Model Checking Pushdown Epistemic Game Structures

Abstract

In this paper, we investigate the problem of verifying pushdown multi-agent systems with imperfect information. We introduce *pushdown epistemic game structures* (PEGSs), an extension of pushdown game structures with *epistemic accessibility relations* (EARs), as the formal model. For the specification, we consider various extensions of alternating-time temporal logics with epistemic modalities, including ATEL, ATEL* and AEMC. We focus on strategies with imperfect recall. For ATEL and ATEL*, we show that size-preserving EARs will render the model checking problem undecidable. We then propose *regular* EARs, and provide automata-theoretic model checking algorithms with matching low bounds, i.e., EXPTIME-complete for ATEL and 2EXPTIME-complete for ATEL*. In contrast, for AEMC, we show the model checking problem is EXPTIME-complete, even in the presence of size-preserving EARs.

1 Introduction

Model checking is a well-studied method for automatic verification of complex systems, and has been successfully applied to verify hardware designs, communication protocols and software, etc (Clarke, Grumberg, and Peled 2001). Recently, it has been extended to verify *multi-agent systems* (MASs). As a model of *finite-state* MASs, Alur et al. proposed *concurrent game structures* (CGSs), whilst alternating-time temporal logics (ATL, ATL*) and alternating-time μ-calculus (AMC) are employed as specification languages, for which model checking algorithms were also provided (Alur, Henzinger, and Kupferman 1997; 2002). Since then, a number of model checking algorithms for MASs have been studied in various models and logics, such as *strategy logic* (SL) and its fragments (Cermák, Lomuscio, and Murano 2015; Mogavero, Murano, and Sauro 2013; 2014; Mogavero et al. 2014).

A noteworthy line of extensions, which is most relevant to the current work, is *pushdown game structures* (PGSs) (Murano and Perelli 2015; Chen, Song, and Wu 2016a; 2016b). PGSs are a model that can represent *infinite-state* MASs, called *pushdown multi-agent systems* (PMASs). PGSs allow, among others, modeling of memory of agents, which is of particular importance in MASs. On the logic side, one considers alternating-time temporal *epistemic* logics (ATEL, ATEL*) (van der Hoek and Wooldridge 2002; Jamroga 2003; Schobbens 2004; Jamroga and Dix 2006; Pilecki, Bednarczyk, and Jamroga 2014), alternating-time *epistemic* μ-calculus (AEMC) (Bulling and Jamroga 2011), and SLK (Cermák 2015). ATEL, ATEL*, AEMC and SLK are respectively extensions of ATL, ATL*, AMC and SL with epistemic modalities for representing knowledge of individual agents, as well as "everyone knows" and common knowledge (Fagin et al. 1995). These logics are usually interpreted over *finite-state* concurrent *epistemic* game structures, which are an extension of CGSs with **epistemic accessibility relations** (EARs), giving rise to a model for representing finite-state MASs with *imperfect information*. Assuming agents only access imperfect information arises naturally in various real-world scenarios, typically in sensor networks, security, robotics, etc. In addition, the extension of logics with epistemic modalities allows one to succinctly express a range of (un)desirable properties of MASs, and has found a wide range of applications in AI, particularly for reasoning about MASs (Fagin et al. 1995; van der Hoek and Wooldridge 2003).

This paper investigates model checking problems for ATEL, ATEL* and AEMC on PMASs with imperfect information. To this end, we propose *pushdown epistemic game structures* (PEGSs), an extension of PGSs with EAR, as a mathematical model for PMASs with imperfect information. To the best of our knowledge, analogous models have not been considered for PMASs *with imperfect information*.

Model checking PEGSs depends crucially on how EARs are defined. A commonly adopted one, called *size-preserving* EARs, was introduced in (Aminof et al. 2013), where two configurations are deemed to be indistinguishable if the two stack contents are of the same size, in addition, neither the pair of control states nor pairs of stack symbols in the same position of the two stack contents, are distinguishable. While this sounds to be a very natural definition, we show, unfortunately, that the model checking problems for ATEL and ATEL* on PEGSs are undecidable in general, even when restricted to imperfect recall strategies. This result suggests that alternative definitions of EARs are needed.

As a solution, we propose EARs that are *regular* and *simple*. Simple EARs are defined over control states of PEGSs and the top symbol of the stack, while regular EARs are simple EARs extended with a finite set of deterministic finite-state automata (DFA), one for each agent, where the states of each DFA divide the set of stack contents into finitely many equivalence classes. To obtain model checking algorithms, we first present an reduction from PEGSs with regular EARs to PEGSs with simple ones. We then provide automata-theoretic algorithms that solve the model checking problem for PEGSs with simple EARs. The algorithm runs in EXPTIME for ATEL and 2EXPTIME for ATEL*, and we show they are optimal by giving matching lower bounds. In contrast, for AEMC, we show that the model checking problem is EXPTIME-complete, even in the presence of size-preserving EARs.

These results suggest that adding appropriate EARs and modalities, which makes model and specification more expressive, does not bring computational overhead to the associated verification problems.

2 Pushdown Epistemic Game Structures

We fix a countable set **AP** of *atomic propositions*. Let $[k]$ denote the set $\{1, ..., k\}$ for some natural number $k \in \mathbb{N}$.

Definition 1. *A* pushdown epistemic game structure *(PEGS) is a tuple $\mathcal{P} = (\mathsf{Ag}, \mathsf{Ac}, P, \Gamma, \Delta, \lambda, \{\sim_i \mid i \in \mathsf{Ag}\})$, where*

- $\mathsf{Ag} = \{1, ..., n\}$ *is a finite set of* agents *(a.k.a. players);*
- Ac *is a finite set of* actions *made by agents; we further define $\mathcal{D} = \mathsf{Ac}^n$ to be the set of* decisions $\mathbf{d} = \langle a_1, ..., a_n\rangle$ *such that $\forall i \in [n]$, $\mathbf{d}(i) := a_i \in \mathsf{Ac}$;*
- *P is a finite set of* control states*;*
- Γ *is a finite* stack alphabet*, which includes a special symbol $\bot$ to denote the bottom of the stack;*
- $\Delta : P \times \Gamma \times \mathcal{D} \to P \times \Gamma^*$ *is a transition function such that, for each $(p, \gamma, \mathbf{d}) \in P \times \Gamma \times \mathcal{D}$, if $\gamma = \bot$, then $\delta(p, \gamma, \mathbf{d}) = (p', \omega\bot)$ for some $p' \in P$ and $\omega \in \Gamma^*$; if $\gamma \neq \bot$, then $\delta(p, \gamma, \mathbf{d}) = (p', \omega)$ for some $p \in P$ and $\omega \in (\Gamma \setminus \{\bot\})^*$* [1]*;*
- $\lambda : P \times (\Gamma^*\bot) \to 2^{\mathbf{AP}}$ *is a valuation that assigns to each configuration (i.e., an element of $P\times(\Gamma^*\bot)$) a set of atomic propositions;*
- $\sim_i \subseteq (P\times(\Gamma^*\bot))\times(P\times(\Gamma^*\bot))$ *is an* epistemic accessibility relation *(EAR) which is an equivalence relation.*

A *concurrent epistemic game structure* (CEGS) is a PEGS where the stack is not used. We can define a CEGS as $\mathcal{P} = (\mathsf{Ag}, \mathsf{Ac}, P, \Delta, \lambda, \{\sim_i \mid i \in \mathsf{Ag}\})$, where $\Delta : P\times\mathcal{D} \to P$. Accordingly, λ and $\sim_i$ are over P solely. A *pushdown game structure* (PGS) is a PEGS $\mathcal{P} = (\mathsf{Ag}, \mathsf{Ac}, P, \Gamma, \Delta, \lambda, \{\sim_i \mid i \in \mathsf{Ag}\})$ in which $\sim_i$ is an identity for every agent $i \in \mathsf{Ag}$. Therefore, a PGS $\mathcal{P}$ is usually denoted as $(\mathsf{Ag}, \mathsf{Ac}, P, \Gamma, \Delta, \lambda)$. A PGS is a pushdown system (PDS) if $|\mathsf{Ag}| = 1$.

A *configuration* of the PEGS $\mathcal{P}$ is a pair $\langle p, \omega\rangle$, where $p \in P$ and $\omega \in \Gamma^*\bot$. We write $C_{\mathcal{P}}$ to denote the set of configurations of $\mathcal{P}$. For every $(p, \gamma, \mathbf{d}) \in P \times \Gamma \times \mathcal{D}$ such that $\Delta(p, \gamma, \mathbf{d}) = (p', \omega)$, we write $\langle p, \gamma\rangle \overset{\mathbf{d}}{\hookrightarrow}_{\mathcal{P}} \langle p', \omega\rangle$ instead.

[1]One may notice that, in the definition of PEGSs, Δ is defined as a *complete* function $P\times\Gamma\times\mathcal{D} \to P\times\Gamma^*$, meaning that all actions are available to each agent. This does not restrict the expressiveness of PEGSs, as we can easily add the transitions to some additional sink state to simulate the situation that some actions are unavailable to some agent.

The transition relation $\Longrightarrow_{\mathcal{P}}: C_{\mathcal{P}}\times\mathcal{D}\times C_{\mathcal{P}}$ of the PEGS $\mathcal{P}$ is defined as follows: for every $\gamma\omega' \in C_{\mathcal{P}}$, if $\langle p, \gamma\rangle \overset{\mathbf{d}}{\hookrightarrow}_{\mathcal{P}} \langle p', \omega\rangle$, then $\langle p, \gamma\omega'\rangle \overset{\mathbf{d}}{\Longrightarrow}_{\mathcal{P}} \langle p', \omega\omega'\rangle$. Intuitively, if the PEGS $\mathcal{P}$ is at the configuration $\langle p, \gamma\omega'\rangle$, by making the decision $\mathbf{d}$, $\mathcal{P}$ moves from the control state p to the control state p', pops γ from the stack and then pushes ω into the stack.

Tracks and Paths. A *track* (resp. *path*) in the PEGS $\mathcal{P}$ is a *finite* (resp. *infinite*) sequence π of configurations $c_0...c_m$ (resp. $c_0c_1...$) such that for every $i : 0 \le i < m$ (resp. $i \ge 0$), $c_i \overset{\mathbf{d}}{\Longrightarrow}_{\mathcal{P}} c_{i+1}$ for some $\mathbf{d}$. Given a track $\pi = c_0...c_m$ (resp. path $\pi = c_0c_1...$), let $|\pi| = m$ (resp. $|\pi| = +\infty$), and for every $i : 0 \le i \le m$ (resp. $i \ge 0$), let π_i denote the configuration c_i, and $\pi_{\le i}$ denote $c_0 \dots c_i$. Given two tracks π and π', π and π' are *distinguishable* for an agent $i \in \mathsf{Ag}$, denoted by $\pi \sim_i \pi'$, if for all $k : 0 \le k \le |\pi|$, $\pi_k \sim_i \pi'_k$. Let $\mathsf{Trks}_{\mathcal{P}} \subseteq C^+_{\mathcal{P}}$ denote the set of all tracks in $\mathcal{P}$, $\prod_{\mathcal{P}} \subseteq C^{\omega}_{\mathcal{P}}$ denote the set of all paths in $\mathcal{P}$, $\mathsf{Trks}_{\mathcal{P}}(c) = \{\pi \in \mathsf{Trks}_{\mathcal{P}} \mid \pi_0 = c\}$ and $\prod_{\mathcal{P}}(c) = \{\pi \in \prod_{\mathcal{P}} \mid \pi_0 = c\}$ respectively denote the set of all the tracks and paths starting from the configuration c.

Strategies. Intuitively a *strategy* of an agent $i \in \mathsf{Ag}$ specifies what i plans to do in each situation. In the literature, there are four types of strategies (Schobbens 2004; Bulling and Jamroga 2011) defined as follows: where **i** (resp. **I**) denotes imperfect (resp. perfect) information and **r** (resp. **R**) denotes imperfect (resp. perfect) recall,

- **Ir** strategy is a function $\theta_i : C_{\mathcal{P}} \to \mathsf{Ac}$, i.e., the action made by the agent i depends on the current configuration;
- **IR** strategy is a function $\theta_i : \mathsf{Trks}_{\mathcal{P}} \to \mathsf{Ac}$, i.e., the action made by the agent i depends on the history, i.e. the sequence of configurations visited before;
- **ir** strategy is a function $\theta_i : C_{\mathcal{P}} \to \mathsf{Ac}$ such that for all configurations $c, c' \in C_{\mathcal{P}}$, if $c \sim_i c'$, then $\theta_i(c) = \theta_i(c')$, i.e., the agent i has to make the same action at the configurations that are indistinguishable from each other;
- **iR** strategy is a function $\theta_i : \mathsf{Trks}_{\mathcal{P}} \to \mathsf{Ac}$ such that for all tracks $\pi, \pi' \in \mathsf{Trks}_{\mathcal{P}}$, if $\pi \sim_i \pi'$, then $\theta_i(\pi) = \theta_i(\pi')$, i.e., the agent i has to make the same action on the tracks that are indistinguishable from each other.

Let Θ^{σ} for $\sigma \in \{\mathbf{Ir}, \mathbf{IR}, \mathbf{ir}, \mathbf{iR}\}$ denote the set of all σ-strategies. Given a set of agents $A \subseteq \mathsf{Ag}$, a *collective σ-strategy* of A is a function $\upsilon_A : A \to \Theta^{\sigma}$ that assigns to each agent $i \in A$ a σ-strategy. We write $\overline{A} = \mathsf{Ag} \setminus A$.

Outcome. Let c be a configuration and υ_A be a collective σ-strategy for a set of agents A. A path π is *compatible* with respect to υ_A iff for every $k \ge 1$, there exists $\mathbf{d}_k \in \mathcal{D}$ such that $\pi_{k-1} \overset{\mathbf{d}_k}{\Longrightarrow}_{\mathcal{P}} \pi_k$ and $\mathbf{d}_k(i) = \upsilon_A(i)(\pi_{\le k-1})$ for all $i \in A$. The *outcome starting from c with respect to υ_A*, denoted by $\mathsf{out}^{\sigma}(c, \upsilon_A)$, is defined as the set of all the paths that are compatible with respect to υ_A, which rules out infeasible paths with respect to the collective σ-strategy υ_A.

Epistemic accessibility relations (EARs). An EAR $\sim_i$ for $i \in \mathsf{Ag}$ over PEGSs is defined as an equivalence relation over configurations. As the set of configurations is infinite in general, we need to represent each $\sim_i$ *finitely*.

A very natural definition of EARs, called *size-preserving* EARs and considered in (Aminof et al. 2013), is formulated as follows: for each $i \in \mathsf{Ag}$, there is an equivalence relation $\simeq_i \subseteq (P\times P)\cup(\Gamma\times\Gamma)$, which captures the indistinguishability of control states and stack symbols. For two configurations $c = \langle p, \gamma_1...\gamma_m\rangle$ and $c' = \langle p', \gamma'_1...\gamma'_{m'}\rangle$, $c \sim_i c'$ iff $m = m'$, $p \simeq_i p'$, and for every $j \in [m] = [m']$, $\gamma_j \simeq_i \gamma'_j$. It turns out that the model checking problem for logic ATEL/ATEL* (cf. Section 3) is undecidable under this type of EARs, even with imperfect recall (cf. Theorem 3). To gain decidability, in this paper, we consider *regular EARs* and a special case thereof, i.e. *simple* EARs. We remark that regular EARs align to the regular valuations (see later of this section) of atomic propositions, and turn out to be useful in practice.

An EAR $\sim_i$ is *regular* if there is an equivalence relation $\approx_i$ over $P \times \Gamma$ and a complete deterministic finite-state automaton[2] (DFA) $\mathcal{A}_i = (S_i, \Gamma, \delta_i, s_{i,0})$ such that for every $\langle p, \gamma\omega\rangle, \langle p_1, \gamma_1\omega_1\rangle \in C_{\mathcal{P}}$, $\langle p, \gamma\omega\rangle \sim_i \langle p_1, \gamma_1\omega_1\rangle$ iff $(p, \gamma) \approx_i (p_1, \gamma_1)$ and $\delta_i^*(s_{i,0}, \omega^R) = \delta_i^*(s_{i,0}, \omega_1^R)$, where δ_i^* denotes the reflexive and transitive closure of δ_i, and ω^R, ω_1^R denote the reverse of ω, ω_1 (recall that the rightmost symbol of ω corresponds to the bottom symbol of the stack). Intuitively, two words ω, ω_1 which record the stack content (excluding the top), are equivalent with respect to $\sim_i$ if the two runs of $\mathcal{A}_i$ on ω^R and ω_1^R respectively reach the same state. Note that the purpose of DFA $\mathcal{A}_i$ is to partition Γ^* into finitely many equivalence classes, hence we do *not* introduce the accepting states. A regular EAR is *simple* if for every word $\omega, \omega_1 \in \Gamma^*$, $\delta_i^*(s_{i,0}, \omega^R) = \delta_i^*(s_{i,0}, \omega_1^R)$, that is, $\mathcal{A}_i$ contains only one state.

Given a set of agents $A \subseteq \mathsf{Ag}$, let $\sim_A^E$ denote $\bigcup_{i\in A} \sim_i$, and $\sim_A^C$ denote the transitive closure of $\sim_A^E$.

Alternating Multi-Automata. To represent potentially infinite sets of configurations finitely, we use alternating multi-automata (AMA) as the "data structure" of the model checking algorithms.

Definition 2. *(Bouajjani, Esparza, and Maler 1997) Given a PDS $\mathcal{P} = (\mathsf{Ag}, \mathsf{Ac}, P, \Gamma, \Delta, \lambda)$, an AMA is a tuple $\mathcal{M} = (S, \Gamma, \delta, I, S_f)$, where S is a finite set of states such that $P \subseteq S$, Γ is the input alphabet, $\delta \subseteq S \times \Gamma \times 2^S$ is a transition relation, $I \subseteq S$ is a finite set of initial states, $S_f \subseteq S$ is a finite set of final states.*

If $(s, \gamma, \{s_1, ..., s_m\}) \in \delta$, we will write $s \xrightarrow{\gamma} \{s_1, ..., s_m\}$ instead. We define the relation $\longrightarrow_\delta \subseteq S \times \Gamma^* \times 2^S$ as the least relation such that the following conditions hold:

- $s \xrightarrow{\epsilon}_\delta \{s\}$, for every $s \in S$;
- $s \xrightarrow{\gamma\omega}_\delta \bigcup_{i\in[m]} S_i$, if $s \xrightarrow{\gamma} \{s_1, ..., s_m\}$ and $s_i \xrightarrow{\omega}_\delta S_i$ for every $i \in [m]$.

$\mathcal{M}$ *accepts* a configuration $\langle p, \omega\rangle$ if $p \in I$ and there exists $S' \subseteq S_f$ such that $p \xrightarrow{\omega}_\delta S'$. Let $\mathcal{L}(\mathcal{M})$ denote the set of all configurations accepted by $\mathcal{M}$. A set $C \subseteq C_{\mathcal{P}}$ is *regular* iff some AMA $\mathcal{M}$ exactly recognizes C.

[2]"complete" means that $\delta(q, \gamma)$ is defined for each $(q, \gamma) \in Q\times\Gamma$.

Proposition 1. *(Bouajjani, Esparza, and Maler 1997) The membership problem of AMAs can be decided in polynomial time. AMAs are closed under all Boolean operations.*

Regular valuations. The model checking problem for PDSs (hence for PEGSs as well) with general valuations λ, e.g., defined by a function l which assigns to each atomic proposition a context free language, is undecidable (Esparza, Kucera, and Schwoon 2003). To gain decidability, we consider valuations specified by a function $l : \mathbf{AP} \to 2^{C_{\mathcal{P}}}$ such that for every $q \in \mathbf{AP}$, $l(q)$ is a *regular* set of configurations. This is usually referred to as a *regular valuation* (Esparza, Kucera, and Schwoon 2003). The function l can be lifted to the valuation $\lambda_l : P \times \Gamma^* \to 2^{\mathbf{AP}}$: for every $c \in C_{\mathcal{P}}$, $\lambda_l(c) = \{q \in \mathbf{AP} \mid c \in l(q)\}$. A *simple valuation* is a regular valuation $l : \mathbf{AP} \to 2^{C_{\mathcal{P}}}$ such that for every $q \in \mathbf{AP}$, $p \in P, \gamma \in \Gamma$, and $\omega, \omega' \in \Gamma^*$, it holds that $\langle p, \gamma\omega\rangle \in l(q)$ iff $\langle p, \gamma\omega'\rangle \in l(q)$. From now on, we assume that regular valuations are given as AMAs. For a PEGS $\mathcal{P}$ with regular valuations, we use $|\mathcal{P}|$ to denote the number of control states of $\mathcal{P}$ and the numbers of states of these AMAs.

3 ATEL, ATEL* and AEMC

In this section, we recall the definition of alternating-time temporal epistemic logics: ATEL (van der Hoek and Wooldridge 2002), ATEL* (Jamroga 2003) and AEMC (Bulling and Jamroga 2011), which were introduced for reasoning about knowledge and cooperation of agents in multi-agent systems. Informally, ATEL, ATLE* and AEMC can be considered as extensions of ATL, ATL* and AMC respectively with *epistemic modalities* for representing knowledge. These include $\mathbf{K}_i$ for $i \in \mathsf{Ag}$ (agent i knows), $\mathbf{E}_A$ for $A \subseteq \mathsf{Ag}$ (every agent in A knows) and $\mathbf{C}_A$ (group modalities to characterise common knowledge).

ATEL$^*_\sigma$ (where $\sigma \in \{\mathbf{Ir}, \mathbf{IR}, \mathbf{ir}, \mathbf{iR}\}$)

Definition 3. *The syntax of ATEL$^*_\sigma$ is defined as follows, where ϕ denotes state formulae, ψ denotes path formulae,*

$$\begin{aligned}\phi ::= &\ q \mid \neg q \mid \phi \vee \phi \mid \phi \wedge \phi \mid \mathbf{K}_i\phi \mid \mathbf{E}_A\phi \mid \\ &\ \mathbf{C}_A\phi \mid \overline{\mathbf{K}}_i\phi \mid \overline{\mathbf{E}}_A\phi \mid \overline{\mathbf{C}}_A\phi \mid \langle A\rangle\psi \mid [A]\psi \\ \psi ::= &\ \phi \mid \psi \vee \psi \mid \psi \wedge \psi \mid \mathbf{X}\,\psi \mid \mathbf{G}\,\psi \mid \psi\,\mathbf{U}\,\psi\end{aligned}$$

where $q \in \mathbf{AP}$, $i \in \mathsf{Ag}$ and $A \subseteq \mathsf{Ag}$.

We use $\mathbf{F}\,\psi$ to abbreviate *true* $\mathbf{U}\,\psi$. An LTL formula is an ATEL$^*_\sigma$ path formula ψ with ϕ being restricted to be atomic propositions.

The semantics of ATEL$^*_\sigma$ is defined over PEGSs. Let $\mathcal{P} = (\mathsf{Ag}, \mathsf{Ac}, P, \Gamma, \Delta, \lambda, \{\sim_i \mid i \in \mathsf{Ag}\})$ be a PEGS, ϕ be an ATEL$^*_\sigma$ state formula, $c \in C_{\mathcal{P}}$ be a configuration of $\mathcal{P}$, the satisfiability relation $\mathcal{P}, c \models_\sigma \phi$ is defined inductively on the structure of ϕ.

- $\mathcal{P}, c \models_\sigma q$, $\mathcal{P}, c \models_\sigma \neg q$, $\mathcal{P}, c \models_\sigma \phi_1 \vee \phi_2$ and $\mathcal{P}, c \models_\sigma \phi_1 \wedge \phi_2$ are defined in a standard way;
- $\mathcal{P}, c \models_\sigma \langle A\rangle\psi$ iff there exists a collective σ-strategy $\upsilon_A : A \to \Theta^\sigma$ such that for all paths $\pi \in \mathsf{out}^\sigma(c, \upsilon_A)$, $\mathcal{P}, \pi \models_\sigma \psi$;
- $\mathcal{P}, c \models_\sigma [A]\psi$ iff for all collective σ-strategies $\upsilon_A : A \to \Theta^\sigma$, there exists a path $\pi \in \mathsf{out}^\sigma(c, \upsilon_A)$, $\mathcal{P}, \pi \models_\sigma \psi$;

- $\mathcal{P}, c \models_\sigma \mathbf{K}_i\phi$ iff for all configurations $c' \in C_\mathcal{P}$ such that $c \sim_i c'$, $\mathcal{P}, c' \models_\sigma \phi$;
- $\mathcal{P}, c \models_\sigma \overline{\mathbf{K}}_i\phi$ iff there is a configuration $c' \in C_\mathcal{P}$ such that $c \sim_i c'$ and $\mathcal{P}, c' \models_\sigma \phi$;
- $\mathbf{E}_A\phi$, $\overline{\mathbf{E}}_A\phi$, $\mathbf{C}_A\phi$ and $\overline{\mathbf{C}}_A\phi$ are defined similar to $\mathbf{K}_i\phi$ and $\overline{\mathbf{K}}_i\phi$, using the relation $\sim_A^E$ and $\sim_A^C$.

The semantics of path formulae ψ is specified by a relation $\mathcal{P}, \pi \models_\sigma \psi$, where π is a path. Since the definition is essentially the one of LTL and standard, we refer the readers to, e.g. (Clarke, Grumberg, and Peled 2001) for details.

ATEL_σ is a syntactical fragment of ATEL_σ^* with restricted path formulae defined by the rule: $\psi ::= \mathbf{X}\,\phi \mid \mathbf{G}\,\phi \mid \phi\,\mathbf{U}\,\phi$, where ϕ are state formulae.

Given an $\text{ATEL}_\sigma/\text{ATEL}_\sigma^*$ formula ϕ, let $||\phi||_\mathcal{P}^\sigma$ denote the set of configurations satisfying ϕ, i.e., $||\phi||_\mathcal{P}^\sigma = \{c \in C_\mathcal{P} \mid \mathcal{P}, c \models_\sigma \phi\}$.

CTL and CTL* are special cases of ATEL_σ and ATEL_σ^* in which no epistemic modalities occur and all the modalities $\langle A\rangle\psi$ and $[A]\psi$ satisfy that $A = \emptyset$.

AEMC_σ (where $\sigma \in \{\mathbf{Ir}, \mathbf{IR}, \mathbf{ir}, \mathbf{iR}\}$)

Definition 4. *AEMC_σ formulae are defined by the following grammar:*

$$\begin{aligned}\phi ::= \; & q \mid \neg q \mid Z \mid \phi \vee \phi \mid \phi \wedge \phi \mid \langle A\rangle\mathbf{X}\phi \mid [A]\mathbf{X}\phi \mid \\ & \mu Z.\phi \mid \nu Z.\phi \mid \mathbf{K}_i\phi \mid \mathbf{E}_A\phi \mid \mathbf{C}_A\phi \mid \overline{\mathbf{K}}_i\phi \mid \overline{\mathbf{E}}_A\phi \mid \overline{\mathbf{C}}_A\phi\end{aligned}$$

where $q \in \mathbf{AP}$, $Z \in \mathcal{Z}$, $i \in \mathsf{Ag}$ and $A \subseteq \mathsf{Ag}$.

The variables $Z \in \mathcal{Z}$ in the definition of AEMC_σ are monadic second-order variables with the intention to represent a set of configurations of PEGSs. An occurrence of a variable $Z \in \mathcal{Z}$ is said to be *closed* in an AEMC_σ formula ϕ if the occurrence of Z is in ϕ_1 for some subformula $\mu Z.\,\phi_1$ or $\nu Z.\,\phi_1$ of ϕ. Otherwise, the occurrence of Z in ϕ is said to be *free*. An AEMC_σ formula ϕ is *closed* if it contains no free occurrences of variables from $\mathcal{Z}$.

The semantics of AEMC_σ can be defined in an obvious way, where temporal modalities $\langle A\rangle\mathbf{X}\phi$ and $[A]\mathbf{X}\phi$ and epistemic modaliteis can be interpreted as in ATEL_σ^* and the fixpoint modalities can be interpreted as in alternating mu-calculus (Alur, Henzinger, and Kupferman 2002). Given a PEGS $\mathcal{P} = (\mathsf{Ag}, \mathsf{Ac}, P, \Gamma, \Delta, \lambda, \{\sim_i \mid i \in \mathsf{Ag}\})$, and a closed formula ϕ, the denotation function $||\circ||_\mathcal{P}^\sigma$ maps AEMC_σ formulae to sets of configurations. A configuration c satisfies ϕ iff $c \in ||\phi||_\mathcal{P}^\sigma$. We include the details as supplement material.

An ATEL_σ (resp. ATEL_σ^*) formula ϕ is a *principal* if ϕ is in the form of $\langle A\rangle\psi$ or $[A]\psi$ such that ψ is an LTL formula. For instance, $\langle 1\rangle Fq$ is a principal formula, while neither $\langle 1\rangle F(q \wedge \langle 2\rangle Gq')$ nor $\langle 1\rangle F(K_2\, q)$ is.

We remark that for AEMC_σ (where $\sigma \in \{\mathbf{Ir}, \mathbf{IR}, \mathbf{ir}, \mathbf{iR}\}$), it makes no difference whether the strategies are perfect recall or not, since each occurrence of the modalities $\langle A\rangle\mathbf{X}\phi$ and $[A]\mathbf{X}\phi$ will "reset" the strategies of agents. Therefore, we will ignore $\mathbf{R}$ and $\mathbf{r}$ and use $\text{AEMC}_\mathbf{I}/\text{AEMC}_\mathbf{i}$ to denote AEMC under perfect/imperfect information.

Remark 1. *In (Bulling and Jamroga 2011), the outcome of a configuration c with respect to a given collective σ-strategy υ_A is defined differently from that in this paper. More specifically, the outcome in (Bulling and Jamroga 2011) corresponds to $\bigcup_{i\in A}\bigcup_{c\sim_i c'} \mathsf{out}^\sigma(c', \upsilon_A)$ in our notation. It is easy to see that for every ATEL_σ or ATEL_σ^* formula $\langle A\rangle\psi$ (resp. $[A]\psi$) and every configuration $c \in C_\mathcal{P}$, $c \in ||\langle A\rangle\psi||_\mathcal{P}^\sigma$ (resp. $c \in ||[A]\psi||_\mathcal{P}^\sigma$) in (Bulling and Jamroga 2011) iff $c \in ||\mathbf{E}_A\langle A\rangle\psi||_\mathcal{P}^\sigma$ (resp. $c \in ||\mathbf{E}_A[A]\psi||_\mathcal{P}^\sigma$) in our notation. Similar differences exist for AEMC. We decide to make the hidden epistemic modalities $\mathbf{E}_A$ explicit in this paper.*

We mention that, although with perfect information $\text{ATEL}_\mathbf{IR}$ and $\text{ATEL}_\mathbf{IR}^*$ can be translated into $\text{AEMC}_\mathbf{I}$, this is *not* the case for imperfect information. Namely, $\text{ATEL}_\mathbf{iR}$, $\text{ATEL}_\mathbf{ir}$, $\text{ATEL}_\mathbf{iR}^*$, and $\text{ATEL}_\mathbf{ir}^*$ cannot be translated into $\text{AEMC}_\mathbf{i}$. The interested reader are referred to (Bulling and Jamroga 2011) for more discussions.

Problem statement. Given a PEGS $\mathcal{P} = (\mathsf{Ag}, \mathsf{Ac}, P, \Gamma, \Delta, \lambda, \{\sim_i \mid i \in \mathsf{Ag}\})$ with regular/simple valuations and size-preserving/regular/simple EARs, $c \in C_\mathcal{P}$, and an $\text{ATEL}_\sigma/\text{ATEL}_\sigma^*$ formula or a closed AEMC_σ formula ϕ, the *model checking problem* is to decide whether $c \in ||\phi||_\mathcal{P}^\sigma$.

The following results are known for model checking PEGSs with perfect information and perfect recall.

Theorem 1 ((Chen, Song, and Wu 2016a))**.** *The model-checking problem on PEGSs is EXPTIME-complete for $\text{ATEL}_\mathbf{IR}/\text{AEMC}_\mathbf{IR}$, and 3EXPTIME-complete for $\text{ATEL}_\mathbf{IR}^*$.*

4 ATEL and ATEL* Model Checking

We first recall the following undecidability result.

Theorem 2 ((Dima and Tiplea 2011))**.** *The model-checking problem of $\text{ATL}_\mathbf{iR}$ and $\text{ATL}_\mathbf{iR}^*$ on CEGSs is undecidable.*

In light of Theorem 1 and Theorem 2, in this section, we focus on the model checking problems for $\text{ATEL}_\mathbf{ir}/\text{ATEL}_\mathbf{ir}^*$. (Results for $\text{ATEL}_\mathbf{Ir}/\text{ATEL}_\mathbf{Ir}^*$ can be obtained as a special case when $\sim_i$ in the PEGS is an identity for each agent i.)

We observe that, when the stack is available, the histories in CEGSs can be stored into the stack, so that we can reduce from the model checking problem for $\text{ATL}_\mathbf{iR}$ on CEGSs to the one for $\text{ATL}_\mathbf{ir}$ on PEGSs. From Theorem 2, we deduce the following result.

Theorem 3. *The model checking problems for $\text{ATL}_\mathbf{ir}/\text{ATL}_\mathbf{ir}^*$ on PEGSs with size-preserving EARs are undecidable.*

Theorem 3 rules out model checking algorithms for $\text{ATEL}_\mathbf{ir}/\text{ATEL}_\mathbf{ir}^*$ when the PEGS is equipped with size-preserving EARs. As mentioned before, we therefore consider the case with regular or simple EARs. We first give a reduction from the former to the latter as, evidently, the latter problem is considerably simpler. The main idea of the reduction, which is inspired by the reduction of PDSs with regular valuations to PDSs with simple valuations in (Esparza, Kucera, and Schwoon 2003), is to store the states of DFAs representing the regular EARs into the stack.

Reduction from Regular to Simple EARs

Assume a PEGS $\mathcal{P} = (\mathsf{Ag}, \mathsf{Ac}, P, \Gamma, \Delta, \lambda, \{\sim_i \mid i \in \mathsf{Ag}\})$ with regular EARs such that, for each $i \in \mathsf{Ag}$, $\sim_i$ is given as the pair of $(\approx_i, \mathcal{A}_i)$, where $\approx_i \subseteq P \times \Gamma$ is an equivalence relation and $\mathcal{A}_i = (S_i, \Gamma, \delta_i, s_{i,0})$ is a DFA.

Let $\vec{\mathcal{A}} = (\vec{S}, \Gamma, \vec{\delta}, \vec{s_0})$ be the product automaton of $\mathcal{A}_i$'s for $i \in \mathsf{Ag}$. Intuitively, the PEGS $\mathcal{P}'$ with simple EARs to be constructed, stores the state obtained by running $\vec{\mathcal{A}}$ over the reverse of the partial stack content up to the current position (exclusive) into the stack, Formally, the PEGS $\mathcal{P}' = (\mathsf{Ag}, \mathsf{Ac}, P, \Gamma', \Delta', \lambda', \{\sim_i' | \ i \in \mathsf{Ag}\})$ where $\Gamma' = \Gamma \times \vec{S}$, and for each $i \in \mathsf{Ag}$, $\sim_i'$ is specified by an equivalence relation $\approx_i'$ on $P \times \Gamma'$ defined as follows: $(p, [\gamma, \vec{s}]) \approx_i' (p', [\gamma', \vec{s'}])$ iff $(p, \gamma) \approx_i (p', \gamma')$ and $\vec{s} = \vec{s'}$, in addition, Δ' is defined as follows: for every state $\vec{s} \in \vec{S}$,

1. for every $\langle p, \gamma \rangle \overset{\mathbf{d}}{\hookrightarrow}_{\mathcal{P}} \langle p', \epsilon \rangle$, we have $\langle p, [\gamma, \vec{s}] \rangle \overset{\mathbf{d}}{\hookrightarrow}_{\mathcal{P}'} \langle p', \epsilon \rangle$,
2. for every $\langle p, \gamma \rangle \overset{\mathbf{d}}{\hookrightarrow}_{\mathcal{P}} \langle p', \gamma_k ... \gamma_1 \rangle$ with $k \geq 1$ and $\vec{\delta}(\vec{s_j}, \gamma_j) = \overrightarrow{s_{j+1}}$ for every $j : 1 \leq j \leq k-1$ (where $\vec{s_1} = \vec{s}$), then $\langle p, [\gamma, \vec{s}] \rangle \overset{\mathbf{d}}{\hookrightarrow}_{\mathcal{P}'} \langle p', [\gamma_k, \vec{s_k}] ... [\gamma_1, \vec{s_1}] \rangle$.

Finally, the valuation λ is adjusted accordingly to λ'.

Theorem 4. *The model checking problem of an ATEL$_{\mathbf{ir}}$ (resp. ATEL$^*_{\mathbf{ir}}$) formula ϕ on a PEGS $\mathcal{P}$, with stack alphabet Γ and regular EARs $\sim_i = (\approx_i, \mathcal{A}_i)$ for $i \in \mathsf{Ag}$, can be reduced to the model checking problem of ϕ on a PEGS $\mathcal{P}'$ with simple EARs $\sim_i'$, such that the state space of $\mathcal{P}'$ is the same as that of $\mathcal{P}$, and the stack alphabet of $\mathcal{P}'$ is $\Gamma \times \vec{S}$, where $\vec{S}$ is the state space of the product of $\mathcal{A}_i$'s for $i \in \mathsf{Ag}$.*

Theorem 4 allows us to focus on the model checking problem over PEGSs with simple EARs.

Model Checking ATEL$_{\mathbf{ir}}$ and ATEL$^*_{\mathbf{ir}}$ on PEGSs with Simple EARs

Assume an ATEL$_{\mathbf{ir}}$ (resp. ATEL$^*_{\mathbf{ir}}$) formula ϕ and a PEGS $\mathcal{P} = (\mathsf{Ag}, \mathsf{Ac}, P, \Gamma, \Delta, \lambda, \{\sim_i | \ i \in \mathsf{Ag}\})$ with regular valuations l such that $l(q)$ is given by an AMA $\mathcal{M}_q$ for each $q \in AP$ and $\sim_i$ is specified by an equivalence relation $\approx_i$ on $P \times \Gamma$ for $i \in \mathsf{Ag}$.

We propose automata-theoretic model checking algorithms for ϕ on $\mathcal{P}$. The idea of the algorithm is to construct, for each state subformula ϕ' of ϕ, an AMA $\mathcal{M}_{\phi'}$ to represent the set of configurations satisfying ϕ'. We will first illustrate the construction in case that ϕ' is a principal formula, then extend the construction to the more general case.

Principal formulae. In the following, we will illustrate the construction for principal formulae $\phi' = \langle A \rangle \psi'$, where ψ' is an LTL formula. The construction for principal formulae $\phi' [A] \psi'$ is similar. We assume that each atomic proposition $q \in AP$ is associated with an AMA $\mathcal{M}_q$ to denote the set of configurations satisfying q.

Our approach will reduce the model checking problem on PEGSs to the problem of PDSs. Note that for $i \in A$, $\approx_i$ is defined over $P \times \Gamma$. It follows that the strategy of any agent $i \in A$ must respect $\approx_i$, i.e., for all $(p, \gamma\omega)$ and $(p', \gamma'\omega')$ with $(p, \gamma) \approx_i (p', \gamma')$, $\upsilon_i(p, \gamma\omega) = \upsilon_i(p', \gamma'\omega')$ for any **ir**-strategy υ_i of i. Therefore, any **ir**-strategy υ_i with respect to $\approx_i$ can be regarded as a function over $P \times \Gamma$ (instead of configurations of $\mathcal{P}$), i.e., $\upsilon_i : P \times \Gamma \to \mathsf{Ac}$.

For each $(p, \gamma) \in P \times \Gamma$, after applying a collective **ir**-strategy $\upsilon_A = (\upsilon_i(p, \gamma))_{i \in A}$ for A, we obtain a PDS $\mathcal{P}_{\upsilon_A} = (P, \Gamma, \Delta', \lambda)$, where $\Delta'(p, \gamma, \mathbf{d}') = \Delta(p, \gamma, \mathbf{d})$, for every $i \in A$ with $\mathbf{d}(i) = \upsilon_i(p, \gamma)$, $\mathbf{d}'$ is the action vector of the agents $\overline{A}$ in $\mathbf{d}$. Then, $\mathsf{out}^{\mathbf{ir}}(c, \upsilon_A) = \prod_{\mathcal{P}_{\upsilon_A}}(c)$, the set of all paths of $\mathcal{P}_{\upsilon_A}$ starting from c. It follows that $\mathcal{P}, c \models_{\mathbf{ir}} \langle A \rangle \psi'$ iff $\exists \upsilon_A$ such that $\mathcal{P}_{\upsilon_A}, c \models_{\mathbf{ir}} \langle \emptyset \rangle \psi'$, where $\langle \emptyset \rangle \psi'$ is a CTL (resp. CTL*) formula if $\langle A \rangle \psi'$ is an ATEL$_{\mathbf{ir}}$ (resp. ATEL$^*_{\mathbf{ir}}$) formula.

- In case that $\langle \emptyset \rangle \psi'$ is a CTL formula, following (Song and Touili 2011), an AMA $\mathcal{M}_{\upsilon_A}$ with $\mathbf{O}(|\mathcal{P}||\psi'|)$ states and $|\Gamma| 2^{\mathbf{O}(|\mathcal{P}||\psi'|)}$ transition rules that recognizes all configurations satisfying $\langle \emptyset \rangle \psi'$.
- In case that $\langle \emptyset \rangle \psi'$ is a CTL* formula, we construct a Büchi automaton B for ψ' and then reduce the problem to the emptiness problem of alternating Büchi pushdown systems. Following (Song and Touili 2011), we can construct an AMA $\mathcal{M}_{\upsilon_A}$ of $|\mathcal{P}| 2^{\mathbf{O}(|\psi'|)}$ states and $|\Gamma| 2^{|\mathcal{P}| 2^{\mathbf{O}(|\psi'|)}}$ transition rules that recognizes all configurations satisfying $\langle \emptyset \rangle \psi'$.

In either case, we have

Lemma 1. $\bigcup_{\upsilon_A} \mathcal{L}(\mathcal{M}_{\upsilon_A}) = ||\langle A \rangle \psi'||^{\mathbf{ir}}_{\mathcal{P}}$.

It is not hard to see that one can construct an AMA $\mathcal{M}_{\phi'}$ such that $\mathcal{L}(\mathcal{M}_{\phi'}) = \bigcup_{\upsilon_A} \mathcal{L}(\mathcal{M}_{\upsilon_A})$. Because there are at most $|\mathsf{Ac}|^{|P||\Gamma||A|}$ collective **ir**-strategies for A and $|A| \leq |\mathsf{Ag}|$, we observe that $\mathcal{M}_{\phi'}$ contains at most $|\mathsf{Ac}|^{|P||\Gamma||\mathsf{Ag}|} \cdot \mathbf{O}(|\mathcal{P}||\psi'|)$ (resp. $|\mathsf{Ac}|^{|P||\Gamma||\mathsf{Ag}|} \cdot |\mathcal{P}| 2^{\mathbf{O}(|\psi'|)}$) states and $|\mathsf{Ac}|^{|P||\Gamma||\mathsf{Ag}|} \cdot |\Gamma| 2^{\mathbf{O}(|\mathcal{P}||\psi'|)}$ (resp. $|\mathsf{Ac}|^{|P||\Gamma||\mathsf{Ag}|} \cdot |\Gamma| 2^{|\mathcal{P}| 2^{\mathbf{O}(|\psi'|)}}$) transition rules if $\langle A \rangle \psi'$ is an ATEL$_{\mathbf{ir}}$ (resp. ATEL$^*_{\mathbf{ir}}$) formula. Since the membership problem of AMAs can be solved in polynomial time (cf. Proposition 1), we deduce that the model checking of ϕ' on $\mathcal{P}$ can be solved in EXPTIME for ATEL$_{\mathbf{ir}}$ and 2EXPTIME for ATEL$^*_{\mathbf{ir}}$.

ATEL$_{\mathbf{ir}}$/ATEL$^*_{\mathbf{ir}}$ formulae. Given an ATEL$_{\mathbf{ir}}$/ATEL$^*_{\mathbf{ir}}$ formula ϕ, we inductively compute an AMA $\mathcal{M}_{\phi'}$ from the state subformulae ϕ' such that $\mathcal{L}(\mathcal{M}_{\phi'}) = ||\phi'||^{\mathbf{ir}}_{\mathcal{P}}$. The base case for atomic propositions is trivial. For the induction step:

- For ϕ' of the form $\neg q$, $\phi_1 \wedge \phi_2$ or $\phi_1 \vee \phi_2$, $\mathcal{M}_\phi$ can be computed by applying Boolean operations on $\mathcal{M}_{\phi_1} / \mathcal{M}_{\phi_2}$.
- For ϕ' of the form $\langle A \rangle \psi'$, we first compute a principal formula ϕ'' by replacing each state subformula ϕ''' in ψ' by a fresh atomic proposition $q_{\phi'''}$ and then saturate λ by setting $q_{\phi'''} \in \lambda(c)$ for every $c \in \mathcal{L}(\mathcal{M}_{\phi'''})$. By Lemma 1, we can construct an AMA $\mathcal{M}_{\phi''}$ from ϕ'' which is the desired AMA $\mathcal{M}_{\phi'}$. The formulae ϕ' of the form $[A]\psi'$ can be handled in a similar way.
- For ϕ' of the form $\mathbf{K}_i \phi''$ (resp. $\mathbf{E}_A \phi''$ and $\mathbf{C}_A \phi''$), suppose that the AMA $\mathcal{M}_{\phi''} = (S_1, \Gamma, \delta_1, I_1, S_f)$ recognizes $||\phi''||^{\mathbf{ir}}_{\mathcal{P}}$. Let $[p_1, \gamma_1], ..., [p_m, \gamma_m] \subseteq P \times \Gamma$ be the equivalence classes induced by the relation $\approx_i$ (resp. $\sim_A^E$ and $\sim_A^C$). We define the AMA $\mathcal{M}_{\phi'} = (P \cup \{s_f\}, \Gamma, \delta', P, \{s_f\})$, where δ' is defined as follows: for every $j \in [m]$, if $\{\langle p, \gamma\omega \rangle \mid (p, \gamma) \in [p_j, \gamma_j], \omega \in \Gamma^*\} \subseteq \mathcal{L}(\mathcal{M}_{\phi''})$, then for all $(p, \gamma) \in [p_j, \gamma_j]$ and $\gamma' \in \Gamma$, $\delta'(p, \gamma) = s_f$ and $\delta'(s_f, \gamma') = s_f$. The AMA $\mathcal{M}_{\phi'}$ for formulae ϕ' of the form $\overline{\mathbf{K}}_i \phi''$ (resp. $\overline{\mathbf{E}}_A \phi''$ and $\overline{\mathbf{C}}_A \phi''$) can be constructed similarly as for $\mathbf{K}_i \phi''$, using the condition $\{\langle p, \gamma\omega \rangle \mid (p, \gamma) \in [p_j, \gamma_j], \omega \in \Gamma^*\} \cap \mathcal{L}(\mathcal{M}_{\phi''}) \neq \emptyset$, instead of $\{\langle p, \gamma\omega \rangle \mid (p, \gamma) \in [p_j, \gamma_j], \omega \in \Gamma^*\} \subseteq \mathcal{L}(\mathcal{M}_{\phi''})$.

From the aforementioned arguments for principal formulae, we know that $\mathcal{M}_\phi$ contains at most $|\mathsf{Ac}|^{|P||\Gamma||\mathsf{Ag}|} \cdot \mathbf{O}(|\mathcal{P}||\phi|)$ (resp. $|\mathsf{Ac}|^{|P||\Gamma||\mathsf{Ag}|} \cdot |\mathcal{P}|2^{\mathbf{O}(|\phi|)}$) states and $|\mathsf{Ac}|^{|P||\Gamma||\mathsf{Ag}|} \cdot |\Gamma|2^{\mathbf{O}(|\mathcal{P}||\phi|)}$ (resp. $|\mathsf{Ac}|^{|P||\Gamma||\mathsf{Ag}|} \cdot |\Gamma|2^{|\mathcal{P}|2^{\mathbf{O}(|\phi|)}}$) transition rules if $\langle A\rangle\psi'$ is an $\mathrm{ATEL}_{\mathbf{ir}}$ (resp. $\mathrm{ATEL}^*_{\mathbf{ir}}$) formula. We then deduce from Theorem 4 and Proposition 1 the following result.

Theorem 5. *The model-checking problems for* $\mathrm{ATEL}^*_{\mathbf{ir}}$ *and* $\mathrm{ATEL}^*_{\mathbf{Ir}}$ *(resp.* $\mathrm{ATEL}_{\mathbf{ir}}$ *and* $\mathrm{ATEL}_{\mathbf{Ir}}$*) on PEGSs with regular/simple valuations and regular/simple EARs are 2EXPTIME-complete (resp. EXPTIME-complete).*

We remark that in contrast to $\mathrm{ATEL}^*_{\mathbf{IR}}$ (cf. Theorem 1), the complexity of $\mathrm{ATEL}^*_{\mathbf{Ir}}$ (as well as $\mathrm{ATEL}^*_{\mathbf{ir}}$) is lower as only imperfect recall is considered.

The lower bound follows from that of model-checking problems for CTL/CTL* on PDSs with simple valuations (Walukiewicz 2000; Bozzelli 2007). Namely, even for PEGSs with a single agent, perfect information, and simple valuations, the model checking problem is already EXPTIME/2EXPTIME-hard for CTL/CTL*.

5 AEMC Model Checking

In this section, we propose algorithms for $\mathrm{AEMC}_{\mathbf{i}}$ model checking on PEGSs with size-preserving/regular/simple EARs. As regular/simple EARs can be tackled in a very similar way to Section 4, we focus on the size-preserving one. At first, we remark that Theorem 3 does *not* hold for $\mathrm{AEMC}_{\mathbf{i}}$ (Recall that $\mathrm{AEMC}_{\mathbf{i}} = \mathrm{AEMC}_{\mathbf{ir}} = \mathrm{AEMC}_{\mathbf{iR}}$). Indeed, we will show that model checking $\mathrm{AEMC}_{\mathbf{i}}$ on PEGSs with size-preserving EARs is EXMPTIME-complete.

Fix a closed $\mathrm{AEMC}_{\mathbf{i}}$ formula ϕ and a PEGS $\mathcal{P} = (\mathsf{Ag}, \mathsf{Ac}, P, \Gamma, \Delta, \lambda, \{\sim_i | \ i \in \mathsf{Ag}\})$ with size-preserving EARs $\sim_i$ for $i \in \mathsf{Ag}$ specified by equivalence relations $\simeq_i \subseteq (P \times P) \cup (\Gamma \times \Gamma)$. We will construct an AMA $\mathcal{A}_\phi$ to capture $||\phi||^{\mathbf{i}}_{\mathcal{P}}$ by induction on the syntax of $\mathrm{AEMC}_{\mathbf{i}}$ formulae.

Atomic formulae, Boolean operators, formulae of the form $\langle A\rangle\mathbf{X}\phi'$ and $[A]\mathbf{X}\phi'$, and fixpoint operators can be handled as in (Chen, Song, and Wu 2016a), where model checking of alternating μ-calculus on PGSs was considered, as imperfect information does not play a role for these operators. In the sequel, we illustrate how to deal with the epistemic modalities.

For the formula $\phi = \mathbf{K}_i\phi'$, suppose the AMA $\mathcal{A}_{\phi'} = (S', \Gamma, \delta', I', S'_f)$ recognizing $||\phi'||^{\mathbf{ir}}_{\mathcal{P}}$ has been constructed. We construct $\mathcal{A}_\phi = (S', \Gamma, \delta, I, S'_f)$ as follows.

- $I = \{p \in P \mid \exists p' \in I'.\ p \simeq_i p'\}$.
- For each $(p, \gamma) \in P \times \Gamma$, let $[p]_{\simeq_i}$ (resp. $[\gamma]_{\simeq_i}$) be the equivalence of p (resp. γ) under $\simeq_i$, and $\overline{S'_{p,\gamma}} := \{S'_{p,\gamma} \mid (p, \gamma, S'_{p,\gamma}) \in \delta'\}$. Then $(p, \gamma, S) \in \delta$ if (1) for all $p' \in [p]_{\simeq_i}$ and $\gamma' \in [\gamma]_{\simeq_i}$, $\overline{S'_{p',\gamma'}} \neq \emptyset$; and (2) $S = \bigcup_{p' \in [p]_{\simeq_i}, \gamma' \in [\gamma]_{\simeq_i}} S''_{p',\gamma'}$, where $S''_{p',\gamma'} \in \overline{S'_{p',\gamma'}}$.
- For every $(s, \gamma, S) \in \delta'$ such that $s \in S' \setminus P$, let $(s, \gamma', S) \in \delta$ for every $\gamma' \in \Gamma$ with $\gamma' \simeq_i \gamma$.

For the formula $\phi = \overline{\mathbf{K}}_i\phi'$, suppose the AMA $\mathcal{A}_{\phi'} = (S', \Gamma, \delta', I', S'_f)$ recognizes $||\phi'||^{\mathbf{ir}}_{\mathcal{P}}$. We construct $\mathcal{A}_\phi = (S', \Gamma, \delta, I, S'_f)$ as follows.

- $I = \{p \in P \mid \exists p' \in I'.\ p \sim_i p'\}$.
- For each $(p, \gamma) \in P \times \Gamma$, if there is $(p', \gamma', S'_1) \in \delta'$ such that $p \simeq_i p'$ and $\gamma \simeq_i \gamma'$, let $(p, \gamma, S'_1) \in \delta$.
- For every $(s, \gamma, S) \in \delta'$ such that $s \in S' \setminus P$, let $(s, \gamma', S) \in \delta$ for every $\gamma' \in \Gamma$ with $\gamma' \simeq_i \gamma$.

The AMA $\mathcal{A}_\phi$ for ϕ of the form $\mathbf{E}_A\phi'$, $\mathbf{C}_A\phi'$, $\overline{\mathbf{E}}_A\phi'$ or $\overline{\mathbf{C}}_A\phi'$ can be constructed in a very similar way, in which the relation $\simeq_i$ is replaced by the relation $\bigcup_{i\in A} \simeq_i$ (resp. the transitive closure of $\bigcup_{i\in A} \simeq_i$).

Lemma 2. *Given a PEGS $\mathcal{P}$ with regular valuations and size-preserving EARs , and a closed* $\mathrm{AEMC}_{\mathbf{i}}$ *formula ϕ, we can construct an AMA $\mathcal{A}_\phi$ recognizing $||\phi||^{\mathbf{i}}_{\mathcal{P}}$ in exponential time with respect to $|\mathcal{P}|$ and $|\phi|$.*

From Lemma 2 and Proposition 1, we have:

Theorem 6. *The model checking problem for* $\mathrm{AEMC}_{\mathbf{i}}$ *on PEGSs with regular/simple valuations and size-preserving/regular/simple EARs is EXPTIME-complete.*

The lower bound follows from that of the model checking problem for AMC on PGSs with simple valuations, which is EXPTIME-complete (Chen, Song, and Wu 2016a).

6 Related Work

Model checking on finite-state CGSs under **IR** setting is well-studied in the literature (Alur, Henzinger, and Kupferman 1997; 2002; Cermák, Lomuscio, and Murano 2015; Mogavero, Murano, and Sauro 2013; 2014; Mogavero et al. 2014). The problem becomes undecidable for ATL on CGSs under **iR** strategies (Dima and Tiplea 2011). Therefore, many works restrict to **ir** strategies (van der Hoek and Wooldridge 2002; Schobbens 2004; Jamroga and Dix 2006; Jamroga 2003; Bulling and Jamroga 2011; Dima and Tiplea 2011; Pilecki, Bednarczyk, and Jamroga 2014; Cermák 2015). The model checking problem on PGSs under **IR** strategies were studied in (Murano and Perelli 2015; Chen, Song, and Wu 2016a; 2016b), but only with perfect information.

Timed (resp. probabilistic) ATLs and timed (resp. probabilistic) CGSs were proposed to verify timed (resp. probabilistic) MASs (Henzinger and Prabhu 2006; Brihaye et al. 2007; Chen and Lu 2007; Chen et al. 2012). These works are, however, orthogonal to the current one.

7 Conclusion and Future Work

In this work, we have proposed PEGSs as a formal model of pushdown MASs with imperfect information, and investigated model checking problems for ATEL, ATEL^*_σ and AEMC_σ. For ATEL and ATEL^*_σ, we showed that model checking is undecidable under size-preserving EARs, and provided automata-theoretic algorithms under regular EARs. For AEMC_σ, we gave model checking algorithms under all three types of EARs. These algorithms are optimal with matching lower bounds.

Future work includes the implementation of our algorithm and investigates the model checking problem for PEGSs against SLK (Cermák 2015).

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