{nf}_x$. Given a state formula φ and a history h, we define $M, h \models^{nf}_x \varphi$ iff $M, \lambda, k \models^{nf}_x \varphi$ for any $\lambda \in \Lambda_M$ such that $\lambda[0, k] = h$. A state formula φ is *valid* in $\mathsf{ATL}^\star_{\mathsf{nf,x}}$ iff $M, q \models^{nf}_x \varphi$ for all models M and states q (note that states can be seen as a special kind of histories); and *satisfiable* if such a pair (M, q) exists.

The new semantics differs from the standard semantics of $\mathsf{ATL}^\star$ only in that it keeps track of the history by incorporating it into each path. Instead of building paths starting in the current state of the game ($\lambda[0]$ in the standard semantics), we look at paths λ that describe the play from the very beginning. $\lambda[0, k-1]$ represents the sequence of past states (excluding the current one), $\lambda[k]$ is the current state, and $\lambda[k+1, \infty]$ is the future part of the play. We illustrate the semantics by the following example.

Example 3.3 (*Shell game ctd.*). Consider the pointed iCGS (M_1, q_0) from Figure 1 again. Whatever the shuffler does in the first two steps, the guesser can adapt its action (in q_2 and q_2') to win the game. In particular, the i-strategy s_2 from Example 2.4 can be used to demonstrate that for all $\lambda \in plays^i(q_0, s_2)$—for every strategy of 2—we have $M_1, \lambda, 0 \models^{nf}_i \Diamond\langle\langle 2\rangle\rangle\Diamond\mathsf{win}$. As a consequence, $M_1, q_0 \models^{nf}_i \neg\langle\langle 1\rangle\rangle\Diamond\neg\langle\langle 2\rangle\rangle\Diamond\mathsf{win}$.

Thus, the no-forgetting semantics gives the intended result for M_1. Note that agents can still forget the *actions* that they have performed. Essentially, our notion of true perfect recall is rooted in the way in which temporal paths are typically defined: as sequences of states rather than sequences of interleaved states and action profiles.

Remark 3.4 (*Forgetting in the truly perfect recall semantics*). In Figure 2, a single-player iCGS M_f is shown. First, we observe that $M_f, q_1 \models^{nf}_i \langle\langle 1\rangle\rangle\Diamond\mathsf{win}$. The player's strategy is s_1 where $s_1(q_1) = \alpha$. The actions assigned to all other histories by s_1 are unimportant. We note that $plays^i_{M_f}(q_1, s_1) = \{q_1 q_2 q_4^\omega\}$ contains a unique path.

However, it also holds that $M_f, q_1 \not\models^{nf}_i \langle\langle 1\rangle\rangle\bigcirc\langle\langle 1\rangle\rangle\bigcirc\mathsf{win}$. When synthesising a strategy s_1' for the subformula $\langle\langle 1\rangle\rangle\bigcirc\mathsf{win}$, we have $plays^i_{M_f}(q_1q_2, s_1') = out_{M_f}(q_1q_2, s_1') \cup out_{M_f}(q_1q_3, s_1')$, since $q_1q_2 \approx_1 q_1q_3$.

The player remembers the history, *but not the actions which have been played, including her own*. This is the case since, in $\mathsf{ATL}^\star$, histories are simply sequences of states and not sequences of interleaved action profiles and states. We consider this as a purely technical issue, as the last performed action profile can be encoded within a state whenever the need arises. Then, the modeler can define explicitly which agents can observe what actions.

3.3. Standard vs. True Perfect Recall: How Are They Different?

As the difference between $\mathsf{ATL}^\star_\mathsf{x}$ and $\mathsf{ATL}^\star_{\mathsf{nf},\mathsf{x}}$ lies in the "forgetting" of past observations when evaluating nested formulae, it comes as no real surprise that the two semantics coincide for perfect information. Agents with perfect information always precisely know the current global state of the system, and thus they cannot be uncertain about anything, including their own observations.

PROPOSITION 3.5. *For all iCGSs M, paths $\lambda \in \Lambda_M$, and $\mathsf{ATL}^\star$ formulae φ we have that $M, \lambda, 0 \models^{nf}_I \varphi$ iff $M, \lambda \models_I \varphi$.*

PROOF. Let $h \circ \lambda$ be an arbitrary path in Λ_M with $k = |h| - 1$ and $|h| \geq 1$. First, we observe that:

$$plays^I_M(h, s_A) = out_M(h, s_A) \tag{1}$$

for arbitrary collective I-strategies s_A. Furthermore, for all collective I-strategies s_A and histories h, there is a collective I-strategy s'_A such that:

$$out_M(h, s'_A) = \{h \circ \lambda \mid last(h) \circ \lambda \in out_M(last(h), s_A)\} \tag{2}$$

and also, for all collective I-strategies s'_A and histories h there is a collective strategy s_A such that:

$$out_M(last(h), s'_A) = \{last(h) \circ \lambda \mid h \circ \lambda \in out_M(h, s_A)\} \tag{3}$$

Informally, in (2) s'_A makes the same decisions as s_A given that history h has already taken place, while in (3) s'_A makes the same decisions after $last(h) \circ \lambda$ as s_A would do after history $h \circ \lambda$.

Now, we prove the stronger statement: $M, h \circ \lambda, k \models^{nf}_I \varphi$ iff $M, last(h) \circ \lambda \models_I \varphi$, for all $h \circ \lambda \in \Lambda_M$ and all $\mathsf{ATL}^\star$-formulae φ, such that $k = |h| - 1$, $|h| \geq 1$. The proof is done by induction over the formula structure of φ.

Base cases: The case for $\varphi = p$ is straightforward. $\varphi = \langle\!\langle A \rangle\!\rangle\gamma$ where γ does not contain cooperation modalities. By (1-3) we have that the following statements are equivalent:

— $M, h \circ \lambda, k \models^{nf}_I \langle\!\langle A \rangle\!\rangle\gamma$
— there exists s_A such that for all $h \circ \lambda' \in plays^I_M(h, s_A)$ we have $M, h \circ \lambda', k \models^{nf}_I \gamma$
— there exists s_A such that for all $last(h) \circ \lambda' \in plays^I_M(last(h), s_A)$ we have $M, last(h) \circ \lambda' \models_I \gamma$
— $M, last(h) \circ \lambda \models_I \langle\!\langle A \rangle\!\rangle\gamma$.

Induction hypothesis: Let φ be a formula. Then, the statement is true for all strict (state) subformulae of φ.
Induction step: The cases $\varphi = \neg\varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\!\langle A \rangle\!\rangle\gamma$ where γ contains cooperation modalities.

Let $\langle\!\langle B_i \rangle\!\rangle\psi_i$ be an outermost $\mathsf{ATL}^\star$-subformula in γ i.e. there is no other cooperation modality in γ which strictly contains $\langle\!\langle B_i \rangle\!\rangle\psi_i$. Let:

$$H_i = \{(h \circ \lambda, k) \mid M, h \circ \lambda, k \models^{nf}_I \langle\!\langle B_i \rangle\!\rangle\psi_i\}$$
$$L_i = \{\lambda \mid M, \lambda \models_I \langle\!\langle B_i \rangle\!\rangle\psi_i\}$$

We observe that the complement of H_i (resp. L_i) with respect to Λ^{fin}_M is precisely the set $\{(h \circ \lambda, k) \mid M, h \circ \lambda, k \models^{nf}_I \neg\langle\!\langle B_i \rangle\!\rangle\psi_i$ (resp. $\{\lambda \mid M, \lambda \models_I \neg\langle\!\langle B_i \rangle\!\rangle\psi_i\}$). By induction hypothesis, $H_i = \{(h \circ \lambda, k) \mid last(h) \circ \lambda \in L_i\}$ (4). Direction "$\Rightarrow$": Let s_A be a witnessing strategy for $M, h \circ \lambda, k \models^{nf}_I \langle\!\langle A \rangle\!\rangle\gamma$, i.e. $\forall \lambda' \in out_M(h, s_A)$ we have $M, \lambda', k \models^{nf}_I \gamma$. By (4) and (2) it follows that $M, last(h) \circ \lambda \models_I \gamma$. Direction "$\Leftarrow$" follows exactly the same argument. □

As a consequence, the logics $\mathsf{ATL}^\star_\mathsf{I}$ and $\mathsf{ATL}^\star_{\mathsf{nf},\mathsf{I}}$ for perfect information are equivalent. However, the two semantics differ in the imperfect information case. To see this, consider model M_1 and state q_0 from Example 3.1. Let $\varphi \equiv \langle\!\langle 1 \rangle\!\rangle\Diamond\neg\langle\!\langle 2 \rangle\!\rangle\Diamond\mathsf{win}$. In Examples 3.1 and 3.3 we have shown that $M_1, q_0 \models_i \varphi$ but $M_1, q_0 \not\models^{nf}_i \varphi$. As a consequence, we obtain the following.

PROPOSITION 3.6. *There is an iCGS M, a state q in M, and an* $\mathsf{ATL}^\star$ *formula* φ *such that* $M, q \models_i \varphi$ *and* $M, q \not\models_i^{nf} \varphi$.

Thus, the standard and truly perfect recall semantics of ability are different for agents with imperfect information. How big is the impact? At the first glance, the change is not necessarily substantial. We will address the question formally in the next sections, and show that assuming "no forgetting" in interpretation of nested modalities changes the class of properties definable by formulae of $\mathsf{ATL}^\star$, as well as the set of valid sentences of the logic.

4. TRULY PERFECT RECALL: EXPRESSIVITY

We now proceed to show that the seemingly small change in semantics has important consequences for the resulting logics. We prove that the forgetting and truly perfect recall variants of $\mathsf{ATL}^\star$ differ in the properties they allow to express. We will look at which properties of iCGSs can be expressed in $\mathsf{ATL}^\star_\mathsf{x}$ and $\mathsf{ATL}^\star_{\mathsf{nf,x}}$, respectively (where $x \in \{i, I\}$). To do this, we briefly recall the notions of distinguishing power and expressive power.[7]

Definition 4.1 (*Distinguishing and expressive power*). Consider two logical systems $\mathsf{L}_1 = (\mathcal{L}_1, \models_1)$ and $\mathsf{L}_2 = (\mathcal{L}_2, \models_2)$ over the same class of models $\mathcal{M}$ (in our case, the class of iCGSs). By $[\![\varphi]\!]_\models = \{(M, q) \mid M, q \models \varphi\}$, we denote the class of pointed models that satisfy φ according to $\models$. Likewise, $[\![\varphi, M]\!]_\models = \{q \mid M, q \models \varphi\}$ is the set of states (or, equivalently, pointed models) that satisfy φ in a given structure M.

We say that L_2 is *at least as expressive as* L_1 (written: $\mathsf{L}_1 \preceq_e \mathsf{L}_2$) iff for every formula $\varphi_1 \in \mathcal{L}_1$ there exists $\varphi_2 \in \mathcal{L}_2$ such that $[\![\varphi_1]\!]_{\models_1} = [\![\varphi_2]\!]_{\models_2}$. Moreover, L_2 is *at least as distinguishing as* L_1 (written: $\mathsf{L}_1 \preceq_d \mathsf{L}_2$) iff for every model M and formula $\varphi_1 \in \mathcal{L}_1$ there exists $\varphi_2 \in \mathcal{L}_2$ such that $[\![\varphi_1, M]\!]_{\models_1} = [\![\varphi_2, M]\!]_{\models_2}$.[8] L_1 and L_2 are equally expressive (resp. equally distinguishing) iff $\mathsf{L}_2 \preceq_x \mathsf{L}_1$ and $\mathsf{L}_1 \preceq_x \mathsf{L}_2$ where $x = e$ (resp. $x = d$). Finally, we say that L_2 is *strictly more distinguishing than* L_1 (written: $\mathsf{L}_1 \prec_d \mathsf{L}_2$) iff L_2 is at least as distinguishing, but not equally distinguishing to L_1. The definition of "strictly more expressive" is analogous.

Thus, expressive power refers to the general definability of properties by formulae of a given logical system. In contrast, distinguishing power captures the ability to discern between particular models. Note that $\mathsf{L}_1 \preceq_e \mathsf{L}_2$ implies $\mathsf{L}_1 \preceq_d \mathsf{L}_2$ but the converse is not true. For example, it is known that CTL has the same distinguishing power as $\mathsf{CTL}^\star$, but strictly less expressive power [Clarke and Schlingloff 2001].

4.1. Comparing Expressivity for Perfect Information

Below is an immediate consequence of Proposition 3.5, again highlighting that both semantics coincide for agents with perfect information.

THEOREM 4.2. $\mathsf{ATL}^\star_\mathsf{I}$ *and* $\mathsf{ATL}^\star_{\mathsf{nf,I}}$ *are equally expressive and have the same distinguishing power.*

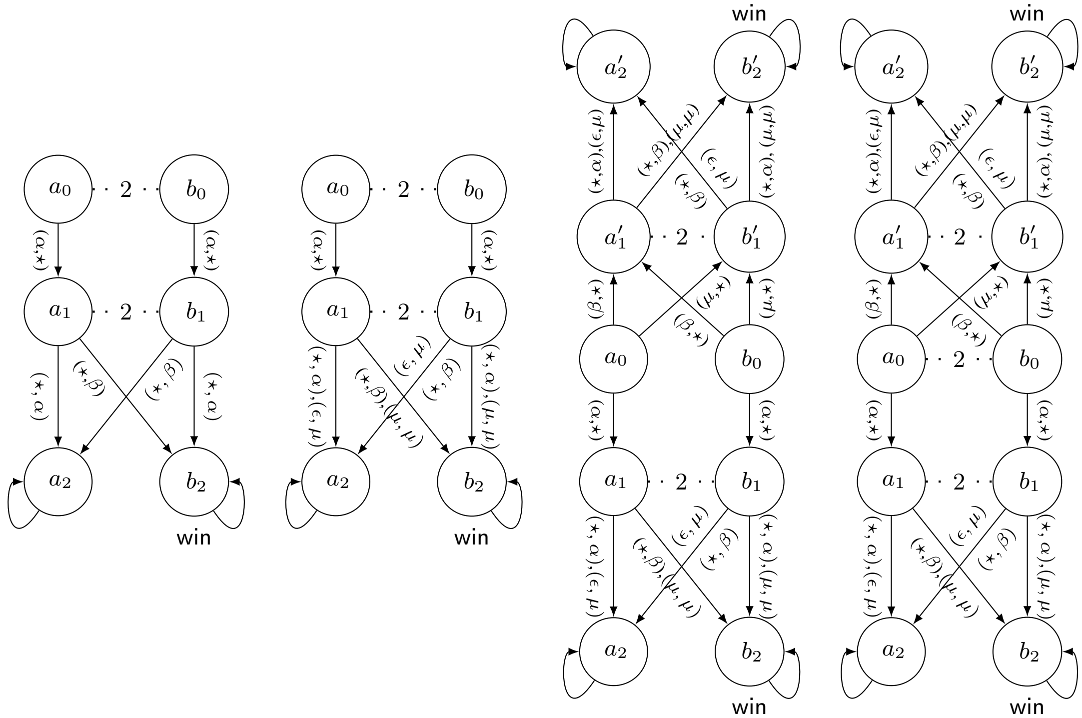
4.2. Imperfect Information

In what follows, we compare the expressiveness of the truly perfect recall variant of $\mathsf{ATL}^\star_{\mathsf{nf,i}}$ with that of its "forgetting" counterpart $\mathsf{ATL}^\star_\mathsf{i}$.

Example 4.3. Consider the models in Figure 3. We have that $M_2, a_0 \models_i^{nf} \langle\!\langle 1 \rangle\!\rangle \bigcirc \langle\!\langle 2 \rangle\!\rangle \bigcirc \mathsf{win}$ but $M_2', a_0 \not\models_i^{nf} \langle\!\langle 1 \rangle\!\rangle \bigcirc \langle\!\langle 2 \rangle\!\rangle \bigcirc \mathsf{win}$. In model M_2, player 2 can learn the state of the game after the first

[7]For more details, see e.g. [Clarke and Schlingloff 2001, Chapter 21].

[8]Equivalently: for every pair of pointed models that can be distinguished by some $\varphi_1 \in \mathcal{L}_1$ there exists $\varphi_2 \in \mathcal{L}_2$ that distinguishes these models.



$\overline{M'}^{\downarrow}_2$ $M'^{\downarrow}_2$ M_2 M'_2

Fig. 3. Submodels $\overline{M'}^{\downarrow}_2$, $M'^{\downarrow}_2$ and models M_2 and M'_2. All models consist of two players 1 and 2. Action tuples (α_1, α_2) give the action of player 1 (α_1) and of player 2 (α_2). The only difference between M_2 and M'_2 is that in model M_2 player 2 can also not distinguish a_0 and b_0.

move (1 plays α in a_0); this is not the case in M'_2. Under the truly perfect recall semantics the two models are distinguishable, however, the models cannot be distinguished in $\mathsf{ATL}^\star_\mathsf{i}$.

To better understand the construction of M_2 and M'_2, let us start with the models $\overline{M'_2}^{\downarrow}$ (Figure 3) and $\overline{M_2}^{\downarrow}$. The latter is simply $\overline{M'_2}^{\downarrow}$ where $a_0 \not\sim_2 b_0$. We note that $(\overline{M_2}^{\downarrow}, a_0)$ and $(\overline{M'_2}^{\downarrow}, a_0)$ can be distinguished in $\mathsf{ATL}^\star_\mathsf{i}$: we have $\overline{M_2}^{\downarrow}, a_0 \models_i \langle\!\langle 1,2\rangle\!\rangle\Diamond\mathsf{win}$ but $(\star)$ $\overline{M'_2}^{\downarrow}, a_0 \not\models_i \langle\!\langle 1,2\rangle\!\rangle\Diamond\mathsf{win}$ — in $\overline{M'_2}$, coalition $\{1,2\}$ has precisely the same (imperfect) information as player 2, and cannot distinguish a_0a_1 from b_0b_1.

We construct $M^{\downarrow}_2$ and $M'^{\downarrow}_2$ by adding transitions (ϵ,μ) and (μ,μ) to both $\overline{M_2}^{\downarrow}$ and $\overline{M'_2}^{\downarrow}$. $M'^{\downarrow}_2$ is shown in Figure 3. Now, $(\star)$ no longer holds: player 2 may play μ in a_0a_1 as well as in b_0b_1. Thus, player 1's action (μ or ϵ) leads to the winning state.

But even in this setup, $(M^{\downarrow}_2, a_0)$ and $(M'^{\downarrow}_2, a_0)$ may still be distinguished in $\mathsf{ATL}^\star_\mathsf{i}$, by e.g. $\langle\!\langle 2\rangle\!\rangle\Box\neg\mathsf{win}$: in $M^{\downarrow}_2$ player 2 can ensure that the winning state is never reached, by playing α in a_0a_1, which is not true in $M'^{\downarrow}_2$, since $a_0a_1 \sim_2 b_0b_1$. Even if the second player does not determine the next-state from a_0, $a_0 \not\sim_2 b_0$ means that he can prevent winning, no matter what 1 does.

To solve this issue, we need to add more options for player 1 in a_0. In M_2, $\langle\!\langle 2\rangle\!\rangle\Box\neg\mathsf{win}$ does not hold in M_2 (nor in M'_2). For instance, if player 1 plays μ, player 2's former strategy no longer prevents winning.

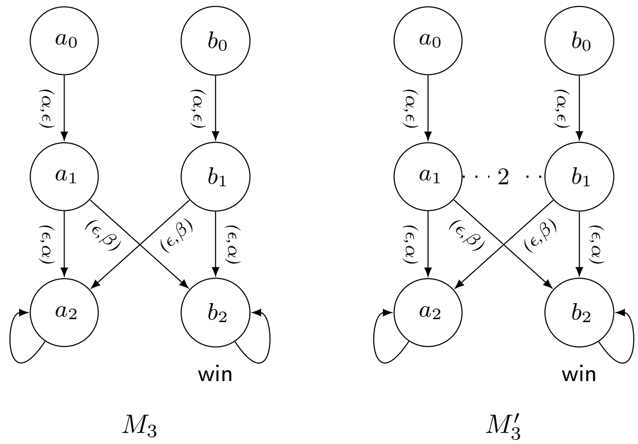


Fig. 4. Models M_3 and M'_3

The following proposition is based on the fact that no $\mathsf{ATL}^\star_\mathsf{i}$ formula distinguishes M_2 and M'_2 from Example 4.3. The complete proof is given in Appendix A.1.

PROPOSITION 4.4. *There are pointed iCGSs which satisfy the same* $\mathsf{ATL}^\star_\mathsf{i}$*-formulae, but can be distinguished in* $\mathsf{ATL}^\star_{\mathsf{nf,i}}$. *Thus,* $\mathsf{ATL}^\star_{\mathsf{nf,i}} \not\preceq_d \mathsf{ATL}^\star_\mathsf{i}$.

Next, we investigate whether $\mathsf{ATL}^\star_{\mathsf{nf,i}}$ is at least as distinguishing as $\mathsf{ATL}^\star_\mathsf{i}$.

Example 4.5. Let us consider the two iCGSs M_3 and M'_3 shown in Figure 4. There is an $\mathsf{ATL}^\star_\mathsf{i}$-formula that can distinguish both models: $M_3, a_0 \models_i \langle\langle 1\rangle\rangle \bigcirc \langle\langle 2\rangle\rangle \bigcirc \mathsf{win}$ and $M'_3, a_0 \not\models_i \langle\langle 1\rangle\rangle \bigcirc \langle\langle 2\rangle\rangle \bigcirc \mathsf{win}$. In the latter case player 2 "forgets" that the game has started in state a_0. Thus, in M_2 the player cannot distinguish the states a_1 from a_2 when evaluating the nested formula. It is easy to see that there is no uniform winning strategy from a_1 and a_2 in M'_2, respectively.

This leads to the following result:

PROPOSITION 4.6. *There are pointed iCGSs which satisfy the same* $\mathsf{ATL}^\star_{\mathsf{nf,i}}$ *formulae, but can be distinguished in* $\mathsf{ATL}^\star_\mathsf{i}$*, i.e.,* $\mathsf{ATL}^\star_\mathsf{i} \not\preceq_d \mathsf{ATL}^\star_{\mathsf{nf,i}}$.

PROOF. We consider models M_3 and M'_3 from Figure 4. We prove $M_3, h \models^{nf}_i \varphi$ iff $M'_3, h \models^{nf}_i \varphi$ for all $h \in \Lambda^{fin}_{M_3}(a_0)$ and all $\varphi \in \mathsf{ATL}^\star$, by induction over the formula structure of φ.

<u>Base cases</u>. Case $\underline{\varphi = p}$ is straightforward. Case $\underline{\varphi = \langle\langle A\rangle\rangle\gamma}$ where γ does not contain cooperation modalities. It is sufficient to observe that: $(\star)$ $\mathit{plays}^i_{M_3}(h, s_A) = \mathit{plays}^i_{M'_3}(h, s_A)$ for any collective strategy s_A and $h \in \Lambda^{fin}_{M_3}(a_0)$.

<u>Induction step</u>. Cases $\underline{\varphi = \neg\varphi'}$ and $\underline{\varphi = \varphi' \wedge \varphi''}$ are straightforward. Case $\underline{\varphi = \langle\langle A\rangle\rangle\gamma}$ where γ contains cooperation modalities.

Let $\langle\langle B_i\rangle\rangle\psi_i$ (with $i = 1 \ldots k$) be an outermost $\mathsf{ATL}^\star$-subformula in γ. By induction hypothesis, we have: $M_3, h \models^{nf}_i \langle\langle B_i\rangle\rangle\psi_i$ iff $M'_3, h \models^{nf}_i \langle\langle B_i\rangle\rangle\psi_i$ — the same histories h satisfy $\langle\langle B_i\rangle\rangle\psi_i$ in both models. It follows (by $(\star)$) that s_A is a witnessing strategy for $M_3, h \models^{nf}_i \langle\langle A\rangle\rangle\gamma$ iff s_A is a witnessing strategy for $M'_3, h \models^{nf}_i \langle\langle A\rangle\rangle\gamma$.

To conclude the proof, we note that both models can be distinguished in $\mathsf{ATL}^\star_\mathsf{i}$ as shown in Example 4.5. $\square$

As an immediate consequence, we obtain the theorem below.

THEOREM 4.7. *The logics* $\mathsf{ATL}^\star_{\mathsf{i}}$ *and* $\mathsf{ATL}^\star_{\mathsf{nf,i}}$ *have incomparable distinguishing and expressive powers.*

5. VALIDITIES

Another way of comparing two logical systems is to compare their sets of valid sentences, that is, the general properties that hold in every model according to the given semantics.

Intuitively, each formula can be interpreted as a game property. Such properties describe the abilities of agents and their groups, that possibly hold for some games, and do not hold in the others. While expressiveness concerns which game properties are definable in the language of the logic, validities are properties that universally hold. Thus, by comparing validity sets of different logical systems, we can compare the general properties of games induced by the underlying semantics (cf. [Bulling and Jamroga 2014]).

Given a semantics *sem*, we use $Val(\mathsf{ATL}^\star_{sem})$ to denote the set of valid sentences of $\mathsf{ATL}^\star_{sem}$, and $Sat(\mathsf{ATL}^\star_{sem})$ to denote the set of satisfiable sentences of $\mathsf{ATL}^\star_{sem}$. Intuitively, each formula $\varphi \in Val(\mathsf{ATL}^\star_{sem})$ describes an *invariant property* or *game rule* of $\mathsf{ATL}^\star_{sem}$. In this section, we investigate the relationship between $Val(\mathsf{ATL}^\star_{x})$ and $Val(\mathsf{ATL}^\star_{x,nf})$ for $x \in \{I, i\}$. In particular, a result of the form $Val(\mathsf{ATL}^\star_{sem_1}) \subsetneq Val(\mathsf{ATL}^\star_{sem_2})$ means that the game rules of $\mathsf{ATL}^\star_{sem_2}$ are a strict *specialization* of those of $\mathsf{ATL}^\star_{sem_1}$.

Finally, we recall that $Sat(\mathsf{ATL}^\star_{sem_2}) \subsetneq Sat(\mathsf{ATL}^\star_{sem_1})$ iff $Val(\mathsf{ATL}^\star_{sem_1}) \subsetneq Val(\mathsf{ATL}^\star_{sem_2})$. Thus, any result comparing the validity sets of $\mathsf{ATL}^\star_{sem_1}$ and $\mathsf{ATL}^\star_{sem_2}$ immediately implies the dual characterization of satisfiable sentences.

5.1. Perfect Information

The following result is a direct corollary of Proposition 3.5.

THEOREM 5.1. $Val(\mathsf{ATL}^\star_{\mathsf{I}}) = Val(\mathsf{ATL}^\star_{\mathsf{nf,I}})$.

5.2. Imperfect Information

Now we will compare the validity sets of $\mathsf{ATL}^\star_{\mathsf{nf,i}}$ and $\mathsf{ATL}^\star_{\mathsf{i}}$. First, we introduce a class $\mathcal{T}$ of iCGSs for which the semantics of $\mathsf{ATL}^\star_{\mathsf{i}}$ and $\mathsf{ATL}^\star_{\mathsf{nf,i}}$ coincide. Members of $\mathcal{T}$ are *infinite epistemic trees* obtained by applying an *unfolding procedure* on arbitrary models M, with respect to a given *initial state* q. For the moment, suppose that all states in M can be distinguished from q. Then, the model M can be unfolded from q to an infinite tree $T(M, q)$ whose states correspond to histories in M. Two nodes h and h' in a tree are linked by an epistemic relation belonging to an agent a, written as $h \sim_a h'$ if, and only if, $h \approx_a h'$ in M.

Now, it might be the case that there are states indistinguishable from q. They have to be considered as well. Let $Q' = \{q' \in M \mid q \sim_{\mathbb{A}\mathrm{gt}} q'\}$ be the set of all states indistinguishable from q for some agent from $\mathbb{A}\mathrm{gt}$. For each state $\hat{q} \in Q'$ we construct the unfolding $T(M, \hat{q})$ as described above. Moreover, we introduce epistemic links *between* these trees. For any two histories h and h' in any of these trees we define $h \sim_a h'$ if, and only if, $h \approx_a h'$ in M. The resulting model, i.e., the collection of all those trees plus the inter-tree epistemic relations, is denoted by $T^{nf}(M, q)$. We note in passing that the unfolding was already considered in [Bulling and Jamroga 2014, Example 7] and shown to be insufficient for $\mathsf{ATL}^\star_{\mathsf{i}}$ with the standard semantics. With respect to the no-forgetting semantics, however, the unfolding does the right thing.

Definition 5.2 (*No-forgetting epistemic tree unfolding*). Consider any iCGS $M = (\mathbb{A}\mathrm{gt}, St, \Pi, \pi, Act, d, o, \{\sim_a \mid a \in \mathbb{A}\mathrm{gt}\})$ and a state q of M. We construct the iCGS $T(M, q)$ as follows:

$$T(M, q) = \langle \mathbb{A}\mathrm{gt}^{T,q}, St^{T,q}, \Pi^{T,q}, \pi^{T,q}, Act^{T,q}, d^{T,q}, o^{T,q}, \{\sim_a^{T,q} \mid a \in \mathbb{A}\mathrm{gt}^{T,q}\}\rangle$$

where:

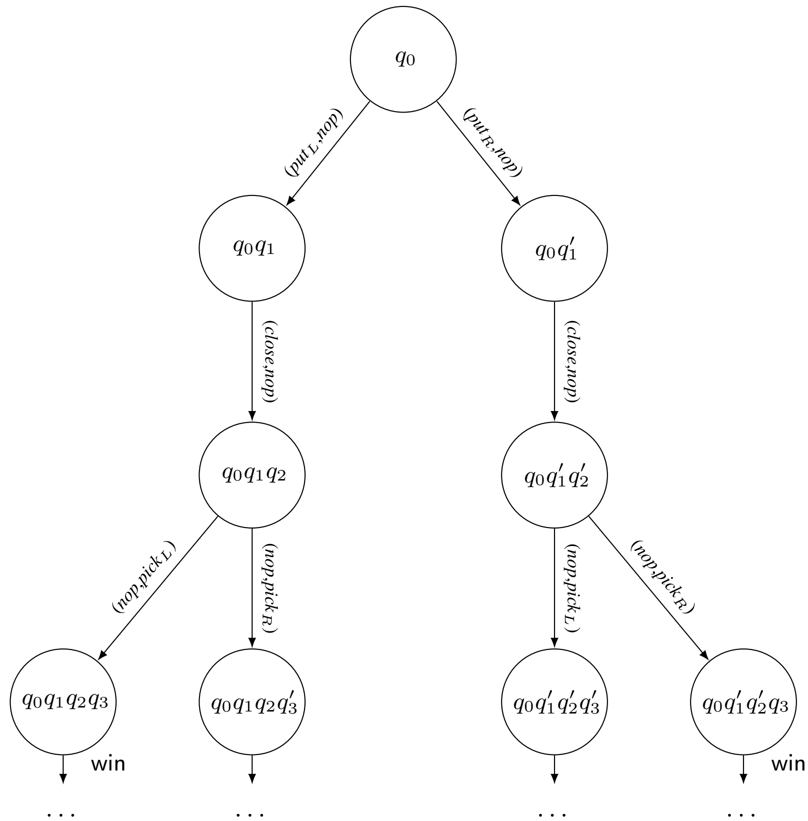


Fig. 5. The no-forgetting epistemic tree unfolding of the iCGS M_1 from Figure 1

— $\mathbb{A}\mathrm{gt}^{T,q} = \mathbb{A}\mathrm{gt}$, $\Pi^{T,q} = \Pi$, $Act^{T,q} = Act$, $St^{T,q} = \{h \mid h \in \Lambda_M^{fin}\}$: the sets of agents, propositions and actions of $T(M,q)$ coincide with those of M. Each state in $T(M,q)$ is a history of M which starts in state q.
— $\pi^{T,q}(p) = \{h \mid \mathit{last}(h) \in \pi(p)\}$, $\forall p \in \Pi$; each state h from $T(M,q)$ is labelled with the same propositions as $\mathit{last}(h)$ in M;
— $d_a^{T,q}(h) = d_a(\mathit{last}(h))$, $\forall h \in St^{T,q}$; the set of allowed actions in each state h of $T(M,q)$ is the same as that in each state $\mathit{last}(h)$ of M;
— $q = o(\mathit{last}(h), \alpha_1, \ldots, \alpha_k) \iff o^{T,q}(h, \alpha_1, \ldots, \alpha_k) = h \circ q$, $\forall h \in St^{T,q}$; each transition from h to $h \circ q'$ in $T(M,q)$ corresponds to a transition from $\mathit{last}(h)$ to q' in M;
— $h \sim_a^{T,q} h'$ iff $h \approx_a h'$ in M, $\forall h, h' \in St^{T,q}$; two states h and h' are indistinguishable for agent a in $T(M,q)$ if the histories h and h' are indistinguishable for a in M.

Let $Q' = \{q' \mid q \sim_{\mathbb{A}\mathrm{gt}} q'\}$. The *no-forgetting epistemic tree unfolding* of M, denoted $T^{nf}(M,q)$, is the iCGS:

$$T^{nf}(M,q) = \langle \mathbb{A}\mathrm{gt}^{T^{nf},q}, St^{T^{nf},q}, \Pi^{T^{nf},q}, \pi^{T^{nf},q}, Act^{T^{nf},q}, d^{T^{nf},q}, o^{T^{nf},q}, \{\sim_a^{T^{nf},q} \mid a \in \mathbb{A}\mathrm{gt}^{T^{nf},q}\}\rangle$$

obtained as follows:

— $\mathbb{A}\mathrm{gt}^{T^{nf},q} = \mathbb{A}\mathrm{gt}^{T,q}$, $\Pi^{T^{nf},q} = \Pi^{T,q}$, $Act^{T^{nf},q} = Act^{T,q}$, $St^{T^{nf},q} = \bigcup_{q \in Q'} St^{T,q}$
— $\pi^{T^{nf}}(p) = \bigcup_{q \in Q'} \pi^{T,q}(p)$, $\forall p \in \Pi$;

— $d_a^{T^{nf},q}(h) = d_a^{T,q}(h), \forall h \in St^{T^{nf},q}$;
— $h \sim_a^{T^{nf},q} h'$ iff $h \approx_a h', \forall h, h' \in St^{T^{nf},q}$;

Example 5.3 (*No-forgetting epistemic tree unfolding*). In Figure 5, we illustrate the no-forgetting epistemic tree unfolding $T^{nf}(M_1, q_0)$ of model M_1 from Figure 1. We note that there are no epistemic links between different states of $T^{nf}(M_1, q_0)$, since all corresponding histories are distinguishable by the second agent in M_1. We also note that, in this particular case: $T^{nf}(M_1, q_0) = T(M_1, q_0)$.

Figure 6 illustrates the submodel $M_2^{\uparrow}$ of M_2 from Example 3 (on the left), as well as $T^{nf}(M_2^{\uparrow}, a_0)$ (right). The unfolding is obtained by first constructing the components $T(M_2^{\uparrow}, a_0)$ and $T(M_2^{\uparrow}, b_0)$, and then adding epistemic links between all indistinguishable states of the latter two models.

The following proposition first establishes that no-forgetting epistemic tree unfoldings are truth-preserving in the no-forgetting semantics. Hence, any sentence which is true with respect to a given state q and model M, is also true with respect to the unfolding of M from q. At the same time, the no-forgetting and standard $\mathsf{ATL}_{\mathsf{i}}^{\star}$-semantics coincide over no-forgetting epistemic tree unfoldings.

PROPOSITION 5.4. *$M, q \models_i^{nf} \varphi$ iff $T^{nf}(M, q), q \models_i^{nf} \varphi$ iff $T^{nf}(M, q), q \models_i \varphi$, for all $\mathsf{ATL}^{\star}$-formulae, iCGSs M and states q.*

The proof is presented in Appendix A.2. This result is key for obtaining the following:

PROPOSITION 5.5. $Val(\mathsf{ATL}_{\mathsf{i}}^{\star}) \subseteq Val(\mathsf{ATL}_{\mathsf{nf,i}}^{\star})$.

PROOF. We prove that $Sat(\mathsf{ATL}_{\mathsf{nf,i}}^{\star}) \subseteq Sat(\mathsf{ATL}_{\mathsf{i}}^{\star})$. Suppose $\varphi \in Sat(\mathsf{ATL}_{\mathsf{nf,i}}^{\star})$. Thus there exists an iCGS M and state q such that $M, q \models_i^{nf} \varphi$. By Proposition 5.4, $T^{nf}(M, q) \models_i \varphi$. Hence $\varphi \in Sat(\mathsf{ATL}_{\mathsf{i}}^{\star})$. $\square$

We now show that there exists a sentence which is valid in $\mathsf{ATL}_{\mathsf{nf,i}}^{\star}$, but not in $\mathsf{ATL}_{\mathsf{i}}^{\star}$. Informally, such a sentence expresses that whenever an agent a has the ability to maintain p, then the agent preserves this ability in some next-state of the game.

PROPOSITION 5.6. $Val(\mathsf{ATL}_{\mathsf{nf,i}}^{\star}) \not\subseteq Val(\mathsf{ATL}_{\mathsf{i}}^{\star})$.

PROOF. We consider the formula $\varphi \equiv \langle\langle a \rangle\rangle \Box \mathsf{p} \rightarrow \mathsf{E} \bigcirc \langle\langle a \rangle\rangle \Box \mathsf{p}$ where $\mathsf{E} \bigcirc \varphi \equiv \neg \langle\langle \emptyset \rangle\rangle \bigcirc \neg \varphi$. Informally, $\mathsf{E} \bigcirc \varphi$ expresses that there is a path on which φ holds in the next moment. We use model M_3' from Figure 4 to show that $\varphi \notin Val(\mathsf{ATL}_{\mathsf{i}}^{\star})$. We interpret $\{1, 2\}$ as a single player and p as win. We have $M_3', a_0 \models_i \langle\langle \{1, 2\} \rangle\rangle \Box \neg \mathsf{win}$ but $M_3', a_0 \not\models_i \mathsf{E} \bigcirc \langle\langle \{1, 2\} \rangle\rangle \Box \neg \mathsf{win}$, hence φ is not valid in $\mathsf{ATL}_{\mathsf{i}}^{\star}$.

Now, suppose that $M, q \models_i^{nf} \langle\langle a \rangle\rangle \Box \mathsf{p}$, let s_a be a witnessing strategy and $\lambda = q q_1 q_2 \ldots \in plays_M^i(q, s_a)$. Since $plays_M^i(q q_1, s_a) \subseteq plays_M^i(q, s_a)$ we have $M, \lambda, 1 \models_i^{nf} \langle\langle a \rangle\rangle \Box \mathsf{p}$ and also $M, \lambda, 0 \models_i^{nf} \bigcirc \langle\langle a \rangle\rangle \Box \mathsf{p}$. Hence $M, \lambda, 0 \models_i^{nf} \mathsf{E} \bigcirc \langle\langle a \rangle\rangle \Box \mathsf{p}$, which concludes the proof. $\square$

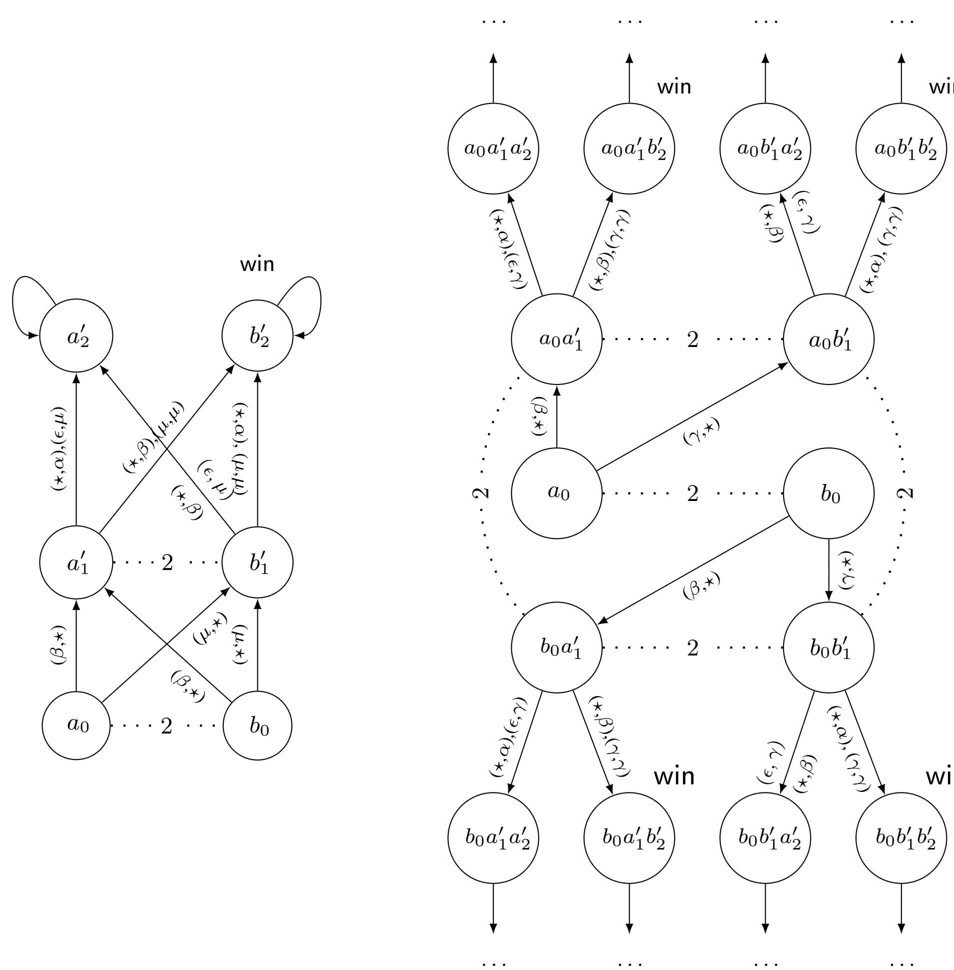
It is immediate from Propositions 5.5 and 5.6 that $\mathsf{ATL}_{\mathsf{nf,i}}^{\star}$ describes a more specific class of games than $\mathsf{ATL}_{\mathsf{i}}^{\star}$ – games in which players do not forget past events:

THEOREM 5.7. *$Val(\mathsf{ATL}_{\mathsf{i}}^{\star}) \subsetneq Val(\mathsf{ATL}_{\mathsf{nf,i}}^{\star})$. Consequently, also $Sat(\mathsf{ATL}_{\mathsf{nf,i}}^{\star}) \subsetneq Sat(\mathsf{ATL}_{\mathsf{i}}^{\star})$.*

Thus, games of truly perfect recall can be seen as a special subclass of games with “standard” perfect recall, as captured by the original semantics of $\mathsf{ATL}_{\mathsf{i}}^{\star}$ in [Schobbens 2004].

6. TRULY PERFECT RECALL UNDER STRATEGIC COMMITMENT

We pointed out in Section 3 that, when proceeding from a higher-level goal to a subgoal, the semantics of $\mathsf{ATL}^{\star}$ “forgets” past observations of agents—even if the agents are assumed to have perfect recall. A similar feature was observed in [Ågotnes et al. 2007; Brihaye et al. 2009] with respect to



$M_2^{\uparrow}$ $\qquad$ $T^{nf}(M_2^{\uparrow}, a_0)$

Fig. 6. The submodel ${M'_2}^{\uparrow}$ of M'_2 (Example 3), is shown on the left. On the right, we have the epistemic tree unfolding $T^{nf}({M'_2}^{\uparrow}, a_0)$. The epistemic links between states $a_0a'_1$, $b_0b'_1$ and $b_0a'_1$, $a_0b'_1$ have been omitted.

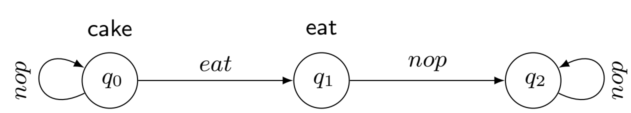


Fig. 7. Have the cake or eat it? The cake dilemma: CGS M_7

agents' strategies: they do not persist from outer to nested strategic modalities. Strategies in ATL* are *revocable* in the sense that an agent is not bound by her strategy anymore when proceeding from the main game to a subgame. In many cases, this makes the meaning of ATL* specifications counterintuitive.

Consider formula $\varphi \equiv \langle\langle a\rangle\rangle\Box\langle\langle a\rangle\rangle\bigcirc\mathsf{eat}$ which says that Alice has a strategy to maintain forever her ability to eat the cake. The formula is easily satisfiable in $\mathsf{ATL}^\star$; a simple model for φ is presented in Figure 7. However, the only way for Alice to keep her ability to eat the cake is by never eating the cake, which is somewhat paradoxical. If Alice executes the strategy, she will deprive herself of the ability she wants to maintain in the first place.

Also, consider the formula $\langle\langle c\rangle\rangle\Box\langle\langle a,b\rangle\rangle\Diamond\mathsf{married_{ab}}$ which expresses that Charlie can provide Alice and Bob with the ability to get married. One can imagine that Charlie can achieve that, e.g., by becoming the local superintendent registrar or a local priest, and granting every marriage request from his friends. However, the $\mathsf{ATL}^\star$ interpretation of the formula is that Alice and Bob must find a strategy for $\Diamond\mathsf{married_{ab}}$ *against every possible behavior of Charlie*, despite the fact that Charlie did select his strategy in order to make $\langle\langle a,b\rangle\rangle\Diamond\mathsf{married_{ab}}$ true.

We will now briefly recall two variants of $\mathsf{ATL}^\star$ that were proposed to handle specifications where persistence of strategies is important [Ågotnes et al. 2007; Brihaye et al. 2009]. We will also extend the variants to the case of imperfect information. After that, we will point out that the "forgetting" phenomenon applies also to $\mathsf{ATL}^\star$ with persistent strategies, and we will look at the formal consequences of the fact.

6.1. ATL* with Long-Term Commitment

6.1.1. Irrevocable $\mathsf{ATL}^\star$*.* The simplest interpretation of $\mathsf{ATL}^\star$ formulae, that assumes "irrevocability" or long-term commitment of agents' strategies, was proposed in [Ågotnes et al. 2007]. The logic takes the syntax of $\mathsf{ATL}^\star$ but changes the semantics to ensure persistence of strategies.[9] In the original version, this is done by unfolding the CGS to a tree and then pruning all the transitions that are inconsistent with the strategies selected by agents. Here, we give an equivalent semantics based on the idea of "strategy contexts", cf. [Brihaye et al. 2009; Jamroga et al. 2005; Walther et al. 2007] or Section 6.1.2 for more details. Formally, the semantics is given in terms of the semantic relation $\models_{x,c}$ that interprets a formula, given an iCGS, a path in it, and a *strategy context* which "stores" the strategies selected so far by the agents. Parameter $x \in \{I, i\}$ indicates whether we assume agents to have perfect or imperfect information. We first recall the definition of *strategy update*.

Definition 6.1 (*Strategy update [Brihaye et al. 2009]*). Given collective strategies s_A and s_B of group A and B, respectively, we define the strategy $s_B \dagger s_A$ as follows: $s_B \dagger s_A|_i(h) = s_A|_i(h)$ if $i \in A$ and $s_B \dagger s_A|_i(h) = s_B|_i(h)$ if $i \in B \setminus A$.

Thus, $s_B \dagger s_A$ combines the strategies s_A and s_B, and the actions specified by s_A will *override* those specified by s_B for agents in $A \cap B$. The semantic relation $\models_{x,c}$ is defined as follows:

$M, \lambda, s \models_{x,c} p$ iff $\lambda[0] \in \pi(p)$ (where $p \in \Pi$);
$M, \lambda, s \models_{x,c} \neg\varphi$ iff $M, \lambda, s \not\models_{x,c} \varphi$;
$M, \lambda, s \models_{x,c} \varphi_1 \wedge \varphi_2$ iff $M, \lambda, s \models_{x,c} \varphi_1$ and $M, \lambda, s \models_{x,c} \varphi_2$;
$M, \lambda, s \models_{x,c} \langle\langle A\rangle\rangle\varphi$ iff there is a collective x-strategy s_A such that for all $\lambda' \in \mathit{plays}^x(\lambda[0], s_A \dagger s)$, we have $M, \lambda', s_A \dagger s \models_{x,c} \varphi$;
$M, \lambda, s \models_{x,c} \bigcirc\varphi$ iff $M, \lambda[1,\infty], s \models_{x,c} \varphi$;
$M, \lambda, s \models_{x,c} \varphi_1 \mathcal{U} \varphi_2$ iff there is $i \in \mathbb{N}_0$ such that $M, \lambda[i,\infty], s \models_{x,c} \varphi_2$ and for all $0 \le j < i$, we have $M, \lambda[j,\infty], s \models_{x,c} \varphi_1$.

Note that the above clauses define two different logics: $\mathsf{ATL}^\star_{\mathsf{I,c}}$ ($\mathsf{ATL}^\star$ with perfect information and strategy commitment) which corresponds to $\mathsf{MIATL}^\star$ from [Ågotnes et al. 2007], and $\mathsf{ATL}^\star_{\mathsf{i,c}}$ ($\mathsf{ATL}^\star$ with imperfect information and strategy commitment) which, to the best of our knowledge has not been considered anywhere yet. We also observe that in $\mathsf{ATL}^\star_{\mathsf{I,c}}$ and $\mathsf{ATL}^\star_{\mathsf{i,c}}$ strategies are indeed

[9]The logic is called $\mathsf{MIATL}^\star$ for "$\mathsf{ATL}^\star$ with irrevocable strategies and memory" in [Ågotnes et al. 2007]. Here, we rather use the acronym $\mathsf{ATL}^\star_{\mathsf{c}}$ to indicate that we deal with the same formulae as in $\mathsf{ATL}^\star$, but strategy commitment is required in the semantics.

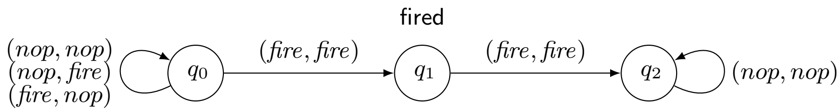


Fig. 8. The cake dilemma with multi-player coordination: CGS M_8

irrevocable. The first strategy selected by an agent is never overridden by a subsequent strategy. This is reflected in the order of strategy updates: the oldest updates are applied last.

Given a state q and a state formula φ, we define $M, q \models_{x,c} \varphi$ iff $M, \lambda, s_\emptyset \models_{x,c} \varphi$ for any $\lambda \in \Lambda_M(q)$, where $s_\emptyset$ refers to the only possible strategy of the empty coalition. Moreover, φ is *valid* in $\mathsf{ATL}^\star_{\mathsf{x,c}}$ iff $M, q \models_{x,c} \varphi$ for every iCGS M and state q in it.

Example 6.2. Consider CGS M_7 in Figure 7. Unlike in the standard semantics of $\mathsf{ATL}^\star$, we have $M_7, q_0 \not\models_{x,c} \langle\langle a\rangle\rangle\Box\langle\langle a\rangle\rangle\bigcirc\,\mathsf{eat}$. Suppose that the formula holds in M_7, q_0. On one hand, if Alice selects the "eat" strategy eat then the formula $\varphi \equiv \langle\langle a\rangle\rangle\Box\langle\langle a\rangle\rangle\bigcirc\,\mathsf{eat}$ can be only satisfied if $M_7, q_0q_1(q_2)^\omega, eat \models_{x,c} \Box\langle\langle a\rangle\rangle\bigcirc\,\mathsf{eat}$, but this is not possible since $M_7, q_1 \not\models_{x,c} \langle\langle a\rangle\rangle\bigcirc\,\mathsf{eat}$. On the other hand, if Alice selects the "do nothing" strategy nop then the formula φ can be only satisfied if $M_7, (q_0)^\omega, nop \models_{x,c} \Box\langle\langle a\rangle\rangle\bigcirc\neg\mathsf{eat}$, which is not possible since $M_7, q_0, nop \not\models_{x,c} \langle\langle a\rangle\rangle\bigcirc\neg\mathsf{cake}$: Alice is already bound to her "do nothing" strategy.

6.1.2. ATL with Strategy Contexts.* "$\mathsf{ATL}^\star$ with strategy contexts" [Brihaye et al. 2009] offers a more elaborate framework for reasoning about strategy commitment. Compared to $\mathsf{ATL}^\star_{\mathsf{I,c}}$ and $\mathsf{ATL}^\star_{\mathsf{i,c}}$, it allows agents to commit, override and revoke their strategies. The syntax extends the language of $\mathsf{ATL}^\star$ with a *strategic release operator* $\rangle A\langle$, and the semantics $\models_{x,sc}$ differs from the one presented in Section 6.1.1 by the following clauses:

$M, \lambda, s \models_{x,sc} \langle\langle A\rangle\rangle\varphi$ iff there is a collective x-strategy s_A such that for each $\lambda' \in plays^x(\lambda[0], s \dagger s_A)$, $M, \lambda', s \dagger s_A \models_{x,sc} \varphi$;
$M, \lambda, s \models_{x,sc} \rangle A\langle\ \varphi$ iff $M, \lambda, s_{\setminus A} \models_{x,sc} \varphi$;

where $s_{\setminus A}$ denotes the strategy context with strategies of agents from A being removed. We emphasize the swap in the update operator in comparison to $\mathsf{ATL}^\star_{\mathsf{x,c}}$: $s \dagger s_A$ versus $s_A \dagger s$. Thus, according to relation $\models_{x,sc}$, newer strategies override older ones.

Again, we define $M, q \models_{x,sc} \varphi$ iff $M, \lambda, s_\emptyset \models_{x,sc} \varphi$ and $M, q, s \models_{x,sc} \varphi$ iff $M, \lambda, s \models_{x,sc} \varphi$ for any $\lambda \in \Lambda_M(q)$. Moreover, φ is *valid* iff $M, q \models_{x,sc} \varphi$ for every M, q.

Example 6.3. Figure 8 depicts a two-player variant of the "cake dilemma." This time, two agents – Alice and Bob – are needed to fire a missile; if they do not coordinate, the game stays in the initial state q_0. Now, we have for example that $M_8, q_0 \models_{x,sc} \langle\langle a\rangle\rangle\mathcal{N}\langle\langle b\rangle\rangle\bigcirc\,\mathsf{fired}$.[10] To achieve this, we select the strategy "*fire the missile whenever the system is in state q_0, and do nop elsewhere*" when evaluating the modality $\langle\langle a\rangle\rangle$ for Alice, and the same strategy later on, when evaluating the modality $\langle\langle b\rangle\rangle$ for Bob. However, $M_8, q_0 \not\models_x \langle\langle a\rangle\rangle\mathcal{N}\langle\langle b\rangle\rangle\bigcirc\,\mathsf{fired}$: when evaluating $\langle\langle b\rangle\rangle\bigcirc\,\mathsf{fired}$ in the standard $\mathsf{ATL}^\star$ semantics, Bob cannot assume that Alice's strategy will persist, and therefore every strategy of his may result in the path q_0^ω. This illustrates that the semantics $\models_{x,sc}$ allows agents to base their decisions on the strategies previously selected by other players, which was not possible in the original semantics of $\mathsf{ATL}^\star$.

Also, $M_8, q_0 \models_{x,sc} \langle\langle a, b\rangle\rangle\Box(\neg\mathsf{fired} \wedge\ \rangle a, b\langle\ \langle\langle a, b\rangle\rangle\bigcirc\,\mathsf{fired})$: the right-hand-side of the conjunction expresses that Alice and Bob release their strategy and then are free to choose a new one. Therefore, "$\mathsf{ATL}^\star$ with strategy contexts" allows agents to explicitly *decommit* previously selected strategies.

[10] Recall that $\mathcal{N}\varphi \equiv \varphi\,\mathcal{U}\,\varphi$ stands for "φ holds in the current moment."

Finally, notice that $M_8, q_0 \models_{x,sc} \langle\langle a,b\rangle\rangle\Box\langle\langle a,b\rangle\rangle\bigcirc \mathsf{fired}$ because Alice and Bob override their former strategy when trying to satisfy the subformula $\langle\langle a,b\rangle\rangle\bigcirc \mathsf{fired}$. Therefore, unlike $\mathsf{ATL}^\star_{\mathsf{x,c}}$, strategies in "$\mathsf{ATL}^\star$ with strategy contexts" are *not* irrevocable.

Remark 6.4. It was shown in [Brihaye et al. 2009, Proposition 3] that the strategic release operator $\rangle A\langle$ adds no expressive power to "$\mathsf{ATL}^\star$ with strategy contexts." For instance, $\rangle A\langle\ \langle\langle B\rangle\rangle\gamma$ can be equivalently rewritten as $\langle\langle B\rangle\rangle\neg\langle\langle A\setminus B\rangle\rangle\neg\gamma$ (B has a strategy such that agents outside A cannot prevent γ). Thus, we can omit strategic release from the syntax without losing generality of our results. In the rest of the paper, we will only use formulae defined by the standard syntax of $\mathsf{ATL}^\star$, exactly like for the other logics studied here.

Assuming that we use only the standard syntax of $\mathsf{ATL}^\star$, the semantic relations $\models_{x,sc}$ define, again, two logical systems: $\mathsf{ATL}^\star_{\mathsf{I,sc}}$ ($\mathsf{ATL}^\star$ with perfect information and strategy contexts), which corresponds to $\mathsf{ATL}^\star_{\mathsf{sc},\infty}$ from [Brihaye et al. 2009], and $\mathsf{ATL}^\star_{\mathsf{i,sc}}$ ($\mathsf{ATL}^\star$ with imperfect information and strategy contexts) which is a new variant of alternating-time temporal logic to our best knowledge.

6.1.3. Comparing Logics of Strategy Commitment. In Sections 6.1.1 and 6.1.2, we have presented four variants of alternating-time logic for reasoning about persistent strategies: $\mathsf{ATL}^\star_{\mathsf{I,c}}$, $\mathsf{ATL}^\star_{\mathsf{i,c}}$, $\mathsf{ATL}^\star_{\mathsf{I,sc}}$, and $\mathsf{ATL}^\star_{\mathsf{i,sc}}$. How do the variants relate? We show that $\mathsf{ATL}^\star_{\mathsf{x,c}}$ can be in fact embedded in $\mathsf{ATL}^\star_{\mathsf{x,sc}}$ for $x \in \{I, i\}$. To this end, we define a translation tr_A such that, for any model M, path λ, strategy context s_A and formula φ of $\mathsf{ATL}^\star$, we have that $M, \lambda, s_A \models_{x,c} \varphi$ iff $M, \lambda, s_A \models_{x,sc} tr_A(\varphi)$:

$$\begin{aligned}
tr_A(p) &= p\\
tr_A(\neg\varphi) &= \neg tr_A(\varphi)\\
tr_A(\varphi_1\wedge\varphi_2) &= tr_A(\varphi_1)\wedge tr_A(\varphi_2)\\
tr_A(\langle\langle B\rangle\rangle\varphi) &= \langle\langle B\setminus A\rangle\rangle tr_{B\cup A}(\varphi)\\
tr_A(\bigcirc\varphi) &= \bigcirc\, tr_A(\varphi)\\
tr_A(\varphi_1\,\mathcal{U}\,\varphi_2) &= tr_A(\varphi_1)\,\mathcal{U}\, tr_A(\varphi_2)
\end{aligned}$$

PROPOSITION 6.5. *For any iCGS M, path $\lambda \in \Lambda_M$, collective strategy s_A where $A \subseteq \mathbb{A}\mathrm{gt}$, and $\mathsf{ATL}^\star_{\mathsf{c}}$ formula φ we have that $M, \lambda, s_A \models_{x,c} \varphi$ iff $M, \lambda, s_A \models_{x,sc} tr_A(\varphi)$ for $x \in \{i, I\}$.*

PROOF. The proof is done by induction over the formula structure of φ.

Base cases: Case $\varphi = p$ is straightforward. Case $\varphi = \langle\langle B\rangle\rangle\gamma$ where γ contains no cooperation modalities. First, we observe ($\star$) $s_B \dagger s_A = s_{B\setminus A} \dagger s_A = s_A \dagger s_{B\setminus A}$: all individual strategies of agents in $A\cap B$ are overridden by those of s_A in $s_B \dagger s_A$ thus if two coalitions are disjoint the update order for their collective strategies is irrelevant; and ($\star\star$) $tr_X(\gamma) = \gamma$ if γ contains no cooperation modalities.

Next, $M, \lambda, s_A \models_{x,c} \langle\langle B\rangle\rangle\gamma$ iff there exists s_B such that $\forall\lambda' \in plays^x(\lambda[0], s_B \dagger s_A)$ we have $M, \lambda', s_B \dagger s_A \models_{x,c} \gamma$. By ($\star$),($\star\star$) $M, \lambda', s_B \dagger s_A \models_{x,c} \gamma$ iff $M, \lambda', s_A \dagger s_{B\setminus A} \models_{x,c} tr_{B\cup A}(\gamma)$. Since $tr_{B\cup A}(\gamma)$ contains no cooperation modalities, the semantics $\models_{x,c}$ and $\models_{x,sc}$ coincide for $tr_{B\cup A}(\gamma)$: $M, \lambda', s_A \dagger s_{B\setminus A} \models_{x,c} tr_{B\cup A}(\gamma)$ iff $M, \lambda', s_A \dagger s_{B\setminus A} \models_{x,sc} tr_{B\cup A}(\gamma)$ iff $M, \lambda, s_A \models_{x,c} \langle\langle B\setminus A\rangle\rangle tr_{B\cup A}(\gamma)$ iff $M, \lambda, s_A \models_{x,c} tr_A(\langle\langle B\rangle\rangle\gamma)$.

Induction step: Cases $\varphi = \neg\varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\langle B\rangle\rangle\gamma$ where γ contains cooperation modalities. Let ξ be an arbitrary outermost occurrence of a formula $\langle\langle B'\rangle\rangle\gamma'$ in γ. We label each state $\lambda'[0]$ such that $M, \lambda', s_B \dagger s_A \models_{x,c} \xi$ (for some $\lambda' \in \Lambda_M$ and each collective strategy $s_B \dagger s_A$), with a new proposition p_ξ which does not occur in Π. We note that any strategy $s_{A\cup B}$ may be written as $s_B \dagger s_A$. By induction hypothesis, $M, \lambda', s_B \dagger s_A \models_{x,c} \xi$ iff $M, \lambda', s_B \dagger s_A \models_{x,sc} tr_{B\cup A}(\xi)$. We replace each occurrence of a subformula ξ by p_ξ in γ. The

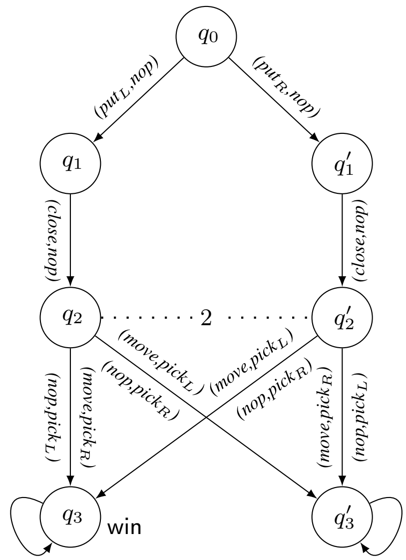


Fig. 9. M_9: the shell game with final shuffling

resulting formula contains no cooperation modalities. We proceed exactly as in the second base case. $\square$

Note that, in particular, $M, q \models_{x,c} \varphi$ iff $M, q \models_{x,sc} tr_\emptyset(\varphi)$. Thus, we obtain the following as an immediate consequence of Proposition 6.5.

COROLLARY 6.6. $\mathsf{ATL}^\star_{\mathsf{x,sc}}$ *is at least as expressive and as distinguishable as* $\mathsf{ATL}^\star_{\mathsf{x,c}}$ *for* $x \in \{i, I\}$. *That is,* $\mathsf{ATL}^\star_{\mathsf{x,c}} \preceq_d \mathsf{ATL}^\star_{\mathsf{x,sc}}$ *and* $\mathsf{ATL}^\star_{\mathsf{x,c}} \preceq_e \mathsf{ATL}^\star_{\mathsf{x,sc}}$.

PROPOSITION 6.7. $\mathsf{ATL}^\star_{\mathsf{x,c}}$ *is not as distinguishable as* $\mathsf{ATL}^\star_{\mathsf{x,sc}}$, *i.e.,* $\mathsf{ATL}^\star_{\mathsf{x,sc}} \not\preceq_d \mathsf{ATL}^\star_{\mathsf{x,c}}$.

The proof is given in Appendix A.3.

Corollary 6.6 and Proposition 6.7 imply the following.

THEOREM 6.8. $\mathsf{ATL}^\star_{\mathsf{x,sc}}$ *is strictly more distinguishing and expressive than* $\mathsf{ATL}^\star_{\mathsf{x,c}}$. *That is,* $\mathsf{ATL}^\star_{\mathsf{x,sc}} \prec_d \mathsf{ATL}^\star_{\mathsf{x,c}}$ *and* $\mathsf{ATL}^\star_{\mathsf{x,sc}} \prec_e \mathsf{ATL}^\star_{\mathsf{x,c}}$.

6.2. Commitment and Truly Perfect Recall

In the previous section, we have presented two variants of how strategic commitment can be added to the standard semantics of $\mathsf{ATL}^\star$. How doe it change the picture? First, we show that persistent strategies *per se* do not rule out counterintuitive effects.

Example 6.9 (*Shell game with final shuffling*). Consider the iCGS M_9 from Figure 9. It depicts a version of the shell game in which the shuffler can switch the shells quickly in the very last moment (after the ball has been enclosed). This is modeled by the action *move* available to the shuffler at states q_2 and q_2'. First, we observe that – in contrast to Example 2.4 – the guesser no longer has a strategy to eventually win in q_0, that is, $M_9, q_0 \not\models_i \langle\langle 2\rangle\rangle\Diamond\mathsf{win}$. On the other hand, if the shuffler is committed to some (arbitrary) strategy s_1, the guesser can secure a win: $M_9, q_0, s_1 \models_{i,sc} \langle\langle 2\rangle\rangle\Diamond\mathsf{win}$. Thus, $M_9, q_0 \models_{i,sc} \neg\langle\langle 1\rangle\rangle\neg\langle\langle 2\rangle\rangle\Diamond\mathsf{win}$: there is no strategy of player 1 that would prevent player 2 from winning, provided that 1 knows the strategy of 2 in advance.[11] So, there are situations where it makes sense to consider strategy commitment.

On the other hand, we still have $M_9, q_2, s_1 \models_{i,sc} \neg\langle\langle 2\rangle\rangle\Diamond\mathsf{win}$ regardless of the strategy s_1 of the shuffler. Thus, again, $M_9, q_0 \models_{i,sc} \langle\langle 1\rangle\rangle\Diamond\neg\langle\langle 2\rangle\rangle\Diamond\mathsf{win}$, which is counterintuitive.

Example 6.9 shows that, when reasoning about persistent strategies of agents, we also need to carefully define the semantics in order to avoid the "forgetting" phenomenon. This leads to the following semantics of strategic ability with commitment.

Definition 6.10 ($\mathsf{ATL}^\star_{\mathsf{nf,x,sc}}$). The semantics $\models^{nf}_{x,sc}$ of $\mathsf{ATL}^\star_{\mathsf{nf,x,sc}}$, with $x \in \{i, I\}$ is defined by changing the clause for $\langle\langle A\rangle\rangle\gamma$ in the following way:

$M, \lambda, k, s \models^{nf}_{x,sc} \langle\langle A\rangle\rangle\varphi$ iff there is a collective x-strategy s_A such that, for each $\lambda' \in$ *plays*$^x(\lambda[0, k], s \dagger s_A)$, we have $M, \lambda', s \dagger s_A \models^{nf}_{x,sc} \varphi$;

The other semantic clauses are defined analogously to Definition 3.2.

Definition 6.11 ($\mathsf{ATL}^\star_{\mathsf{nf,x,c}}$). The semantics of $\mathsf{ATL}^\star_{\mathsf{nf,x,c}}$ can be defined in two alternative (equivalent) ways: either we directly update the semantic clauses from Section 6.1.1, or we apply the translation from Section 6.1.3 and use the semantics of $\mathsf{ATL}^\star_{\mathsf{nf,x,sc}}$ from Definition 6.10. To simplify the presentation, we chose the latter option. Thus, for an $\mathsf{ATL}^\star$ formula φ, we define $M, \lambda, k, s_A \models^{nf}_{x,c} \varphi$ iff $M, \lambda, k, s_A \models^{nf}_{x,sc} tr_A(\varphi)$.

Remark 6.12. The way in which we defined the semantics of $\mathsf{ATL}^\star_{\mathsf{nf,x,c}}$ immediately implies that $\mathsf{ATL}^\star_{\mathsf{nf,x,c}} \preceq_e \mathsf{ATL}^\star_{\mathsf{nf,x,sc}}$, and hence also $\mathsf{ATL}^\star_{\mathsf{nf,x,c}} \preceq_d \mathsf{ATL}^\star_{\mathsf{nf,x,sc}}$.

The notions of truth of a state formula in a pointed model and validity of a formula are defined as in the previous sections.

Example 6.13 (*Commitment and truly perfect recall*). Consider the iCGS M_9 again. Similarly to Example 3.3, we now have that $M_9, q_0 \models^{nf}_{i,sc} \neg\langle\langle 1\rangle\rangle\Diamond\neg\langle\langle 2\rangle\rangle\Diamond\mathsf{win}$. If the shuffler commits to doing nothing (action nop) in q_2 (resp. q_2'), the guesser uses the history-based strategy from Example 2.4. If the shuffler intends late shuffling (action $move$), the guesser uses the "swap" strategy of picking, i.e. in comparison to the previous strategy selects $pick_L$ instead of $pick_R$, and $pick_R$ for $pick_L$.

Remark 6.14. Another variant of alternating-time temporal logic with imperfect information, truly perfect recall, and strategy contexts has been already proposed and studied in [Guelev and Dima 2012]. The differences to our work are as follows. First, the variant of $\mathsf{ATL}^\star$ in [Guelev and Dima 2012] features very ornate syntax, including past tense operators, collective knowledge operators, and strategic modalities that indicate which members of the coalition are allowed to revise their strategies, and which are not. This makes comparative analysis rather difficult to conduct. Secondly, their semantics differs from the standard approach by assuming runs to be interleaved sequences of states and action profiles, which affects indistinguishability relations. Thirdly, the interaction between the strategic and the epistemic aspects is further complicated by extending epistemic relations to indistinguishability over strategies. All of this is for a reason: the focus of [Guelev and Dima 2012] is on providing a rich logical framework where all aspects of persistent play under imperfect information can be modeled, described, and studied. In contrast, we start with a simple update of the standard semantics of persistent play from [Ågotnes et al. 2007; Brihaye et al. 2009], and focus on a comparison between different semantics of perfect recall.

[11] Note that $\neg\langle\langle 1\rangle\rangle\neg\langle\langle 2\rangle\rangle\Diamond\mathsf{win}$ is a formula of $\mathsf{ATL}^\star$ but not ATL. The same property can be equivalently expressed with the ATL formula $\neg\langle\langle 1\rangle\rangle\mathcal{N}(\neg\langle\langle 2\rangle\rangle\Diamond\mathsf{win})$.

6.3. Expressivity of ATL* with Commitment and Truly Perfect Recall

In this section we study the relation between the "forgetting" and "no forgetting" variants of $\mathsf{ATL}^\star$ with strategy commitment, i.e., $\mathsf{ATL}^\star_{\mathsf{x,c}}$ vs. $\mathsf{ATL}^\star_{\mathsf{nf,x,c}}$ and $\mathsf{ATL}^\star_{\mathsf{x,sc}}$ vs. $\mathsf{ATL}^\star_{\mathsf{nf,x,sc}}$.

6.3.1. Perfect Information. Similarly to Section 4.1 we have the following results for the perfect information case. The proof is done analogously to Proposition 3.5 (see Appendix A.3 for details).

PROPOSITION 6.15. *For all M, λ, s and every $\mathsf{ATL}^\star$ formula φ, we have that $M, \lambda, 0, s \models^{nf}_{I,sc} \varphi$ iff $M, \lambda, s \models_{I,sc} \varphi$.*

Thus, similarly to the setting with standard non-persistent strategies, the truly perfect recall and standard perfect recall semantics are equivalent under perfect information. We obtain the following as an immediate corollary.

COROLLARY 6.16. *$\mathsf{ATL}^\star_{\mathsf{I,sc}}$ and $\mathsf{ATL}^\star_{\mathsf{nf,I,sc}}$ are equally expressive and have the same sets of validities. By Proposition 6.5, the same holds for $\mathsf{ATL}^\star_{\mathsf{I,c}}$ vs. $\mathsf{ATL}^\star_{\mathsf{nf,I,c}}$.*

6.3.2. Imperfect Information. The next theorem shows that, under imperfect information, the truly perfect recall semantics for persistent strategies differs from the standard one.

PROPOSITION 6.17. *There is a pointed iCGS (M, q) and an $\mathsf{ATL}^\star$ formula φ such that $M, q \models_{i,sc} \varphi$ and $M, q \not\models^{nf}_{i,sc} \varphi$.*

PROOF. The result follows from Examples 6.9 and 6.13 for $M = M_9$, $q = q_0$ and $\varphi \equiv \langle\!\langle 1 \rangle\!\rangle \Diamond \neg \langle\!\langle 2 \rangle\!\rangle \Diamond \mathsf{win}$. $\square$

The following propositions compare the distinguishing power of the truly perfect recall vs. "forgetting" semantics for $\mathsf{ATL}^\star_{\mathsf{i,sc}}$, and $\mathsf{ATL}^\star_{\mathsf{i,c}}$. The proofs are given in Appendix A.3.

PROPOSITION 6.18. *There are iCGSs which satisfy the same formulae of $\mathsf{ATL}^\star_{\mathsf{i,sc}}$, but can be distinguished in $\mathsf{ATL}^\star_{\mathsf{nf,i,c}}$. That is, $\mathsf{ATL}^\star_{\mathsf{nf,i,c}} \not\preceq_d \mathsf{ATL}^\star_{\mathsf{i,sc}}$.*

PROPOSITION 6.19. *There are iCGSs which satisfy the same formulae of $\mathsf{ATL}^\star_{\mathsf{nf,i,sc}}$, but can be distinguished in $\mathsf{ATL}^\star_{\mathsf{i,c}}$. That is, $\mathsf{ATL}^\star_{\mathsf{i,c}} \not\preceq_d \mathsf{scATL}^\star_{\mathsf{nf,i}}$.*

Combining Propositions 6.18 and 6.19 with Proposition 6.5 and Remark 6.12, we get that:

THEOREM 6.20. *For any $\mathsf{L}_1 \in \{\mathsf{ATL}^\star_{\mathsf{i,sc}}, \mathsf{ATL}^\star_{\mathsf{i,c}}\}$ and $\mathsf{L}_2 \in \{\mathsf{ATL}^\star_{\mathsf{nf,i,sc}}, \mathsf{ATL}^\star_{\mathsf{nf,i,c}}\}$, the logics L_1 and L_2 are incomparable with respect to distinguishing and expressive power.*

6.4. Comparing Validity Sets

Finally, we compare the sets of valid sentences of the "no forgetting" logics $\mathsf{ATL}^\star_{\mathsf{nf,x,c}}$ and $\mathsf{ATL}^\star_{\mathsf{nf,x,sc}}$ with those of their "forgetting" counterparts.

6.4.1. Perfect Information. The following is an immediate consequence of Propositions 6.15 and 6.5.

PROPOSITION 6.21. *$\mathsf{ATL}^\star_{\mathsf{I,sc}}$ and $\mathsf{ATL}^\star_{\mathsf{nf,I,sc}}$ have the same sets of validities. The same holds for $\mathsf{ATL}^\star_{\mathsf{I,c}}$ vs. $\mathsf{ATL}^\star_{\mathsf{nf,I,c}}$.*

6.4.2. Imperfect Information. For imperfect information, we can obtain the result below analogously to Proposition 5.5, i.e., by using epistemic tree unfoldings.

PROPOSITION 6.22. *$Val(\mathsf{scATL}^\star_{\mathsf{i}}) \subseteq Val(\mathsf{scATL}^\star_{\mathsf{nf,i}})$. Thus, also $Val(\mathsf{ATL}^\star_{\mathsf{i,c}}) \subseteq Val(\mathsf{ATL}^\star_{\mathsf{nf,i,c}})$.*

A detailed proof is presented in Appendix A.3.

PROPOSITION 6.23. *$Val(\mathsf{ATL}^{\star}_{\mathsf{nf,i,c}}) \not\subseteq Val(\mathsf{ATL}^{\star}_{\mathsf{i,c}})$. Thus, also $Val(\mathsf{ATL}^{\star}_{\mathsf{nf,i,sc}}) \not\subseteq Val(\mathsf{ATL}^{\star}_{\mathsf{i,sc}})$.*

PROOF. We can reuse formula φ from the proof of Proposition 5.6 because it does not contain nested modalities—apart from $\emptyset$—and over such formulae the commitment and no-commitment semantics coincide. □

Similarly to Theorem 5.7, we immediately obtain the following characterization:

THEOREM 6.24. *$Val(\mathsf{scATL}^{\star}_{\mathsf{i}}) \subsetneq Val(\mathsf{scATL}^{\star}_{\mathsf{nf,i}})$ and $Val(\mathsf{ATL}^{\star}_{\mathsf{i,c}}) \subsetneq Val(\mathsf{ATL}^{\star}_{\mathsf{nf,i,c}})$*

Thus, games of truly perfect recall can be seen as a special subclass of games with "standard" perfect recall also in the case of persistent strategies, as captured by the semantics of strategic ability proposed in [Ågotnes et al. 2007; Brihaye et al. 2009].

7. MODEL CHECKING

Model checking is the problem of establishing whether a logical formula is satisfied in a given structure. The complexity of model checking is especially insightful for the development of verification tools for particular logics. Decision problems for games with imperfect information and perfect recall are known to be computationally hard even for 2-player games, and undecidable when coalitional strategies are considered [Peterson and Reif 1979; Pnueli and Rosner 1990; Peterson et al. 2001; Berwanger and Kaiser 2010; Dima and Tiplea 2011]. On the other hand, some decidable cases have also been identified [Chatterjee et al. 2007a; Berwanger and Kaiser 2010; Guelev et al. 2011; Berwanger and Mathew 2014; Berwanger et al. 2015], most notably the verification of abilities of single individual agents. In this section, we point out that the truly perfect recall semantics shares (un)decidability of model checking with the standard perfect recall variant, but it makes verification more costly in the decidable cases.

Note: the purpose of this section is not to establish new technical results, but to shortly review the impact of "no forgetting" from the computational perspective.

Since the two semantics coincide for perfect information, we only consider the case of imperfect information. We begin by quoting a rather pessimistic result for verification of alternating-time specifications with imperfect information and perfect recall.

THEOREM 7.1 ([DIMA AND TIPLEA 2011]). *Model checking $\mathsf{ATL}_{\mathsf{i}}$ (and hence also $\mathsf{ATL}^{\star}_{\mathsf{i}}$) is undecidable.*

The undecidability result was proved by a reduction of the halting problem that employed a 3-player iCGS and a formula with no nested cooperation modalities [Dima and Tiplea 2011]. We recall that, when no nested cooperation modalities are present, the $\mathsf{ATL}^{\star}_{\mathsf{nf,i}}$ semantics coincides with that of $\mathsf{ATL}^{\star}_{\mathsf{i}}$. Thus, we get the following as a consequence.

COROLLARY 7.2. *Model checking of $\mathsf{ATL}_{\mathsf{nf,i}}$ (and hence also $\mathsf{ATL}^{\star}_{\mathsf{nf,i}}$) is undecidable.*

What about decidable fragments of the problem? Restricting the class of models to turn-based structures will not work, as the proof in [Dima and Tiplea 2011] can be adapted to yield a turn-based model in the reduction of the halting problem. On the other hand, [Guelev et al. 2011] proposed an effective algorithm for model checking ATL with truly perfect recall and coalitions whose strategies are based on the distributed knowledge relation within the coalition. This is in turn equivalent to model checking the "singleton fragment" of ATL with truly perfect recall, i.e., the fragment of $\mathsf{ATL}_{\mathsf{nf,i}}$ with strategic modalities restricted to coalitions of size at most one [Kaźmierczak et al. 2014]). More formally:

THEOREM 7.3 ([GUELEV ET AL. 2011]). *Model checking of the singleton fragment of $\mathsf{ATL}_{\mathsf{nf,i}}$ is decidable and can be done in nonelementary time with respect to the size of the model and the* *length of the formula. For formulae of strategic depth*[12] *of at most k, the model checking problem is in* $\mathbf{kEXPTIME}$.

The lower bounds can be derived from the following result for model checking of temporal-epistemic logic with perfect recall.

THEOREM 7.4 ([SHILOV ET AL. 2004; SHILOV ET AL. 2006]). *Model checking* CTLK *with perfect recall is decidable with nonelementary upper and lower bounds with respect to the size of the model and the length of the formula. For formulae of knowledge depth*[13] *of at most k, the model checking problem is* $\mathbf{kEXPTIME}$*-complete.*

From this, we obtain the following characterization of complexity for the singleton fragments of $\mathsf{ATL_i}$ and $\mathsf{ATL_{nf,i}}$.

THEOREM 7.5. *Model checking of the singleton fragment of* $\mathsf{ATL_{nf,i}}$ *is complete in nonelementary time with respect to the size of the model and the length of the formula. For formulae of strategic depth of at most k, the model checking problem is* $\mathbf{kEXPTIME}$*-complete.*

PROOF. Straightforward from Theorems 7.3 and 7.4.[14] □

THEOREM 7.6. *Model checking of the singleton fragment of* $\mathsf{ATL_i}$ *is* $\mathbf{EXPTIME}$*-complete with respect to the size of the model and the length of the formula. It remains* $\mathbf{EXPTIME}$*-complete for formulae of bounded length.*

PROOF. We proceed recursively (bottom-up), starting with subformulae that contain no nested strategic modalities, and replacing them with fresh atomic propositions that hold in exactly the same subset of states. By Theorem 7.3, the procedure runs in exponential time. The lower bound follows from Theorem 7.4. □

Thus, assuming truly perfect recall under imperfect information changes the verification complexity for worse. On the other hand, it simplifies the underlying tree unfoldings (cf. Section 5.2), which can potentially make verification easier, especially for simple (and short) formulae of ATL. Moreover, model checking $\mathsf{ATL_{nf,i}}$ is no harder than verification of temporal-epistemic logic with perfect recall [van der Meyden and Shilov 1999; Shilov et al. 2004; Shilov et al. 2006]. Thus, $\mathsf{ATL_{nf,i}}$ buys the expressivity of strategic operators for no extra computational price. We also note that the increase in complexity is due to the perfect recall assumption itself, and *not* due to the interaction of strategic modalities with truly perfect recall.

8. CONCLUSIONS

In this paper, we formally study a semantics of strategic ability which propagates agents' observations to nested strategic modalities. Thus, unlike the standard semantics of alternating-time logic $\mathsf{ATL}^\star$, it models agents who *never* forget their past observations. Most importantly, we investigate the relationship between the two approaches, encoded by the "forgetting" and the "truly perfect recall" semantics. Both approaches turn out to be equivalent for agents with perfect information of the global state of the system. In the more interesting case of incomplete information, however, the two kinds of semantics are significantly different. In particular, they yield logical systems that are incomparable with respect to their expressive as well as distinguishing power. Equally interesting is the comparison of general properties of games induced by the different semantics. Formally, this means to compare the sets of validities generated by the alternative semantics. We show that the validities according to the truly perfect recall semantics form a strict superset of the "forgetting" validities. Thus, they capture a more specific class of games than the standard $\mathsf{ATL_i^\star}$.

[12]The maximal number of nested strategic operators.
[13]The maximal number of nested epistemic operators.
[14]The idea of the proof was hinted to us by Catalin Dima in a personal communication.

The same pattern of results carries over to the setting where agents are assumed to persist with their strategies by some kind of (irrevocable or revocable) strategic commitment.

From the computational point of view, reasoning about agents with perfect recall is always complex, but assuming truly perfect recall makes it even harder, as exemplified by the nonelementary complexity of model checking even for abilities of individual agents. That seems to suggest that, in practice, using the truly perfect recall semantics puts one at a disadvantage. However, when doing verification, the most important thing is to *verify the right properties of the right system*. In particular, the input model must match the relevant aspects of the system, and the formula together with its semantic interpretation must capture the property that we want to verify. The same applies to any other kind of reasoning and analysis. We believe that, when reasoning about agents who are supposed to memorize their observations, the truly perfect recall semantics of ability is the right one. Our technical results show that it cannot be replaced by the standard, more compositional semantics of $\mathsf{ATL}^\star_\mathsf{i}$, despite the latter offering somewhat lower complexity of related computational tasks. This is because the properties definable in both frameworks are essentially different, and because $\mathsf{ATL}^\star_\mathsf{i}$ allows for paradoxical specifications that should not be satisfiable for agents with real perfect memory.

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A. PROOFS

A.1. Proofs of Section 4

PROPOSITION 4.4. *There are pointed iCGSs which satisfy the same* $\mathsf{ATL}^\star_{\mathsf{i}}$*-formulae, but can be distinguished in* $\mathsf{ATL}^\star_{\mathsf{nf,i}}$. *Thus,* $\mathsf{ATL}^\star_{\mathsf{nf,i}} \not\preceq_d \mathsf{ATL}^\star_{\mathsf{i}}$.

PROOF. Let M_2 and M_2' be the iCGSs shown in Figure 3 and φ be any $\mathsf{ATL}^\star$-formula. We define $M_2^\uparrow$ (resp. $M_2^\downarrow$) as the sub-model of M_2 obtained by keeping only the states $x \in \{a_0, b_0, a_1', b_1', a_2', b_2'\}$ (resp. $x \in \{a_0, b_0, a_1, b_1, a_2, b_2\}$) and removing all other states and subsequent transitions. The model $M_2'^\uparrow$ and $M_2'^\downarrow$ are defined analogously as sub-models of M_2'.

First, we observe that $M_2^\uparrow$ and $M_2^\downarrow$ (resp. $M_2'^\uparrow$, $M_2'^\downarrow$) are bisimilar. As a consequence, we have:

$$M_2, x_j \models_i \varphi \text{ iff } M_2, x_j' \models_i \varphi \text{ for } x \in \{a, b\} \text{ and } j \in \{1, 2\} \tag{1}$$

and analogously for M_2'. Moreover, we have

$$M_2, x_j \models_i \varphi \text{ iff } M_2', x_j \models_i \varphi \text{ for } x \in \{a, b, a', b'\} \text{ and } j \in \{1, 2\} \tag{2}$$

We prove:

$$(\star)\ M_2, a_0 \models_i \varphi \text{ iff } M_2', a_0 \models_i \varphi$$

by induction over the formula structure of φ.

Base cases: Case $\varphi = \mathsf{p}$. Straightforward. Case $\varphi = \langle\!\langle A \rangle\!\rangle \gamma$ where γ contains no cooperation modalities. For $A \in \{\emptyset, \{1\}\}$ the proposition follows immediately as each strategy of A generates the same set $plays^i_M(a_0, s_A)$ in both models.

Suppose $A = \{2\}$. Direction "$\Leftarrow$": as $plays^i_{M_2'}(a_0, s_A) = out_{M_2'}(a_0, s_A) \cup out_{M_2'}(b_0, s_A)$ and $plays^i_{M_2}(a_0, s_A) = out_{M_2}(a_0, s_A)$, we have $plays^i_{M_2}(a_0, s_A) \subseteq plays^i_{M_2'}(a_0, s_A)$. $(\star)$ follows immediately.

Direction "$\Rightarrow$": let s_2 be an arbitrary uniform strategy of player 2 in M_2. We investigate $plays^i_{M_2}(a_0, s_2)$. First: (i) we observe that player 2 cannot prevent any of a_1', b_1' from being possible next-states of the game. Second: (ii) for any action in $\{\alpha, \beta, \mu\}$ which player 2 may play in both $a_0 a_1'$ and $a_0 b_1'$ (the same action must be planned for both histories, since they are indistinguishable to 2 and s_2 is uniform), $plays^i_{M_2}(a_0, s_2)$ contains a path on which win eventually holds (e.g. $a_0 b_1' (b_2')^\omega$ if α is played) and one on which win never holds ($a_0 a_1' (a_2')^\omega$ if α is played). The statements (i),(ii) also hold in M_2'. Player 2 can neither prevent win nor ensure win from $a_0 a_1'$ and $a_0 b_1'$, in either model. This concludes case $A = \{2\}$.

Suppose $A = \{1, 2\}$. Direction "$\Leftarrow$": we again note $plays^i_{M_2}(a_0, s_A) \subseteq plays^i_{M_2'}(a_0, s_A)$.

Direction "$\Rightarrow$": Let $s_A = (s_1, s_2)$ be an arbitrary uniform strategy for A. We first observe that $plays^i_{M_2}(a_0, s_A)$ contains a unique path, however $plays^i_{M_2'}(a_0, s_A)$ contains two paths. Starting from s_A we build $s_A' = (s_1', s_2')$ such that both paths from $plays^i_{M_2'}(a_0, s_A')$ are propositionally equivalent to that from $plays^i_{M_2}(a_0, s_A)$: win is either maintained false or eventually (and thereafter always) fulfilled. The construction is as follows: $s_j'(a_0) = s_j(a_0)$ for $j \in \{1, 2\}$ (s_A' replicates s_A in the initial state);

for histories $h \in \{a_0 a_1, a_0 a_1'\}$:
- if $s_2(h) = \alpha$ then $s_1'(h) = \epsilon$ and $s_2'(h) = \mu$;
- if $s_2(h) = \beta$ then $s_1'(h) = \mu$ and $s_2'(h) = \mu$;
- otherwise $s_1'(h) = s_1(h)$ and $s_2'(h) = s_2(h)$;

for history $h = a_0 b_1'$:
- if $s_2(h) = \alpha$ then $s_1'(h) = \mu$ and $s_2'(h) = \mu$;
- if $s_2(h) = \beta$ then $s_1'(h) = \epsilon$ and $s_2'(h) = \mu$;
- otherwise $s_1'(h) = s_1(h)$ and $s_2'(h) = s_2(h)$;

For all other histories, the assigned actions are unimportant. We note that s_2 is uniform, and also that in the absence of actions μ and ϵ we would, e.g, have that $M_2, a_0 \models_i \langle\!\langle 1, 2 \rangle\!\rangle \Diamond \mathsf{win}$ but $M_2', a_0 \not\models_i \langle\!\langle 1, 2 \rangle\!\rangle \Diamond \mathsf{win}$.

Induction step: The cases $\varphi = \neg \varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\!\langle A \rangle\!\rangle \gamma$ where γ contains cooperation modalities. Let ξ be an arbitrary occurrence of an outermost formula $\langle\!\langle A' \rangle\!\rangle \gamma'$ in γ. We note that $M_2, x \models_i \xi$ iff $M_2', x \models_i \xi$ by induction hypothesis if $x = a_0$ and by (1-2), otherwise. We label each state x of M_2 and M_2' where ξ holds by a new proposition p_ξ. The resulting models retain properties (1-2). We replace each ξ by p_ξ in γ and obtain a formula without cooperation modalities. We proceed as in the second base case. This concludes the proof of $(\star)$.

In Ex. 4.3 we have shown that both pointed models can be distinguished in $\mathsf{ATL}^\star_{\mathsf{nf,i}}$. For every $\mathsf{ATL}^\star_{\mathsf{i}}$-formula φ we have $a_0 \in [\![\varphi, M_2]\!]_{\models_i}$ iff $a_0 \in [\![\varphi, M_2']\!]_{\models_i}$ but $a_0 \in [\![\varphi', M_2]\!]_{\models_i^{nf}}$ and $a_0 \notin [\![\varphi', M_2']\!]_{\models_i^{nf}}$ for some φ'. Thus, we have that $\mathsf{ATL}^\star_{\mathsf{nf,i}} \not\preceq_d \mathsf{ATL}^\star_{\mathsf{i}}$. $\square$

A.2. Proofs of Section 5

PROPOSITION 5.4. *$M, q \models_i^{nf} \varphi$ iff $T^{nf}(M,q), q \models_i^{nf} \varphi$ iff $T^{nf}(M,q), q \models_i \varphi$, for all* ATL*-*formulae, iCGSs M and states q.*

PROOF. To simplify the notations, we write T^{nf} instead of $T^{nf}(M,q)$. We observe the following, from the construction of T^{nf}:

(1) $\lambda \in \Lambda_M$ iff $\hat{\lambda} = h_0 h_1 \dots h_i \dots \in \Lambda_{T^{nf}}$ where $h_i = \lambda[0,i]$ and $i \in \mathbb{N}$;
$h \in \Lambda_M^{fin}$ iff $\hat{h} = h_0 h_1 \dots h_{k-1} \in \Lambda_{T^{nf}}^{fin}$ where $h_i = h[0,i]$ for $i < k$ and $k = |h|$.
Moreover, λ and $\hat{\lambda}$ are propositionally equivalent.
(2) $h \approx_a^M h'$ iff $\hat{h} \approx_a^{T^{nf}} \hat{h}'$ for all $a \in \mathbb{A}\mathrm{gt}$.
(3) for each collective strategy s_A in M there exists a collective strategy $\hat{s}_A$ in T^{nf} such that $\lambda \in out(h, s_A)$ iff $\hat{\lambda} \in out(\hat{h}, \hat{s}_A)$ and vice-versa.

Note that we assume that it is clear from context how the different histories are concatenated, e.g. $(q_0), (q_0 q_1), (q_0 q_1 q_2), \dots$. To increase the readability, we omit parentheses.
We prove both equivalences separately.

(i). $M, q \models_i^{nf} \varphi$ iff $T^{nf}, q \models_i^{nf} \varphi$. We prove the stronger statement: $M, h \circ \lambda, k \models_i^{nf} \varphi$ iff $T^{nf}, \hat{h} \circ \hat{\lambda}, k \models_i^{nf} \varphi$, where $k = |h| - 1$. Our claim follows for $k = 0$. The proof is by induction over the formula structure of φ.
Base cases: Case $\varphi = p$. It is sufficient to note that $(h \circ \lambda)[k] \in \pi^M(p)$ iff $(\hat{h} \circ \hat{\lambda})[k] \in \pi^{T^{nf}}(p)$. Case $\varphi = \langle\langle A \rangle\rangle \gamma$ where γ contains no cooperation modalities. From (2-3) it follows that $\lambda \in plays_M^i(h, s_A)$ iff $\hat{\lambda} \in plays_{T^{nf}}^i(\hat{h}, \hat{s}_A)$. Since λ and $\hat{\lambda}$ are propositionally equivalent (c.f. (1)): $M, h \circ \lambda \models_i^{nf} \langle\langle A \rangle\rangle \gamma$ iff $T^{nf}, \hat{h} \circ \hat{\lambda} \models_i^{nf} \langle\langle A \rangle\rangle \gamma$.
Induction step: Cases $\varphi = \neg\varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\langle A \rangle\rangle \gamma$ where γ contains cooperation modalities. Let $\{\langle\langle B_i \rangle\rangle \psi_i \mid i = 1, \dots, k\}$ be the set of outermost (positive) ATL*-subformulae in γ. We define the following two sets:
$H_i = \{(h \circ \lambda, k) \mid M, h \circ \lambda, k \models_i^{nf} \langle\langle B_i \rangle\rangle \psi_i\}$
$\hat{H}_i = \{(\hat{h} \circ \hat{\lambda}, k) \mid T^{nf}, \hat{h} \circ \hat{\lambda}, k \models_i^{nf} \langle\langle B_i \rangle\rangle \psi_i\}$
By induction hypothesis, we have that $\hat{H}_i = \{(\hat{h} \circ \hat{\lambda}, k) \mid (h \circ \lambda, k) \in H_i\}$ as all formulae have a simpler structure that φ ($\star$).
We now consider the two directions of (i): "$\Rightarrow$": Let s_A be a witnessing strategy for $\langle\langle A \rangle\rangle \gamma$ from history $h \circ \lambda$ and index k, i.e. $\forall \lambda \in plays_M^i(h, s_A) : M, \lambda, k \models_i^{nf} \gamma$. By (2) and (3), there exists a strategy $\hat{s}_A$ in T^{nf} such that:

$$\{\hat{\lambda} \mid \hat{\lambda} \in plays_M^i(h, s_A)\} = \{\lambda \mid \lambda \in plays_{T^{nf}}^i(\hat{h}, \hat{s}_A)\}$$

We note that for all h it holds that $\hat{s}_A(\hat{h}) = s_A(h) = s_A(last(\hat{h}))$. The claim now follows by ($\star$). The direction "$\Leftarrow$" follows exactly the same argument.

(ii). $T^{nf}, q \models_i^{nf} \varphi$ iff $T^{nf}, q \models_i \varphi$. As before, we prove the stronger claim $T^{nf}, \hat{h} \circ \hat{\lambda}, k \models_i^{nf} \varphi$ iff $T^{nf}, last(\hat{h}) \circ \hat{\lambda} \models_i \varphi$ where $k = |\hat{h}| - 1$, by induction over the formula structure of φ. Then, the claim follows for $k = 0$.
Base cases: Case $\varphi = p$ is straightforward. Case $\varphi = \langle\langle A \rangle\rangle \gamma$ where γ contains no cooperation modalities. From (1-3) we observe that for each uniform collective strategy s_A and history $\hat{h} \in \Lambda_{T^{nf}}^{fin}$, there exists a uniform collective strategy s'_A such that $plays_{T^{nf}}^i(\hat{h}, s_A) = \{\hat{h} \circ \hat{\lambda} \mid last(\hat{h}) \circ \hat{\lambda} \in plays_{T^{nf}}^i(last(\hat{h}), s'_A)\}$. Informally, each state $last(\hat{h})$ encodes a history. Hence, if coalition A has a perfect-recall strategy w.r.t. $\hat{h}$, then cf. (2-3) it can implement a strategy with the same effects from $last(\hat{h})$ alone. Similarly, for each collective strategy s'_A and $\hat{h} \in \Lambda_{T^{nf}}^{fin}$ there exists a

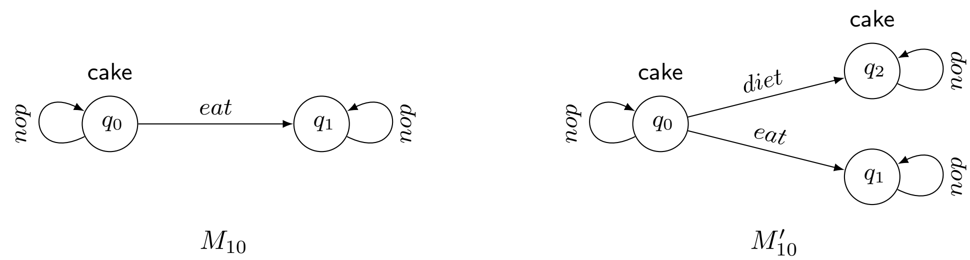

Fig. 10. Simplified cake dilemma: with and without diet

collective strategy s_A such that $plays^i_{T^{nf}}(last(\hat{h}), s'_A) = \{last(\hat{h}) \circ \hat{\lambda} \mid \hat{h} \circ \hat{\lambda} \in plays^i_{T^{nf}}(\hat{h}, s_A)\}$. Thus $T^{nf}, \hat{h} \circ \hat{\lambda}, k \models^{nf}_i \langle\langle A\rangle\rangle\gamma$ iff $T^{nf}, last(\hat{h}) \circ \hat{\lambda} \models_i \langle\langle A\rangle\rangle\gamma$.

Induction step: Cases $\varphi = \neg\varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\langle A\rangle\rangle\gamma$ where γ contains cooperation modalities.

Let $\{\langle\langle B_i\rangle\rangle\psi_i \mid i = 1, \ldots, k\}$ be the set of outermost (positive) $\mathsf{ATL}^\star$-subformulae in γ. We also define the following sets:

$\hat{H}_i = \{(\hat{h} \circ \hat{\lambda}, k) \mid T^{nf}, \hat{h} \circ \hat{\lambda}, k \models^{nf}_i \langle\langle B_i\rangle\rangle\psi_i\}$ and

$\hat{L}_i = \{\hat{\lambda} \mid T^{nf}, \hat{\lambda} \models_i \langle\langle B_i\rangle\rangle\psi_i\}$.

By induction hypothesis: $\hat{H}_i = \{(\hat{h} \circ \hat{\lambda}, k) \mid last(\hat{h}) \circ \hat{\lambda} \in \hat{L}_i\}$ $(\star)$. We now consider the two directions of (ii): "$\Rightarrow$": Let s_A be a witnessing strategy for $\langle\langle A\rangle\rangle\gamma$ from $\hat{h} \circ \hat{\lambda}$ and index k, i.e. $\forall\hat{\lambda}' \in plays^i_{T^{nf}}(s_A, \hat{h})$, we have $T^{nf}, \hat{\lambda}', k \models^{nf}_i \gamma$. By (2) and (3) there exists a s'_A such that:

$$plays^i_{T^{nf}}(s_A, \hat{h}) = \{\hat{h} \circ \hat{\lambda} \mid last(\hat{h}) \circ \hat{\lambda} \in plays^i_{T^{nf}}(s'_A, last(\hat{h}))\}$$

The claim now follows by $(\star)$. Direction "$\Leftarrow$" follows exactly the same argument.

□

A.3. Proofs of Section 6

PROPOSITION 6.7. $\mathsf{ATL}^\star_{\mathsf{x,c}}$ *is not as distinguishable as* $\mathsf{ATL}^\star_{\mathsf{x,sc}}$*, i.e.,* $\mathsf{ATL}^\star_{\mathsf{x,sc}} \not\preceq_d \mathsf{ATL}^\star_{\mathsf{x,c}}$.

PROOF. We show that $M_{10}, q_0 \models_{x,c} \varphi$ iff $M'_{10}, q_0 \models_{x,c} \varphi$ for all $\varphi \in \mathsf{ATL}^\star_{\mathsf{x,c}}$, for models M_{10} and M'_{10} shown in Figure 10. We first observe that:

$$M_{10}, q_1, s \models_{x,c} \varphi \text{ iff } M'_{10}, q_1, s \models_{x,c} \varphi \text{ for all } \varphi \in \mathsf{ATL}^\star_{\mathsf{x,c}} \text{ and all strategies } s \neq s_\emptyset \quad (1)$$

$$plays^x_{M_{10}}(q, s) = out_{M_{10}}(q, s) \text{ for all states } q \text{ of } M_{10} \text{ and all strategies } s. \quad (2)$$

We prove: $(\star)$ $M_{10}, q_0, s \models_{x,c} \varphi \implies M'_{10}, q_0, s \models_{x,c} \varphi$ for all strategies s in M_{10} and all $\varphi \in \mathsf{ATL}^\star_{\mathsf{x,c}}$.

Base cases: Case $\varphi = p$ is straightforward. Case $\varphi = \langle\langle A\rangle\rangle\gamma$ where γ does not contain cooperation modalities. Since $\mathbb{A}\mathrm{gt} = \{1\}$ in M_{10}, we have only two possible coalitions: $A = \emptyset$ or $A = \{1\}$. We consider each possiblity in turn. First, suppose $A = \{1\}$, $M_{10}, q_0, s \models_{x,c} \langle\langle 1\rangle\rangle\gamma$ and let s_1 be a witnessing strategy i.e. $M_{10}, \lambda[0], s_1 \dagger s \models_{x,c} \gamma$ for all $\lambda \in out_{M_{10}}(q_0, s_1 \dagger s)$. Since $s_1 \dagger s$ is a $\{1\}$-strategy (if $s = s_\emptyset$ then $s_1 \dagger s = s_1$ otherwise: $s_1 \dagger s = s$), it follows that $out_{M_{10}}(q_0, s_1 \dagger s) = out_{M'_{10}}(q_0, s_1 \dagger s)$ by construction of the models. Then $M'_{10}, \lambda[0], s_1 \dagger s \models_{x,c} \gamma$ and therefore $M'_{10}, q_0, s \models_{x,c} \langle\langle 1\rangle\rangle\gamma$.

Second, suppose $A = \emptyset$. If $s \neq s_\emptyset$ the argument is exactly as above. If $s = s_\emptyset$ we observe that $plays^x_{M_{10}}(q_0, s_\emptyset) = \{q_0^+ q_1^\omega, q_0^\omega\}$ and that $plays^x_{M'_{10}}(q_0, s_\emptyset) = \{q_0^+ q_2^\omega, q_0^+ q_1^\omega, q_0^\omega\}$. It is sufficient to note that each path in $plays^x_{M_{10}}(q_0, s_\emptyset)$ is propositionally equivalent to one in $plays^x_{M'_{10}}(q_0, s_\emptyset)$ and vice-versa.

Induction step: Cases $\varphi = \neg\varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\langle A\rangle\rangle\gamma$ where γ contains cooperation modalities and $A = \{1\}$. Suppose $M_{10}, q_0, s \models_{x,c} \langle\langle 1\rangle\rangle\gamma$ and let s_1 be a witnessing strategy. For each outermost $\mathsf{ATL}^\star$-subformula $\langle\langle B_i\rangle\rangle\psi_i$ in γ (with $i = 1, \dots, k$), strategy s^* and state q such that $M_{10}, q, s^* \models_{x,c} \langle\langle B_i\rangle\rangle\psi_i$, we observe that s^* can only be of the form $s' \dagger s_1$. Hence $s^* = s_1$ since s_1 is irrevocable. Then $M_{10}, q, s^* \models_{x,c} \langle\langle B_i\rangle\rangle\psi_i$ iff $M_{10}, q, s_1 \models_{x,c} \psi_i$. Therefore we can treat φ exactly as in the base case. The same holds for $A = \emptyset$.

We prove $(\star\star)$ $M'_{10}, q_0, s' \models_{x,c} \varphi \implies M_{10}, q_0, t(s') \models_{x,c} \varphi$ where $t(s')$ is the strategy s such that $s(h) = \mathit{nop}$ if $s'(h) = \mathit{diet}$ and $s(h) = s'(h)$, otherwise. Also, t leaves the empty strategy unchanged: $t(s_\emptyset) = s_\emptyset$.

Base cases: Case $\varphi = p$ is straightforward. Case $\varphi = \langle\langle A\rangle\rangle\gamma$ where γ does not contain cooperation modalities. As before, we first consider $A = \{1\}$. Suppose $M'_{10}, q_0, s' \models_{x,c} \langle\langle 1\rangle\rangle\gamma$, $s = t(s')$ and s_1 be a witnessing strategy of the former, i.e. $M'_{10}, \lambda[0], s_1 \dagger s' \models_{x,c} \gamma$ for all $\lambda \in out_{M_{10}}(q_0, s_1 \dagger s')$. We note that $s_1 \dagger s'$ is a $\{1\}$-strategy and that $out_{M'_{10}}(q_0, s')$ and $out_{M_{10}}(q_0, s)$ contain unique paths which are propositionally equivalent. For instance, if s' executes *diet* for some history yielding path $q_0^+ q_2^\omega$ then s will execute *nop* for that same history resulting in the path q_0^ω. Therefore we have $M_{10}, \lambda[0], s_1 \dagger s \models_{x,c} \gamma$ thus $M_{10}, q_0, s \models_{x,c} \langle\langle 1\rangle\rangle\gamma$. For $A = \emptyset$ we follow exactly the same argument.

Induction step: Cases $\varphi = \neg\varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\langle A\rangle\rangle\gamma$ where γ contains cooperation modalities and $A = \{1\}$. Suppose $M'_{10}, q_0, s' \models_{x,c} \langle\langle 1\rangle\rangle\gamma$ and let s_1 be a witnessing strategy. For each outermost $\mathsf{ATL}^\star$-subformula $\langle\langle B_i\rangle\rangle\psi_i$ in γ (with $i = 1, \dots, k$), strategy s^* and state q such that $M_{10}, q, s^* \models_{x,c} \langle\langle B_i\rangle\rangle\psi_i$, we observe that s^* can only be of the form $s' \dagger s_1$. Hence $s^* = s_1$ since s_1 is irrevocable. Then $M'_{10}, q, s^* \models_{x,c} \langle\langle B_i\rangle\rangle\psi_i$ iff $M'_{10}, q, s_1 \models_{x,c} \psi_i$. Therefore we can treat φ exactly as in the base case. The same holds for $A = \emptyset$.

From $(\star), (\star\star)$ it follows that: $M_{10}, q_0, s_\emptyset \models_{x,c} \varphi$ iff $M'_{10}, q_0, s_\emptyset \models_{x,sc} \varphi$, which concludes this part of the proof.

Let $\varphi = \langle\langle 1\rangle\rangle\Box(\mathsf{cake} \wedge \neg\langle\langle 1\rangle\rangle\bigcirc\neg\mathsf{cake})$. The formula expresses that player 1 has a strategy to maintain cake, however given that strategy it is not possible for him/her to achieve $\neg\mathsf{cake}$ in the next state. We have $M_{10}, q_0 \not\models_{x,sc} \varphi$ but $M'_{10}, q_0 \models_{x,sc} \varphi$, which concludes the proof. $\Box$

PROPOSITION 6.15. *For all M, λ, s and every $\mathsf{ATL}^\star$ formula φ, we have that $M, \lambda, 0, s \models^{nf}_{I,sc} \varphi$ iff $M, \lambda, s \models_{I,sc} \varphi$.*

PROOF. For each history $\eta \in \Lambda^{fin}_M$, and strategy s, we define:

$$s^{+\eta}(h) = \begin{cases} s(\mathit{last}(\eta) \circ h') & \text{iff } h = \eta \circ h' \\ s(\mathit{last}(\eta)) & \text{iff } h = \eta \\ \text{undefined} & \text{otherwise} \end{cases} \qquad s^{-\eta}(h) = \begin{cases} s(\eta) & \text{iff } h = \mathit{last}(\eta) \\ s(\eta \circ h') & \text{iff } h = \mathit{last}(\eta) \circ h' \\ \text{undefined} & \text{otherwise} \end{cases}$$

Informally, $s^{+\eta}$ reproduces s starting from history $\eta[0, |\eta| - 1]$, while $s^{-\eta}$ reproduces s as if history $\eta[0, |\eta| - 1]$ did not occur. We recall from the proof of Proposition 3.5, that:

$$\mathit{plays}^I_M(h, s_B \dagger s_A) = \mathit{out}_M(h, s_B \dagger s_A) \tag{1}$$

for all $h \in \Lambda_M$ and all collective-$A \cup B$ strategies $s_B \dagger s_A$. Also for all collective strategies s_A and histories $h \in \Lambda_M$, there is a collective strategy s'_A such that:

$$\mathit{out}_M(h, s \dagger s_A) = \{h \circ \lambda \mid \mathit{last}(h) \circ \lambda \in \mathit{out}_M(\mathit{last}(h), s^{-h} \dagger s'_A)\} \tag{2}$$

and also, for each collective strategy s'_A there is a collective strategy s_A such that:

$$\mathit{out}_M(\mathit{last}(h), s \dagger s'_A) = \{\mathit{last}(h) \circ \lambda \mid h \circ \lambda \in \mathit{out}_M(h, s^{+h} \dagger s_A)\} \tag{3}$$

We show $(\star) M, h \circ \lambda, k, s \models^{nf}_{I,sc} \varphi$ iff $M, last(h) \circ \lambda, s^{-h} \models_{I,sc} \varphi$, for all paths $h \circ \lambda \in \Lambda_M$, such that $k = |h| - 1$, $|h| \geq 1$ and all scATL*-formulae φ. The proof is by induction over the formula structure of φ. The proposition follows from $(\star)$ for $k = 0$.

Base cases: The case $\varphi = p$ is straightforward. Case $\varphi = \langle\langle A\rangle\rangle\gamma$ where γ does not contain cooperation modalities.

$M, h \circ \lambda, k, s \models^{nf}_{I,sc} \langle\langle A\rangle\rangle\gamma$ iff

— $\exists s_A$ such that $\forall h \circ \lambda' \in plays^I_M(h, s \dagger s_A)$ we have $M, h \circ \lambda', k, s \dagger s_A \models^{nf}_{I,sc} \gamma$ iff (by (1-3))
— $\exists s'_A$ such that $\forall last(h) \circ \lambda' \in plays^I_M(last(h), s^{-h} \dagger s'_A)$ we have $last(h) \circ \lambda', s^{-h} \dagger s_A \models_{I,sc} \gamma$ iff
— $M, last(h), s^{-h} \circ \lambda \models_{I,sc} \langle\langle A\rangle\rangle\gamma$.

Induction step: The cases $\varphi = \neg\varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\langle A\rangle\rangle\gamma$ where γ contains cooperation modalities.

We consider the two directions of $(\star)$:"$\Rightarrow$": Suppose $M, h \circ \lambda, k, s \models^{nf}_{I,sc} \langle\langle A\rangle\rangle\gamma$ and let s_A be a witnessing strategy, i.e. $M, h \circ \lambda', k, s \dagger s_A \models^{nf}_{I,sc} \gamma$ for all $h \circ \lambda' \in out_M(h, s \dagger s_A)$. Also, let $\langle\langle B_i\rangle\rangle\psi_i$ with $i = 1, \ldots, k$ be an outermost ATL*-subformula in γ. As before, we define:

$H_i = \{(h \circ \lambda, k) \mid M, h \circ \lambda, k, s \dagger s_A \models^{nf}_{I,sc} \langle\langle B_i\rangle\rangle\psi_i\}$
$L_i = \{\lambda \mid M, \lambda, s \models_{I,sc} \langle\langle B_i\rangle\rangle\psi_i\}$

By induction hypothesis, $H_i = \{(h \circ \lambda, k) \mid last(h) \circ \lambda \in L_i\}$ (4). By (4) and (2) our claim follows immediately. Direction "$\Leftarrow$" follows exactly the same argument. $\square$

PROPOSITION 6.18. *There are iCGSs which satisfy the same formulae of* $\mathsf{ATL}^\star_{\mathsf{i,sc}}$*, but can be distinguished in* $\mathsf{ATL}^\star_{\mathsf{nf,i,c}}$*. That is,* $\mathsf{ATL}^\star_{\mathsf{nf,i,c}} \not\preceq_d \mathsf{ATL}^\star_{\mathsf{i,sc}}$.

PROOF. The proof is similar to that of Proposition 4.4. First, we use the models M_2 and M'_2 from Figure 3 to show that $M_2, a_0 \models_{i,sc} \varphi$ iff $M'_2, a_0 \models_{i,sc} \varphi$. We start by observing that a strategy is uniform in M_2 iff is also uniform in M'_2. This holds since player 2 has a unique available action in a_0 (and b_0) respectively.

We define the strategy $t(s_A)$ with respect to s_A as follows:

$$t(s_A) = \begin{cases} s'_A & \text{if } A = \{1, 2\} \\ s_A & \text{otherwise} \end{cases}$$

where s'_A denotes the collective strategy whose construction was illustrated in the proof of Proposition 4.4. We observe that, s and $t(s)$ differ only in those actions assigned to histories prefixed by a_0. Thus:

$$M_2, x_j, s \models_{i,sc} \varphi \text{ iff } M_2, x'_j, t(s) \models_{i,sc} \varphi \tag{4}$$

for $x \in \{a, b\}$, $j \in \{1, 2\}$ and any strategy s (the same is the case for M'_2)

$$M_2, x_j, s \models_{i,sc} \varphi \text{ iff } M'_2, x_j, t(s) \models_{i,sc} \varphi \tag{5}$$

for $x \in \{a, b, a', b'\}$, $j \in \{1, 2\}$ and any strategy s.

We prove:

$$(\star)\ M_2, a_0, s_B \models_{i,sc} \varphi \text{ iff } M'_2, a_0, t(s_B) \models_{i,sc} \varphi, \text{ for all } B \in \{\emptyset, 1, 2, \{1, 2\}\}$$

by induction over the formula structure of φ. Note that our claim follows for $B = \emptyset$, since $t(s_\emptyset) = s_\emptyset$. Base cases:The case $\varphi = p$ is straightforward. Case $\varphi = \langle\langle A\rangle\rangle\gamma$ where γ contains no strategic modalities.

Suppose $A \cup B \in \{\emptyset, \{1\}\}$. The proposition follows immediately as $\mathit{plays}^i_{M_2}(a_0, s_B \dagger s_A) = \mathit{plays}^i_{M_2'}(a_0, s_B \dagger s_A)$.

Suppose $A \cup B = \{2\}$. We consider each direction of $(\star)$ in turn: "$\Leftarrow$": Since $\mathit{plays}^i_{M_2'}(a_0, s_B \dagger s_A) = \mathit{out}_{M_2'}(a_0, s_B \dagger s_A) \cup \mathit{out}_{M_2'}(b_0, s_B \dagger s_A)$ and $\mathit{plays}^i_{M_2}(a_0, s_B \dagger s_A) = \mathit{out}_{M_2}(a_0, s_B \dagger s_A)$, we have $\mathit{plays}^i_{M_2}(a_0, s_B \dagger s_A) \subseteq \mathit{plays}^i_{M_2'}(a_0, s_B \dagger s_A)$. $(\star)$ follows immediately.

Direction "$\Rightarrow$" follows the very same reasoning as that from the proof of Proposition 4.4, where s_2 is replaced by $s_{A\cup B}$. Note that $s_{A\cup B} = t(s_{A\cup B})$, since $A \cup B = \{2\}$.

Suppose $A \cup B = \{1, 2\}$. Direction "$\Leftarrow$": we note $\mathit{plays}^i_{M_2}(a_0, s_B \dagger s_A) \subseteq \mathit{plays}^i_{M_2'}(a_0, s_B \dagger s_A)$.

Direction "$\Rightarrow$": the set $\mathit{plays}^i_{M_2}(a_0, s_B \dagger s_A)$ contains a unique path, while $\mathit{plays}^i_{M_2}(a_0, t(s_B \dagger s_A))$ contains two paths. By the construction of t, the latter two paths are propositionally equivalent to the former.

Induction step: The cases $\varphi = \neg\varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\langle A\rangle\rangle\gamma$ where γ contains cooperation modalities. Let ξ be an arbitrary occurrence of an outermost formula $\langle\langle A'\rangle\rangle\gamma'$ in γ. We note that $M_2, x, s \models_{i,sc} \xi$ iff $M_2', x, t(s) \models_{i,sc} \xi$ by induction hypothesis if $x = a_0$ and by (1-2), otherwise. We label each state x of M_2 and M_2' where ξ holds by a new proposition p_ξ. The resulting models retain properties (1-2). We replace each ξ by p_ξ in γ and obtain a formula without cooperation modalities. We proceed as in the second base case. This concludes the proof of $(\star)$.

Second, we show that the models from Figure 3 can be distinguished by an $\mathsf{ATL}^\star_{\mathsf{nf,i,c}}$-formula. Recall that $\mathsf{E}\varphi \equiv \neg\langle\langle\emptyset\rangle\rangle\neg\varphi$. Then, we have $M_2, a_0 \models^{nf}_{i,sc} \mathsf{E}\bigcirc\langle\langle 2\rangle\rangle\bigcirc\mathsf{win}$ but $M_2', a_0' \not\models^{nf}_{i,sc} \mathsf{E}\bigcirc\langle\langle 2\rangle\rangle\bigcirc\mathsf{win}$: in M_2, there is a path on which player 2 can ensure win by itself in the next state. The latter does not hold in M_2'. $\square$

PROPOSITION 6.19. *There are iCGSs which satisfy the same formulae of* $\mathsf{ATL}^\star_{\mathsf{nf,i,sc}}$*, but can be distinguished in* $\mathsf{ATL}^\star_{\mathsf{i,c}}$*. That is,* $\mathsf{ATL}^\star_{\mathsf{i,c}} \not\preceq_d \mathsf{scATL}^\star_{\mathsf{nf,i}}$.

PROOF. The proof is similar to that of Proposition 4.6. First, we use models M_3 and M_3' from Figure 4 to show $(\star)$ $M_3, h, s \models^{nf}_{i,sc} \varphi$ iff $M_3', h, t(s) \models^{nf}_{i,sc} \varphi$ for all histories $h \in \Lambda^{fin}_{M_3}(a_0)$ and all $\mathsf{ATL}^\star$-formulae φ. We start by defining the strategy $t(s_A)$ with respect to s_A, as follows:

$$t(s_A) = \begin{cases} s_A' & \text{if } A = \{2\} \\ s_A & \text{otherwise} \end{cases}$$

where $s_A'(b_1) = s_A(a_1)$ and $s_A'(h) = s_A(h)$ for all $h \in \Lambda^{fin}_{M_3}$. Note that, for all strategies s, $t(s)$ is a uniform strategy in M_3'.

The proof is by induction over the formula structure of φ.

Base cases: Case $\varphi = p$ is straightforward. Case $\varphi = \langle\langle A\rangle\rangle\gamma$ where γ contains no strategic modalities. It is sufficient to observe:

$$\mathit{plays}^i_{M_3}(h, s \dagger s_A) = \mathit{plays}^i_{M_3'}(h, t(s) \dagger t(s_A)) \tag{1}$$

for any collective strategy s_A and $h \in \Lambda^{fin}_{M_3}(a_0)$. We note that s and $t(s)$ produce the same effects when the initial state is a_0, for any uniform strategy s. However, we use $t(s)$ instead of s since the latter may not be uniform in M_3' — the transformation t enforces uniformity by ensuring that agents play the same action in indistinguishable states a_1 and b_1 of M_3'.

Cases $\varphi = \neg\varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\langle A\rangle\rangle\gamma$ where γ contains cooperation modalities. Suppose $M_3, h, s \models^{nf}_{i,sc} \langle\langle A\rangle\rangle\gamma$ and let s_A be a witnessing strategy. Let $\langle\langle B_i\rangle\rangle\psi_i$ (with $i = 1 \dots k$) be an outermost $\mathsf{ATL}^\star$-subformula in γ. By induction hypothesis, we have:

$M_3, h, s \dagger s_A \models^{nf}_{i,sc} \langle\langle B_i \rangle\rangle \psi_i$ iff $M'_3, h, t(s \dagger s_A), \models^{nf}_{i,sc} \langle\langle B_i \rangle\rangle \psi_i$. Moreover, $t(s \dagger s_A) = t(s) \dagger t(s_A)$ — applying the uniformity constraint on $s \dagger s_A$ is equivalent to applying it on s and s_A individually.

It follows by (1) that s_A is a witnessing strategy for $M_3, h \models^{nf}_{i} \langle\langle A \rangle\rangle \gamma$ iff $t(s_A)$ is a witnessing strategy for $M'_3, h \models^{nf}_{i} \langle\langle A \rangle\rangle \gamma$.

Finally, to show that M_3 and M'_3 can be distinguished by an $\mathsf{ATL}^\star_{\mathsf{i,c}}$-formula we consider the formula $\varphi \equiv \mathsf{E}\bigcirc \langle\langle 2 \rangle\rangle \bigcirc \mathsf{win}$ from Proposition 6.18. We have that $M_3, a_0 \models \varphi$ and $M'_3, a'_0 \not\models \varphi$. $\square$

PROPOSITION 6.22. $Val(\mathsf{scATL}^\star_{\mathsf{i}}) \subseteq Val(\mathsf{scATL}^\star_{\mathsf{nf,i}})$. *Thus, also* $Val(\mathsf{ATL}^\star_{\mathsf{i,c}}) \subseteq Val(\mathsf{ATL}^\star_{\mathsf{nf,i,c}})$.

PROOF. We proceed similarly to the proof of Proposition 5.4. First, we show:

$$M, q, s \models^{nf}_{i,sc} \varphi \text{ iff } T^{nf}(M,q), q, t(s) \models^{nf}_{i,sc} \varphi \text{ iff } T^{nf}(M,q), q, t(s) \models_{i,sc} \varphi$$

for all $\mathsf{ATL}^\star$-formulae, iCGSs M, states q, strategies s which are uniform w.r.t. $\mathbb{A}\mathrm{gt}$ and where $t(s)$ is the strategy s' in $T^{nf}(M,q)$, such that $s'(\hat{h}) = s(h)$, for all $h \in \Lambda^{fin}_M(q)$ and $t(s_\emptyset) = s_\emptyset$. We first observe that $t(s \dagger s') = t(s) \dagger t(s')$. The proof also relies on the observations (1-3) from the proof of Proposition 5.4. We prove both equivalences separately.

(i). $M, q, s \models^{nf}_{i} \varphi$ iff $T^{nf}, q, t(s) \models^{nf}_{i} \varphi$. We prove the stronger statement: $M, h \circ \lambda, k, s \models^{nf}_{i} \varphi$ iff $T^{nf}, \hat{h} \circ \hat{\lambda}, k, t(s) \models^{nf}_{i} \varphi$, where $k = |h| - 1$. Our claim follows for $k = 0$. The proof is by induction over the formula structure of φ.

Base cases: Case $\varphi = p$. It is sufficient to note that $(h \circ \lambda)[k] \in \pi^M(p)$ iff $(\hat{h} \circ \hat{\lambda})[k] \in \pi^{T^{nf}}(p)$. Case $\varphi = \langle\langle A \rangle\rangle \gamma$ where γ contains no cooperation modalities. From (2-3) it follows that $\lambda \in \mathit{plays}^i_M(h, s \dagger s_A)$ iff $\hat{\lambda} \in \mathit{plays}^i_{T^{nf}}(\hat{h}, t(s) \dagger t(s_A))$. Since λ and $\hat{\lambda}$ are propositionally equivalent cf. (1), we have: $M, h \circ \lambda, s \models^{nf}_{i} \langle\langle A \rangle\rangle \gamma$ iff $T^{nf}, \hat{h} \circ \hat{\lambda}, t(s) \models^{nf}_{i} \langle\langle A \rangle\rangle \gamma$.

Induction step: Cases $\varphi = \neg\varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\langle A \rangle\rangle \gamma$ where γ contains cooperation modalities. We consider the two directions of (i) in turn: "$\Rightarrow$": Suppose $M, h \circ \lambda, k, s \models^{nf}_{i,sc} \langle\langle A \rangle\rangle \gamma$ and let s_A be a witnessing strategy i.e. $M, h \circ \lambda', k, s \dagger s_A \models^{nf}_{i,sc} \gamma$ for all paths $h \circ \lambda' \in \mathit{plays}^i_M(h, s \dagger s_A)$. Also, let $\langle\langle B_i \rangle\rangle \psi_i$ with $i = 1, \ldots, k$ be an outermost $\mathsf{ATL}^\star$-subformula in γ and define the sets:

$H_i = \{(h \circ \lambda, k) \mid M, h \circ \lambda, k, s \dagger s_A \models^{nf}_{i,sc} \langle\langle B_i \rangle\rangle \psi_i\}$

$\hat{H}_i = \{(\hat{h} \circ \hat{\lambda}, k) \mid T^{nf}, \hat{h} \circ \hat{\lambda}, k, t(s) \dagger t(s_A) \models^{nf}_{i,sc} \langle\langle B_i \rangle\rangle \psi_i\}$

By induction hypothesis, $\hat{H}_i = \{(\hat{h} \circ \hat{\lambda}, k) \mid (h \circ \lambda, k) \in H_i\}$ (4). Also, from (2-3) we have:

$$\{\hat{\lambda} \mid \hat{\lambda} \in \mathit{plays}^i_M(h, s \dagger s_A)\} = \{\lambda \mid \lambda \in \mathit{plays}^i_{T^{nf}}(\hat{h}, t(s) \dagger t(s_A))\}$$

The claim now follows by (4). Direction "$\Leftarrow$" follows exactly the same argument.

(ii). $T^{nf}, q, s \models^{nf}_{i} \varphi$ iff $T^{nf}, q, s \models_{i} \varphi$. As before, we prove the stronger claim $(\star)$ $T^{nf}, \hat{h} \circ \hat{\lambda}, k, s \models^{nf}_{i} \varphi$ iff $T^{nf}, \mathit{last}(\hat{h}) \circ \hat{\lambda}, s \models_{i} \varphi$ where $k = |\hat{h}| - 1$, by induction over the formula structure of φ. Then (ii) follows for $k = 0$.

Base cases: Case $\varphi = p$ is straightforward. Case $\varphi = \langle\langle A \rangle\rangle \gamma$ where γ contains no cooperation modalities. From (1-3) we observe that for each uniform collective strategy s_A and history $\hat{h} \in \Lambda^{fin}_{T^{nf}}$, there exists a uniform collective strategy s'_A such that:

$$\mathit{plays}^i_{T^{nf}}(\hat{h}, s \dagger s_A) = \{\hat{h} \circ \hat{\lambda} \mid \mathit{last}(\hat{h}) \circ \hat{\lambda} \in \mathit{plays}^i_{T^{nf}}(\mathit{last}(\hat{h}), s \dagger s'_A)\}$$

Informally, each state $\mathit{last}(\hat{h})$ encodes a history. Hence, the action of coalition A assigned by a perfect-recall strategy to $\hat{h}$ can be executed with the same effects from $\mathit{last}(\hat{h})$ alone. Similarly, for each collective strategy s'_A and history $\hat{h} \in \Lambda^{fin}_{T^{nf}}$ there exists a collective strategy s_A such that:

$$plays^i_{T^{nf}}(last(\hat{h}), s \dagger s'_A) = \{last(\hat{h}) \circ \hat{\lambda} \mid \hat{h} \circ \hat{\lambda} \in plays^i_{T^{nf}}(\hat{h}, s \dagger s_A)\}$$

We conclude that $T^{nf}, \hat{h} \circ \hat{\lambda}, k, s \models^{nf}_i \langle\langle A \rangle\rangle \gamma$ iff $T^{nf}, last(\hat{h}) \circ \hat{\lambda}, s \models_i \langle\langle A \rangle\rangle \gamma$.
Induction step: Cases $\varphi = \neg\varphi'$ and $\varphi = \varphi' \wedge \varphi''$ are straightforward. Case $\varphi = \langle\langle A \rangle\rangle \gamma$ where γ contains cooperation modalities.
Direction "$\Rightarrow$": Supose $T^{nf}, \hat{h} \circ \hat{\lambda}, k, s \models^{nf}_{i,sc} \langle\langle A \rangle\rangle \gamma$ and let s_A be a witnessing strategy i.e. $T^{nf}, \hat{h} \circ \hat{\lambda}', k, s \dagger s_A \models^{nf}_{i,sc} \gamma$ for all paths $\hat{h} \circ \hat{\lambda}' \in plays^i_{T^{nf}}(\hat{h}, s \dagger s_A)$. Also, let $\langle\langle B_i \rangle\rangle \psi_i$ with $i = 1, \ldots, k$ be an outermost $\mathsf{ATL}^\star$-subformula in γ and define:

$\hat{H}_i = \{(\hat{h} \circ \hat{\lambda}, k) \mid T^{nf}, \hat{h} \circ \hat{\lambda}, k, s \dagger s_A \models^{nf}_{i,sc} \langle\langle B_i \rangle\rangle \psi_i\}$

$\hat{L}_i = \{\hat{\lambda} \mid T^{nf}, \hat{\lambda}, s \dagger s_A \models_{i,sc} \langle\langle B_i \rangle\rangle \psi_i\}$

By induction hypothesis: $\hat{H}_i = \{(\hat{h} \circ \hat{\lambda}, k) \mid last(\hat{h}) \circ \hat{\lambda} \in \hat{L}_i\}$ (5). Finally, by (2,3) we have:

$$plays^i_{T^{nf}}(s \dagger s_A, \hat{h}) = \{\hat{h} \circ \hat{\lambda} \mid last(\hat{h}) \circ \hat{\lambda} \in plays^i_{T^{nf}}(t(s \dagger s_A), last(\hat{h}))\}$$

The claim now follows by (5). Direction "$\Leftarrow$" follows exactly the same argument.

We show $Sat(\mathsf{scATL}^\star_{\mathsf{nf,i}}) \subseteq Sat(\mathsf{ATL}^\star_{\mathsf{i,sc}})$. Suppose $\varphi \in Sat(\mathsf{scATL}^\star_{\mathsf{nf,i}})$. Thus there exists an iCGS M and state q such that $M, q, s_\emptyset \models^{nf}_{i,sc} \varphi$. By (i, ii) and since $t(s_\emptyset) = s_\emptyset, T^{nf}(M, q), s_\emptyset \models_i \varphi$. Hence $\varphi \in Sat(\mathsf{ATL}^\star_{\mathsf{i,sc}})$. $\square$