FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION OF HIGHER EDUCATION ITMO UNIVERSITY

Report

on the practical task No. 2

"Algorithms for unconstrained nonlinear optimization. Direct methods"

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Accepted by

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Brief theoretical part:

In this task need to do a lot of things:

- 1) Implement three one-dimensional methods
 - a. Exhaustive search: check all available dots, choose one which satisfy condition and return this point.
 - b. Dichotomy method, this method has something common with ternary search. For this method function must be convex. And after each iteration border narrow.
 - c. Golden section method. This method has similar algorithms with method above. But here left and right borders narrowing with other coefficients.

Then compare them on three different functions and provide corresponded plots.

2) In second part this homework must find approximated line coefficients. (y = a * x + b) a and b coefficients.

Data set must be generated with according formula and additional noise based on normal distribution.

Here must compare two different approximant functions.

First – linear function y = a * x + b

Second – rational function y = a / (1 + x * b)

For locate the best approximate parameters must use method of least squares. In simple terms: required found such coefficient a and b in which $D(a, b) = sum(F(x_k, a, b) - y_k)^2$, k = 0...100 the least. Function F(x, a, b) described above.

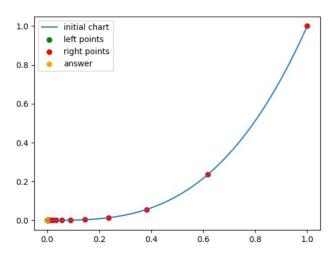
For founding a and b must be used to three different methods:

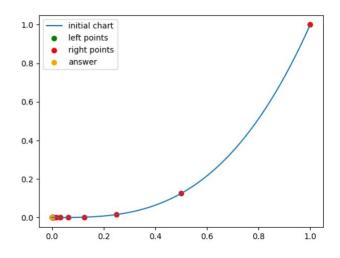
- a) Exhaustive search: the same analogy with one-dimensional method
- b) Gauss method: in this method we need to choose initial point (random for example) then fixate all except first coordinate, minimize this unfixed coordinate (use any of one-dimensional method), substitute obtained coordinate except old coordinate and move to other point. Repeat these actions while stop conditional will not be executed.
- c) Nelder-mead method: in this method we need choose three points and build triangle, then we may figure out which point has less value then other two and in dependence of this point continue move in decrease direction. (We investigate location around THE BEST point and in last action replace the worst point by freshly found)

Result:

For implementing I use python3, bellow results are given:

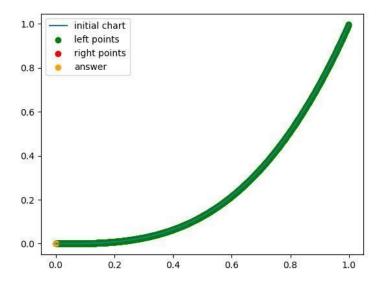
Function body	Method name	Iteration by convergences
$f = x \wedge 3$	Golden search	15
$f = x \wedge 3$	Exhaustive search	1100
$f = x ^ 3$	Dichotomy	12





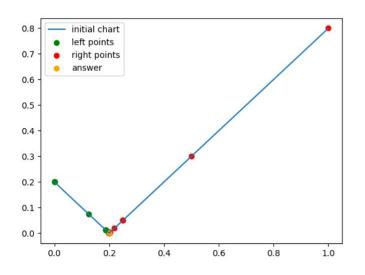
Dichotomy

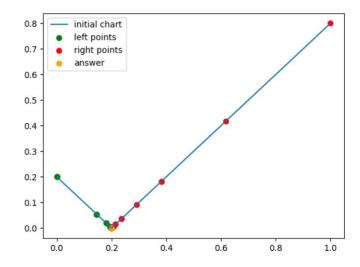
Golden search



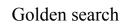
Exhaustive search

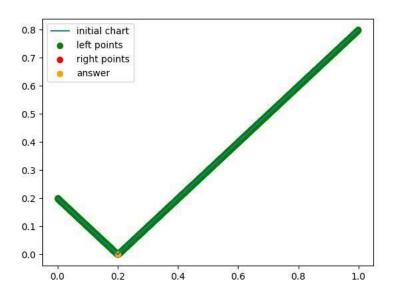
Function body	Method name	Iteration by convergences
f = x - 0.2	Golden search	15
f = x - 0.2	Exhaustive search	1100
f = x - 0.2	Dichotomy	12





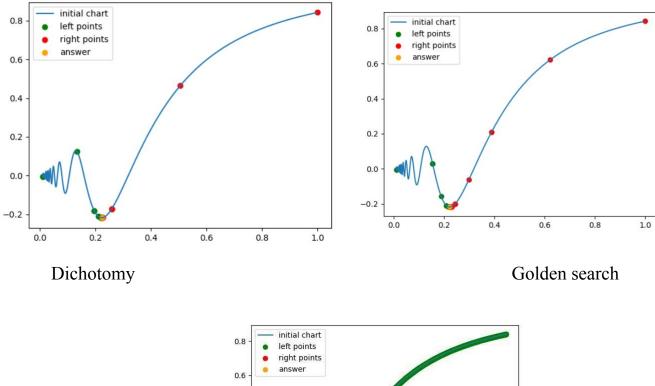
Dichotomy





Exhaustive search

Function body	Method name	Iteration by convergences
$f = x * \sin(1/x)$	Golden search	15
$f = x * \sin(1/x)$	Exhaustive search	1100
$f = x * \sin(1/x)$	Dichotomy	12

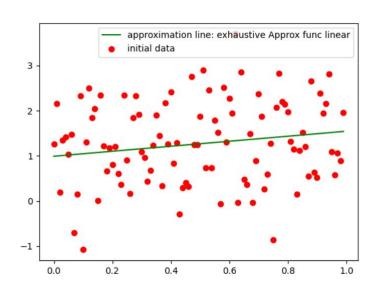


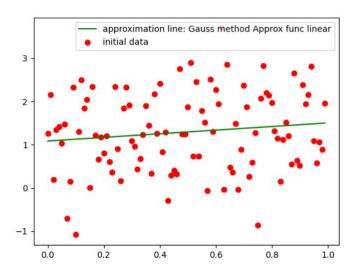
Exhaustive search

In images above clearly seen, that all three methods convergences to minimum root, and Dichotomy methods spent less iterations rather than Golden search, because at first steps first method made more step then second method and come to "critical" section faster than opponent. in consequence of what Dichotomy convergences faster than Golden search.

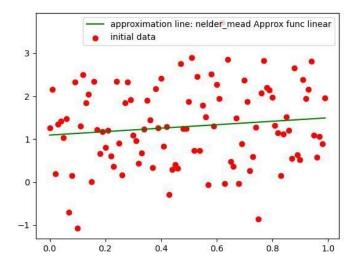
Part two.

Approximate method	Method name	Less square
linear	Exhaustive	83.35994604410344
Linear	Gauss	83.0910900634
Linear	Nelder-mead	83.08861963457687



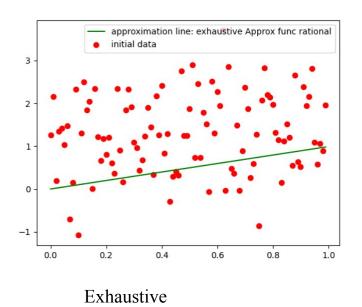


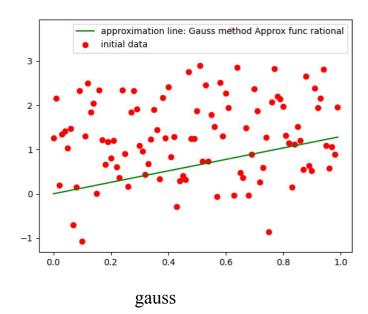
Exhaustive gauss

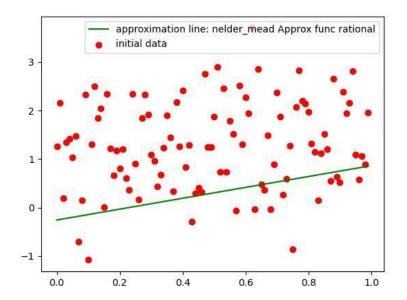


Nelder-mead

Approximate method	Method name	Less square
Rational	Exhaustive	93.57577973349801
Rational	Gauss	84.43528779165963
Rational	Nelder-mead	83.14554132206874







Nelder-mead

In section results clearly seen that all method works approximately equals. But Exhaustive method provide worse result because step was chosen equal EPS, and because some better points missed.

Conclusion:

In this task was implemented a lot of methods: for searching one dimensional function minimum, for searching coefficient of approximate line.

In conclusion may summarize that methods more difficult then exhaustive method give result better and faster than brute-force methods.