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Finding a Nash equilibrium in spatial games is an NP-complete problem⋆

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Summary. We consider the class of (finite) spatial games. We show that the problem of determining whether there exists a Nash equilibrium in which each player has a payoff of at least k is NP-complete as a function of the number of players.

Keywords and Phrases: Spatial games, NP-completeness, Graph k-colorability.

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1 Introduction

Nash (1950, 1951) has shown that every finite game in normal form has at least one equilibrium point, if mixed strategies are considered. Since his seminal contributions, attention has shifted from existence to the number and determination of equilibria. Rosenmüller (1971), Wilson (1971), and Harsanyi (1973) showed that the number of Nash equilibria in any finite normal form game is generically finite with respect to payoffs. A more recent literature derives generic upper bounds for the number of equilibria of such games; see Keiding (1997), McKelvey and McLennan (1997), McLennan (1997), among others. In addition to individual rationality which underlies Nash equilibrium, Harsanyi and Selten (1988) invoke further principles (payoff dominance, risk dominance, symmetry) to arrive at a unique solution for every game.

Showing existence of a Nash equilibrium or of a Nash equilibrium with particular properties and finding one are very different tasks. In general, computation

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of a Nash equilibrium in mixed strategies of a finite game poses a numerical challenge for the following reason. If there are at least six players and at least two pure strategies for each player, then determining the Nash equilibria in mixed strategies amounts to solving a system of multivariate polynomial equations of order five or higher which, as a rule, does not have an explicit solution. The task becomes much less demanding, if one is content with searching for Nash equilibria in pure strategies, since then it suffices to compare certain numbers given by the payoff functions (payoff vectors) of the game. The problem is further simplified, if one restricts oneself to the task of verifying whether some randomly selected (pure or mixed) strategy profile is a Nash equilibrium. In case this verification can be done in polynomial time, the task at hand is called an **NP**-problem. An **NP**-problem that in a sense (to be specified in Section 2) encompasses any other **NP**-problem is called an **NP**-complete problem.

Polynomial time refers to the asymptotic growth in computing time with respect to a size parameter as the size of the game increases. The size parameter is a positive integer variable that determines the size of the game when all other parameters affecting size are kept constant. For example, it can be the number of pure strategies per player or the number of players. Computing time refers to the maximal running time it would take a Turing machine to perform the task. In the literature on computational complexity, polynomial growth of computing time is considered preferable to (more acceptable than, less complex than) non-polynomial or "exponential" growth. Indeed, for sufficiently large size parameters, a problem with exponential computing time takes much longer than one with polynomial computing time. The growth of computing time can be crucial when a problem of practically relevant size already requires considerable computing time. Then the rate of growth of computing time determines whether somewhat larger problems, that is problems of the same category with slightly larger size parameters can be handled in reasonable or affordable time.

In addition to its importance in computer science, computational complexity provides one of the main motivations for theories of bounded rationality in economics since the pioneering work of Simon (1955). Needless to say that computational complexity need not concern us if analysts and players alike are omniscient, as is frequently assumed in economics and game theory. For then Nash equilibria would be determined in an instant. In contrast, theories of bounded rationality assume that decision makers are goal-oriented, that is rational in a broader sense, but not necessarily omniscient. Perhaps the most general definition of bounded rationality is rationality exhibited by decision makers with limited abilities. Behavioral models of such agents have been studied in economics, psychology, and artificial intelligence. For a survey of evidence and models of bounded rationality in economics, see Conlisk (1996). According to Simon (1987), "the term 'bounded rationality' is used to designate choice that takes into account the cognitive limitations of the decision-maker – limitations of both knowledge and computational capacity." Selten (2001) stresses that bounded rationality has to be distinguished from irrationality. Like a fully rational person, the boundedly rational individual uses his mind as well as possible to attain goals. In this respect, computers may simply be considered extensions of the human mind and limited computational capacities (of the brain and the computer) constitute one instance of bounded rationality. Suppose in a game or market experiment, some subjects are unable to perform a particular task, like finding a Nash equilibrium, whereas other subjects and the experimentalist prove capable of performing the task. For lack of analytic or numerical skills, some individuals might have been unable to perform a particular task, regardless how much time they were given. Others would have been able to perform the task, if only given sufficient time, but could not complete the assignment under pressure. That decision-making or, more specifically, computation consumes time and resources and that time is scarce forms one of the underlying premises of theories of bounded rationality. This can prevent decision-makers from finding the ideal solution(s) to a problem, for example a Nash equilibrium or all Nash equilibria of a game, if the task cannot be carried out within the set time limit.

Now let us replace subjects in an experiment by players or analysts using a computer program. Disregarding momentarily the rough classification offered by computational complexity theory – polynomial time problems, exponential time problems, **NP**-problems, and so forth – that theory deals with classes of finite problems for which a finite computer program can be written that solves in finite time any problem for any size parameter. Thus if only given sufficient time, the economic agent, player or theorist equipped with a suitable computing device can solve any such problem, like deciding whether a Nash equilibrium with specific properties exists. But if decisions must be made within a certain time limit, then the computer may take too long to come up with an answer and a decision would have to be taken without the sought answer. In such a case, it is not very likely that a solution recommended or predicted by theory, say a Nash equilibrium, would be the outcome. If the maximal running time of the program tends to grow fast with the size parameter, it is more likely that one encounters such a situation as the size parameter increases. Therefore, if the rational agent or expert cannot come up with a polynomial time algorithm, one might consider the larger ones in the corresponding class of problems computationally complex and, perhaps, *a priori* intractable. In any case, the question arises what outcomes might be chosen by players who would like to coordinate on a Nash equilibrium, but fail to achieve their goal (or expect to fail) because of computational limitations. Bounded rationality theories aim to provide models of how players would proceed in such a situation. Myopia is a conceivable manifestation of bounded rationality and a common trait of agents in the literature on spatial games cited below.

Specifically, this paper investigates the complexity of computing Nash equilibria for finite games in strategic form. We show that the problem of determining, for any given k, whether an n-player spatial game has a Nash equilibrium in which each player receives at least k is **NP**-complete (as a function of the number of players). Sahni (1974) is the first to show that computing a pure Nash equilibrium in an arbitrary finite n-player game in normal form is **NP**-complete as a function of the number of players. More recently, Gilboa and Zemel (1989) considered the related problem of computing a mixed Nash equilibrium. They showed that given a finite two-player game $\mathcal{G}$ and an integer k, deciding whether there exists a mixed Nash equilibrium where each player gets a payoff of at least k is **NP**-complete as a function of the number of strategies while existence of a correlated equilibrium where each player gets a payoff of at least k is decidable in polynomial time. These results are the most basic complexity results concerning solution concepts for non-cooperative games in normal form.[1] They suggest that one should not expect to find a polynomial time algorithm for determining a Nash equilibrium in an arbitrary finite game in normal form. Thus if we expect to be able to compute a Nash equilibrium without an exponential growth in computing time as the size of the game increases, we will have to restrict the instances to classes of games other than the class of arbitrary finite games in normal form. We have at our disposal a very large number of classes from which to choose.

An n-player spatial game is a game in normal form where the payoff of each player is the weighted sum of the payoffs from playing each of his neighbors, with the set of neighbors of a player given by the spatial structure of the game. As n, the number of players increases and x, the number of strategies per player remains constant, the information needed to determine payoffs is given by the base game payoff bi-matrix of dimension $2x^2$ plus an $(n^2 - n)$-dimensional weight matrix, hence grows polynomially in n. In contrast, the dimension of payoff vectors for an arbitrary n-player game with $x > 1$ is nx^n, hence grows exponentially in n. Insofar, the spatial games constitute a lower dimensional subclass of games.

Spatial games are studied by Blume (1993), Ellison (1993), Young (1998), and Baron et al. (2002), among others. This type of strategic interaction is of interest because it arises quite naturally in economic and social situations. Consider the following example. Players are firms where each firm is regarded as a repository of competencies. Represent each firm by the vertex of a graph and join two firms by an edge if they cooperate in a process of production. Thus the firms are organized in a network of two-firm cooperative agreements where a firm may cooperate with several firms. We say that two firms are neighbors if they cooperate in a process of production. Each firm has to choose an activity among a finite set of activities. Activities are complementary in the sense that they represent different phases of a process of production. Thus each firm prefers to choose an activity unlike the activities chosen by its partners. The total payoff of a player is the sum of the payoffs he gets from all his cooperative agreements. An obvious goal is to compute the maximum payoff the players can get at an equilibrium of this spatial game. We show that the problem of computing a pure Nash equilibrium, where each player gets a total payoff of at least k, is **NP**-complete as a function of the number of players. On the positive side, this problem becomes solvable in polynomial time in several interesting cases where the number of strategies is two for each player. The presentation of the paper is as follows. In Section 2 we define the spatial game and give a brief introduction to the theory of computational complexity. In Section 3 we prove the **NP**-completeness result. In Section 4 we make some qualifying remarks.

[1] For complexity considerations in extensive form games see Koller and Megiddo (1992), Koller, Megiddo, and von Stengel (1996), Chu and Halpern (2001). Conitzer and Sandholm (2002) strengthen and extend the results by Gilboa and Zemel. Furthermore, they obtain hardness results for games of incomplete information and stochastic games.

2 Preliminaries

2.1 Spatial games

The basic building block is a two-player game G, called the *base game*, with a common finite action set X, and combined payoff function $\pi : X \times X \to I\!R^2$ which assigns to each strategy pair $x = (x_1, x_2)$ the pair $\pi(x) = (\pi_1(x), \pi_2(x))$ of payoffs. We associate with G a *spatial game*

$$\mathcal{G} = (I, \Gamma_n, (S_i)_{i \in I}, (u_i)_{i \in I})$$

in the following way. The player set is $I = \{1, \ldots, n\}$. Every player $i \in I$ has strategy set $S_i = X$. Moreover, the elements of I form the vertices of a weighted graph Γ_n of order n, with the interpretation that player i is located at vertex i. E denotes the set of edges of Γ_n. Two vertices or players i and j are neighbors, if $\{i, j\} \in E$. $N(i)$ denotes the set of *neighbors* of i. Γ_n is undirected in the sense that $j \in N(i)$ if and only if $i \in N(j)$. Furthermore, we assume that $N(i) \neq \emptyset$ for all i, i.e. none of the vertices is isolated. Each $\{i, j\} \in E$ has a weight w_{ij} that measures its relative importance. Note that w_{ij} is not necessarily equal to w_{ji}. Payoffs in the spatial game $\mathcal{G}$ are given by

$$u_i(s) = \sum_{j \in N(i)} w_{ij} \pi_i(s_i, s_j)$$

for $i \in I$, $s = (s_j)_{j \in I} \in \prod_{j \in I} S_j \equiv \mathcal{S}$, that is, a player collects the aggregate weighted payoffs from playing with each of his neighbors.
Let $s^* \in \mathcal{S}$ be a strategy profile of the spatial game $\mathcal{G}$. The profile s^* is a *Nash equilibrium* if for all $i \in I$, $s_i \in S_i$,

$$u_i(s^*) \geq u_i(s_i, s^*_{-i})$$

where $s^*_{-i} = (s^*_j)_{j \neq i}$. Clearly, $w_{ij} = w_{ji} = 0$ has the same effect on payoffs as $\{i, j\} \notin E$. We can set $w_{ij} = w_{ji} = 0$ for $\{i, j\} \notin E$ and assume without loss of generality that $w_{ij} > 0$ or $w_{ji} > 0$ for $\{i, j\} \in E$. Then $\{i, j\} \in E$ if and only if $w_{ij} + w_{ji} > 0$. That is E is determined by the weight matrix $W = [w_{ij}]$. Hence for given X and n, the variable inputs for an instance of the spatial game consist of the entries of the weight matrix $W = [w_{ij}]$.

2.2 NP-completeness

For a comprehensive introduction to NP-completeness the reader is referred to Garey and Johnson (1979) or Papadimitriou (1994). Let **P** denote the class of problems that can be solved on a deterministic Turing machine by a polynomial time algorithm, that is, polynomial in the length of inputs for an instance of the problem. Let **NP** denote the class of all decision problems which can be solved in polynomial time by a nondeterministic Turing machine. Instead of using the notion of nondeterminism, one can define the class **NP** in terms of the concept of *polynomial-time verification*. A verification algorithm is an algorithm which takes as input an instance of the problem and a candidate solution to the problem, called a certificate, and verifies in polynomial time whether the certificate is a solution to the given instance. Thus the class **NP** is the class of problems which can be verified in polynomial time.

The fundamental open question in computational complexity is whether $\mathbf{P} = \mathbf{NP}$. By definition $\mathbf{P} \subset \mathbf{NP}$. It is not known, however, whether all problems in $\mathbf{NP}$ can, in fact, be solved in polynomial time by a deterministic Turing machine. The generally accepted belief is that $\mathbf{P} \neq \mathbf{NP}$. In an effort to determine whether $\mathbf{P} = \mathbf{NP}$, the class of NP-complete problems has been introduced. We say that a problem P_1 is *polynomial-time reducible* to a problem P_2, written $P_1 \preceq_p P_2$, if

(i) there exists a function f which maps any instance of P_1 to an instance of P_2 in such a way that I_1 is a "yes" instance of P_1 if and only $f(I_1)$ is a "yes" instance of P_2.
(ii) for any instance I_1, the instance $f(I_1)$ can be constructed in polynomial time.

If P_1 is polynomial-time reducible to P_2, we can say that any algorithm for solving P_2 can be used to solve P_1. Intuitively, problem P_1 is "no harder" to solve than problem P_2. A problem P is said to be **NP**-*complete* if (i) $P \in \mathbf{NP}$, and (ii) for every problem $P' \in \mathbf{NP}$, $P' \preceq_p P$. If a problem satisfies condition (ii) but not necessarily condition (i), then we say that it is **NP**-*hard*. Let **NPc** denote the class of NP-complete problems.
The binary relation $\preceq_p$ is transitive on the set of decision problems. Because of this, a method frequently used in demonstrating that a given problem is NP-complete is the following:

(i) show that $P \in \mathbf{NP}$, and
(ii) show there exists a problem $P' \in \mathbf{NPc}$, such that $P' \preceq_p P$.

It follows from the definition of NP-completeness that if any problem in **NPc** can be solved in polynomial time, then every problem in **NPc** can be solved in polynomial time, and $\mathbf{P} = \mathbf{NP}$. On the other hand, if there is some problem in **NPc** that cannot be solved in polynomial time, then no problem in **NPc** can be solved in polynomial time.

3 The NP-completeness result

We show that the problem of determining a Nash equilibrium in a spatial game is NP-complete as a function of the number of players. To state this problem in the accepted format, we convert it to a decision problem in considering the problem of deciding whether the Nash profile gives a payoff of at least $k \in I\!N$.

NASH FOR SPATIAL GAMES (NSG)
INSTANCE: A finite spatial game $\mathcal{G} = (I, \Gamma_n, (S_i)_{i \in I}, (u_i)_{i \in I})$ and a positive integer k. The finite spatial game is given by X, n, and a weight matrix $W = [w_{ij}]$.
QUESTION: Does there exist a Nash equilibrium in $\mathcal{G}$ in which each player obtains the payoff of at least k?

Proposition 1. *NSG* $\in$ **NPc**.

Proof. We follow the method described in Section 2. We must do two things. First we must show that NSG $\in$ **NP**. The nondeterministic Turing machine just guesses an arbitrary strategy profile $s \in \mathcal{S}$ and has to consider $n \cdot (|X| - 1)$ deviations and to take the $n(n-1)$ weights into account in the computation in order to verify whether the Nash equilibrium and payoff conditions are satisfied at s where the time of computing a single payoff is of the order n.
Second we must construct a reduction from a known NP-complete problem to NSG. We use the GRAPH k-COLORABILITY problem.

GRAPH k-COLORABILITY INSTANCE: A graph $\Gamma_n = (V, E)$ and a positive integer $k \leq |V|$.
QUESTION: Is Γ_n k-colorable, i.e. does there exist an assignment of k colors $\{1, 2, \ldots, k\}$ to the vertices of Γ_n so that neighboring vertices are assigned different colors ? This problem is NP-complete for an arbitrary k.

Given an instance $\Gamma_n = (V, E)$ and k of GRAPH k-COLORABILITY, we construct an instance $\mathcal{G}$ of NSG as follows. In a FIRST STEP, we restrict ourselves to graphs $\Gamma_n = (V, E)$ without isolated points. The set of vertices constitutes the set of players. The neighborhood of player i is exactly the set of vertices adjacent to vertex i, i.e. $N(i) = \{j \in V : \{i, j\} \in E\}$ and $N(i) \neq \emptyset$ if i is not isolated. For distinct players i and j in V, define the weights w_{ij} as follows:

$$w_{ij} = \begin{cases} 1/|N(i)| & \text{if } \{i, j\} \in E; \\ 0 & \text{otherwise.} \end{cases}$$

The finite set of strategies is $S_i = X = \{1, \ldots, k\}$ for every player $i \in V$. The base game payoff of player i when he encounters a player $j \in N(i)$ is given by

$$\pi_i(s_i, s_j) = \begin{cases} k \text{ if } s_i \neq s_j \\ 0 \text{ otherwise} \end{cases}$$

We must show that an instance of GRAPH k-COLORABILITY is a "yes" instance if and only if the constructed game $\mathcal{G}$ has a Nash equilibrium in which each player gets a payoff of at least k.

First assume that Γ_n is k-colorable. Then no two neighboring vertices are assigned the same color. Create a strategy profile s^* as follows : $s_i^* = c_i$ where c_i is the color assigned to vertex i. The strategy profile s^* is a Nash equilibrium in which each player gets a payoff of at least k for $\mathcal{G}$ because (i) each player i gets the maximum payoff $k/|N(i)|$ in each bilateral encounter, (ii) his total payoff is k.

Conversely, suppose that $\mathcal{G}$ has a Nash equilibrium s^* in which each player gets a payoff of at least k. Notice that the maximum total payoff the players can get in this game is in fact k. Each player gets a total payoff of k only if he gets $k/|N(i)|$ in each bilateral encounter i.e. only if for each pair of neighbors, the players choose a different strategy. We can therefore create a "yes" instance of GRAPH k-COLORABILITY.

It should be clear that our construction for creating an instance of NSG from an instance of GRAPH k-COLORABILITY, can be carried out in polynomial time.

The length of an instance of GRAPH k-COLORABILITY is $\mathcal{O}(n + |E| + k)$. The graph of the constructed spatial game is exactly Γ_n, the cardinality of the common strategy set is k, and the base game payoff can be determined with the elements of Γ_n. In particular, each value $1/|N(i)|$ can be computed by means of n elementary operations when the graph structure is given. Therefore, an instance of NSG can be constructed from an instance of GRAPH k-COLORABILITY in polynomial time. Let f_2 denote the above mapping that associates instances of GRAPH k-COLORABILITY for graphs without isolated points and NSG.

In a SECOND STEP we associate to any instance of GRAPH k-COLORABILITY an instance of GRAPH k-COLORABILITY without isolated points. Denote this mapping f_1. f_1 is constructed as follows. k remains unchanged. If $\Gamma_n = (V, E)$ is any graph, its image assumes the form $\Gamma_n^* = (V, E^*)$. In case Γ_n has no isolated points, set $E^* = E$. In this case, $\Gamma_n = \Gamma_n^*$ and Γ_n is k-colorable if and only if Γ_n^* is. In case Γ_n does have isolated points, let J denote the set of isolated points and label them $j_1, \ldots, j_m$. In the special case $J = V$, set $E^* = \{\{j_1, j_2\}, \{j_2, j_3\}, \ldots, \{j_{m-1}, j_m\}\}$. Then both Γ_n and Γ_n^* are k-colorable in the following way: Choose color $c_r \equiv r \mod k$ for node j_r, $r = 1, \ldots, m$. In the subcase $J \neq V$, choose any $j_0 \in V \setminus J$ and set $E^* = E \cup \{\{j_0, j_1\}, \{j_1, j_2\}, \ldots, \{j_{m-1}, j_m\}\}$. Obviously, if Γ_n^* is k-colorable, then Γ_n is also k-colorable, since $E \subset E^*$. Suppose Γ_n is k-colorable. Then Γ_n^* is k-colorable as well. Namely, fix any k-coloring of Γ_n and denote by c_0 the color given to node j_0 in that coloring. Keep the colors of nodes in $V \setminus J$. Assign color $c_r \equiv c_0 + r \mod k$ to node j_r, $r = 1, \ldots, m$. Then the new coloring is a k-coloring of Γ_n^*. Hence we have constructed a mapping f_1 that assigns to any graph Γ_n a graph Γ_n^* without isolated points such that Γ_n is k-colorable if and only if Γ_n^* is.

To show that the mapping f_1 can be performed in polynomial time, we provide a polynomial-time algorithm which relies on the following numerical representation of the graph $\Gamma_n = (V, E)$. The vertices are represented by the set $V = \{1, \ldots, n\}$. The edges are represented by the symmetric adjacency matrix $A = [A(i, j)]$ such that $A(i, j) = A(j, i) = 1$ if $\{i, j\} \in E$ and $A(i, j) = A(j, i) = 0$ if $\{i, j\} \notin E$. An algorithm that transforms the matrix A into the adjacency matrix $A^* = [A(i, j)^*]$ of the graph $\Gamma_n^* = (V, E^*)$ would consist in the following steps:

Algorithm:

Step 1: `compute the number` N `of isolated vertices and build a list of these vertices`

Step 2: `if` $N = n$, `then connect the vertices to form a chain`

Step 3: `if` $0 < N < n$, `then connect the isolated vertices to form a chain, and connect this chain with the formerly non-isolated vertices`

The algorithm can be carried out using the adjacency matrix. Creating a new edge amounts to changing the two corresponding entries of the adjacency matrix from 0 to 1. To check whether a particular vertex i is isolated or not requires to check if any of the entries $A(i, j), j \neq i$, differs from 0 and therefore can be done by means of $n-1$ elementary numerical comparisons. *Step 1* requires the examination of all entries of each line in the adjacency matrix, that is, a number of operations of the order n^2. *Step 2* creates the edges $\{1,2\},\ldots,\{n-1,n\}$ if all vertices turned out to be isolated. *Step 3* constructs the edges $\{j_0,j_1\},\ldots,\{j_{m-1},j_m\}$ where $j_1,\ldots,j_m$ were the isolated vertices in canonical order of the list built in *Step 1*, if some but not all vertices turned out to be isolated. Note that if $N=0$, that is if none of the vertices turned out to be isolated, the algorithm leaves the adjacency matrix unchanged. Clearly, *Step 2* and *Step 3* require a number of operations of the order n. Consequently, the mapping f_1 can be performed in polynomial time. The composition $f = f_2 \circ f_1$ constructs in polynomial time a corresponding NSG instance from any GRAPH k-COLORABILITY instance.

End of Proof.

4 Comments

1. The GRAPH k-COLORABILITY problem is known to be in **P** when $k=2$. Therefore, if we restrict attention to the maximum payoff problem, then the NSG problem is in **P** when each player has two strategies, the weight matrix is up to normalization a row-stochastic matrix and the base game is an anti-coordination game (in which, as in the example of the introduction, players prefer to choose a strategy unlike the strategy chosen by their opponents) where up to normalization players receive payoff 0 for identical choices and payoff 2 for different choices. The reason is that this special NSG problem is equivalent to the GRAPH 2-COLORABILITY problem.

2. In the main step of the proof, the constructed NSG instance satisfies symmetry of the weights, $w_{ij}=w_{ji}$ if the graph Γ_n is regular. One also obtains symmetry of the weights if different players can have different base game payoff functions, by setting $w_{ij}=1$ in case $\{i,j\}\in E$ and $\pi_i(s_i,s_j)=k/|N(i)|$ in case $s_i \neq s_j$.

3. The construction in the last step of the proof can be modified to show that GRAPH k-COLORABILITY remains **NP**-complete when restricted to connected graphs.

4. Proposition 1 forms a contrast to the results obtained by Gilboa and Zemel (1989) and Mailath, Samuelson and Shaked (1997). As underlined in the introduction, Gilboa and Zemel (1989) showed that computing a correlated equilibrium where each player gets a payoff of at least k is decidable in polynomial time. On the other hand Mailath, Samuelson and Shaked (1997) showed that a correlated equilibrium is equivalent to a Nash equilibrium of a specific spatial game. This equivalence can be implemented in polynomial time by simple manipulation of a set of equations. Proposition 1 still remains valid, since the Mailath-Samuelson-Shaked model of local interaction is quite different from ours.

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