Index

@, 238	accessible, 6, 9
$\mathcal{G}_{\mathcal{A},t},$ 260	action
T(A), 252	free, 788
$T(\varphi)$, 270	action algebra, 681
$\mathcal{T}^{\omega}_{A,k}$, 251	action lattice, 682
$\mathcal{T}_{\Sigma}^{\mathrm{unr}}$, 237	Adam, 257
AC^{0} , 449	adjacency matrix, 898
ACC^{0} , 449	Alëshin group, 804, 808
CC^0 , 449	is free, 815
D-class, 932	algebraic power series, 845
rank, 932	algebraic extension, 771
regular, 932	algorithm, see McNaughton-Yamada
structure group, 932	almost finite type shift, 931
NC, 449	almost prime, 833
NC^{1} , 449	alphabet, 4
TC^0 , 449	canonical, 860
dom(u), 589	involutive, 760
cuts(u), 589	alphabetic
inner-cuts(u), 589	matrix, 908
ω -semigroup, 648	amalgamated free product, 782
\sim_s , 591	amenable group, 807
*-semiring, 657	analytic sets, 630
left handed inductive, 677	anticipation, block map, 898
partial, 657	aperiodic, see identities
right handed inductive, 677	aperiodic tiling, 89
symmetric inductive, 678	aperiodic monoid, 771
dom(t), 222	Arden's lemma, 46, 49, 64, 66
ext(t), 238	arena, 257
fcns(t), 238	arithmetic
k-automaton, 827	integer and real, 1083
2-renewing sequence, 486	Presburger, 867, 1072
	progression, 859
Ap , 512	arithmetical hierarchy, 716
abelianisation map, 871	array
abelian group	periodic, 885
free, 781, 792	pseudo-periodic, 885
abstract numeration system, 870	ultimately periodic, 885
AC^{0} , 515	Artin-Schreier polynomial, 851
accepted, see automaton	Artin group, 795
accepting run	automata
of a tree automaton, 252	desert, 705

. 1 . 7	. 760
equivalent, 7	inverse, 762
automata group, 761, 801–818	involutive, 762
[regular] [weakly] branch, 809	Kari, 492
contracting, 807	Krieger, 914
nuclear, 806	labeled out-merge, 910
word problem, 806	labeled out-split, 910
automatic real numbers, 835	labeled in-merge, 908
automatic sequence, 827	labeled in-split, 908
automatic set, 829	language accepted by -, 43
automatic group, 791–799	language recognised by -, 43
biautomatic, 794	left delay, 930
normal forms, 791	local, 26, 334, 926
quadratic isoperimetric inequality, 796	looping tree
right/left, 794	independent (ILTA), 695
word problem in quadratic time, 796	minimal, 18, 320, 913, 917
automatic structure, 792	minimal, 10, 320, 513, 517 minimal complete, 18
geodesic, 793	minimal complete, 16
with uniqueness, 793	Nerode, 17
=	
automaton, 6, 43, 60, 663	one-cluster, 493
α -extensible, 489	quotient, 320
Černý, 484	real number, 882
accessible, 9	real vector, 882
almost finite type, 334	reduced, 913
behaviour of -, 43, 61, 65	reduction, 917
bipartite, 921	reversal, 322
bounded, 806	right delay, 930
coaccessible, 9	semi, 324
complete, 9, 928	series accepted by -, 65
conjugate, 908	shift recognized, 906
contained in another, 926	simple, 337
cyclic, 332	slow, 324
decomposition, 922	slow for Hopcroft, 331
depth, 323	slow for Moore, 331
deterministic, 8, 913	splitter, 321
dimension of –, 43	Stallings, 764
edge, 906	standard, 10
essential, 906	standard –, 55
Eulerian, 490	standard local, 928
expansion, 935	standard weighted –, 67
extended, 33	state, 906
extension, 910	subset accepted by –, 61
finite truncated, 783	symbolic conjugate, 919
Fischer, 916	synchronized, 913
flower, 763	synchronizing, 475
in-split, 924	trie, 335
inherently weak, 1089	trim, 907
iniciently weak, 1009	umi, 907

weak, 1083	Blikle net, 671
weighted –, 64	block, 898
automaton (of an expression)	substitution, 898
derived term –, 69, 70	block decomposition, see matrix
derived-term –, 59	block map, 883
equation –, 70	anticipation, 898
follow -, 70	memory, 898
Glushkov -, 54, 67, 70	sliding, 898
position -, 54, 70	Boolean operators, 1068
standard –, 55, 61, 67, 70	Borel hierarchy, 627
Thompson –, 57, 61	Borel set, 627
automorphism	boundary
orbits, 783	hyperbolic, 772, 775
Whitehead, 774	BOUNDED-SYNCHRONIZING-
,	Coloring, 501
backward, see closure	bounded gap, 863
Baire space, 627	bounded group, 806
base complement, 1069, 1081	is amenable, 807
base-k expansion, 827	is contracting, 807
Basilica group, 805, 807	Bowen-Franks group, 905
is amenable, 807	branched covering, 811
is regular weakly branch, 809	combinatorially equivalent, 811
presentation, 808	branch group, 808–809
basis, 449	broken derivation, <i>see</i> derivation
complete, 449	,
numeration, 860	canonical
standard, 449	beta-polynomial, 862
Baumslag-Solitar group, 800, 804	canonical alphabet, 860
Beatty sequence, 91	Cantor normal form, 626
behaviour, see automaton	Cantor space, 627
Bertrand numeration basis, 861	
	cardinal arithmetic, 626
beta-polynomial	
beta-polynomial canonical, 862	cardinal number, 624, 625
	cardinal number, 624, 625 Cayley automaton, 813
canonical, 862 extended, 862	cardinal number, 624, 625
canonical, 862 extended, 862 biautomatic group, 794	cardinal number, 624, 625 Cayley automaton, 813 Cayley graph, 781, 788 cellular automaton
canonical, 862 extended, 862 biautomatic group, 794 conjugacy problem, 797	cardinal number, 624, 625 Cayley automaton, 813 Cayley graph, 781, 788 cellular automaton one-way real-time, 701
canonical, 862 extended, 862 biautomatic group, 794	cardinal number, 624, 625 Cayley automaton, 813 Cayley graph, 781, 788 cellular automaton
canonical, 862 extended, 862 biautomatic group, 794 conjugacy problem, 797 bicombing, 799	cardinal number, 624, 625 Cayley automaton, 813 Cayley graph, 781, 788 cellular automaton one-way real-time, 701 Černý conjecture, 485
canonical, 862 extended, 862 biautomatic group, 794 conjugacy problem, 797 bicombing, 799 bifix code, 768	cardinal number, 624, 625 Cayley automaton, 813 Cayley graph, 781, 788 cellular automaton one-way real-time, 701 Černý conjecture, 485 Černý function, 485
canonical, 862 extended, 862 biautomatic group, 794 conjugacy problem, 797 bicombing, 799 bifix code, 768 bipartite automaton, 921	cardinal number, 624, 625 Cayley automaton, 813 Cayley graph, 781, 788 cellular automaton one-way real-time, 701 Černý conjecture, 485 Černý function, 485 characteristic
canonical, 862 extended, 862 biautomatic group, 794 conjugacy problem, 797 bicombing, 799 bifix code, 768 bipartite automaton, 921 components, 922	cardinal number, 624, 625 Cayley automaton, 813 Cayley graph, 781, 788 cellular automaton one-way real-time, 701 Černý conjecture, 485 Černý function, 485 characteristic polynomial, 861
canonical, 862 extended, 862 biautomatic group, 794 conjugacy problem, 797 bicombing, 799 bifix code, 768 bipartite automaton, 921 components, 922 bireversible Mealy automaton, 803, 814–	cardinal number, 624, 625 Cayley automaton, 813 Cayley graph, 781, 788 cellular automaton one-way real-time, 701 Černý conjecture, 485 Černý function, 485 characteristic polynomial, 861 sequence, 865
canonical, 862 extended, 862 biautomatic group, 794 conjugacy problem, 797 bicombing, 799 bifix code, 768 bipartite automaton, 921 components, 922 bireversible Mealy automaton, 803, 814— 818	cardinal number, 624, 625 Cayley automaton, 813 Cayley graph, 781, 788 cellular automaton one-way real-time, 701 Černý conjecture, 485 Černý function, 485 characteristic polynomial, 861 sequence, 865 characteristic mapping, 355
canonical, 862 extended, 862 biautomatic group, 794 conjugacy problem, 797 bicombing, 799 bifix code, 768 bipartite automaton, 921 components, 922 bireversible Mealy automaton, 803, 814– 818 bisimulation	cardinal number, 624, 625 Cayley automaton, 813 Cayley graph, 781, 788 cellular automaton one-way real-time, 701 Černý conjecture, 485 Černý function, 485 characteristic polynomial, 861 sequence, 865 characteristic mapping, 355 characteristic vector, 491

depth, 448	cone type, 790, 798
size, 448	configuration in Hopcroft's algorithm, 326
uniformity, 448	confluent states, 335
width, 448	congruence, 761, 917
class of languages, 520	syntactic, 918
Classification Theorem, 904	
•	Conjecture
clock constraint, 1131 k-bounded, 1132	Sakarovitch, 780
	conjugacy labeled, 908
diagonal, 1131	
diagonal-free, 1131	of shifts, 899
closure	conjugacy problem, 760
deterministic, 540	conjugate automata, 908
polynomial, 541	symbolic, 919
unambiguous, 540	conjugates, 709
closure (backward), 54	conjugate elements, 760
CNF, 626	consistency problem, 363
co-induction, 71	constant term
coaccessible, 6, 9	of a language, 43
coarser partition, 319	of a series, 64
Cobham's theorem, 859, 1071, 1083	of an expression, 43
code	constraint
bifix, 768	equality, 1072, 1084
sliding block, 883	inequation, 1075, 1087
coding, 828, 864	linear, 1072, 1075, 1076, 1084, 1087
coefficient, see series	modular, 1072, 1076
cofinality, 637	context
coincidence condition, 478	of a word, 917
coloring (of a graph), 493	context, right, 913, 914
column division matrix, 902	context-free
combing of group, 794	word problem submonoid, 781
commensurator of group, 771	context-free language, 84
common prefix, 340	continuity, 629
commutator in groups, 760	continuous, 629
complement, 1069	continuous operation, see language opera-
complete, 629	tion, continuous
complete automaton, 928	contracting automaton/group, 806, 807, 812
complexity function, 865	contraction
components, 922	graph, 905
compressible pair, 496	symbol, 904
compressible set, 496	Conway, see semiring
computable operation, see language opera-	Conway's leap, 658
tion, computable	couple, 481
computable real number, 834	Coxeter group, 795
computation, 79	Cramér's model, 832
concatenation, 5, 453	Cross-Section Theorem, 83
mod_p -concatenation, 453	Curtis-Lyndon-Hedlund Theorem, 898
r '	, , , , , , , , , , , , , , , , , , , ,

cut, 589	domain, 589
inner, 589	of a tree, 251
cyclic automaton, 332	dominating eigenvalue, 872
cylinder, 883	dot-depth, 462
ey imaer, ose	DTD, 242
D0L language, 100	Dyck language, 781
decidable, 35	dynamical system, 882
decision problem	factor, 882
HD0L (ultimate) periodicity problem,	isomorphism, 883
888	minimal, 882
decomposition, 922	periodic, 882
prefix-suffix, 340	periodie, 002
Decomposition Theorem, 903	edge, 898
delay	consecutive, 898
left, 930	initial vertex, 898
right, 930	source, 898
denoted, see expression	target, 898
depth, see expression	terminal vertex, 898
depth of Moore's algorithm, 323	edge in-merging map, 902
derivation (of an expression), 57, 61	edge in-splitting map, 902
broken –, 70	edge shift, 898
K, 68	Ehrenfeucht-Fraïssé game, 533
derivative (of an expression), 70	Eilenberg's Theorem, 522
partial –, 70	elementary equivalent matrices, 903, 911
=	emptiness problem, 1134
derived term (of an expression), 58, 68 true –, 58, 68	emptiness problem, 226
derived-term, <i>see</i> automaton (of)	empty word, 76
deterministic transducer, 78	encoding
determinacy, 258, 630, 631	dual, 1081
determined game, 631	extension, 238
deterministic automaton, 913	first-child-next-sibling, 238
determinization	fractional part, 1081, 1084
NDD, 1075	integer numbers, 1069
digit, 1069	integer part, 1081, 1084
sign, 1070, 1081	integer vectors, 1070
direct sum, 659	real numbers, 1081
Dirichlet's theorem, 833	real vectors, 1081
disjunctive rational subset, 780	relation, 1068
distance 769	serialized, 1070, 1075, 1079, 1082
geodesic, 768	valid, 1069, 1070, 1081
prefix metric, 772	endomorphism
profinite, 516	extension, 775
word metric, 790	virtually injective, 775
divisible group, 852	entropy, 900
division matrix, 902	equation
document type definitions, 242	profinite, 518

explicit, 518	extended
symmetrical, 520	beta-polynomial, 862
equation automaton, see automaton (of)	extension
equations	algebraic, 771
with rational constraints, 783	finite-index, 771
equivalence	HNN, 782
Moore, 322	extension automaton, 910
Nerode, 320	
equivalent, see expression	factor, 76
equivalent matrices	factor map, 883
elementary, 911	FCNS, 238
strong shift, 911	fellow traveller property, 792
symbolic elementary, 921	final state, 78
symbolic strong shift, 921	F_{∞} group, 796
erasing substitution, 873	finite automata
ergodic measure, 883	non-uniform, 453
essential	partially ordered, 470
automaton, 906	two way, 470
graph, 898	finite automaton
euclidean division, 626	alternating (AFA), 694
Eva, 257	Boolean, 694
even shift, 897	nondeterministic (NFA), 694
existential theory of equations, 783	universal, 704
expansion	finite substitution, 77, 100
α -expansion, 861	finite-state transduction, 79
automaton, 935	finite-valued transduction, 80
graph, 905	finite order element, 760
symbol, 904	first-order logic, 270
explicit	Fischer automaton, 916
profinite equation, 518	fixed point, 673, 864
expression, 42	fixed point induction, 677, 678
constant term of –, 43	fixed point of morphism, 828
depth of $-$, 42	fixpoint, 932
derivation of –, see derivation	flow equivalent, 904
equivalent –s, 42	FO, 270
language denoted by -, 42	follow automaton, see automaton (of)
literal length of –, 42	forbidden factors, 897
rational –, 42	forbidden pattern, 530
reduced -, 46, 66	formula
regular, 453	first-order, 532
regular –, 42	monadic second-order, 531
series denoted by –, 64	free
star height of –, 52	monoid, 5
star-normal form, see star-normal form	semigroup, 5
valid –, 64	variable, 867
weighted rational –, 64, 71	free ω -semigroup, 648
, ,	C 1,

free monoid, 57	essential, 898
free group, 761, 808, 816	Eulerian, 490
basis, 761	expansion, 905
free factor, 770	higher edge, 899
generalized word problem, 765	in-merge, 901
is residually finite, 773	in-split, 901
rank, 762	morphism, 901
free product, 816	of a letter, 496
amalgamated, 782	of constant out-degree, 493
full shift, 897	path, 898
function	primitive, 494
basis, 449	Schreier, 768, 779, 789, 807
functorial star, 668	state, 898
Fundamental theorem	strongly connected, 490
of finite automata, 42, 43, 61, 65	vertex, 898
fundamental group, 788, 790	Wielandt, 500
of 3-fold is automatic, 795	graphs of groups, 782
of negatively curved manifold, 799	greedy algorithm
fusion of states, 334	compression, 486
future of a state, 320	extension, 489
	Green relations, 932
Gale-Stewart game, 631	Grigorchuk group, 800, 802, 804, 810
game, 258	conjugacy problem, 806
gate, 448	generalized word problem, 806
generalized input, 448	has intermediate growth, 810
input, 448	is regular branch, 809
output, 448	is torsion, 808
genealogical order, 870	presentation, 808
generalized power series, 851	group, 451
generalized word problem, 761, 799	<i>p</i> -group, 451
generating set, see monoid	affine, 803, 813, 814
generator, 629	Alëshin, 804
geodesic distance, 768	amenable, 807
Glushkov, see automaton (of)	Artin, 781, 795
golden mean shift, 897	asynchronously automatic, 794
good substitution, 877	automata, 761, 803
grammar	automatic, 791
Boolean, 699	of an automaton, 801
conjunctive, 698	ball of radius n in, 790
linear, 701	Basilica, 805
context-free, 697	Baumslag-Solitar, 800, 804, 814
graph, 480, 898	biautomatic, 794
adjacency matrix, 898	bounded, 806
Cayley, 788	braid, 795
contraction, 905	branch, 808, 809
edge, 898	conjugacy separable, 806

Coxeter, 795 is regular branch, 809 F_{∞} , 796 is torsion, 808 finite, 780 finitely presented, 761, 782, 788 free, 761 free abelian, 781, 792 hedge, 238 free partially abelian, 781 functionally recursive, 803 graph, 781, 783 forigorchuk, 804 height, see star height, 222 growth, 809–811 fourtiender, 804 heisenberg, 800 hierarchy iterated monodromy, 812 Kazhdan, 818 lamplighter, 804 mapping class, 795 is regular branch, 809 is torsion, 808 is torsion, 808 from iterated, 808 header series, 851 hedge, 238 hedge, 238 hedge automaton language of, 239 nondeterministic, 239 height, see star heigh
finite, 780 finitely presented, 761, 782, 788 free, 761 free abelian, 781, 792 free partially abelian, 781 functionally recursive, 803 graph, 781, 783 Grigorchuk, 804 growth, 809–811 Gupta-Sidki, 804 Heisenberg, 800 iterated monodromy, 812 Kazhdan, 818 lamplighter, 804 mapping class, 795 Hahn's power series, 851 HD0L (ultimate) periodicity problem, 888 hedge, 238 hedge automaton language of, 239 nondeterministic, 239 height, see star height, 222 height of a state, 336 Heisenberg group, 800 hierarchy dot-depth, 462 higher block shift, 899 edge graph, 899
finitely presented, 761, 782, 788 free, 761 free, 761 HD0L (ultimate) periodicity problem, 888 free abelian, 781, 792 free partially abelian, 781 functionally recursive, 803 graph, 781, 783 Grigorchuk, 804 Grigorchuk, 809–811 Gupta-Sidki, 804 Heisenberg, 800 iterated monodromy, 812 Kazhdan, 818 lamplighter, 804 mapping class, 795 Hahn's power series, 851 HD0L (ultimate) periodicity problem, 888 hedge, 238 hedge automaton language of, 239 nondeterministic, 239 height, see star height, 222 height of a state, 336 Heisenberg group, 800 hierarchy dot-depth, 462 higher block shift, 899 edge graph, 899
free, 761 free abelian, 781, 792 free partially abelian, 781 functionally recursive, 803 graph, 781, 783 Grigorchuk, 804 growth, 809–811 Gupta-Sidki, 804 Heisenberg, 800 iterated monodromy, 812 Kazhdan, 818 lamplighter, 804 mapping class, 795 HD0L (ultimate) periodicity problem, 888 hedge, 238 hedge, 238 hedge automaton language of, 239 nondeterministic, 239 height, see star height, 222 height of a state, 336 Heisenberg group, 800 hierarchy dot-depth, 462 higher block shift, 899 edge graph, 899
free abelian, 781, 792 free partially abelian, 781 functionally recursive, 803 graph, 781, 783 Grigorchuk, 804 growth, 809–811 Gupta-Sidki, 804 Heisenberg, 800 iterated monodromy, 812 Kazhdan, 818 lamplighter, 804 mapping class, 795 hedge, 238 hedge automaton language of, 239 nondeterministic, 239 height, see star height, 222 height of a state, 336 Heisenberg group, 800 hierarchy dot-depth, 462 higher block shift, 899 edge graph, 899
free partially abelian, 781 functionally recursive, 803 graph, 781, 783 Grigorchuk, 804 growth, 809–811 Gupta-Sidki, 804 Heisenberg, 800 iterated monodromy, 812 Kazhdan, 818 lamplighter, 804 mapping class, 795 hedge automaton language of, 239 nondeterministic, 239 height, see star height, 222 height of a state, 336 Heisenberg group, 800 hierarchy dot-depth, 462 higher block shift, 899 edge graph, 899
functionally recursive, 803 graph, 781, 783 Grigorchuk, 804 growth, 809–811 Gupta-Sidki, 804 Heisenberg, 800 iterated monodromy, 812 Kazhdan, 818 lamplighter, 804 mapping class, 795 language of, 239 nondeterministic, 239 height, see star height, 222 height of a state, 336 Heisenberg group, 800 hierarchy dot-depth, 462 higher block shift, 899 edge graph, 899
graph, 781, 783 Grigorchuk, 804 growth, 809–811 Gupta-Sidki, 804 Heisenberg, 800 iterated monodromy, 812 Kazhdan, 818 lamplighter, 804 mapping class, 795 nondeterministic, 239 height, see star height, 222 height of a state, 336 Heisenberg group, 800 hierarchy dot-depth, 462 higher block shift, 899 edge graph, 899
Grigorchuk, 804 growth, 809–811 height of a state, 336 Gupta-Sidki, 804 Heisenberg, 800 hierarchy iterated monodromy, 812 Kazhdan, 818 lamplighter, 804 mapping class, 795 height, see star height, 222 height, see star height, 222 height of a state, 336 Heisenberg group, 800 hierarchy idot-depth, 462 higher block shift, 899 edge graph, 899
growth, 809–811 height of a state, 336 Gupta-Sidki, 804 Heisenberg group, 800 Heisenberg, 800 hierarchy iterated monodromy, 812 dot-depth, 462 Kazhdan, 818 higher lamplighter, 804 block shift, 899 mapping class, 795 edge graph, 899
Gupta-Sidki, 804 Heisenberg, 800 Heisenberg, 800 iterated monodromy, 812 Kazhdan, 818 lamplighter, 804 mapping class, 795 Heisenberg group, 800 hierarchy dot-depth, 462 higher block shift, 899 edge graph, 899
Heisenberg, 800 hierarchy iterated monodromy, 812 dot-depth, 462 Kazhdan, 818 higher lamplighter, 804 block shift, 899 mapping class, 795 edge graph, 899
iterated monodromy, 812 dot-depth, 462 Kazhdan, 818 higher lamplighter, 804 block shift, 899 mapping class, 795 edge graph, 899
Kazhdan, 818 higher lamplighter, 804 block shift, 899 mapping class, 795 edge graph, 899
lamplighter, 804 block shift, 899 mapping class, 795 edge graph, 899
mapping class, 795 edge graph, 899
nilpotent, 774, 782, 800 HNN extension, 782
1
<i>p</i> -, 774, 804, 808 Hopcroft
relatively hyperbolic, 799 automaton slow, 324
relators of a, 788 Hopcroft's algorithm
residually finite, 773 configuration, 326
right angled Artin, 781 splitter, 325
self-similar, 803, 811 waiting set, 325
semi-hyperbolic, 799 hyper-arithmetical set, 716
surface, 789 hyperbolic
VH-, 817 boundary, 772, 798
virtually abelian, 782 space, 797
virtually free, 781, 783 hyperbolic graph, 807
word-hyperbolic, 783, 797 hyperbolic plane, 790
growing
letter, 875 ideal, 657
substitution, 876 idempotent, 932
growth identities
maximal, 876 aperiodic –, 47, 70
growth type, 875, 876 natural –, 46, 66
less, 875 rational –, 46
growth function, 790 trivial –, 46, 66, 67
growth of groups, 809–811 identity, 510
exponential, 810 classical, 676
intermediate, 810 group, 667
non-uniformly exponential, 810 matrix star, 660
polynomial, 809 permutation, 660
growth series, 790 product star, 657
gsm, 78 sum star, 657
Gupta-Sidki group, 800, 802, 804 identity of unary algebras, 479

heterotypical, 479	k-kernel, 864
homotypical, 479	Kleene algebra, 678
in-merge	left handed, 677
graph morphism, 901	residuated, 681
labeled, 908	right handed, 677
of graph, 901	with domain, 679
in-split	with tests, 679
labeled, 908	Kleene lattice, 679
of graph, 901	Kleene monoid, 70
in-splitting map, 919	Kleene's theorem, 40, 44, 61, 65
incidence matrix, 871	Krieger automaton, 914
inclusion problem, 1150	Krohn-Rhodes Theorem, 543
incompressible pair, 496	Kronecker's theorem, 859
incompressible set, 496	Thomesen s theorem, 653
index of a partition, 319	label, 6
initial state, 78	labeled
initializable, 643	conjugacy, 908
injective morphism, 88	in-merge, 908
input alphabet, 78	in-split, 908
integer	out-merge, 910
•	out-split, 910
multiplicatively dependent, 859 multiplicatively independent, 859	lamplighter group, 804
* * *	language, 5
integer and real arithmetic, 1083	
intersection, 1068	$\Sigma_1, 526$
invariant	accepted, <i>see</i> automaton
measure, 883	conjunctive, 698–700
inverse morphism, 77	linear, 701–702
irreducible	constant term of –, 43
matrix, 871	context-free, 697–698
substitution, 871	denoted, see expression
irreducible automaton, 833	dense, 528
irreducible shift, 915	Dyck, 701, 781
isomorphism problem, 761, 799	fragile, 711
isoperimetric inequality, 796, 799	group, 714
	polynomial of, 714
J , 512	left, 320
$J_1, 510$	local, 25
\mathcal{J}^{+} , 515	non-counting, 460
\mathcal{J} -trivial, 512	of a tree automaton, 252
join, 537	P-complete, 702
Julia set, 812	piecewise-testable, 511, 524
	prime, 702, 711
k-block, 898	quotient, 514
k-recognizable set, 858	rational, 775
Kazhdan group, 818	rational –, 42
kernel, 81, 828, 831	recognisable –, 44

recognised, see automaton	linear numeration basis, 800
recognizable, 7, 775	linear ordering, 589
recognized, 7	linear recurrence, 842
recursive, 703, 706–712	linear time computation, 834
recursively enumerable (r.e.), 706–712	linked pair, 649
right, 320	Liouville number, 836
sequence, limit of, 691	Liouville's inequality, 835
slender, 528	literal length, see expression
sparse, 528	local automaton, 334
star-free, 454, 538	local automaton, 926
super-turtle, 471	standard, 928
turtle, 471	loop
unambiguous, 470	complexity, 53
with zero, 527	index, 53
language equation, 690	Lyndon word, 338
explicit, 696	Zynaon wora, 220
resolved, 696	Mal'cev product, 539
strict, 696	map
language operation	abelianisation, 871
computable, 706	edge in-merging, 902
continuous, 691–692	edge in-splitting, 902
monotone, 692–693, 714	edge out-merging, 902
word-based, 693	edge out-splitting, 902
lattice	in-merging, 920
generated by a set, 537	in-splitting, 919
of languages, 518	out-splitting, 920
Laurent series, 850	Parikh, 871
leaf	sliding block, 898
	_
transitions, 223 learner, 355	mapping
	automatic, 801
learning	marker, 86
from given data, 363	marking, 86
in the limit, 355	matrix
MAT-learning, 360	adjacency, 898
through a minimally adequate teacher,	alphabetic, 908
360	block decomposition of –, 51
left	column division, 902
delay, 930	elementary equivalent, 903, 911
left language, 320	incidence, 871
length, 5	irreducible, 871
length of a word, 76	primitive, 871
letter	row division, 902
neutral, 456	similar, 921
letters, 4	strong shift equivalent, 904, 911
limit space, 812	symbolic elementary equivalent, 921
Lindenmayer system, 716	symbolic strong shift equivalent, 92

transition, 908	automatic, 794
transition –, 44	countably factorizable, 671
matrix representation, 84	finitely factorizable, 661
matrix embedding, 802	finitely generated –, 60, 63
max-regular, 650	free, 5, 450
maximal growth, 876	free profinite, 517
word, 876	generating set of –, 60
McNaughton's and Papert's Theorem, 535	graded -, 46, 63
McNaughton-Yamada algorithm, 45, 49, 65,	idempotent and commutative, 510
70	inverse, 763, 771
Mealy machine, 78	Kleene –, 70
Mealy automaton, 801	non-solvable, 451
bireversible, 803, 814–818	partial, 661
Cayley automaton, 813, 814	rational –, 71
contracting, 807	solvable, 451
dual, 803	syntactic, 450, 779
nuclear, 806	transition, 763, 771
reset machine, 813	monotone operation, see language operation
reversible, 803, 813	monotone
measure	Moore
ergodic, 883	automaton slow, 324
invariant, 883	Moore equivalence, 322
uniquely ergodic, 883	Moore algorithm
membership game, 260	depth, 323
membership problem, 761	morphic composition, 77
rational subset, 782	morphic equivalence problem, 99
memory, block map, 898	morphism, 76, 828
merge of states, 334	fixed point, 864
mergeable states, 334	graph, 901
method	length multiplying, 516
recursive, 45, 51, 65	non-erasing, 515, 525
state-elimination, 45, 47, 65, 70	of shifts, 898
system-solution, 45, 48, 65, 66	prolongable, 864
military order, 870	recognizing, 37
minimal	semigroup, 904
automaton, 913, 917	uniform, 864
dynamical system, 882	morphism of automata, 18
nondeterministic automaton, 345	morphism of deterministic automata, 763
strongly connected component, 916	Morse–Hedlund's theorem, 865
minimal automaton, 320	MSO, 229, 270
minimization, 1069	-definable, 229
monadic second-order logic, 270	multidimensional automatic set, 831
monadic second-order logic, 229	multidimensional automaton, 831
monoid, 4, 450	multiplication (exterior), 63
aperiodic, 512, 771	multiplicative labelling, 589
aperiodic, group-free, 451	multiplicatively

dependent, 859	closed, 354
independent, 859	consistent, 354
multitape automaton, 79	for words, 354
Myhill-Nerode theorem, 353	occurrence, 5
Wijimi Werode theorem, 355	omega
NDD, 858, 1071	ω -power, 517
completion, 1079	operation
construction, 1072, 1075, 1076	boolean, 453
determinization, 1075	
expressive power, 1071	star, 453 order
operations, 1076	
projection, 1078	genealogical, 870
quantification, 1077	military, 870
size, 1074	radix, 870
negative examples, 356	order-type, 625
Nerode congruence, 320	ordered semigroup, 917
Nerode equivalence, 19	order problem, 761, 799
NFHA, 239	ordinal number, 624
NFTA, 223	out-merge, 902
nilpotent group, 774, 800	labeled, 910
Nivat's conjecture, 867	out-split
non-self dual, 636	labeled, 910
non-self dual class, 637	out-splitting map, 920
non-uniformity, 448	output alphabet, 78
nondeterministic automaton	
minimal, 345	p-group, 774, 804, 808
nonerasing morphism, 76	p-substitution, 885
normalization, 860	Parikh map, 871
normalized transducer, 80	parity game, 258
nucleus of a Mealy automaton, 807	Parry number, 861
number	partial signature, 335
beta-number	partition, 319, 901
β -number, 861	coarser, 319
Parry, 861	refinement, 319
Perron, 872	past of a state, 320
Pisot, 861	path, 6
Number Decision Diagram, 1071	accepting, 6
Number Decision Diagram, 858	end, 6
numeration basis	final, 6
Bertrand, 861	initial, 6
numeration base, 1069, 1081	length, 6
numeration basis, 860	origin, 6
linear, 860	successful, 6
numeration system	path in a graph, 898
abstract, 870	perfect field, 847
abstract, 070	period, 865, 900
observation table, 354	periodic

array, 885	isomorphism, 761
dynamical system, 882	membership, 761
inside X , 885	order, 761
locally periodic, 885	word, 760
word, 865	generalized, 761
periodic tiling, 89	product, 4, 5, 451
Perron	2-sided semidirect, 543
number, 872	deterministic, 540
theorem, 871	prefix, 451
Perron-Frobenius' theorem, 872	semidirect, 542
Pisot	suffix, 451
number, 861	unambiguous, 540
numeration system, 861	wreath, 542
play, 257	profinite
pointed ω -semigroup, 649	C-identity, 521
polish space, 627	ordered, 521
polynomial, 63	distance, 516
positional strategy, 258	equality, 520
positionally determined, 258	equation, 518
positive examples, 356	inequality, 520
power series, see series	monoid, 517
pre-fixed point, 673	
prebase, 70	program branching, 453
prefix, 76	<u> </u>
prefix code, 477	super-turtle, 471
maximal, 477	turtle, 470
	progression arithmetic, 859
synchronized, 477	
prefix tree acceptor, 353 prefix-closed, 352	projection, 76, 458, 630
±	projective hierarchy, 630
prefix suffix decomposition, 340	prolongable morphism, 864
prefix metric, 772	prolongable morphism, 828
preorder, 917	proper, see series
stable, 917	property
preperiod, 865	fellow traveller, 792
Presburger arithmetic, 867, 1072, 1077	geometric, 791
decision, 1079	(T), 818
quantifiers, 1077	pseudovariety, 510, 936
transformation, 1080	C-pseudovariety of stamps, 522
primitive	of finite semigroups, 515
matrix, 871	pumping lemma, 7, 832
substitution, 871	[p-]pure subgroup, 771
substitutive w.r.t. σ , 872	pure induction, 682
problem	purely substitutive word, 864
Post correspondence, 782	
problem, decision, 760–761, 789	QA, 515
conjugacy, 760	QA , 515

quantale, 671	recognizable set, 859
quasi-automatic function, 852	recognizable language, 775
quasi-geodesic, 798	recognizing, 37
quasi-identity, 669	recursive, see method
quasi-isometry, 781, 791	reduced automaton, 913
quasi-variety, 669, 679	reduced expression, see expression
quotient	reduction of automata, 917
of a language, 57	reduction relation, 630
of a series, 67	reflexive-transitive closure, 658, 680
of an automaton, 54	region, 1136
quotient automaton, 320	region automaton, 1138
	region graph, 1138
radix order, 870	regular, 5
random walk, 807	sequence, 881
self-similar, 807	tree, 268
rank of a \mathcal{D} -class, 932	tree language, 253
rational, 5	regular \mathcal{D} -class, 932
closure, 45, 64	regular expression, see expression
element, 663	regular tree grammar, 230, 242
expression, see expression, 667	Reiterman's Theorem, 522
identities, see identities	representation, 65
language, 42	S-, 870
monoid, 71	normal, 860
operations, 663	U-, 860
series, 64	reset threshold, 483
subset, 60, 69	reset word, 475
rational composition, 86	residual automaton, 346
rational expression	residually finite group, 773
linear, 27	residuation, 681
rational function, 80, 844	return word, 878
rational relation, 82	reversal
rational subset, 82	automaton, 322
rational transduction, 79	reversible Mealy automaton, 803, 813
rational constraint, 783	rewriting system
rational cross-section, 793	confluent, 761
rational language, 775	length-reducing etc., 779
reachability relation, 500	Ridout's theorem, 838
Real Number Automaton, 882	right
Real Vector Automaton, 882, 1082	context, 913, 914
realize, 84	delay, 930
recognisable	right language, 320
language, 44	ring, 4
subset, 61, 69	RNA, 882
recognised, see automaton	Road Coloring Conjecture, 495
recognizable, 37	Road Coloring Problem, 493
element, 663	root, 251

Roth's theorem, 836	boolean, 657
row division matrix, 902	commutative, 656
run	complete, 670
of a tree automaton, 251	complete iteration, 671
of an alternating tree automaton, 256	completely idempotent, 670
of an NFHA, 239	continuous, 672
of an NFTA, 223	continuous iteration, 673
of a TWA, 234	Conway, 657
RVA, 882, 1082	Conway –, 63, 70
construction, 1084, 1087	countably complete, 670
decision, 1090	countably complete iteration, 671
expressive power, 1083	countably idempotent, 670
minimization, 1090	dual, 662, 670
operations, 1089	formal series, 661, 669
projection, 1090	idempotent, 656
size, 1085	iteration, 668
,	matrix, 659, 669
saturated subset, 319	of binary relations, 657
Schützenberger's Theorem, 538	of languages, 657
Schreier graph, 768, 779, 789, 807	ordered, 672
self dual, 636	partial Conway, 657
self dual class, 637	partial iteration, 668
self-similarity biset, 811	partial iterative, 658
Semenov's theorem, 1071, 1083	polynomial, 662
semi-automaton, 324	quasi-Conway –, 63
semigroup, 4	sum ordered, 672
automatic, 794	symmetric partial iterative, 662
of an automaton, 801	tropical, 657
free, 5	tropical/Min-Plus, 602
inverse, 783	zero-sum free, 658
left-zero, 525	Sénizergues, Géraud, 779, 780, 782
ordered, 713, 917	•
	sentence, 229, 532
simple, 713	separated states, 321
stable, 526	separator symbol, 1081
syntactic, 515, 918	sequence
transition, 918	automatic, 864
semigroup morphism, 904	sequential transducer, 78
semilattice, 510	serialization, 1070, 1082
semilattice order, 672	series, 63, 661
semilinear, 37	characteristic, 662
semiring, 4, 62, 656	coefficient in a –, 63
ω -continuous, 672	constant term of –, 64
ω -continuous iteration, 673	denoted, see expression
ω -idempotent Conway, 657	proper, 661
ω -idempotent iteration, 668	proper –, 64
Boolean, 4	rational, 663

	rational –, 64	partial, 335
	recognisable –, 65	tree, 335
	recognizable, 663	signature of a state, 334, 336
	support of –, 63	similar matrices, 921
set	,	simple
	k-recognizable, 858	automaton, 337
	difference, 1068	simple transducer, 78
	k-recognizable, 866, 882	Skolem–Mahler–Lech theorem, 842
	locally periodic, 885	sliding block code, 883
	periodic inside X , 885	sliding block map, 898
	projection, 1078	slow automaton, 324
	purely substitutive, 869	slow automaton
	recognizable, 859	for Moore, 331
	S-recognizable, 870	for Hopcroft, 331
	substitutive, 869	slow for Hopcroft, 324
	syndetic, 863	slow for Moore, 324
	<i>U</i> -recognizable, 860	small cancellation, 790, 795
	ultimately periodic, 859	sofic shift, 897
set c	onstraints, 695	source of an edge, 898
shift	space	spanning tree, 766
	forbidden factors, 897	split, 591, 652
shift	transformation, 883	normalised, 591
shift	space, 897	Ramsey (for a multiplicative labelling),
	almost finite type, 931	592
	conjugacy, 899	splitter, 321
	edge, 898	splitter in Hopcroft's algorithm, 325
	entropy, 900	spontaneous, see transition
	even, 897	stability relation, 495
	finite type, 897	stable, 68
	flow equivalent, 904	stable pair, 495
	full, 897	stable preorder, 917
	golden mean, 897	Stallings' construction, 768
	higher block, 899	amalgamated free products etc., 782
	in-splitting, 920	graph groups, 782
	morphism, 898	Stallings automaton, 764
	recognized by an automaton, 906	Stallings construction, 763
	sofic, 897	stamp, 522
	transformation, 897	ordered, 522
	zeta function, 900	quasi-aperiodic, 525
	irreducible, 915	standard, see automaton
	RT-RESET-WORD, 481	standard local automaton, 928
	RTEST-RESET-WORD, 482	star, 36
	sentence, 513	star height
	header, 1070, 1082	of a rational language, 70
_	symbol, 1070, 1081	of expression, 52
sign	ature	problem, 53, 70

star-free languages, 538	subspace theorem, 839
star-normal form	substitution, 478, 864
expression in –, 56, 70	block, 898
of an expression, 56, 67	erasing, 873
starable, 63	good, 877
state, 78, 898	growing, 876
accessible, 6	irreducible, 871
coaccessible, 6	of finite length, 478
confluent, 335	ω_{α} -substitution, 880
final, 6	primitive, 871
fusion, 334	projection, 873
future, 320	sub-substitution, 877
height, 336	substitutive w.r.t. S , 872
initial, 6	substitutive w.r.t. σ , 872
merge, 334	substitutive word, 864
mergeable, 334	subtree, 222
partial signature, 335	subword, 511
past, 320	successor, 867
separated, 321	suffix, 76
signature, 334, 336	suffix-closed, 352
state-elimination, see method	sum, 4
states, 6	sum order, 672
strategy, 258	superchain, 646
strictly connected component, 642	support, see series, 661
strong shift equivalent matrices, 904, 911,	supremum, 641
921	surface group, 789
strongly	growth series, 791
connected component, minimal, 916	symbol
strongly connected component, 500	contraction, 904
structure group, 932 Sturmian	expansion, 904
	symbolic
tree, 333	conjugacy of automata, 919
sub-substitution, 877	elementary equivalent matrices, 921
subautomaton, 914	strong shift equivalent matrices, 921
subautomaton rooted at a state, 320	symbolic representation, 1068
subgroup	synchronized automaton, 913
finitely generated, 760	synchronizing word, 913
fixed point, 774	synchronizing word of a code, 477
index of a, 760	syndetic set, 863
intersection, 770	syntactic, 649
normal, 769	congruence, 918
[<i>p</i> -]pure, 771	monoid, 21
quasi-convex, 797	morphism, 21
subset automaton, 480	semigroup, 918
subshift, 883	syntactic semigroup, 918
generated, 883	syntactic congruence, 22

syntactic graph, 932	trie, 335
syntactic monoid, 779	trim, 9
system-solution, see method	trim automaton, 907
	TWA, 234
target of an edge, 898	
test set, 94	U_1 , 509
Theorem	ultimately periodic, 859
Benois', 777	array, 885
theorem	word, 865
Classification, 904	unambiguous automaton, 273
Curtis–Lyndon–Hedlund, 898	unambiguous transducer, 83
Decomposition, 903	unary term, 479
Franks', 905	underlying graph (of an automaton), 480
Kronecker, 859	uniform morphism, 76, 828
Perron, 871	uniformity
Perron–Frobenius, 872	DLOGTIME, 448
Thompson, see automaton (of)	polynomial-time, 448
Thue–Morse sequence, 827	union, 1068
Thue–Morse word, 865	uniquely ergodic
tiling, 89	measure, 883
timed automaton, 1132	universality problem, 1150
deterministic, 1154	• •
	universal cover, 788
diagonal-free, 1132	
with invariants, 1132	valuation, 1131
topological class, 629	variable
topology	free, 867
pro- V , 773	variety, 656, 682
transformation	C-, 521
automatic, 801	positive, 521
transition	generated by a set, 537
matrix, 908	positive, 516
matrix, 44	variety of languages, 513
semigroup, 918	vertex, 898
spontaneous, 54	initial, 898
transitions, 6, 78	ramified, 497
consecutive, 6	terminal, 898
transition monoid, 763, 771	VH-group, 817
tree, 251	virtually free group, 781, 783
signature, 335	
Sturmian, 333	Wadge Borel determinacy, 635
tree-walking automaton, 234	Wadge class, 629
tree automaton	Wadge degree, 638
nondeterministic, 223	Wadge game, 634
trellis automaton, see cellular automaton,	Wadge hierarchy, 629
one-way real-time	Wadge ordering, 629
Tribonacci word, 872	waiting set in Hopcroft's algorithm, 325

```
Wang tile, 89
weak automaton, 1083
weak monadic second-order logic, 270
weakly branch group, 806, 808-809
     satisfies no identity, 809
well quasi-order (wqo), 712-715
well-ordered set, 851
winning condition, 258
winning region, 258
winning strategy, 258
wire, 448
WMSO, 270
word, 4
    accepted, 6
     almost periodic, 888
     automatic, 864
     characteristic, 865
     cyclically reduced, 762
     empty, 5
     maximal growth, 876
    \omega_{\alpha}-substitutive, 880
     periodic, 865
     primitive substitutive w.r.t. \sigma, 872
    reduced, 761
     return, 878
     substitutive, 864
       purely, 864
     substitutive w.r.t. S, 872
     substitutive w.r.t. \sigma, 872
     Thue-Morse, 865
     Tribonacci, 872
     ultimately periodic, 865
word equation, 712
word problem over monoid, 451
word semigroup, 76
word-based operation, see language opera-
          tion, word-based
word-hyperbolic group, 783, 797–800
     is biautomatic, 798
     linear isoperimetric inequality, 799
word metric, 790
word problem, 760, 790, 796, 799
     in automata groups, 806
     submonoid, 780
       context-free, 781
       rational, 780
```

wreath product principle, 542 wreath product, 802, 804

XML Schema, 242

zero in a semigroup, 932 zeta function, 900