Strategy Logic with Randomised Strategies

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Abstract

Strategy Logic (SL) is a logical formalism for strategic reasoning in multi-agent systems that generalises most proposals, including ATL^*. It's main feature is that it has variables for pure strategies. Motivated by the fact that the canonical strategies in game-theory are randomised (i.e., mixed), we generalise SL by allowing variables to vary over randomised strategies. The resulting *strategy-logic with randomised strategies*, SLR, can express many standard solution concepts, include Nash Equilibria, Subgame-Perfect Equilibria, and Secure Equilibria. We study the model-checking and satisfiability problems for SLR.

1 Introduction

Outline:

1. syntax and semantics of SLR- strategy logic with randomised uniform strategies. perhaps allow probabilistic observations and effects in the CGS.
2. model-checking SLR: undecidable for memfull; tarski for memless
3. fragments SLR[1], SLR[BG], SLR[NG] ?
4. pushdown structures?
5. qualitative vs quantitative
6. applications: mixed equilibria, mixed rational synthesis.

Deciding existence of mixed NE is undecidable!!! Is there an interesting orthogonal angle?

1.1 Related Work

Standard logics for strategic reasoning only allow pure strategies, i.e., ATL, ATL^*, SL.

There are a number of dimensions of extensions of strategic logics by probabilities:

- deterministic CGS vs probabilistic CGS (MDP),
- boolean probabilistic operators OP(X,Y):

  the coalition A, using the joint-strategy S from X, for all counter strategies from Y, can ensures that property ϕ holds with probability at least α.

- quantitative (c can be any constant) vs qualitative ($c = 0, 1$)
- deterministic (D) vs. randomised (R) strategies
- memoryless (M) vs. history-dependent (H) strategies

Probabilistic extensions of ATL

1. PATL and PATLS [4]: extends ATL by X = memoryless, Y = memoryless. Labeling algorithm. Two player concurrent stochastic parity game.
2. PATL with predicted behaviours [3]: extends ATL by X = memless, Y = memless/behavioural.
3. PATL and PATL* [5]: extend ATL^* by X = mixed memoryfull, Y = ??
4. Stochastic Game Logic SGL [1] is a very expressive logic: it is like ATL with strategy contexts. It is built over the ω-regular languages (rather than LTL as in SL), it allows strategies to be propogated to subformulas (ATL^* does not allow this, but SL does), and it has probabilistic operators stating "for all (mixed, memoryfull) strategies of agents that have not been assigned strategies, the probability that a run (actually, the projection of a run with respect to given subformulas) is in a given ω-regular language is at least (at most, etc.) α". The logic is interpreted over (discrete) probabilistic turn-based game structures. Question: what about deterministic structures? or continuous probability distributions on the structures? There are six types of strategies in this paper: a strategy is either mixed or pure, and either memoryfull, memoryless of finite-memory. SGL can express the existence of mixed Nash Equilibria, i.e., $\langle\langle A\rangle\rangle \bigwedge_{a\in A} (\langle\langle\{a\}\rangle\rangle\phi_a \to \phi_a)$, where A is the set of agents, and ϕ_a is the objective of agent a. EXPAND! RELATE!

   Results: RH undecidable; DM PSPACE-complete; RM between PSPACE and EXPSPACE (automata, then Tarski); RM qualitative PSPACE-complete; RH qualitative unknown.

Extensions of ATL in non-probabilistic setting and other stuff

1. Extension of ATL by strategy contexts and memory requirements [2],

2. BSIL [11] has PSPACE MC (turn based) and is orthogonal to ATL, ATL*, GL, AMC what fragment of SL does it correspond to?, similarly [7].

3. [10]: QAPI is like ATL* with strategy contexts. The "strategy choice" variables S_i are bound to quantifiers, i.e., $\langle\langle A_i, S_i\rangle\rangle$, and S_i is a function such that $S_i(\sigma, \phi)$ is a memoryless uniform strategy for agent σ that depends on a "goal" ϕ. Thus the logic is interpreted wrt a CGS and an assignment of the variables. In particular, the meaning of $\langle\langle\langle A_i, S_i\rangle\rangle\rangle\phi$ is that agent $a \in A_i$ uses strategy $S_i(a, \phi)$ (and agents not in the A_is are free, and don't have to play uniform). The operator is probabilistic.

   The logic QAPI is actually of the form:

   $$\forall S_1 \exists S_2 \ldots \forall S_n \Psi$$

   where Ψ has no bound variables, and the S_i are the free variables in Ψ. The logic is interpreted over deterministic and probabilistic concurrent game structures. Strategies are pure and memoryless (i.e., strategies are functions from states to actions). The logic also has epistemic operators.

   The point is that one can express that the countercoalition continues for its own goal, or that it counteracts A's goal, or that it plays arbitrary (non-uniform) actions.

   Decidability follows from the memoryless assumption. The perfect recall case is undecidable.

4. [9] PCTL+LAP, is like PCTL with long-run average properties, MDPs, synthesis. "the percentage of time spent in bad states is at most 3percent". encode in FO(R).

5. [8]: with one player(!), incomplete information and sync perfect recall you can code the emptiness problem for probabilistic automata, which is undecidable.

Tools [6] two-player game structure;

This work

In our work we consider concurrent deterministic game structures, mixed strategies, and we extend SL (which itself extends ATL*). Thus, a property that we can express, than can't be expressed in any of the cited models, is the existence of mixed Nash Equilibria in a concurrent game structure. DO THIS!

- SGL can almost express this, but it is turn based.

- example. matching pennies. each player has a strategy enforcing a value of at least a half.

-not clear how to model check this! can't use automata. and we have free variables. so maybe only some ATL* like fragment. so maybe the only contribution is to concurrent games. FOCUS ON A LOGIC THAT CAN REASON ABOUT NE IN CONCURRENT GAMES, e.g., matching pennies. What is known about stochastic concurrent games?

Undecidability Fact: Existence of mixed NE is undecidable for terminal-reward payoffs on concurrent games.

Question: what, exactly, are the assumptions here? does one get decidability by, e.g., looking at strategies in which actions are interleaved, e.g., by restricting to BC?

extend QCTLstar by Probabilities?

2 Strategy Logic with Randomised Strategies

Sentences of SLR are interpreted over *concurrent game structures*, just as for ATL and SL [**?**; **?**].

Definition 1. *A* concurrent game structure (CGS) *is a tuple* $\mathsf{S}{:=}\langle AP, Ag, Ac, St, s, \lambda, tr\rangle$, *where*

- AP *is a finite set* atomic propositions*,*
- Ag *is a finite set of* agents*,*
- Ac *is a finite set of* actions*,*
- St *is a finite set of* states*,*
- $s \in St$ *is the* initial state*,*
- $\lambda : St \to 2^{AP}$ *is the* labeling function *mapping each state to the set of atomic propositions true in that state.*
- *Let* $Dec{:=}Ag \to Ac$ *be the set of* decisions*, i.e., functions describing the choice of an action by every agent. Then,* $tr : Dec \to (St \to St)$*, a* transition function*, maps every decision* $d \in Dec$ *to a function* $tr(d) : St \to St$*.*

Theorem 1. *The model-checking problem for SLR is undecidable for randomised strategies, and ?? for randomised memoryless strategies.*

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